The FOCUS Framework: Characterising Productive Noticing During Lesson Planning, Delivery and Review

Ban Heng Choy, Michael O.J. Thomas and Caroline Yoon

Abstract Enacting the work of diagnostic teaching is challenging and demands that teachers pay attention to mathematical details when designing tasks, orchestrating discussions and reflecting on their lessons. This chapter presents the FOCUS Framework on teacher noticing, which can be used to characterise teachers' efforts to notice productively during all three phases of diagnostic teaching: lesson planning, delivery and review. Using the two key components of the framework, the focus and its focusing, we provide snapshots of a teacher's mathematical noticing in each of the phases. The findings from this research suggest that productive noticing in all the three phases is highly consequential, and illustrates how the FOCUS Framework can be used to analyse a teacher's mathematical noticing.

Keywords Productive teacher noticing · Lesson planning · Orchestrating discussions · Lesson review · Fractions

Introduction

Teaching mathematics well does not just depend on what you teach but on what and how you notice. Mathematics teacher noticing—what mathematics teachers see and how they understand instructional events or details in classrooms (Mason, 2002; Sherin, Jacobs, & Philipp, 2011a)—is central to mathematics teaching practices and is considered necessary for improving teaching (Mason, 2002;

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Schoenfeld, 2011). The processes of noticing help teachers break down and analyse their practice in order to learn from their teaching (Mason, 2002; Sherin, Jacobs, & Philipp, 2011b). Placing noticing in the context of developing students' mathematical thinking, there are three productive classroom practices that are of interest in this chapter: designing a task that reveals students' thinking; listening and responding to students' thinking during the lesson; and reflecting about students' thinking after the lesson. If noticing is considered to be productive when teachers respond with instructional decisions that promote student thinking, then although all teachers may notice, it can be argued that not all noticing is productive. For example, it can be difficult for teachers to notice the mathematical features of learning tasks (Star, Lynch, & Perova, 2011; Vondrová & Žalská, 2013), or teachers may be distracted by noticing features that are not useful for enhancing mathematical thinking (Ball, 2011; Star & Strickland, 2008). Furthermore, it is possible for teachers to describe the specific strategies that students use to solve problems but have difficulties relating these strategies to important characteristics of the problems (Fernandez, Llinares, & Valls, 2012). Therefore, the crux of enhancing instruction to promote mathematical thinking lies in what teachers attend to, and how they think about instructional events (Ball, 2011).

Despite the apparent simplicity of the construct of teacher noticing, the ability to notice productively during mathematics teaching can be both difficult to master and complex to study (Jacobs, Philipp, & Sherin, 2011, p. xxvii). Moreover, what teachers deem productive may be highly subjective and dependent on one's views about teaching and learning of mathematics (Clarke, 2001). Nevertheless, if teachers want to teach in a way that enhances students' reasoning, they may need to attend to relevant aspects of student thinking evidenced in classroom artefacts and students' explanations, and interpret them using a mathematical perspective before, during and after a lesson.

Most researchers who study and support mathematics teachers' noticing do so by examining what teachers observe from video clips of lessons (Star et al., 2011; van Es, 2011), while others (Sherin, Russ, & Colestock, 2011) try to capture what teachers notice in the moment during lessons. One limitation of these approaches is the lack of focus on preparation to notice. As Mason (2002) put it, 'noticing is an act of attention, and as such is not something you can decide to do all of a sudden. It has to happen to you, through the exercise of some internal or external impulse or trigger' (p. 61). More specifically, Mason (2002) highlights the importance of advanced preparation to notice, and the use of prior experience to enhance noticing in order to have a different act in mind in the moment. Therefore, it is critical for researchers to examine the role of noticing during lesson planning.

However, examining what teachers notice is non-trivial. Most research generally focuses on developing teachers' ability to notice a wide range of classroom features—classroom environment; classroom management; tasks; mathematical content; communication; mathematical thinking, and so forth—without specifying what teachers should notice (Jacobs, Lamb, & Philipp, 2010; Star et al., 2011). A study by Star and Strickland (2008), as well as a replication study by Star et al. (2011), found that teachers seemed to notice more instructional events, *both* mundane and important, after participating in professional development that involved viewing video clips of actual teaching. But, neither study provided a focus for noticing, nor tested the usefulness of an explicit focus. Moreover, even when teachers are given a focus it can still be challenging for them to sieve out and reflect upon critical incidents amongst the ‘buzz’ in the classroom.

On the other hand, the ability to describe specific details when planning, teaching and reviewing mathematics lesson is seen as the distinguishing mark of a proficient teacher in China (Yang & Ricks, 2012). They detail how Chinese teachers think about teaching using the Three Points Framework: the ‘Key Point’, the ‘Difficult Point’, and the ‘Critical Point’ (p. 54). The Key Point of a lesson is the mathematical concept to be learned during the lesson. The Difficult Point refers to the difficulty or confusion students have when learning the Key Point. By having a strong grasp of these two points (the concept and its associated confusion), teachers can design tasks that address specific difficulties that students may have when learning the concept. The teaching approach or the main considerations used by teachers when designing the task is then the Critical Point, which forms the ‘heart of the lesson’ (Yang & Ricks, 2012, p. 43). Noticing that the Critical Point is targeted at the Difficult Point related to the Key Point is essential if teachers want to promote students’ reasoning.

In addition, how teachers notice also matters. Many researchers focus on the specificity of what teachers have noticed as an indicator of noticing expertise, but specificity is not sufficient for noticing to be productive. In a study involving seven prospective secondary school mathematics teachers, Fernandez et al. (2012) found that most were unable to relate the strategies used by students to the characteristics of the problem, even though they were all able to describe the specific strategies at the beginning of the study. Choy (2014b) also highlights the role of pedagogical reasoning, beyond giving teachers an explicit focus, as a means to promote more productive noticing when they plan their lessons.

The research presented here addresses the challenge of noticing student thinking, building upon previous research to bring task design into the realm of teacher noticing. The research was guided mainly by the following question:

What makes teachers’ mathematical noticing, during planning, teaching and reviewing of lessons, productive for enhancing students’ mathematical reasoning?

This question reflects the importance of preparation in noticing, and draws attention to the ways teachers can plan to anticipate student thinking as they engage with the tasks (Smith & Stein, 2011). In this chapter, we describe the FOCUS Framework, developed from part of a larger doctoral study (Choy, 2015), which pinpoints specific focal points and actions teachers can take to attend to, make sense of and respond to students’ thinking when planning, teaching and reviewing a mathematics lesson. More importantly, we demonstrate how the FOCUS Framework can be used to characterise, analyse and support teacher noticing.

Research Design

Design Research Paradigm

The FOCUS Framework was developed from a design-based research project (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), which addressed the twin challenges of theoretical development and practical application (Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008). Using an iterative and highly interventionist approach (Cobb et al., 2003), a design-based research project aims to generate usable knowledge (Design-Based Research Collective, 2003) that is grounded in complex real-world settings (McKenney & Reeves, 2012). Data collection for the doctoral study, which consisted of three phases, took place in Singapore over a period of eight months in 2012 and 2013. A total of 36 teachers from three schools, a primary school and two secondary schools, participated in the study. The three schools volunteered for the study when their principals responded to our advertisement seeking research participants. All three schools had processes in place to support learning communities and the teacher participants had used Lesson Study as a professional development activity. Hence, they were familiar with the Lesson Study protocol.

We engaged teachers in a systematic investigation of their teaching (Hiebert, Morris, & Glass, 2003) as they participated in the five key tasks of Lesson Study (Lewis, Friedkin, Baker, & Perry, 2011). First, teachers began by clarifying pedagogical research questions (Hiebert et al., 2003), whereby they articulated their own hypotheses that connect the task design with the intended learning goals. Next, teachers reasoned about their choice of instructional strategies, and specified how these tasks can help change students' thinking before they designed the lesson. This shift from 'spontaneous' decision-making to one in which teachers plan and consider possibilities is the essence of the discipline of noticing—'to be methodical without being mechanical' in order to be more sensitised to notice in the moment (Mason, 2002, p. 61). Teachers then collected data on students' thinking, which helped to inform future revisions to the lesson design. Finally, teachers interpreted the data, and drew conclusions about the effectiveness of the task on student learning.

This systematic investigation of teaching via Lesson Study was useful since it provided a theoretical justification for, and an operationalisation of, the design study methodology adopted in this research. Lesson Study, which situates the systematic investigation of teaching within a cycle of activities to make teachers' thinking visible, focuses on improving teaching, instead of improving teachers. Hence, Lesson Study was adopted for this study because it not only encapsulates the essence of the design research paradigm, but also provides a lens, both to examine the noticing of groups of teachers, and to zoom in on a single teacher.

Participants and Setting

This chapter recounts vignettes of what, and how, six teachers from Greenhill Primary School (a pseudonym) in Singapore noticed as they collaboratively designed a lesson on Fraction of a Set for Primary Four students (age 10), given three explicit focal points: mathematical concept, students' confusion, and teachers' instructional decisions. Six teachers, who volunteered for this research, were involved in this Lesson Study group: Kirsty (facilitator); Cindy; Flora; Anthony; Rani; and James (research teacher). All teachers had at least five years of experience teaching mathematics.

In our study, we incorporated Mason's (2002, p. 95) practices of noticing—systematic reflection; recognising; preparing and noticing; validating with others—into the Lesson Study protocol. This modified protocol provided a way for teachers to discuss the mathematical aspects of teaching and learning. The first author primarily took on a participant observer role, shifting between observational and participatory roles during the seven lesson study sessions. During the discussions, he used questions to prompt and direct the teachers' attention to explicit focal points and provided necessary mathematical content knowledge when needed.

Data Collection, Condensation and Analysis

Data were collected and generated through voice recordings of the lesson study discussions and video recording of the lessons observed. A key challenge during data analysis was to deal with the huge amount of data generated from the recordings. In order to condense the data to a level that was manageable, the following procedure was followed:

1. All recordings were reviewed with the field notes taken;
2. The voice recordings were marked for discussion segments that dealt with the five key tasks of Lesson Study. Segments that were focused on logistical issues, administrative matters, and other unrelated incidents were not marked for further analysis;
3. These selected segments were then reviewed again, and initially classified using the framework for noticing (van Es, 2011) student thinking, which is shown in Table 1;
4. Mathematically noteworthy segments were then selected for transcription. Care was taken to ensure a wide spread of segments ranging from baseline noticing to extended noticing (van Es, 2011).

The classification of noticing segments as productive or otherwise, and the selection of noteworthy segments were potentially biased, but this issue was negotiated partially through the use of the five key tasks in the Lesson Study (Lewis et al., 2011), and the aims related to enhancing student reasoning (Hiebert et al., 2003). Segments were characterised as productive using our defining characteristic of

Table 1
Framework for noticing students' thinking adapted from van Es (2011, p. 139)

	What teachers notice	How teachers notice
Level 1 Baseline	Attend to generic aspects of teaching and learning, e.g. seating arrangement, student behaviour, etc.	Provide general descriptive comments with little or no evidence from observations
Level 2 Mixed	Begin to attend to particular instances of students' mathematical thinking and behaviours	Provide mostly evaluative comments with a few references to specific instances or interactions as evidence
Level 3 Focused	Attend to particular students' mathematical thinking	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence
Level 4 Extended	Attend to the relationships between particular students' mathematical thinking, mathematical concepts and teaching approaches	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence, make connections to principles of teaching and learning, and propose alternative pedagogical solutions

whether teachers responded with instructional decisions that promote student thinking. The selected segments were of mathematical or pedagogical interest, and were characterised mainly by discussions surrounding issues related to the Three Points (Cohen, Raudenbush, & Ball, 2003; Yang & Ricks, 2012).

After we selected and condensed the huge amount of data, we began the process of transcribing the selected segments to facilitate further analysis. The selected episodes were transcribed word for word, including pauses (…), and ungrammatical or colloquial language, which were not edited. Words added into the transcript to enhance clarity were given in angled parentheses [ ], and actions, if any, were indicated within round parentheses ( ). Findings related to teachers' noticing were developed through identifying categories, codes and themes related to the elements of productive mathematical noticing. To aid analysis, the Three-Point Framework (Yang & Ricks, 2012) and the processes of noticing (Jacobs et al., 2010) were used to code instances in the selected episodes. A 'thematic approach' was used to develop patterns within the instances of these selected episodes (Bryman, 2012, p. 578).

The FOCUS Framework

The FOCUS framework characterises two important components of noticing by teachers who engage in productive classroom practices:

1. An explicit focus: The three focal points, and their alignment;

2. Focusing: The active process of pedagogical reasoning that aligns the instructional decisions to the observations made.

An explicit focus reflects the notion that noticing is more likely to be productive when teachers use a frame to guide what they attend to (Levin, Hammer, & Coffey, 2009). The second component of the FOCUS framework stems from the idea that it is not trivial to direct one's noticing, but this may be realised through teachers' pedagogical reasoning, which connects what they observe to how they respond to classroom situations. Together, attention to these two components of the FOCUS Framework can support a teacher's efforts to enact productive classroom practices that can enhance students' reasoning.

An Explicit Focus

The FOCUS framework uses three specific mathematically significant aspects of learning and teaching as explicit foci for noticing. These three focal points are (1) Concept; (2) Confusion; and (3) Course of action. These points parallel the Three Point Framework suggested by Yang and Ricks (2012). The teaching of *fraction of a set* at Primary 4 (age 10) can be used to illustrate these focal points: a teacher may identify the key concept as the fact that the relationship between the number of elements (items) in a subset and the set can be represented as a fraction (Concept); recognise students' confusion with this concept in terms of their inability to see a set of objects as the whole (Confusion); and propose to create tasks where students can partition a set of items and explain how their partitions relate to fractions (Course of action). The three focal points also provide a *language* for teachers to describe and analyse the relationships between specific aspects of the concept (Concept and Confusion) to the design of the task (Course of action).

Besides these three focal points, the FOCUS Framework also highlights the crucial notion that aligning these three points is challenging. A teacher, for instance, may be able to identify the concept and students' confusion around the concept, but may not be able to respond appropriately during the planning, delivery or review of a lesson (Choy, 2014a, b). Ensuring that the teacher's response targets the confusion associated with the concept can increase the likelihood of a more productive stance in noticing. Therefore, the alignment of the three focal points forms part of the explicit focus for noticing.

Focusing Noticing

The process of focusing attention in order to bring the three focal points into alignment may not come naturally to teachers. This highlights the critical role of pedagogical reasoning as a mechanism to connect the process of attending to the process of responding in noticing. The alignment of the three focal points thus depends on how teachers connect their responses to what they see or attend to. The FOCUS framework proposes that teachers' responses can be better aligned with the other two focal points when they base their instructional decisions on the interpretation of what they attend to. This can be achieved by justifying responses using specific details from observations, and by considering other possible courses of action. To a large extent, this component of the FOCUS Framework resonates with what van Es (2011) termed as focused or extended noticing.

Together, the explicit focus (the Three Points and their alignment) and the pedagogical reasoning (focusing), not only provide a way to examine at the macro level what makes noticing productive, but more importantly, can capture a micro view of what happens during the planning, teaching and reflection of a lesson. These perspectives can be combined to build a theoretical or ideal model of the noticing process, which describes and decomposes noticing at a more fine-grained level, as demonstrated in Figure 1.

The theoretical model from the FOCUS Framework (see Figure 1) describes what, and how, a teacher can notice productively when learning from practice. It maps a teacher's noticing processes (attending, making sense and responding) through three stages of learning from practice (planning, teaching, and reviewing) to the three key productive practices for mathematical reasoning (designing lessons to reveal thinking; listening and responding to student thinking; and analysing student thinking). In other words, the model describes an idealised process of productive noticing, where teachers make instructional decisions that promote student thinking. The model explicitly highlights the three crucial focal points, and how the alignment between these three points can be achieved. Referring to the planning portion in Figure 1 as an example, a teacher is more likely to design a task that targets and reveals student thinking when he or she:

1. Identifies specifics of the mathematical concept(s) for the lesson;
2. Recognises what students may find difficult or confusing about the concept;
3. Analyses why students might find the concept difficult or confusing;
4. Analyses possible ways to address students' confusion about the concept; and
5. Develops and implements a high-level cognitive demand task (Smith & Stein, 1998) to target students' potential confusion about a concept.

The explicit focus (Steps 1 and 2) helps support teachers in their systematic reflection of student thinking. Teachers' analysis of students' confusion (Step 3) and possible ways to address the identified sources of confusion (Step 4) prepares the teachers to consider possibilities so that they can respond with a better designed task (Step 5), which targets students' confusion to support them in their learning of the concept.

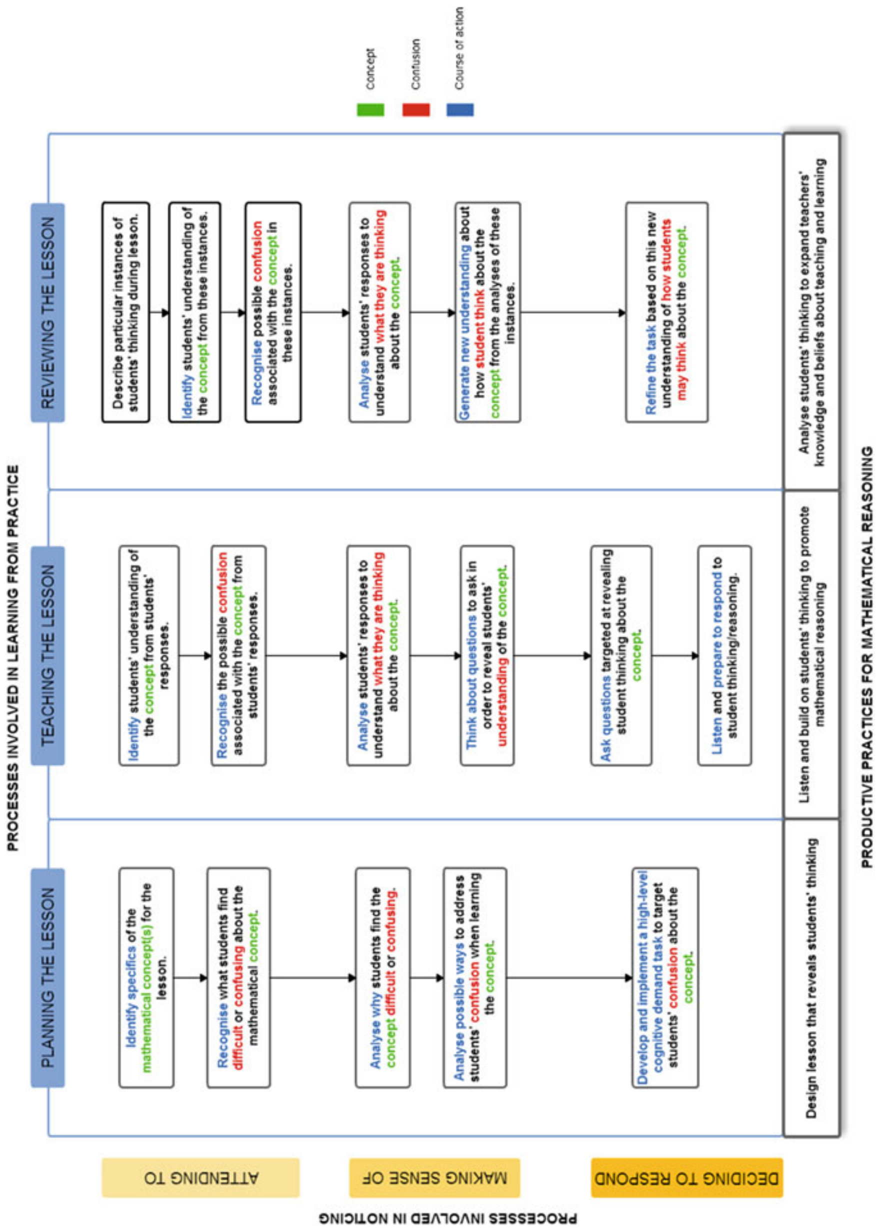


Figure 1. Theoretical model for productive noticing.

Snapshot of Noticing: From Planning to Reviewing

This section illustrates how the two components of the FOCUS Framework—an explicit focus and focusing noticing—can be used to provide snapshots of teacher noticing by analysing and characterising teachers' noticing. Three vignettes, which centre on James (the teacher who taught the lesson) are presented and analysed: The first focuses on a few discussion episodes that happened during the planning; the second highlights what James noticed in the moment during the lesson; and the third recounts what teachers noticed during the post-lesson discussion. Each vignette is then followed by a discussion on how the FOCUS Framework can be used to support teacher noticing before, during and after the lesson.

Vignette 1: Analysing James' Noticing During Task Design

The role of analysing and justifying in aligning the Three Points can be seen in James' explanation of how a *met-before* (McGowen & Tall, 2010) of 'fraction as part of a whole' may hinder students' understanding of 'fraction of a set'. During the first Lesson Study discussion, James highlighted the targeted concept and possible student confusion:

> I think the objective for fraction of a set is for students to see, to interpret fraction as part of a set of objects. Previously, the fraction [concept] they learnt is more of part of a whole. They are very used to thinking about part out of a whole. Now that we give them a lot of whole things, they cannot link that actually these fractional parts can refer to a set of whole things also. So I think, to me, I feel that the connection that is missing, is that, how this fraction concept—which is part of one whole, which they have learnt so far—can be linked to [a set of] whole things. For example, previously we used to teach fractions as parts of a cake or pizza. From that, how can it be that we have many pizzas, we don't cut out the pizza, there is a fraction of the pizzas. I think they cannot make a link there.

In this episode, James not only described specific details about the Concept ('… to interpret fraction as part of a set of objects'.) and the Confusion ('They are very used to thinking about part out of a whole'.), but he was also able to relate these aspects to his knowledge and experience. James then suggested that students may only possess an image of fractions as 'part of a whole' (see Figure 2); and highlighted how the type of examples used by teachers to teach fractions ('…previously we used to teach fractions as parts of a cake or pizza…') may have been stuck in the students' minds. Thus, according to James, students' notion of fraction as 'part of a whole' might have conflicted with the notion of fractions as part of a set of objects:

> For me, the main difficulty is to relate part of a whole into items that are "whole" but you take a fraction out of it. So, I think that's where the confusion comes… [After some time] For example, if you say ¾ of the cats are… [Imitating the students] Ah… you cut the cat into three quarters? [Laughter] Cut each cat into four parts. So, yeah, but based on what they learnt so far, that may be the first thought they might have. To them, fraction could still be cutting up into parts. Whereas, fractions of a set, we leave the things as a whole entity but we look it as a collection of things.

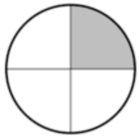


Figure 2. Concept image of fraction as ‘part of a whole’.

The link between students’ image of fraction as ‘part of a whole’ and their difficulty grasping the idea of ‘fraction of a set’ was further elaborated by James with the use of two examples—the pizza and the cat. Particularly, he drew teachers’ attention to students’ ways of thinking about fractions with his vivid example of ‘cutting up the cat’ to illustrate how students might be thinking of fractions as ‘cutting up into parts’. In the next session, James highlighted an example from the textbook to reiterate what students were confused about:

> I think that the difficulty is putting the things into the sets, and imagining that each of this set is one part. The textbook makes it look like a very good way to teach this, they arrange the items very neatly into visible lines like this, for example, like this one, 2 fifths of the circles are yellow [See Figure 3]. It is very clear and you can see two sections. But without the pictures, the children cannot imagine neatly like that.

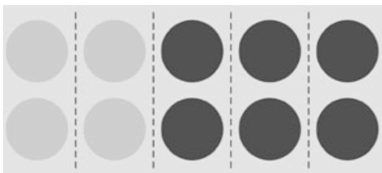


Figure 3. ‘*Visible lines* ’to show equal partitions of a set of items.

As seen from these instances of noticing, James was able to direct his colleagues to consider possible reasons for students’ difficulties by maintaining a focus on, and reasoning about, the Concept and Confusion. He stressed the diagrams might have made it obvious for the students to see the partition, and students possibly find it difficult when the diagrams were removed. James’s noticing prompted Flora to suggest getting students to ‘arrange’ the items into the partitions and explain why they arranged it that way. James then suggested a possible teaching approach that made explicit links between the three focal points:

> I think the confusion part also comes when… we tell [them] that … ¼ of the cups are yellow and then the answer is 4 cups. Huh… ¼ and then why got 4 in the 1/4? They cannot link between the… the ¼ in their mind is still ¼ of a whole… and then there is these four cups, four whole things… and so they cannot link… I was thinking whether we can put it into… something more familiar because… eh… they have learnt models [referring to the Singapore Model Method], how to represent questions in model also, so, I was just looking at this… could we box the whole thing up instead… These lines can be the partitioning of the whole model… they [students] can still see that the 4 items are still inside the parts.

James' suggested approach was directly linked to students' image of 1/4 as 'part of a whole'. He attempted to use the part-whole model (see Figure 4), which the students were familiar with, as a scaffold to help them see that there could be 'whole items' inside a 'part'. This provided a bridge for students to extend their notion of fractions by emphasising fraction as a way to express the relationship between a part and its whole.

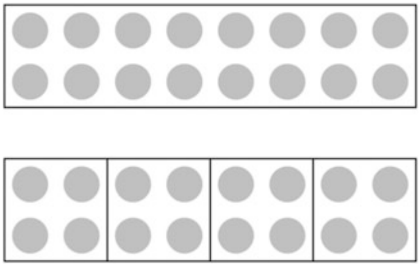


Figure 4. James' use of the part-whole model.

By directing students' attention to the number of discrete items in a partition of the whole, James hoped to create a way for students to see that fractions can be used to refer to 'whole things'. James' noticing would be characterised as productive in this case because he directed his noticing to the three focal points and justified how the suggested approach might target students' confusion about the notion of fraction of a set. Hence, what James attended to and analysed provided some design considerations for the task. What distinguished James' noticing as more productive was not the workability of the approach suggested, but rather the justification that reinforces the alignment between the three points. Justification based on what was noticed not only helped the teachers maintain their attention on specific concepts and students' confusion, but also lessened the likelihood of generating a course of action that does not provide opportunities to enhance students' reasoning.

Productive Noticing in Task Design. James' noticing, as analysed by the FOCUS framework, illustrates that both the focus and the focusing are crucial for designing a task that reveal student thinking. Engaging students with appropriate tasks is critical for developing students' mathematical reasoning (Brodie, 2010; Sullivan, Clarke, & Clarke, 2013) and, hence, the design of mathematics tasks plays a key role in facilitating and encouraging student thinking (Ball & Bass, 2003; Mason & Johnston-Wilder, 2006; Smith & Stein, 1998). Therefore, teachers need to design, select and adapt tasks thoughtfully so that they can provide ample opportunities for students to generalise, explain and justify their mathematical ideas (Ball & Bass, 2003; Smith & Stein, 1998; Sullivan et al., 2013). The FOCUS Framework can support teachers to do this work by offering them a language to explain their task design with regard to the three focal points. This helps to direct teachers' attention to how students think about the concept so they can prepare to notice when they teach the lesson.

Vignette 2: Analysing James' Noticing During Lesson Delivery

After the initial warm-up activity, James then went on to explain the proposed task using 12 physical cubes with a colour configuration of 2 green, 4 blue, 3 red and 3 yellow. In the following interaction, James engaged Student S5, perceived as competent in mathematics, in an interesting conversation (Figure 5):

In this episode, James attended to S5's use of the cubes to reveal how S5 thought about the partitioning. He realised that S5's idea of partition was different from what he had in mind (Line 15). James then tried to ask S5 some questions to understand what S5 was thinking with regard to the six groups (Lines 9 and 11). S5 seemed to have understood about the 'six parts' and counted each cube (Line 9) in one of the rows he created. James could see that S5 understood that 1/6 of the total number of cubes in the first row is green ('And the green is what? 1 out of? 6, is it?'). S5's answer of 'still the same' in Line 12 indicated that he perceived the grouping as two equal groups of 6 cubes, with 1 green cube in each group or possibly a different partition. James's expected answer—that the two green cubes form one out of the six equal partitions—was thus different from S5's. Therefore, James tried to get Student S5 to see his expected answer by putting the two rows of cubes together (Line 13).

James' question (Line 13 and 15) indicated he was trying to get students see his expected arrangement of the cubes. His use of the cubes as a way to hint at the intended arrangement did not seem to convince S5 (Line 13). S5's hesitation pointed to a possible confusion and showed he did not attend to the same structural features (e.g. imaginary lines) as his teacher. This was evident from S5's arrangement of the cubes that did not show the six partitions clearly (Line 15).

Sensing that S5 might not have caught his expected answer, James then asked another student, S7, to arrange the cubes and he came up with a configuration meeting James' expectations. It appears that James noted and interpreted specifically what S5 was thinking with regard to the partitioning, but his response was limited in revealing explicitly what S5 was thinking. James tried to direct S5 to see the intended arrangement through a series of questions to funnel his thinking. This approach did not seem to work and S5 was confused at the end of this episode. An alternative approach would have been for James to ask S5 to explain his own reasoning for his arrangement, so that James could then make sense of what S5 was thinking (Burns, 2005; Davis & Renert, 2014). His response during the interaction (Critical Point) did not help S5 to overcome his difficulty in seeing the proposed partition, and James missed an opportunity to find out what S5 was thinking. It appears that James did not have, at his disposal, other ways of responding when the

1. James: What fraction of my cubes is green? OK, [S5]?

2. S5: 1 out of 6.

3. James: 1 out of 6.... 1 sixth. Let me shift it up a bit (James shifts the cubes on the table so that everyone can see on the projector). Anybody disagree with [S5]? He said it's 1/6. Hey... [S6]? No? Do you agree or disagree with [S5]?

4. S6: No.

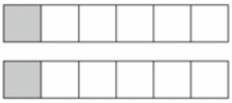
5. James: Don't agree. Then what would be your answer then?

6. S6: 2 out of 12.

7. James: Ok. We have two answers here. 2 out of 12 and S5 said 1 out of 6. (Writes the fractions on the white board) Do you think they are related?

8. Students: [Chorus] Yes…

9. James: Ok. First, [S5]. Can you come and show us how you got 1 part out of 6 when there are so many cubes here. (S5 comes out and arranges the cubes.)



Ok. [S5], stay there... stay there. Where's your six parts? (S5 points to the cubes and counts 1, 2, 3, 4, 5, 6...)

And the green is what? 1 out of? 6, is it?

10. S5: Yeah.

11. James: Then what about the remaining cubes?

12. S5: Still the same.

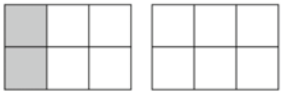
13. James: Still the same, ok? If I put it this way? (James puts the two groups of cubes together.)

Would you all be able to see the six parts?

Figure 5. Transcript for Vignette 2.

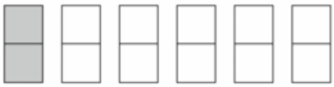
14. Students: [Chorus] Yes...

15. James: Yes... So, [S5], where are the six parts? (S5 points to the cubes again, and shrugs his shoulders.) Ok. Can you imagine the imaginary lines between the cubes? OK. How can you have put this better? (S5 rearranges the cubes.)



How many parts can you see now? Anybody wants to give [S5] a hand?

Yes, [S7]. Ok. Thank you, [S5]. (S7 comes out to do another arrangement.) Mmm ... Something different from what [S5] did. (S7 rearranges the cubes to be 6 groups of 2. See Figure 5.)



Ok. Let’s shift this a bit. Ok. Do you see 6 parts now?

16. Students: [Chorus] Yes…

17. James: A bit clearer?

18. Students: [Chorus] Yes…

19. James: Thank you, [S7]. I was asking for the fraction of…

20. Students: One out of six…

21. James: Green cubes right? So, it’s one part out of…

22. Students: Six.

23. James: Six parts. Same thing, yeah? Has my number of cubes changed?

24. Students: No…

25. James: So, actually, is [S6] right to say that it’s actually 2 parts of 12 also?

26. Students: Yes.

27. James: Actually, he’s correct also? But how did I get from 2/12 to 1/6?

28. Students: Divide… Simplify…

29. James: Yes… we could have simplified it, right? They are equivalent fractions, right?

Figure 5. (continued)

student gave an unexpected explanation. The lack of alignment between his response and S5's confusion reflects a lapse in James' awareness of the student's thinking. Hence, his noticing would be classified as non-productive, according to the FOCUS Framework, even though his attention was focused and his interpretation might be accurate.

Noticing in responding to critical incidents. James' encounter with Student S5 is an example of a critical incident. Critical incidents are events that occur during a lesson, and which have the potential to deepen our understanding of students' mathematical thinking (Goodell, 2006; Yang & Ricks, 2012). These incidents can involve students' unexpected responses to teachers' questions (Yang & Ricks, 2012); those that raise questions about teaching approaches or students' understanding (Goodell, 2006); or events that change the direction of the lesson from what was planned (Fernandez, Cannon, & Chokshi, 2003). Reflecting on critical incidents is important for developing teaching practices that enhance students' mathematical thinking (Fernandez et al., 2003; Goodell, 2006).

The ability to see and interpret these incidents in the moment can impact how teachers decide to respond to these events. The FOCUS Framework highlights that the key to respond productively to enhance student reasoning lies in the ability of the teacher to adopt a more interpretive stance in listening, and allow students' responses or answers to modify the flow of the lesson (Davis & Renert, 2014). Moreover, the teacher has to think on the spot to attend selectively to the myriad responses from the students. The alignment between the three focal points is helpful for directing teachers' attention to the mathematically significant details in the midst of a lesson. By being more sensitive to students' Confusion, and maintaining a focus on the Concept, teachers might be able to raise their own awareness of how they listen to students' responses that make aspects of their thinking visible. In so doing, they might have a better chance of generating a Course of action that enhances students' mathematical thinking. On the contrary, as illustrated by James during the critical incident, when teachers fail to maintain a keen awareness of student thinking, they are more likely to think about their own thinking, instead of the students'.

Vignette 3: Analysing James' Noticing During Post-lesson Discussion

In the post-lesson discussion, the first thing that James brought to the attention of the teachers was students' inability to partition, which he hypothesised was because they counted and simplified the fraction. For example, students might have counted 2 green cubes out of 12 and written the answer as 2/12 before they simplified to 1/6:

> The glaring thing that I noticed about my pupils is that too many of them, they didn't get their fraction by partitioning … they got it more by counting and then simplifying… so that was the easy option to them. Which was why later when I got them to explain, "How did you get this fraction for example?"… "one sixth of the cubes were red" or something like that. Some of them were not able to show the six parts or to group the objects into six parts.

So they were a bit lost. Because how they did it was, count the number of red cubes over the total number of cubes, then simplify. When they cannot put it in parts, right … it was very clear what their thought process was – simplify …

James highlighted that the students ‘didn’t get their fraction by partitioning’, but instead by ‘counting and then simplifying’. He explained how that prompted him to try asking students to reason how they arrived at the fraction. James was able to give very specific details about students’ difficulty in showing the partitioning of the cubes (‘they were not able to show the six parts…’), and interpreted that as a manifestation of their ‘thought processes’. James’ noticing was not only specific and focused on the three focal points, but also more importantly, it set the stage for the teachers to learn about another possible student confusion not previously discussed. James attended to the Course of action—getting students to show their understanding by representing the fractions through partitioning of the cubes—and realised that students had difficulties doing that (a new Confusion), and supported his claim using his observations from the critical incident. He reasoned that there could be a gap in students’ understanding even though they might give the correct answers.

Even though the teachers did not decide precisely how to respond, they suggested different possible interpretations that could potentially generate new understanding of how students think. The teachers attended to specific instances, and made connections between their observations, and that of others to their own knowledge and experience. The process of detailed interpretation further encouraged teachers to examine these observations more deeply. For example, the teachers argued that getting students to explain their partitioning, even when they were able to give the correct answers, could have given teachers insight into students’ thinking. Flora articulated the need to listen and referred to Student S5 as an example:

[Student S5] is very complex when he does maths. I’ve had him to explain to me. He can get an answer just like that – without workings or anything. The boy is very complex up here (pointing to her head). And I don’t fault him for doing things a bit differently – as long as I understand what he is trying to say like, I can imagine how he does things. I think it’s okay. Like for him, he may arrange it that way, but he may mean it like the second way…

James agreed and also highlighted that it is important to be more specific in the questioning with regard to the three focal points. The emphasis on getting students to explain more specifically in order to reveal their reasoning suggests a shift from explaining to listening as a result of teachers’ noticing. As Mason (2002) suggests, the purpose of noticing is to bring to mind the possibility of a different decision. James’ noticing, throughout the post-lesson discussion, sensitised his awareness and helped him think more deeply about students’ thinking beyond giving the right answers:

I was just thinking the danger of – during the design of this lesson, we didn’t see that maybe they may skip the partitioning part of it … they didn’t show how the answer is found. It is something we need to recognise. It is good that we now know that if they missed the partitioning part… this may cause a problem later. Missing the partitioning part will be fine

> until we show them they have a problem. Even though they can do a fraction of a set, and they can solve fraction of a set problems – it will pose learning problems in future when they move on… I think we need to look at it more carefully.

James' noticing can be largely characterised as productive because his suggestion was targeted at what he saw and understood about students' confusion when learning the concept. What he noticed about the critical incident helped other teachers to gain insights into students' thinking: students' difficulties in partitioning and how that is related to understanding fractions. More importantly, he recognised the 'blind spot'—that students might skip the partitioning—for their initial lesson plan and suggested that they should look at the task design more carefully. James was able to see how this gap in students' understanding could have implications beyond the lesson to find 'the general meaning of such incidents' (Yang & Ricks, 2012, p. 46). The use of specific instances to support his claims or suggestions also indicates that James has begun to gain a heightened awareness of student thinking when viewing the critical incidents that happened during the lesson.

Noticing to zero in on student thinking during reflection. Although James' in-the-moment noticing in Vignette 2 was less productive, his noticing during reflection was productive as described in the preceding paragraphs. Fruitful post-lesson discussion occurs when the points raised help teachers to refine their ideas about students' thinking or lesson design. They should go beyond vague or broad statements to focus on supporting or refuting claims made by teachers about students' learning. In this way, the discussions can move towards a more generative position when these claims are supported or refuted based on teachers' observations of specific instances. For this study, we supported teachers' systematic investigation of their practice using the three focal points and their alignment to frame the post-lesson discussions.

As seen from James' responses, his focus on the three focal points seemed to help him zero in on the mathematical features of the critical incident. In particular, James and other teachers were able to draw on specific instances from the lesson to analyse and explain whether the planned Course of action targeted students' confusion. As a result, the discussion, as described in Vignette 3, generally centred on students' strategies related to the incident, and the implications for the design of the task. More importantly, the two components of the FOCUS Framework enable teachers to think about what they observed from the lesson, which led the teachers to gain new insights about students' thinking as seen in Vignette 3.

Concluding Remarks

The FOCUS framework highlights that teachers' noticing is more productive when they direct their attention to the *mathematically significant* aspects of engaging in all three phases of diagnostic teaching—the planning, delivery and review. The three focal points offer a focus for teachers to attend to, and make sense of, in order to respond with an instructional decision that can potentially enhance student reasoning. Our findings support the use of an explicit focus to frame noticing (Goldsmith & Seago, 2013), rather than not directly specifying a focus for teacher noticing (Star et al., 2011).

The snapshots of noticing, presented here, demonstrate how the FOCUS Framework is useful for researchers when analysing what, and how, teachers notice during the planning, teaching and reflecting of mathematics lessons. The three focal points and their alignment, together with the pedagogical reasoning processes to align the three points, provide a means to describe and characterise both more productive and less productive noticing in terms of the instructional decisions undertaken by the teachers. These snapshots paint a detailed portrait of a teacher's noticing, and can be used to point out the strengths and areas for improvement to promote more productive noticing. Such characterisation can provide researchers with a language to decompose and analyse complex interactions between the processes of noticing before, during and after a lesson.

With regard to its practical implications, the framework provides a means to support teachers in reflecting systematically, suspending one's habitual reactions to classroom events, in order to have a different act in mind (Mason, 2002). By emphasising both specificity (van Es, 2011) and *alignment* of the three focal points, the framework can be used to create opportunities for teachers to focus on mathematically relevant details when planning to teach a lesson. In this chapter, the two components—explicit focus and pedagogical reasoning—of the FOCUS Framework were used to support teachers in their planning and reasoning about the evidence from observations, in order to target their instructional decisions at enhancing student reasoning.

While this research aimed to characterise the notion of productive mathematical noticing, it is important to acknowledge that our study was limited to investigating the mathematical and pedagogical aspects of enhancing students' reasoning. This study, for example, did not investigate what teachers notice about classroom management (van den Bogert, van Bruggen, Kostons, & Jochems, 2014) even though it may play a role in carrying out mathematical activities to enhance reasoning. Moreover, what students notice mathematically about a task, and how, was not examined. Since teacher noticing and student noticing are 'two sides of the same coin', it could be fruitful for future researchers to explore the relationships between teacher and student noticing. Finally, despite the measures taken to reduce researcher's bias, our interpretation of the data only constitutes one possible emerging narrative about productive teacher noticing. It remains to be seen whether the FOCUS Framework is robust enough to be applied, and adapted, for other contexts and in other studies.

Notwithstanding the limitations of this study, our findings demonstrate how the framework can support teachers in the 'practices' of noticing, so as to enhance their 'sensitivity to notice opportunities to act freshly in the future' (Mason, 2002, p. 59). Therefore, this study suggests the potential of incorporating the framework into the design of professional development activities. In conclusion, the FOCUS Framework, as a research and practical tool, can afford opportunities for both researchers and teachers to investigate the high-leverage practice of teacher noticing.

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