Investigating the Relationship Between Professional Noticing and Specialized Content Knowledge

Lara K. Dick

Abstract Professional noticing of children's mathematical thinking as conceptualized by Jacobs et al. (J Res Math Educ 41(2):169–202, 2010) includes attention to and interpretation of children's mathematical thinking, and deciding how to respond instructionally. In order to interpret a child's mathematical thinking, a teacher draws on her or his mathematical knowledge for teaching (MKT) (Ball et al. in J Teach Educ 59(5):389–407, 2008). This research study focuses on the relationship between elementary preservice interns' development of MKT and their engagement with professionally noticing their students' mathematical thinking through analysis of their students' work samples. An integrated professional noticing and MKT framework for simultaneous measurement is applied to the research study. Four preservice interns placed in a first-grade classroom participated in the study. A sequence of three professional learning tasks (PLTs) focused on the preservice interns' analysis of their students' multi-digit addition and subtraction work was developed. Results show specialized content knowledge (SCK), a subset of MKT, as an integral part of professional noticing. The results suggest that in situated contexts focused on developing SCK, preservice interns can increase their engagement with professionally noticing their students' mathematical thinking.

Keywords Professional noticing · Mathematical knowledge for teaching · Specialized content knowledge · Preservice teachers · Student work

The professional noticing of children's mathematical thinking framework (Jacobs, Lamb, & Philipp, 2010) emphasizes attention to and interpretation of children's mathematical thinking, as well as deciding how to respond instructionally. To diagnose children's mathematical thinking, a teacher draws on her or his mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008). Thus, to engage in professional noticing, teachers must rely, at least partially, on their MKT. While others have addressed the intersection between MKT and

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E.O. Schack et al. (eds.), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks*, Research in Mathematics Education,
DOI 10.1007/978-3-319-46753-5_20

professional noticing either by referring directly to MKT in the context of teacher noticing or by designing noticing interventions focused on developing aspects of MKT (Ball, 2011; Fernandez, Llinares, & Valls, 2013; Flake, 2014; Kazemi et al., 2011; Schack et al., 2013; Vondrova & Žalská, 2013), I have chosen to focus on the integration of the measurement of MKT and teacher noticing. I contend that because MKT is content specific, measurement of teacher noticing is situated both in mathematical content and in the context of interventions designed around professional noticing. For this chapter, one aspect of a research study in which preservice elementary interns' specialized content knowledge (SCK), a subset of MKT, and professional noticing were measured simultaneously will be presented. A discussion of further ideas for integrating the measurement of SCK and professional noticing of children's thinking is included.

Conceptual Frameworks

This study examines the relationship between the development of preservice elementary interns' MKT (Ball et al., 2008) and their engagement with professionally noticing their children's mathematical thinking as conceptualized by Jacobs et al. (2010). For this study, SCK, one component of the MKT framework was integrated into the professional noticing framework. Below, the individual frameworks will be discussed and then the integrated framework guiding the study will be presented.

Mathematical Knowledge for Teaching

For the past 30 years, researchers have sought to determine the types of knowledge necessary for the teaching and learning of mathematics. In Shulman's (1986) seminal paper, he described subject matter knowledge as different domains of knowledge comprised content knowledge, pedagogical content knowledge, and curricular knowledge. Marks (1990) adapted Shulman's concept of pedagogical content knowledge to elementary mathematics. He defined four highly connected subsets of pedagogical content knowledge: subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter (i.e., texts and materials), and instructional processes for the subject matter (p. 4). Manouchehri (1997) proposed having teacher educators infuse content knowledge and pedagogical content knowledge into their preservice teacher preparation programs. She explained that subject specific knowledge should not be taught separately from knowledge of students, and knowledge of teaching and learning.

In 2000, Ball identified three concerns that needed to be addressed for successful merging of mathematics content and pedagogy. She explained, "the [first] problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how much knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice" (p. 244). During the past two decades, Ball and her colleagues have built on Shulman's (1986) work to develop a comprehensive framework for describing mathematical knowledge as it is used in the practice of teaching. Their framework defines MKT and breaks it into two main domains: Pedagogical Content Knowledge is comprised of Knowledge of Content and Students, and Knowledge of Content and Teaching. Subject Matter Knowledge is comprises of Common Content Knowledge along with Specialized Content Knowledge (Hill, Ball, & Schilling, 2008).

Knowledge of Content and Students includes the ability to predict how students will respond to mathematical topics, what they will find interesting and which topics will be the most difficult. Knowledge of Content and Teaching includes making appropriate choices for examples or representations, knowing how to best sequence a topic, and guiding classroom discussions. Common Content Knowledge comprises the ability to solve mathematical problems, provide definitions of mathematical terms, and compute correct answers; this type of knowledge is not specific to teachers. SCK is the knowledge teachers draw upon when evaluating students' invented definitions, interpreting their developed algorithms, and asking questions to press students' thinking. SCK is the MKT subset focus of this study.

Professional Noticing of Children's Mathematical Thinking

Professional noticing in mathematics education is a more recent area of interest for mathematics teacher educators. Noticing student thinking is a teaching practice that requires active real-time engagement from the teacher (Mason, 2002; Sherin, Jacobs, & Philipp, 2011). Past research with preservice and practicing teachers has shown that teachers need support in learning to notice students' mathematical thinking and is therefore a practice to purposefully develop (Santagata, 2011; Star & Strickland, 2008; van Es, 2011). Jacobs et al. (2010) conceptualize professional noticing of children's mathematical thinking as comprised three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings.

Goldsmith and Seago (2011) utilized the professional noticing framework to study practicing teachers' analysis of mathematics student work. They explained that professional noticing of student work "involves attending to both the mathematical content of the task and students' mathematical thinking" (p. 170). Similar to the work of Goldsmith and Seago (2011), for this study, the professional noticing framework was used to study elementary preservice interns' engagement with noticing their students' mathematical thinking via analysis of their students' addition and subtraction work samples.

Integration of SCK and Professional Noticing

For this study, preservice interns were engaged with the teaching practice of analyzing student work. Researchers have found that preservice teachers must possess a special type of content knowledge in order to connect what they learn from student work analysis to their teaching practice (Bartell, Webel, Bowen, & Dyson, 2013; Fernandez et al., 2013; Hiebert, Morris, Berk, & Janson, 2007; Jacobs et al., 2010). Hiebert et al. (2007) explained that making connections between analysis of student work and instructional practice "requires a set of competencies or skills that draw directly on subject matter knowledge combined with knowledge of student thinking" (p. 52). They discussed how teachers must (a) observe and predict types of strategies students will use to solve a problem; and (b) know what a particular response implies about the student's thinking. Similarly, Jacobs et al. (2010) explained "to interpret children's understandings, one must not only attend to children's strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts" (p. 195). I consider these specific types of content knowledge to be in line with Ball and colleagues' concept of SCK (Ball et al., 2008; Hill et al., 2008).

In 2013, Vondrova and Žalská conjectured that the "ability to notice was possibly a manifestation of mathematical knowledge for teaching" (p. 361). For this study, I directly considered their claim. I looked specifically at how an intervention designed with a focus on the development of SCK related to preservice interns' engagement with professionally noticing their students' mathematical thinking. Ball et al.'s (2008) "Mathematical Tasks of Teaching" that draw on SCK (p. 10) as well as their explanation that SCK encompasses teachers' knowing "features of mathematics that they may never teach to students, such as a range of non-standard methods or the mathematical structure of student errors," was used to map SCK to each of the components of the professional noticing framework. For example, the mathematical teaching task "using mathematical notation and language and critiquing its use" was considered part of the attend component since it deals with noticing mathematically significant details, while the teaching task "evaluating the plausibility of students' claims" was considered a part of the interpret component since it is used when interpreting a student's work sample. Table 1 contains the subset of Ball et al.'s (2008, p. 10) teaching tasks requiring SCK that were mapped to the professional noticing components. This mapping served as the integrated SCK and professional noticing framework for this study.

Table 1
Mathematical tasks requiring SCK as related to professional noticing

Mathematical tasks requiring SCK Ball, Thames and Phelps (2008)	Related professional noticing component
Critique notation and language	Attend
Evaluating plausibility of student claims	Interpret
Evaluate math expressions	
Know non-standard methods and common errors	
Ask productive math questions	Decide

To summarize, for this study, an intervention was designed for preservice elementary interns to develop their SCK regarding multi-digit addition and subtraction, as well as their engagement with professionally noticing their students' mathematical thinking. In what follows, I address the following research question: how does SCK relate to elementary interns' engagement with professionally noticing their students' work in the context of multi-digit addition and subtraction?

Description of Study

Four preservice interns completing their culminating licensure requirement, a semester-long student teaching field experience in first-grade classrooms, participated in the research study. The interns completed a set of three carefully sequenced professional learning tasks (PLTs) facilitated by a college supervisor. The PLTs were focused on analyzing the interns' first-grade students' written work on multi-digit addition and subtraction story problems. Each PLT included a set of directions for the interns and a facilitation guide for the college supervisor. Prior to each of the three PLT sessions, the interns were provided directions on the types of problems to pose to their students as well as how to choose student work samples to bring to the sessions. Figure 1 contains the facilitation guide for PLT #1. See Dick (2016) for the full PLT directions. Each of the PLTs was focused on developing the interns' SCK around a particular aspect of multi-digit addition and subtraction. Table 2 contains information about the PLT sessions and their SCK focus. During the implementation of the PLT sessions, I took on the role of participant observer (Yin, 1998). As a participant observer, I sometimes interjected comments or questions when the conversation veered off course or when my expertise was needed in analyzing a student work sample; my participation was rare.

Begin by having the student teachers share their task and reasons behind their choices as to which of their students' work to bring to the session.

1. What strategies did your students use to solve the task?
2. What did you find surprising or unexpected in your students' work?

Lead a discussion about their initial anticipation of the ways their students would approach the problems

3. What is the mathematics embedded in each of their strategies?

Lead a discussion about different types of addition/subtraction problems.

4. What questions could you ask to help your student reflect on their strategy?

Lead a discussion on how to probe student thinking without guiding their work and on how to describe student work without projecting their knowledge onto the solution. Suggest that the student teachers take notes while monitoring their students as they complete tasks.

Figure 1. PLT #1 facilitation guide.

Table 2
PLT sessions with description of SCK focus

PLT session	PLT specialized content knowledge focus
One	Different types of addition and subtraction story problems (NGACBP, 2010, p. 88)
Two	Multi-digit addition and subtraction problems levels of sophistication: 1. Direct modeling 2. Counting 3. Number fact strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson, 2003; NRC, 2001)
Three	Developing questioning techniques based on student's mathematical thinking: probing versus extending questions (Jacobs & Ambrose, 2008)

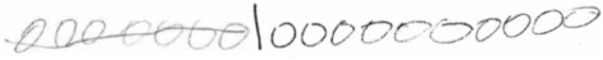
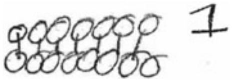
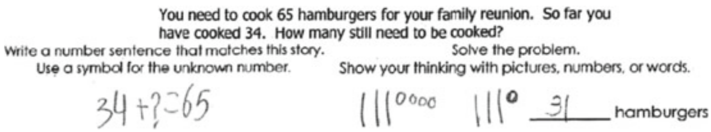
Methodology

The case study presented in this paper is a subset of a larger design research study. Primary data sources were the interns' students' work samples and transcriptions of the three PLT sessions. The interns' discussions during the PLT sessions were divided into "distinct shifts in focus or change in topic" known as "idea units" (Jacobs, Yoshida, Stigler, & Fernandez, 1997). The choice was made to code discourse idea units rather than individual talk turns because collective analysis emerged as an important aspect of the situated learning (Greeno, 1991; Lave & Wenger, 1991) that occurred during the PLT sessions. Each idea unit, when applicable, was coded for the noticing components: attend, interpret, and/or decide. As a means of reliability, the college supervisor was asked to apply the professional noticing codes to the idea units related to student work samples; on the second iteration of analysis, 88% reliability was reached and thereafter, I continued to code alone. After each idea unit was coded for the three noticing components, each component was assigned a level within that component (0: Lacking, 1: Limited, 2: Robust). Table 3 contains the full codebook which was based on Jacob's professional noticing scheme (personal communication, April 3, 2013).

Table 3
Codebook

Level	Description
Attend: mentioning mathematically significant details	
2	Robust evidence • Mentions CORRECT specifics of mathematics they notice – Discussion of strategy type
1	Limited evidence • Mentions some INCORRECT specifics of mathematics they notice • General mention of mathematics including naming strategy
0	Lacking evidence • Missed opportunity for mentioning mathematics
Interpret: what is known about students' mathematical thinking based on student work	
2	Robust evidence • Draws on evidence when giving a plausible interpretation what a student understands – Coherent discussion of students' mathematical thinking
1	Limited evidence • Draws on some evidence when interpreting what a student understands – Making assumptions – Implausible interpretation – Interpretation hard to follow (vague and/or incomplete)
0	Evidence lacking – Missed opportunity for mathematical interpretation – Interpretation lacking mathematical evidence
Decide: next question based on students' mathematical thinking	
2	Robust evidence • Draws on specifics to develop a potentially useful probing or extending question
1	Limited evidence • Develops a probing or extending question – Question not useful or vague – Question potentially useful but not drawing on specifics
0	Decision lacking • No question developed • No mention of students' mathematical thinking

Table 4
Examples of discourse exchanges coded as interpret-2

No evidence of SCK	L: So her counting and marking out the 7 from 17  (Exchange around a correctly answered work sample for 17–7)
No explicit evidence of SCK	CS: 2, 4, 6, 8, 10, 12, 14, 15 circles K: He just didn't finish, it looks like D: 'Cause I've taught them that when you have a certain number, you can make partners to see how many are left over  (Exchange around an incorrectly answered work sample for 18–7) *CS: College Supervisor*
Explicit evidence of SCK	T: Yes, with an unknown number and he used addition so he separated the tens and ones and did a tens stick and one circles and just counted up. So, I guess it's still…it's kind of counting but it's also decomposing in a way K: When I looked at this, I thought it was interesting that he…So he did the 34 here. He knew that he had to get to 65 so instead of counting up to 34–50, he knew…the way he did it was interesting, like he went in and did tens first D: So he just held 34 in his head and did…Like 44, 54, 64  (Exchange around a correctly answered work sample for 65–34)

For each of the three noticing components, within each of the three levels of coding, the designation "with evidence of SCK" was used to highlight instances where the idea unit provided explicit evidence of the interns applying their SCK. The integrated SCK and professional noticing framework presented in Table 1 was used as a guide for coding each of the professional noticing components for evidence of SCK. Ball et al. (2008) described evaluating students' strategies and explanations as teaching tasks that require SCK, which implies that any analysis of student work requires some level of SCK. However, for this study, the decision to code for SCK within the professional noticing components required that an idea unit contain explicit evidence of the interns' drawing on SCK. There most likely were instances where the interns drew on SCK to analyze student work samples, but if they did not explicitly apply SCK to their professional noticing in the discourse exchanges, evidence of SCK was not coded. To illustrate, Table 4 contains examples of excerpts from idea units coded as Interpret 2: Robust Evidence. The first is a plausible interpretation where there is no evidence of the intern drawing on SCK, the second is an exchange where SCK may have been present, but was not explicit and therefore was not coded as evidence of SCK. The third is an exchange where evidence of SCK was explicit.

In the first example, the student colored 17 circles and crossed out 7. The intern developed a plausible interpretation of the student's strategy, but the interpretation lacked evidence that the intern drew on SCK. Interpreting the work sample did not require any expert teacher knowledge; thus, all she needed was common content knowledge for her interpretation. In the second example, the interns interpreted the student's work and determined that the student was employing a familiar strategy, but made a mistake and did not finish drawing the 18 circles. While a plausible interpretation, it is unclear as to whether the interns drew on SCK to interpret the students' thinking. The interns did not explicitly discuss how the student's method made sense with the subtraction problem type or what could have caused the error. Thus, while the interns may have actually drawn on SCK regarding their understanding of non-standard methods to interpret the students' work, there was not explicit evidence of them doing so and therefore this exchange was not coded as exhibiting evidence of SCK. In the final example, the interns interpreted a students' counting up strategy. Their discussion began with one of the interns connecting the strategy to the concept of decomposition. Decomposition was one of the strategies discussed during PLT #2 as part of the SCK focus; an understanding of decomposition is an example of SCK. Because the interns explicitly connected their interpretation of the students' mathematical thinking to SCK regarding decomposition, this exchange was coded as evidence of SCK. To reiterate, explicit evidence was required in order for an idea unit to be coded as "with evidence of SCK."

Results

Overall Professional Noticing Results

From the leveled coding that occurred during analysis, it is evident that, for the most part, the interns increased their engagement with professional noticing throughout the PLT sessions. The Appendix contains a table of the professional noticing leveled coding results for all idea units. Blank entries are for idea units that included discussion, but the discussion did not lend itself toward the particular noticing component; only when a noticing component had the potential to occur, was it coded. The interns' growth in engagement with professional noticing can be seen in two directions. The horizontal line represents the change from limited evidence toward robust evidence ($0 \rightarrow 1 \rightarrow 2$) for the three individual noticing components, visible in each row of the table. The diagonal line represents the interns' development in their overall engagement with the three components ($A \rightarrow I \rightarrow D$) as they progressed through the PLT sessions. In general, the interns engaged first with attending to the mathematics found within their students' work samples, then demonstrated an increase of plausible interpretations of their students' mathematical thinking and eventually began to engage with deciding on productive mathematical questions to pose to their students. As the PLT sessions progressed, the intern's analysis of their students' work exhibited increased coherence of the three components of noticing.

Instances with Evidence of SCK

Explicit instances of SCK, as they appeared throughout the PLTs, are discussed before results of the integrated SCK and professional noticing analysis are presented. All names are pseudonyms.

Evidence of SCK attributed to prior knowledge. Specialized content knowledge possessed by some of the interns prior to PLT #1 was exhibited during the first session. During PLT #1, Kelli drew on previously held SCK. One of her students invented a strategy for solving his "put together/total unknown" (NGACBP, 2010, p. 88) problems (Figure 2-sample 1). Kelli recognized the uniqueness of his non-standard method and asked him to explain the strategy while she audio-recorded his explanation. The student explained

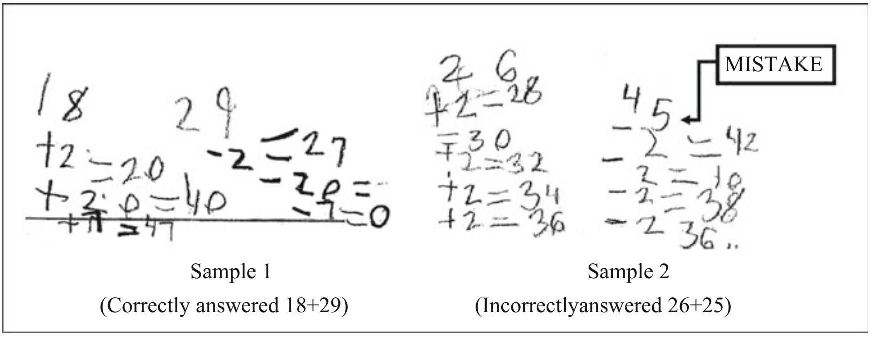


Figure 2. Kelli's student work sample from PLT #1.

> I had 18 and 29 so I subtracted 2 from 29 and it equaled 27, then I plussed that 2 for the 18 and it equaled 20 and I kept doing the same thing and the same thing over and over and I finally got the answer and I found out that the strategy was working really good.

Kelli realized that if the student continued to use this strategy, it could be quite inefficient with larger quantities. She stated

> I knew that he was gonna get stuck on the next one 'cause the next answer was 71, so I was like, 'How is he going to do this when we get to the 45 oak leaves and 26 maple leaves?' So I just left him alone for a little bit.

On the subsequent problem, her student made an error subtracting 3 at the beginning when he wrote to subtract 2, which caused him to get an incorrect answer

(Figure 2-sample 2 contains a subset of the student's work on this problem). Kelli did not catch the mistake, and instead of helping her student find it, she pushed him toward a tens-and-ones strategy. It is evident that Kelli attended to the mathematics behind her student's solution by focusing on the language of his explanation and developed an interpretation of his surprising, non-standard strategy. Kelli illustrated her SCK through both her attention to the student's mathematical thinking, and her interpretation and recognition of the inefficiency of his strategy. While Kelli drew on SCK when noticing her students' work, because she did not look for the student's mistake, the exchange was coded as interpret level-1, with evidence of SCK.

Also during PLT #1, Donna and the college supervisor had an exchange about her students' task. Donna had created a worksheet for labeling an appropriate strategy for single-digit addition problems with the choices: "make-a-ten," "counting on," and "doubles plus one" (Fuson, 2009). While not at the highest levels of noticing, Donna's attention to mathematically significant details and interpretations of her students' mathematical thinking showed evidence of previously held SCK. For example, in part of the idea unit, she stated

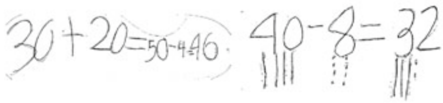
> I found that they really don't have a concept of making a ten or using the double strategy or the doubles plus one, but they all can count on. Like if they have 5 + 4, they go: 1, 2, 3, 4, ***5***, 6, 7, 8, 9 like that, that's how they do it. They don't see that it's 5 + 5 − 1 or that it's 4 + 4 +1 or doubles plus one. They just count on" (emphasis in original).

She then shared individual conferences she had with her students following the assignment. She asked them, "You know, counting on does work but it's not going to work every time for you. What's a more efficient strategy for you to use?" During the exchange, Donna showed explicit evidence of her SCK regarding different strategies for single-digit addition. She was able to evaluate her students' explanations and recognize that she wanted to push her students toward a more efficient strategy. Donna possessed this SCK prior to the PLT session, and she applied it to her interpretation of her students' mathematical thinking. Differently from PLT #1, during PLT #2 and #3, instances of explicit evidence of SCK all related to the SCK foci (see Table 2) and thus, were attributed to the SCK that was being developed through the PLTs.

Evidence of SCK related to the PLT sessions' SCK foci. The only instances of explicit evidence of SCK during PLT #1 were the two instances previously discussed. While PLT #1 focused on developing SCK, the discussions surrounding the development of SCK occurred after the interns' collective analysis of their student work samples. The PLT #1 SCK focus was on different types of multi-digit addition and subtraction problems (NGACBP, 2010) through which the interns were exposed to a variety of different strategies. The interns drew on both of these areas of SCK during PLT #2. At the start of PLT #2, Donna shared the two different types of story problems she posed to her students: an "addend unknown-put together/take apart problem" and a "bigger unknown-compare problem" (NGACBP, 2010, p. 88). She provided evidence of her SCK in her explanation of how finding the missing partner using addition was easier for her students than her traditional view of subtraction as take-away. For the compare problem, when interpreting her students' work, Donna explained how the word "fewer" affected her students' choice of strategy, which drew on the SCK discussion during PLT #2; many of her students were confused as to whether to add or subtract for the "compare problem" (NGACBP, 2010, p. 88). While either operation can be used to solve the problem, the situation equation that would model the story used addition.

In the following idea unit excerpt, two of the interns showed evidence of drawing on SCK while developing an interpretation for one of Kelli's student's approaches to two different subtraction problems (see Figure 3). The interns recognized that the student has "got the concept of tens and ones," then discussed why he directly modeled for one problem and not another problem, though the problems were similar. In the exchange, the interns exhibited SCK regarding different methods and the effect of the actual quantities in the story problem.

K: And there we can see he took away the eight by adding…by drawing out the ones (Sample 2).
D: Um hmm. But it's interesting that he did it there (sample2) are not up there (Sample 1).
K: I know.
D: I mean I guess four is a smaller number so you could probably just count back (Sample 1).



Sample 1 Sample 2

Figure 3. Kelli's student work samples from PLT #2.

Toward the end of PLT #2, the college supervisor introduced the SCK focus for the session: levels of sophistication of different strategies (See Table 2). During their discussion, the group had a conversation about Tammy's students' work on a story problem for 13 + 5. Donna made the comment that she was happy to see Tammy's students "hold the thirteen, and count on from there." The interns were asked if based on the students' work they had analyzed for 13 + 5 they could come up with different strategies the students might employ for 13 + 8. This led to a discussion about making a new 10. Donna shared her experience with students' wanting to work from 10. For example, she discussed a student who, when adding 9 + 8, changed the problem to 10 + 7. She was able to apply her SCK relating to her student's algorithm to this new problem which led to a whole group discussion about different level 3 compensation strategies during which some of the interns exhibited further evidence of SCK.

PLT #3 was designed to develop the interns' ability to decide on appropriate probing and/or extending questions (Jacobs & Ambrose, 2008) to ask their students. Differently from PLT #1 and #2, this SCK focus on asking productive math questions was part of the interns' pre-PLT assignment. Thus, it was discussed toward the beginning of the session with the hope that the interns would draw on their newfound SCK while engaging with noticing the work samples. During PLT #3, the interns exhibited many instances of drawing on different aspects of SCK that they gained throughout the PLT sessions.

For example, one of Donna's students had solved an "add to/change unknown" (NGACBP, 2010, p. 88) problem in a manner that would have been very difficult to interpret without questioning him (Figure 4). During the session, Tammy noted that she was not sure what the student was doing. Donna was able to explain the student's work in light of her asking him probing questions while in class which assisted her interpretation of his mathematical thinking. Throughout the idea unit, the interns collectively discussed the student's solution and drew on their SCK to identify the level of sophistication of the strategy as a level 3 (see Table 3). Kelli explained her reasoning, "I would think so [level 3 strategy]. Since he could explain it. Looking at this I couldn't really tell his work but he could explain it. I mean, I think explaining it is the hardest part. And the fact that he knew he needed less than 50 because he already had over 50." Kelli's comment is focused on the mathematics and illustrates her realization of the importance of asking students questions.

"I didn't ask him about that during but afterwards when I looked at it, I said, 'Why did you draw five on top and on bottom?' and he said, 'Well there were 54 so I drew 54 first' and then he said, 'And then I realized that I needed to get to 100 so I was drawing…I was making partners on top to make sure they added up,' and then he realized that 50 and 50 was 100. So he said, 'And I knew then that I needed less than 50 to make my problem true.' And he can count back and not many of my students can count back, especially with two-digit numbers. So that really surprised me."

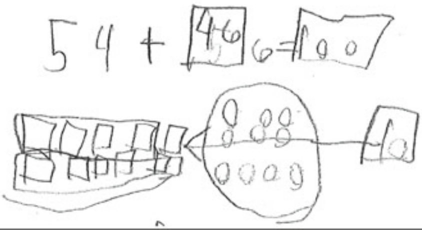


Figure 4. Donna's student work from PLT #3.

As another example, when the interns discussed Kelli's student's work as shown in Figure 5 they realized that while Kelli's student exhibits a higher level of sophistication of strategy for the first problem (sample 1), the student needed help applying her strategy to problems requiring making a new ten (sample 2). In the idea unit exchange, the interns exhibited SCK regarding non-standard algorithms and common student reactions to problems that require making a new ten.

K: It’s decomposition (Sample 1) but it’s also creating equivalent but easier problems because it’s easier to add 20 and 40, and a 6 and 3 together. I mean even I personally use this method when I’m checking their work in my head because I know that it’s pretty efficient.

D: The problem that I’ve seen with my kids is that when they do have to break down their numbers like this, it’s great when the ones is the highest that it goes in that line, but when it gets to like 20 + 40 = 60 and they’ve got like 6 and 7 and that’s 13, then you have 60 + 13 and they’re like, “Okay….” and then they draw up 13 circles and they don’t understand it’s a 10 and a 3…

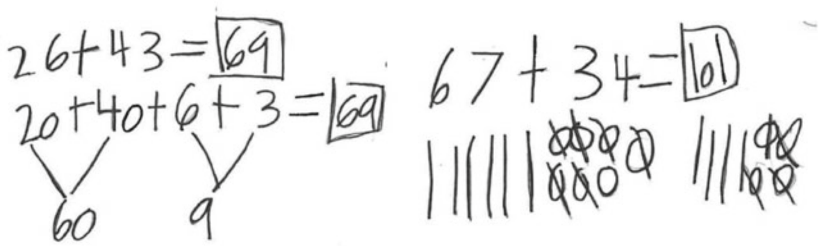
⋮

T: I like how the student used…that’s what I try to do with my students, is add the tens first, then add the ones so that it’s getting away from the standard algorithm…it’s level 3.

⋮

K: I wonder if she knew that she was gonna have to make a new 10 here (sample 2) so she chose to do it this way ‘cause she made a new 10.

D: I think I would agree with that too ‘cause when my kids know that they have to make a new 10, they’ll normally just go ahead and do tens and ones and then they’ll circle their new ten rather than doing the decomposing method.



Sample 1 Sample 2

Figure 5. Kelli’s first student work example from PLT #3.

SCK Analysis

Overall analysis of explicit evidence of SCK shows that both the number of instances, as well as percentages of the total idea units containing evidence of SCK, increased throughout the PLT sessions (see Table 5). It is evident that the interns drew more on SCK to engage with professionally noticing their students’ thinking as the PLT sessions progressed. By PLT #3, the interns provided evidence of their SCK during 10 exchanges which was just under half of the 23 total idea units from the session.

Table 5
Idea unit containing explicit evidence of intern SCK

	# of idea units containing evidence of SCK	% of PLT session’s idea units (%)
PLT #1	2	8
PLT #2	4	18
PLT #3	10	43

To delve into the relationship between professional noticing and explicit evidence of SCK, Table 6 includes all of the idea units that contained explicit evidence of SCK together with the coded level of professional noticing. The interns' SCK assisted them throughout their engagement with all three components of the professional noticing framework. For the idea units where the interns exhibited evidence of SCK, their SCK led to greater levels of either attend, interpret and/or decide. Of the 25 documented instances showing evidence of SCK, 20 of them were for the highest level (level 2) of professional noticing.

Table 6
Professional noticing codes for idea units that contain evidence of SCK throughout the PLT sessions

	PLT session #1		PLT session #2				PLT session #3									
Idea unit	2	12	1	7	12	13	1	2	3	4	5	11	12	13	17	22
Attend	1*	2	2*	2	2	2*	2*	2*	2*	2*	2	2	2*	2*	2*	2
Interpret	1*	1*	2*	2*	2*	2*	1	1*			1*	2*	1	2*	2*	2*
Decide		2											2*	1	2*	0

* Indicates the "explicit evidence of SCK" designation

For the individual components of professional noticing, the number of instances of explicit evidence of SCK increased throughout the PLT sessions for both attend and interpret. Refer to Table 1 for the types of SCK which will now be discussed. The interns provided explicit evidence of SCK while attending to mathematically significant details in their students' work samples via the teaching task of critiquing notation and language during 10 exchanges. For interpret, SCK regarding different types of non-standard multi-digit addition and subtraction algorithms was the most prevalent type seen throughout the PLT sessions. The interns also drew upon SCK about different strategies when evaluating their students' claims and when evaluating efficiency of strategies. Furthermore, the interns drew on SCK regarding non-standard methods and common ways students approach different multi-digit addition and subtraction problems when interpreting their students' mathematical thinking. Perhaps expectedly, there was not explicit evidence of the interns drawing on SCK to ask productive mathematical questions for the decide component during the first two PLT sessions. However, during PLT #3, likely due to the SCK focus, the interns exhibited SCK regarding productive math questions in two instances. Overall, the results show a relationship between SCK and professional noticing. Namely, SCK was related to an increase in the interns' engagement with professional noticing for each of the three individual noticing components: attend, interpret, and decide.

Discussion

The results presented above illustrate that there was indeed a relationship between exhibited SCK and increased professional noticing. As part of the design of the PLT sessions, the preservice interns were developing their SCK around multi-digit addition and subtraction. They showed increasing evidence of their SCK as the PLTs progressed, and as their SCK increased, they engaged more with professionally noticing their students' mathematical thinking.

In 2011, Sherin et al. (2011) called for research on how noticing of practicing teachers compares to that of preservice teachers. They asked "What trajectories of development related to noticing expertise exist for prospective and practicing teachers?" (Sherin et al., 2011, p. 11). The results from this study illustrate preservice interns' development of noticing expertise throughout a relatively short intervention. Their growth required support via the situated PLTs facilitated by the college supervisor and discussed with their peers. Like others have shown, support is a necessity when developing preservice teachers' engagement with noticing their students' mathematical thinking (Jacobs et al., 2010; Star & Strickland, 2008; Vondrova & Žalská, 2013), but growth can and does occur with purposefully developed interventions. The SCK focus of this study's intervention assisted the preservice interns as they engaged with professional noticing through the teaching practice of analyzing their students' work.

It is hoped that preservice teachers exposed to professional noticing will take what they learn from their preservice experiences and apply it to their own teaching. Franke, Carpenter, Levi, and Fennema (2001) found that inservice teachers who learn by interpreting their students' mathematical thinking continue to learn after designed interventions. Upon following up on the interns from this study, one intern explained

> As a result of my participation in this study, I began looking deeper at student work and trying to figure out why students did what they did instead of just looking and seeing what strategy they used or the mistakes they made. By thinking about why they did what they did when solving an equation, I was able to help the students better.

The intern's response shows her application of SCK when considering why her students solved problems in different manners. Both the preservice interns' SCK and professional noticing skills assisted her as she continued to grow in her own teaching practice. Thus, working with preservice interns to professionally notice their students' thinking has the potential to better their teaching practice as they continue to engage with noticing in their own classrooms.

Implications

This study provided an example of one method for measuring teachers' professional noticing of children's mathematical thinking and SCK simultaneously within a content and context specific situation. Professional noticing can and should be measured differently depending on the content of focus and the particular context of the designed intervention or teaching practice under study. To develop measures, the integrated SCK and professional noticing framework presented in this paper can be adapted to other situations.

Ball et al.'s (2008, p. 10) list of teaching tasks requiring SCK can serve as a basis for mapping different mathematical concepts and interventions to the components of professional noticing. For example, say a mathematics teacher educator was interested in working with practicing secondary mathematics teachers as they engage with professionally noticing their students' proof writing for non-standard or unfamiliar geometric situations. First, a professional development intervention could be designed that focused on developing secondary teachers' SCK regarding the underlying structure, and connections between proofs about circles that require knowledge about triangles [for example, Euclid's Intersecting Chords Theorem III.35 (Euclid, 2002)]. Referring to Ball et al. (2008, p. 10) list of teaching tasks, in order to engage with professionally noticing their students' proofs, teachers would need to "recognize what is involved in using a particular representation" which could be considered part of the attend component. Teachers might also need to evaluate the ways their students "choose and develop useable definitions" within their proofs, which could be considered part of the interpret component. For making instructional decisions, the teachers might choose to "modify the task to be easier or harder" depending on the needs of their individual students. In order to study the teachers' engagement with professional noticing throughout or as a result of this fabricated intervention, a full mapping of teaching tasks requiring SCK or even extended to teaching tasks that require additional aspects of MKT would be completed. Then analysis of the teachers' professional noticing with evidence of MKT could occur.

The above is just one example of a teaching situation containing different mathematical content and professional development context that could be used to simultaneously measure both teachers' professional noticing and their SCK. The example shows how the integrated SCK and professional noticing framework can be transferred across areas of preservice and inservice teacher education as well as across mathematical topics. Furthermore, the integration of content knowledge for teaching and professional noticing can be measured in other disciplines. Work to expand the MKT and noticing frameworks separately is underway in the field of science education (Barnhart & van Es, 2015; Johnson & Cotterman, 2015). Integrating these frameworks is an area of future work for teacher educators across disciplines.

Regardless of the particular content or context, the practice of teaching requires a set of knowledge specific to teachers. Engaging with professionally noticing how students' think about content is a teaching practice that must be developed. Yet, the practice of noticing cannot occur without discipline specific knowledge for teaching. Thus, the measurement of noticing while simultaneously measuring discipline specific knowledge for teaching is worthwhile and should continue to be addressed throughout different fields of teacher education.

Appendix: Professional Noticing Codes for Idea Units Focused on Student Work Analysis Throughout the PLTs

First Grade Session One																								
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Attend	0	1	1			2		1	1	1	1	2				2			1					
Interpret		1				2		1	0	1	1	1	2						0					
Decide												2												

First Grade Session Two																						
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Attend	2	2	0	1	1	2	2	2	2	1	2	2	2	2	1	2	2	1	1			
Interpret	2	2		1	1	2	2	2	1	0	1	2	2		1	1	1	2				
Decide						2																

First Grade Session Three																							
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Attend	2	2	2	2	2	1	2	2				2	2	1		1	2	2	2	2	2	2	
Interpret	1	2			1				1	1	2	2	2	1		1	2	2	2	1	2	2	
Decide												2	1	1		2	2	1	0	1	2	0	2

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