Research in Mathematics Education
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Edna O. Schack
Molly H. Fisher
Jennifer A. Wilhelm *Editors*

Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks

Springer

Research in Mathematics Education

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Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks

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Foreword

Mathematics teachers are continually bombarded with information streaming from many different sources. These streams are punctuated by time intervals (periods, weeks, terms), by critical moments of student (mis)conceptualization, and by curricular packaging, each of which impact the others in the strange dance of their professional lives. The subject of *noticing* is a useful perspective on this dance. We use this metaphor purposefully. First, we use it because John Mason, the acknowledged pioneer in this area in mathematics education, is so eloquent with his metaphors. Second, we use it because dance is always nuanced; involving actors, music, tempo, partner(s), and occasion, the metaphor captures much of a mathematics teacher's experience, and these highly situated factors require the attention of the dancer—the teacher—to understand them in such a manner as to improve the dance—the lesson, the learning opportunity—and, importantly, future opportunities. Thus, noticing in mathematics education research is more than just a technique—it is interpretive, projecting a new pattern more meaningfully engaging for students as partners.

In this volume, the authors provide not only rigorous analyses of cases as well as large-scale studies, but also examples from practice that can serve as anchors for discussion among teachers and researchers both intent upon the same goal: improving mathematics teaching and learning. Its beauty is not that the authors answer all of our questions about how teachers attend to information, interpret it (multiple streams and all), and then decide what to do, but that it challenges the reader to re-frame what is *meant* by attention, interpretation, and decision in the context of improvement of one's own teaching practice. Instead of a method of noticing, the authors provide insight into a *discipline* of noticing (again, Mason's words) requiring *mindfulness* and *decision* as well as *adaptation*. After all, merely reflecting on practice does not change it.

There are clearly different norms for what is considered pedagogically productive and what is considered counterproductive. Such relativism is difficult when our goals are to develop coherent methodological frameworks. However, the advantage noticing as a perspective provides is just this kind of nuanced look at what it means to improve practice and how local conditions, broader political aims, tools, and resources contribute to local pedagogical innovation.

This volume is an excellent resource for both doctoral seminars in mathematics teaching, and, we think, for teacher professional development programs. As we indicated in the Foreword of other books in the series, we have designed the solicitation, review, and revision process of volumes in the series to produce thematic volumes, allowing researchers to access numerous studies on a theme in a single, peer-reviewed source. Our intent for this series is to publish the latest research in the field in a timely fashion. This design is particularly geared towards highlighting the work of promising graduate students and junior faculty working in conjunction with senior scholars. The audience for this monograph series consists of those at the intersection between researchers and mathematics education leaders—those who need the highest quality research, methodological rigor, and potentially transformative implications ready at hand to help them make decisions regarding the improvement of teaching, learning, policy, and practice. With this vision, our mission of this book series is:

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3. To support the usefulness and wide-spread adoption of research-based innovation.

Like other volumes in the series, this volume serves the mission well. We would like to thank the editors, Edna O. Schack, Molly H. Fisher, and Jennifer Wilhelm, for their efforts in shepherding this volume. We also commend the authors of the empirical articles and commentaries for providing such excellent examples of research and reflection.

Jinfa Cai
James A. Middleton
Co-Editors in Chief, RME Book Series

Preface

Is it Noticing or is it . . .?

What is teacher noticing? Is teacher noticing different than *simply good teaching*? Is teacher noticing only an *in-the-moment* occurrence within a whole class setting or is it also akin to a one-on-one clinical interview? Is the framing of this practice as teacher noticing affording us an opportunity to reexamine, or more deeply examine, a crucial aspect of teaching? These questions continue to be discussed and passionately debated among our research team and colleagues. When one colleague was discussing *professional noticing* within mathematics, another was interpreting it as *professional vision,* while still another was attempting to envision it beyond a mathematics content perspective. These discussions caused us pause and we had to ask ourselves, "How IS all of this different? Or, is it?" So, "**is it *noticing*, or is it….**?"

In Sherin, Jacobs, and Philipp's (2011) edited volume, several authors described the foundations of noticing. The roots are varied and the interpretations of the construct are many. Foundational research in professional vision (Goodwin, 1994) and the discipline of noticing (Mason, 2002) set the stage for additional research in this field. Further frameworks of noticing have surfaced, such as noticing (van Es, 2011; Sherin & Star, 2011), teacher noticing (Sherin, Jacobs, & Philipp, 2011), and professional noticing of children's mathematical thinking (Jacobs, Lamb, & Philipp, 2010). As a result, one goal of this monograph is to seek clarification of the construct and its related branches. Our monograph explores recent developments in noticing and responds, in part, to the challenges Alan Schoenfeld put forth in the final commentary of the aforementioned volume by Sherin et al.

In his commentary, Schoenfeld (2011) left us with multiple questions to pursue with the goal of applying our researched knowledge to the development of effective teachers. Sherin et al.'s and Schoenfeld's questions and our own successes and struggles with defining and measuring teacher noticing led us to propose this monograph as a product of the 2013 Psychology of Mathematics Education-North America and 2014 International Group of the Psychology of Mathematics Education Working Groups, *Teacher Noticing: A Hidden Skill of Teaching.*

Many have contributed to its fulfillment. This work would not have been possible without the encouragement and guidance of Jinfa Cai, co-editor of the Research in Mathematics Education book series, the thoughtful contributions of the commenting authors, the compilation of years of work by the many researcher-authors whose work appears in this monograph, and the valuable feedback from the reviewers. Each proposal was submitted to a blind review process of at least three reviewers and full chapters were subsequently reviewed and edited by additional reviewers and the editors.

In planning the monograph, we sought to address not only some of the questions raised by previous authors, (Sherin et al., 2011; Schoenfeld, 2011), but also those questions that continued to emerge in the working sessions attended by researchers worldwide. The question, "What are the key components of teacher noticing?" led to the section, *Exploring the Boundaries of Teacher Noticing.* A related question is, "Can key components be isolated for study?" The chapters in the section, *Measuring Teacher Noticing*, illustrate multiple methods used by researchers to study the components. The section, *Noticing in Various Grade Bands and Contexts,* is in response to the question, "Is teacher professional noticing situation specific?" Of course, the study of teacher noticing is ultimately focused on improving student learning. The effects on student learning are addressed in the *Examining Student Thinking through Teacher Noticing* section. Finally, the section, *Extending Equitable Practices in Professional Noticing,* developed after the chapter proposals were submitted and this new and exciting theme emerged that we had not anticipated. Interestingly, the addition of this section itself illustrates a point regarding noticing that is raised by Mason in the introductory chapter herein, that of the distinction between listening-to and listening-for. Had we limited ourselves to the areas originally conceived for this monograph and only listened-for, we may have missed this important addition to the teacher noticing dialog.

Whether noticing is its own new construct or whether it is a rose by another name, the goal is to study, to learn, and to contribute to the increased effectiveness of teachers. Ultimately, if we are to prepare effective teachers, knowing what makes an effective teacher must guide this preparation. The research on teacher noticing attempts to define, describe, and capture that which is essentially invisible to the observer. In doing so, we hope to learn how to develop this ability in prospective and practicing teachers in order to build robust learning environments for all students.

Most importantly, this book is the result of the thought and work of many before us, including those whom we have never met but whose writing in diverse areas has influenced our work and the work of the author-researchers within. Like a good conversation, good research builds upon the thinking of others to develop into a deeper understanding by all. Our hope is that the chapters in this volume contribute to the conversation.

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Lexington, KY, USA Molly H. Fisher
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About the Editors

Dr. Edna O. Schack is a professor of education and Co-Director of MSUTeach at Morehead State University in Morehead, KY where she has taught prospective elementary teachers since 1987. She is the recipient, along with several Kentucky colleagues, of two National Science Foundation grants investigating prospective elementary teacher professional noticing of children's early numeracy and early algebraic thinking. Her interests also include investigating key practices prospective teachers need to develop a foundation for growth as an effective teacher. Involved in the Kentucky Committee for Mathematics Achievement since its inception in 2005, she served as the Chair (2010–2012) and is currently the Assistant to the Chair. She has published in both research and practitioner journals.

Dr. Molly H. Fisher is an Associate Professor of Mathematics Education in the STEM Education Department at the University of Kentucky. She is also the Director of Graduate Studies where she directs the M.S. in STEM Education program as well as the STEM Education strand of the Ph.D. program in Education Sciences. She holds a B.A. in Mathematics, M.A. in Mathematics Education, and a Ph.D. in Curriculum Instruction (with an Urban Mathematics Education specialization) from the University of North Carolina at Charlotte. She is a former high school Mathematics and Computer Science teacher with a myriad of experiences teaching in the classroom and online. Her research focuses on the professional noticing of children's mathematical thinking of preservice elementary teachers through the lens of the Stages of Early Arithmetic Learning (SEAL) and Algebraic Thinking. Additionally, she is passionate about the support and mentoring of new teachers and she studies the stress, burnout, and retention of inservice teachers, especially secondary mathematics teachers.

Dr. Jennifer A. Wilhelm is a Professor of Science and Mathematics Education at the University of Kentucky. She holds a Ph.D. in Science/Mathematics Education from the University of Texas at Austin and a M.S. in Physics from Michigan State University. Dr. Wilhelm's primary research interest involves the design of project-enhanced, interdisciplinary learning environments. She investigates how people understand science and mathematics concepts as they participate in project work that demands the integration of multiple content areas. Dr. Wilhelm's research focuses on project pieces that are inherently interdisciplinary and fruitful for contextualized student learning. Some examples include examining the development of students' science and mathematics content understanding as they engage in studies of motion and rate of change; sound waves and trigonometry; and the moon's phases, the moon's motion, and spatial geometry.

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Probing Beneath the Surface of Experience

John Mason

Abstract Using a phenomenological stance which values lived experience, I probe beneath the surface of data presented as descriptive accounts-of incidents by focusing on attention. This includes both what is being attended to, and the form of that attention. Practical actions are proposed which can afford access into the lived experience of others, by asking oneself what someone would need to be attending to, and how, in order to say what they say and do what they do. This pedagogic action can function as a research tool for analysis of what subjects say and do.

Keywords Noticing · Phenomenological · Mindfulness · Distinctions · Origins of research questions

> *It is only after you come to know the surface of things that you venture to see what is underneath; but the surface of things is inexhaustible.*
>
> Italo Calvino: *Mr Palomar* (1983)

Drawing on my outline of a philosophically well-founded approach to qualitative research into one's own practice (Mason, 2002), I look more closely at the process of analysing qualitative data (whether focussing on oneself or on others). My 'data' in this instance, apart from extensive experience working on myself with the help of others, makes use of experience gained during a term spent at the University of Calgary where we met to discuss how to interpret and make sense of data that various people had collected.

From a phenomenological foundation in the discipline of noticing (Mason, 2002) which values lived experience, not only as an ideal, but as a method both of enquiry and of reporting that enquiry, it is necessary and valuable to probe beneath the surface of what others say and do, by seeking resonance and dissonance with one's

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own experience. Practical actions are proposed which can afford access into the lived experience of others, by asking oneself what someone would need to be attending to, and how, in order to say what they say and do what they do. This pedagogic action can function as a research tool for analysis of what subjects say and do.

Introduction

In order to work myself round to the core of this paper, namely the power of a disciplined approach to interpreting educational data, I begin with a justification of my phenomenological stance. I then consider the origins of research questions, and how they are intimately interconnected with philosophical and ethical stances, access to methods, and sensitisation to particular theoretical distinctions, which arise from personal disposition, from past experience and from reading the literature. I then introduce some important distinctions, which for me are more than theoretical, arising as they do from observations of lived experience and confirmed by resonance with disparate communities.

On the Purpose of Research

If the purpose of educational research is to further a career and/or to appease institutional requirements concerning research, then it makes sense to administer questionnaires, conduct interviews, observe teachers and students and to declare these as data. Analysis can then consist of distinctions from the literature in order to categorise or classify current data. All too rarely it is possible to apply theoretical constructs to make predictions or to explain rather than simply to classify. Often people find that they want to refine those distinctions to form an even finer grain theoretical frame for making distinctions. With a great deal of educational research, certainly in mathematics education, it can be difficult to decide whether the data provided (usually a fragment of a larger collection) is illustrating distinctions made by the author, or whether the data is supposed to be the source and origin of those distinctions. Put another way, if a collection of distinctions is going to be of use to the research community, they must be robust against misinterpretation and misapplication. Robustness depends not so much on formal definitions, but rather on an analogue of concept images: a collection of agreed examples. How distinctions are discerned needs to be negotiated through the offering of potential examples and discussion about whether these really do instantiate the distinctions being claimed (Mason, 1998).

If, however, the purpose of educational research is to improve the experience of learners and teachers, then theoretical distinctions must somehow inform practice, preferably leading to an improvement of learners' experience, appreciation and comprehension of concepts, procedures and thinking. This involves establishing mutual trust and respect through evident caring both for the learner and for the mathematics, so that learners feel empowered to take initiative and so to develop a positive relationship with mathematics (Noddings, 1996; Mason, 2009; Handa, 2012).

In order to improve the lived experience of others, whether researchers, teachers or learners, it makes sense to concentrate on that lived experience, and the best starting point for the lived experience of others is to work on refining your awareness of your own lived experience. That is precisely the aim of the discipline of noticing (Mason, 2002, 2011; Bennett, 1976), namely to inform future practice through enriching what is noticed. To do this requires activating and developing an inner witness or inner executive (Schoenfeld, 1985), a ‘person on the shoulder’ who asks questions such as ‘why are we doing this?’ and ‘might there be a better way?’, in short, who observes without participating. This could be what an early stanza of the Rg Veda is pointing to:

> Two birds, close yoked companions,
>
> Both clasp the self same tree,
>
> One eats of the sweet fruit,
>
> The other looks on without eating.
>
> (Bennett, 1964, p. 108; see also Radhakrishnan, 1953, p. 623)

The inner witness is perhaps the ‘still small voice of calm’ of the Quaker poem (Whittier, 1872). Ouspensky (1950) described the action of the inner witness as *self-observation*, and Burke (1905) described a state in which the witness is awake and functioning, as *cosmic consciousness*.

Another discourse recently popular and taken from Buddhist sources is *mindfulness* (Langer, 1997; see also oxfordmindfulness.org). Neville (1989) describes the *education of psyche* by calling upon emotion and imagination in order to access the unconscious, and this aligns with an image of the human psyche as a chariot found in several of the Upanishads (Radhakrishnan, 1953, p. 623; Mason, 1994), as well as with a framework of human energies which distinguishes between sensation, conscious and creative energies (Bennett, 1964). This all unfolds into a sixfold structure of the human psyche consisting of enaction, affect, cognition, attention, will and witness. Notice that already I am making use of distinctions, which have deep and ancient origins, yet, which in this paper are dependent on immersion in a discourse rather then being expounded theoretically as definitions.

I take the phenomenological notion of *lived experience* seriously, to the extent that I prefer to offer people immediate experience via task exercises rather than expounding theoretical distinctions. I then invite them to relate that to past experience as preparation for future experience so that in the future they may not only be sensitised to notice and discern details not previously attended to but also so that they may find that fresh actions become available to be enacted as a result. This is what Gattegno (1970) meant by *only awareness is educable*, because for him awareness is what enables action (the enaction or behaviour of Western psychology). The results of such an enquiry are the task exercises, which people may then modify and use with others in a similar fashion (Mason, 2002, p. 91).

Readers who find themselves at least partly swayed towards this stance may wish to undertake some self-observation of their own. For example:

How do you change from lying in bed to standing up beside your bed in the morning when you get up?

Who or what is the "I" that claims to be acting whenever you use the word "I" in a sentence?

If you are tempted to give an immediate reaction, then I invite you to park that, and reconsider on the basis of freshly collected 'data'. Take your time! These are non-trivial tasks. If you engage seriously with them, you will get a taste of what it means to be sensitised to notice, and you will learn something about the origins of your own behaviour. The challenge then is to extend such enquiries into professional practice.

On the Arising of Research Questions

Editors of journals and the reviewers on whom they rely are very keen that authors should draw upon papers previously published in the journal, present the theoretical basis for their research, describe the research question clearly, present relevant data and analyse that data according to methods consistent with and in alignment with a methodological stance which draws on the theoretical underpinnings. Students undertaking research method courses often get the impression that you are supposed to start with research questions, that theoretical frameworks and methodological stances are chosen to be appropriate to the research questions, data is then collected, and analysis proceeds using the theoretical framework. But is this realistic?

Most of the colleagues whom I have supervised have found themselves learning as they go. The research question with which they begin is transformed by fresh experience and deep thought, as well as by practicalities such as the methods that they tend to favour, and the actual opportunities they have for carrying them out. In addition to what is possible in terms of getting access to classrooms or teachers, there is the influence of the kind and quantity of data that they imagine is manageable. This too tends to change as they start acting. The scope and extent of their reading expands and contracts at various times, stimulated by new developments or insights into their own data collection, leading to fresh ways of thinking about that data and even to fresh data collection; sometimes they are immersed in making sense of their own data and do not need outside stimulation.

In *Researching Your Own Practice Using The Discipline of Noticing*, I proposed that the standard components of a thesis: research question, theoretical basis, methods, data collection and analysis are all intimately interrelated. Methodological preferences will influence the formulation of the research questions, as will underlying theoretical frameworks and ethical stances. Many years ago, we used to encourage students to try to articulate their own theoretical framework early on, in order to be able to be self-critical about their data interpretation. More frequently now I hear students ‘searching for a theoretical framework’ that will enable them to analyse their data, often after collecting that ‘data’. But the act of declaring counts and accounts to be data is already influenced by theoretical stance and preferences as to method. It seems to me most unlikely, if you have a preference for qualitative data that you will pose a research question that suggests counting things and doing a statistical analysis. Nor is it likely that with a statistical preference, you will pose research questions that can only be addressed by thick descriptions. If you have a preference for video, or for audio, or for interviews, then your research questions are likely to reflect this preference. More significantly, as your questions shift, or if you try to use someone else’s data, which has been collected in connection with one set of research questions, great care is then needed in using that same data to address a different research question, precisely because it may not be representative in the relation to the new question.

Even more significantly, I ask myself who benefits and in what way? I know some schools of thought want to find out ‘what works’ and ‘what works best’; my own stance is that it is the researcher who learns the most from research (Mason, 1998), because during the activity called ‘research’ their sensitivity to notice is altered and sharpened. They notice things previously overlooked, or they notice finer detail. Even a researcher using only statistical methods is sensitised by detected correlations to think differently in the future, not to say picking up fresh ways to assemble, analyse and present data. The much-desired objectivity is, in my experience, best sought through acknowledging and taking into account the subjectivity of all enquiries. If the ‘research’ is entirely at arms length and detached, the researcher is unlikely to locate anything more than a possibility worthy of more detailed study.

I chose to develop a phenomenological stance to working with colleagues, whether in classes, workshops or supervision of dissertations. It seemed to me that what really matters is that people become aware of something previously overlooked or downplayed. This might involve making new (for them or for all of us) connections; it might involve reworking and bringing to the surface competing pulls and pushes, locating tensions that are usually buffered from being exposed and so debilitating us. It certainly entails internalising actions and associating them with particular events, whether in the classroom, during interview or when contemplating data.

Recognition of a Phenomenon

An incident becomes a phenomenon when it resonates with or triggers access to other related incidents and to ways of describing or referring to such incidents. A *phenomenon* is a space of examples of incidents with perceived common qualities. At issue is how precisely those common qualities are specified or negotiated with and by colleagues. The significance lies in the perception of common qualities, and this is what needs to be made available for negotiation with others, to see if they discern the same similarities and differences.

Example: Reversal

> I asked my 7-year-old son, what is three times four? He instantly replied "seven". I then asked, "what is three plus four?" (emphasis on the 'plus') and he said "Oh, twelve".

A few weeks later I read in a book the following account which was triggered by the author's colleague reporting that he was noticing students making some classic errors such as $4 \times 4 = 8$; $2^3 = 6$; $6 \div 1/2 = 3$.

… in every case, the student was giving a correct answer–but a correct answer to a different question. The student had not answered the original question …. [The colleague] proposed, and tested a remediation procedure. He recommended that the teacher figure out the question that the student had answered; the teacher should then ask [that question]. [He] predicted that, in nearly every case, the student would not answer that question but would immediately correct the answer to the original question.

> Teacher: how much is seven times seven?
>
> Student (grade seven): fourteen
>
> Teacher: how much is seven plus seven?
>
> Student: Oh! It should be forty-nine!

The frequency of dialogues on this pattern suggests that there is some kind of echoic 'second hearing' or … 'instant replay capability'. … it reveals something interesting about the student's control structure, and about the student's understanding of the teacher's goals, that the student does NOT bother answering the second question, assuming (correctly) that what was really wanted was a correct answer to the original question (Davis, 1984, pp. 100–101).

Comments. There are several aspects to this multi-layered account. First there is the phenomenon of noticing what we are sensitised to notice. When you return from holiday, it is common to notice multiple references to where you have been; if you buy a car, you may start to notice other cars like it on the road; if you encounter and use a new word, you may start to notice the word being used quite frequently; if you have had a sharp experience recently, you may start to notice other people reporting similar experiences. It all has to do with sensitivity to notice, to discerning what was available to be discerned but not previously discerned. Note that a sensitisation to notice can be overlaid by the next sharp noticing. On the boundary of this phenomenon is the effect of someone using a label for some experience that, when described, you recognise, but which you had not previously articulated for yourself. Here, I was sensitised by my own experience with my son not only to recognise what Davis (1984) was reporting, but for it to make a lasting impression, which comes back to me vividly years later. I can still ‘see’ where my son and I were at the time. Noticing is not necessarily a cumulative process, unless worked at explicitly, which is what the Discipline of Noticing offers (Mason, 2002).

Second, there is the pedagogic awareness that Davis (1996) calls *listening-to,* as distinct from *listening-for*. Being sensitised to the notion that students are sense-making organisms, it is reasonable to conjecture that when what they say or do is not what a teacher expects, there must be some reason behind it. Rather than treating the utterance as ‘wrong’, it can be treated with respect. Often we do not manage to say what we see, to express articulately what we are experiencing, and the same is likely to be true for learners. Malara and Navarra (2003) introduced the term (mathematical) *babbling* to describe learners’ inexpert efforts to express what they are thinking. The metaphor is based on babies in their cot making sentence-like sounds without actually saying anything. The notion of babbling as a label for error-full utterances (spoken or written) has been taken up by others (Berger, 2006; Scataglini-Belghitar & Mason, 2011), and contrasted with *gargling*, which labels learners throwing technical terms into assignments in the hope that the teacher will recognise them and award marks for them.

The pedagogic action of asking the question that has been answered may at least sometimes trigger students to attend to the full question. Students rarely give wilfully wrong answers to probes, especially when those probes take place in a supportive and conjecturing atmosphere where the intention is always to try something out and then modify what one says or does on the basis of further consideration by yourself and by others (Mason, Burton & Stacey, 2010, pp. 233–234).

Thus there are two actions to enact when you become aware of a mis-answered question: asking the question just answered, and asking yourself what someone would have to be attending to, and how might they be attending to it, in order to say what they say and do what they do. This is no mean feat but it can be the origin of significant sensitisation and insight into learner behaviour. It is the result of a constructivist stance, an assumption that all human beings are sense-makers who try to glue together their fragmentary experience by the construction of narratives. Here, the researcher blends disparate acts together to construct some story to account for contrasting or changed behaviour.

Third, notice that we are offered a brief-but-vivid context, then a sample dialogue (an account-of what was said), then a comment about frequency which suggests that the incident is not isolated but rather an instance of a phenomenon and then some theorising to account-for what has been observed.

Someone sensitised to the distinction made in Dual Process Theory (Kahneman, 2012) might account-for the student’s first utterance as generated by System 1, an immediately enacted reaction; the teacher’s revised question might then be seen as initiating System 2 to consider, to process cognitively, and to respond with an alternative answer to the original question (Mason, 2009). This is consistent with my earlier suggestion to *park* the first thought, action or emotion that arises and to probe more deeply over time.

Alternatively, as Davis suggests, the teacher's second question might have been taken by the student not as a question but as a prompt to try a different answer. However, this might also be a reaction, System 1 again. This would reflect a socialisation into the rubric of that teacher's classroom informed by past experience.

Another way of accounting-for the student's final answer is to probe beneath the surface of enacted action, reaction and response, and to consider what the student might have been attending to. The student's immediate utterance suggests that the number seven was dominant. Elsewhere in his book, Davis proposes that because children usually (certainly in the late 1970s) spend a long time on the action of addition before encountering subtraction more briefly, then multiplication even more briefly, then division ever so briefly, they are most likely to enact the most familiar, most deep-seated of actions, namely addition. Either way, the teacher's second utterance shifts the student's attention to the operation, triggering the correct answer to the initial question.

Every attempt to probe the experience of someone else comes up against the question of what the researcher assumes drives subjects' behaviour. Are their actions reactions to the situation, (Kahneman's System S1), unconsidered automatic actions displaying habits, or are they considered responses (System S2)? Interpretation of questionnaires' 'responses', interviews and even observed behaviour needs to take into account whether the data consists of reactions or responses, or even *remoting* (a contracted form of re-emoting, which describes alienation of the subject from the probe as an affective state).

Attending to what the speaker might be attending to when enacting some action turns out to be a very fruitful line of enquiry, as we found during sessions of data inspection in a seminar series in Calgary in 2014. Assumptions about reacting, responding and remoting can be considered as alternatives, preserving the complexity of human psyche by accumulating contrasting and even contradictory interpretations without choosing between them. Furthermore, it can be even more enlightening to try to detect evidence not simply of what subjects are attending to, but *how* they are attending to it. The notion of different ways of attending is elaborated later.

The *reversal* phenomenon was further enriched for me when I encountered Malara and Navarra (2003) using the term *babbling* to describe learners' attempts to express their comprehension of some mathematical situation. In both situations the learner can be seen as trying to express, to bring to articulation thoughts, which have not yet crystallised. Again, I was able to relate it to my own experience with my son:

> On a car journey I asked my 5-years-old son, "what is three plus four?" and then when he answered that, "what is four plus three?". After only two or three of these, he suddenly announced, "anything plus anything is anything plus anything".

Of course this is, on the surface, mathematical nonsense. But almost certainly he was trying to express the generalisation that order does not matter when you are adding. He was babbling a bit, because he had not learned, perhaps even had not encountered ways to speak about general, or as-yet-unknown, but different numbers. This is in contrast to some of the essays I used to have to mark at the end of a mathematics education course, where students seemed to be *gargling*: throwing lots of technical terms on the page in the hope that the marker would recognise them and reward their presence, even though what was written made little or no sense.

The association of the distinction between *babbling* and *gargling* with the reversal phenomenon and with the distinction between listening-to and listening-for, anchored in my own experiences, creates a rich collection of possible metonymic triggers and metaphoric resonances to bring to the surface possible pedagogic actions in a range of circumstances. This, for me, is what professional development is about.

What is Gained? Arising from these two accounts and various ways of accounting-for them there is an acknowledgement that human experience is highly complex, so there is unlikely to be one single 'correct' interpretation or reading of someone else's behaviour. Therefore it is valuable to seek multiple readings, to seek various interpretations. When some of these are contradictory, you can feel that you are indeed probing beneath the surface, because no matter how coherent the narrative, underneath there are very often conflicting sentiments and feelings, conflicting cognitions, conflicting awarenesses with associated actions.

Put more analytically, the more specific and detailed the analysis of some event, the more that is revealed about the sensitivities of the researcher. I proposed (Mason, 2002, pp. 181–182) that there is a sort of Heisenberg's uncertainty principle acting, in which the ratio of the degrees of precision in the description of the event and in the sensitivities of the observer is usually more or less constant. This aligns with, 'The universe is a mirror in which we can contemplate only what we have learned to know about ourselves' (Calvino, 1983). and 'Every scientific discovery is in a sense the autobiography of the [person] who made it' (Korzybski, 1994).

This is another reason for trying to articulate one's own beliefs, assumptions and stances when conducting research. Indeed, an extreme position might be that since what is learned from a research report is as much about the researcher as about the situation being analysed, research itself is a process of self-exploration and self-enquiry!

Pertinent Distinctions

As is my preference, a number of technical terms have been used in the analysis above, and deserve elaboration. To do this here will enable me to sketch in some of the historical roots of those distinctions.

Reacting, Re-emoting and Responding

It has been well known for thousands of years that in many situations, human beings tend to react without 'thinking'. It is as if an action is enacted before any conscious cognitive processing takes place, even though much of the time people think that they are responding after due consideration when in fact they are reacting spontaneously and out of habit and prejudice. For example, Shakespeare's *Much Ado About Nothing* can be seen as an exploration of the humour and potential tragedy from unconsidered reaction.

Reaction and response seen as two different 'systems' or 'types of action' has been adumbrated by Stanovich and West (2000, quoted in Kahneman 2012, p. 48). Under the title *Dual Systems* theory (and also *two type* theory), it has been exploited and developed as a way of explaining a great deal of human behaviour by Kahneman (2002, 2012; also Kahneman & Frederick, 2005). In *Dual Systems* theory, System S1 is reactive and quick to act, while System S2 is responsive and considered. S1 provides S2 with possible actions using assumptions based on whatever limited data is available, in line with past experience, a process characteristic of abduction (Eco, 1983) and consonant with the notion of 'frames' which act as soon as all parameters have assigned values (Minsky, 1975, 1986). If parameters have default values, the frame will fire without waiting for more data. Of course it is often essential to act immediately, without considering pros and cons, especially in a classroom, but it is sometimes valuable, even essential, to park the first actions proposed and to respond rather than react. The notion of dual systems has also been used to account for phenomena in mathematics education by Leron and Hazzan (2006).

Physiological action (behaviour or enaction) tend to provoke emotional shifts, which are then accounted for by narratives generated by the frontal cortex, the intellect (Norretranders, 1998), sometimes blaming affect arising from past experiences. For example, Mandler (1989) pointed out how in mathematics, emotions that block actions and thoughts can be triggered before cognition registers any difficulty. Emotional arousal could often be described as re-emoting because those emotions are triggered by current situations making metonymic connections with previously experienced emotions. Metonymies are well known to run at but below the surface of consciousness (Jakobson, 1951; Lacan, 1985). Re-emoting can usefully be contracted to *remoting* as a reminder that negative emotions tend to alienate the individual from the situation, creating emotional, behavioural and cognitive distancing (Duffin and Simpson, 1993). Rather than directing activity, the intellect is usually well behind affect and enaction, and this ancient observation is attested to by neural science experiments (Norretranders, 1998). Thus, we have three words for reactions based in each of the three 'centres' of the psyche in western psychology: enaction (*reacting*), affect (*remoting*) and cognition (*responding*). The gerunds are used because they indicate an on-going process rather than a single thing.

Accounts-of and Accounting-for

In the 1980s, my colleagues and I were commissioned by the UK government to make video recordings of 'best practice' for distribution to every secondary school in the country. Initially sceptical, we soon encountered two aspects of teachers watching videos of other teachers: (1) viewers focussed initially on the mathematics (if it was not familiar they started to think it through for themselves); it was almost as if they did not 'see' the pedagogy until the mathematics was sorted out; (2) viewers were very likely to react with statements such as 'I wouldn't let that teacher in my classroom' and 'But my attainers are lower than those [low attainers]'.

The first phenomenon is in alignment with experience at Open University mathematics summer schools. I used to ask students who were themselves teachers, to describe what they noticed about the way various sessions were conducted at the summer school. This, they almost universally found really difficult to do, and my explanation is that their attention was absorbed by the content, the mathematics that they were learning. For me there is also a parallel with course teams at the Open University. We were expected to comment in detail on drafts of teaching materials written by colleagues on our team. However, it was difficult to comment on what was not there, on alternatives; attention tended to be focussed on tinkering with what was written. Similarly, young learners are not always entirely articulate about the teaching they are receiving, and may not be able to comment 'from outside', because they are at most aware of what is, but unaware of alternatives. This means that what 'is' may not be visible, as in the adage, 'if you want to know about water, don't ask a fish'.

In order to counteract the tendency to react negatively, we drew upon experience of working in groups on mathematical posters and animations, which we picked up experientially in sessions led by experienced educators, and that can be traced back to ways of working developed for working on Nicolet films (Tahta, 1981). The idea is to show a short video and then to invite participants to try to reconstruct what they saw, mentally. It is not a competition and it is not a memory test, so it does not matter whether you know in advance about being asked to reconstruct. Then as a group you try to reconstruct what you saw, step by step. An alternative we also used is to invite participants to describe briefly but vividly some incidents that struck them. When the description has been refined so that others recognise the incident being referred to (possibly through reviewing the video), everyone is asked to describe an incident from their own, usually recent experience that they think is similar. Again brief-but-vivid accounts are wanted, which leave out judgements, explanations and theorising. The emphasis here is on what was seen, rather than on what you think you saw. This enables recognition to be based on the incident rather than on theory and jargon. Out of this practice grew the distinction between an *account-of* an incident, and *accounting-for* an incident (Mason, 2002, pp. 40–42; Pimm, 1993).

An *account-of* tries to eliminate judgements and emotional content, valuing brevity and vividness. It promotes identification of *fragments* (Mason, 1988) or what James Stewart (1983) called 'little pieces of time'. These are the fragments, which for some reason stick in our memory and can be almost 're-entered' mentally later. Stewart was aware of them from people who used to come up to him and comment on a particular incident (fragment) from a film. This led me to realise and offer as a conjecture that not only is experience recalled in fragments, or reconstructed from fragments, but, that experience itself is actually fragmentary. This is the one place where I disagree with James (1890) who put into circulation the notion of the *stream of consciousness*. Even the briefest of attempts at self-observation will confirm that in one moment, attention is attracted to something, but then the intensity decays over time, only to be overlaid by some fresh stimulation, whether stemming from the 'chattering monkeys' in our heads which constitute our 'consciousness', or from sensory stimulation connected with current experience. The work we did with posters and animations, the work we did on prompting teachers when watching videos and the work we did at giving accounts-of salient fragments are actually a contribution to the development and empowering of a different part of the human psyche, namely the inner witness.

Collecting several instances of similar incidents under one label develops a rich base of experience which can, in the moment, trigger metonymically and/or resonate metaphorically with a developing situation, so that possible actions become available (Mason, 1999, 2002). This is how opportunities to act freshly come to be noticed in the flow of a lesson, or when preparing a lesson, and how fresh insights arise when reflecting back on a lesson.

Forms of Attention

In studying children's geometric thinking, van Hiele-Geldof (1957) distinguished five different states, described as levels. Her husband developed and elaborated these to apply beyond geometry, but sticking to the notion of levels as a form of progression (van Hiele, 1986). Subsequent research has developed tests to determine at which level individuals are operating (Burger and Shaunessy, 1986; Usiskin, 1982), despite self-observations that our attention usually leaps about between these various 'levels'. Independently, drawing on the system of Systematics of Bennett (1993; see also Shantock Systematics Group, 1975). I developed a very similar scheme approached not from the point of view of levels, but more specifically, different ways of attending to something at the micro level (Mason, 1982). Although states may sometimes be stable for a period of time, mostly attention darts around from form to form as can be confirmed through self-observation on appropriate task-exercises. Figure 1 provides a more complete description of forms of attention combines these with a macro view (Mason and Davis, 1989) and a meso view (Watson, 2008, 2010).

Macro	*Meso*	*Micro*
Locus Focus Multiplicity Breadth or scope	Personal focus, axis, centre of gravity, absolute. Self Social Sex Purpose, role, meaning Ethics	Holding Wholes Discerning Details Recognising Relationships Perceiving Properties Reasoning on the basis of agreed properties

Fig. 1. Forms of attention

The macro view builds on the metaphor of one or more searchlights illuminating the sky, which can be single or several, focused or diffuse, broad or narrow in scope and experienced as based inside the front or back of the head, on one shoulder or the other, or from behind, to the side, or to the front of the physical body.

The meso view captures the sense of on-going concern that we experience as we go through the human phases, as in the riddle of what walks on four legs in the morning, two legs at noon and three or more legs in the evening? Gattegno (1970) used the term *absolutes* for that which occupies attention as a default when attention is released or drifts, the axis around which everything currently revolves. These change over time of course.

The micro view is of most interest to teachers because it relates to shifts in perception, ways of thinking and ways of reasoning that take place fleetingly in the moment, and which are part of the curriculum.

Being aware of the macro, meso and micro views of attention so as to be able to take them into account when interpreting other people's behaviour as data can provide a rich starting point for making sense of what subjects say and do.

What distinguishes these articulations from those of the van Hiele's is that attention tends to be in flux, to flit between these very quickly. Yet if a teacher is attending in one way while learners are attending differently, communication is not likely to be effective (Mason, 2003).

It is not always immediately evident what form of attention a learner is experiencing, and, of course, conjectures about attention cannot be verified through observation or even interview or questionnaire because any relevant probe will immediately influence the subject's attention. But it turns out to be informative to be aware of the structure of your own attention, and to use this to inform possibilities.

The technique is to ask yourself both what you would have to be attending to, and how, so that you would do and say what the other has done and said. Of course, it will be necessary to make assumptions about past experience, but what is known are the person's actions: what was done and said. Once you bring into play observed gesture, voice tones, facial expressions and posture, you enter the realm of multiple interpretations and idiosyncrasies. But restricting attention (sic!) to what is said and done can open up possible appreciation of the focus, locus, multiplicity and structure of the person's attention. This can then inform further analysis using any other chosen framework of distinctions.

One caution about trying to appreciate what is being attended to and how: absence of evidence is not evidence of absence: just because there is no evidence that someone is or was attending to something (some detail, say) or in some way (perceiving properties being instantiated, say), it does not mean that the individual was not attending in this way, simply that it did not come to expression. Perhaps they did not consider it relevant, perhaps it was attended to only peripherally or in passing, and perhaps the necessary words to express it were not forth-coming.

A Delicate Point for Educational Research

In order to make sense of questionnaire 'responses', interviews and observations of events in or associated with classrooms, it is important to pay attention to underlying assumptions. For example, do you assume that each subject has responded to each question by considering it carefully, seeking relevant instances from past experience? Or, do you assume that the data collected is wholly, largely or partly reactive, influenced by current state and conditions, and with recourse only to what 'comes to mind' (really, what comes to action to be enacted)? Might it even be predominantly 'remoting', triggered by memories or by the situation of being 'tested'. One of the reasons for follow-up interviews with selected subjects is that questionnaire returns are often difficult to construe, much less provide a basis for generalisation. But in interview, it is not always easy to reach a state of *interview* (Kvale, 1996) in which both parties are able to work with 'taken as shared' constructs (Cobb, Wood, Yackel & McNeal, 1992). Great care is required in staying with accounts-of for as long as possible, generating multiple interpretations through considering what someone would have to be attending to, and how, in order to say and do what the data indicates, and so moving to other theoretical terms in providing a rich accounting-for the data.

Summary

Attractive as it may be to analyse surface features of the grammar of utterances, whether questionnaire returns, interviews or taped interactions between people, it is valuable to probe beneath the surface, as implied by Calvino (1983) in the opening quote. One way to do this is by assuming that the subjects are narrative-building human beings who construct and reconstruct meaning for themselves, under the influence and direction of peer groups and respected 'others'. Such an assumption opens the way to listening to what students are trying to express rather than listening for an expected or intended utterance, for respecting babbling and dismissing gargling.

A productive beginning to analysing descriptive accounts-of incidents is to ask yourself what you would need to be attending to, and how you would be attending, in order to say and do what the data describes subjects as saying and doing. This is one way to listen to what people are saying, rather than being trapped into listening for what you want to hear. Once you have at least one conjecture about what someone might be attending to, and how, you are in a good position to invoke other frameworks of distinctions in order to characterise or classify the data.

Observation, whether of oneself or of the behaviour of others is always governed by what you are sensitised to notice. Even the research question and the methods used to enquire into those questions are subject to personal preferences, dispositions and available actions. Furthermore, in order to be of use, the results of enquiry have to inform the future actions of the researcher and other readers. To this end, the results have to sensitise people to notice possibilities for action that go beyond former habits and propensities.

It is absolutely vital to distinguish between accounts-of, which are brief-but-vivid incidents that afford entry into people's past experience and which can serve both as data and as a medium for relating to other people's experience, and accounting-for those incidents, which involves theorising, labelling and perhaps even justifying. When theory-based descriptive language is presented as data (accounting-for), it is impossible for the reader to agree or disagree with the labelling, making the analysis empty.

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Part I
Teacher Noticing in Various Grade Bands and Contexts

Teacher Noticing in Various Grade Bands and Contexts: Commentary

Brett Criswell and Rebecca McNall Krall

Abstract The chapters in this section explore professional noticing in contexts that include both middle and high school pre- and in-service teachers. The authors employ different theoretical and conceptual frameworks, and examine the professional noticing activities of their participants using different research methodologies. To try to tie together these different intellectual contributions, we present and provide initial considerations of six overarching questions related to this important field of scholarship. The hope is that readers will also consider these questions, and will reflect on the way the authors within this section address them as we work as a field to deepen our understanding of professional noticing.

Keywords Professional noticing · Purposeful reflection · Pedagogical content knowledge · Gestalt psychology · Inattention blindness

> *So Hilbert's strategy, one that we might do well to learn from, was to predict ignorance and not answers. He put no timeline on when the major problems might be solved, but nonetheless there are few mathematicians who would not agree that Hilbert's little speech at the opening of the 20th century was a positive influence on mathematics that effectively set much of the field's agenda for more than a hundred years.*
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> —Stuart Firestein in *Ignorance: How it drives science* (2012, p. 46), referring to David Hilbert's speech at the Second International Congress of Mathematicians held in Paris, 1900

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Framing

Firestein's book *Ignorance* suggests that it is not *finding answers* that drives science (or mathematics), but it is in fact *establishing ignorance* that moves the field (s) ahead. He points to David Hilbert's identification of the 23 most vexing problems of mathematics as proof of his central thesis (Hilbert's full speech can be found at http://www2.clarku.edu/∼djoyce/hilbert/). Given the growth in research in the field of professional noticing, it seems appropriate to accept Firestein's assertion and follow Hilbert's model of how to address that point. As such, while chapters in this section generated many insights regarding professional noticing across contexts, we will focus the discussion on four significant questions that emerged from our review.

Question 1: What *Is* Noticing? And How Is It Different from/Related to *Reflecting*?

What is noticing? In her chapter, Males defines professional noticing by drawing on Mason's (2002) work, explaining that, "noticing is something that we do all the time, but in a profession 'we are sensitized to notice certain things,' (p. xi)," (p. 91). She asserts, "the ability to notice is often perceived to develop over time as it requires extended opportunities to focus on aspects of practice and make connections between teaching and learning," (p. 91). Krupa, Huey, Lesseig, Casey, and Monson elaborate on Mason's (2011) construct, referring to his notion of *awareness,* describing it as a consequence of noticing, the ability to direct teachers' attention toward relevant teaching events. In contrast, Floro and Bostic employ Luna, Russ, and Colestock's (2009) definition, which describes teacher noticing as "a means for teachers to engage in formative assessment practices because 'teachers must recognize students' thinking … as it happens and make … instructional choices in response to what they notice,'" (p. 76).

If "noticing is something that we do all the time," (Mason, 2002, p. xi), how is professional noticing as Luna et al. (2009) describe it different from noticing in general? Mason's (2002) construct suggests there may be generic skills of noticing that translate to all professions, while specific skills and coding schemes (Goodwin, 1994) demarcate noticing in specific professions. Could it be that professional noticing is analogous to the layers of an onion where the outer layers symbolize general skills of noticing and the inner layers represent the increasingly complex coding schemes salient to a profession?

This progression from generic skills to more specific coding schemes seems to correlate with the shift from outer attention – focus on the superficial features of that which is being examined – to inner attention – focus on the deeper underlying structure discussed by Dewey (Mason, 2011). Applied to teaching, this could be conceptualized as a shift from those things easily observed – student behaviors and actions – to those things that must be meaningfully inferred – student thinking about problems and phenomena. Preservice teachers (PSTs) in several of the studies included in this section of papers demonstrated movement along this progression. For example, PSTs in Males' study tended to maintain a focus on outer attention, rather than on student thinking (inner attention). Krupa et al.'s study illustrated the difficulty secondary mathematics PSTs had in applying their abilities to attend to and interpret classroom events to formulate instructional responses that support student mathematical thinking. In comparison, PSTs in Teuscher, Leatham, and Peterson's study demonstrated that, with extensive experience analyzing videos for student mathematical thinking, they were able to attend, interpret, and respond to student thinking in the moment during student teaching.

Taken together these studies suggest a progression of professional noticing from general noticing skills to more specific skills for given contexts. Research is needed to determine whether there may be general skills that support noticing and serve as a foundation for more specific coding schemes characteristic to a profession and, if so, what such a "learning progression" for noticing looks like.

A related issue is, "How is professional noticing different from reflection?" In their chapter, Teuscher et al. reference Stockero's (2014) notion of *mathematical important moments* (MIMs). Although the authors do not explicitly define this notion, it is clear from the discussion that these represent significant events within classroom activity in which there are pedagogical opportunities for a teacher to respond to and build on students' thinking (Leatham, Peterson, Stockero, & Van Zoest, 2015). In this sense, then, this notion seems closely connected to the idea of *critical incidents*, which has been a focus of research on teacher reflection for over a quarter of a century (Farrell, 2008). If a goal of work in professional noticing is to help teachers be better able to recognize mathematically important moments (e.g., Stockero &Van Zoest, 2013), then it is necessary to explain how it is different from/related to assisting teachers in becoming more capable of identifying critical incidents in their reflective practice.

One way to distinguish between professional noticing and reflecting might be to suggest that professional noticing is directed toward the recognition of and response to key teaching events *in the moment*, while reflecting is more focused on making sense of such events *after the moment*. However, Teuscher et al. note that one of the features that distinguishes their work from that of Stockero (2014) is

> that [student teachers] applied a framework to the analysis of videos of lessons, the latter experience wherein we measure their noticing skills was based on real-time observations, where student teachers reported the details of their noticing without the ability to replay video in order to aid their analysis (p. 35).

Moreover, Schön (1983) described the difference between reflection-*on*-action and reflection-*in*-action, where the former indicated the capacity to reflect in retrospect and the latter the capacity to reflect in the immediacy of the moment.

This suggests that reflection and noticing could be conceived as a dialectic pair of processes that could be mutually reinforcing, indicating that professional noticing might help teachers better identify what to reflect on (attending) and assist them in determining how to act upon the outcomes of the reflective process (deciding). Conversely, reflection could push teachers to more critically analyze their sense making within professional noticing (interpreting) to recognize when their biases and beliefs are impinging on those interpretations. It is crucial that future research empirically examines the possible relationships so teachers can be supported in developing synergy between these processes.

Question 2: What Are the Psychological Mechanisms of Noticing, and How Could Collaborations Between STEM Education Researchers and Educational Psychologists Elucidate These Mechanisms?

In the theoretical framework, Krupa et al. interconnect Goodwin's (1994) notion of *highlighting* with Mason's (2002) idea of *awareness* and Jacobs, Lamb, and Philipp's (2010) principle of *attending*. Further, Teuscher et al. begin their chapter noting that teachers must sift through the minutia of sensory data in order "to make in-the-moment decisions that will support student learning," (p. 31). They observe that while some expert teachers are able to monitor the complexity of the classroom, many teachers resort to cognitive tunneling (Miller, 2011). Implicit in these discussions is the recognition of psychological processes that underlie practices of professional noticing, and the potential value collaborations with educational psychologists could bring in investigating those processes.

In this vein, it seems important to remember that one of the main foci of Gestalt psychology since its inception has been in determining how humans perceive figure and ground (Koffka, 1935)—very much related to a notion of highlighting, awareness, or attending. In the area of perception, additional research on visual seeing and perception has also indicated that inattentional blindness (Mack & Rock, 1998) is common during high perceptual load conditions (Most, 2013). Schoenfeld (2011) asserts that teachers' orientations to teaching greatly affect their perceptions, and therefore attention, on specific events in the classroom. Gestalt psychology has helped to build understanding in such areas as object recognition in computers (Wu & Zhang, 2013), and in delineating the parallels between foregrounding and backgrounding bodily feelings, and pre-reflective and reflective bodily awareness (Colombetti, 2011). Furthermore, research on perception and inattentional blindness can help identify ways to make objects—or salient classroom events—more apparent while teachers are attending to other stimuli (Schnuerch, Kreitz, Gibbons, & Memmert, 2016). All these studies seem to hold potential insights for those trying to understand how teachers with various levels of experience and operating in different contexts (1) determine what to focus upon, (2) connect the action of the classroom to the desired learning outcomes for their students, and (3) use professional noticing to inform reflection in and on teaching episodes.

The importance of this second question also surfaced in Krupa et al.'s discussion of various examples of teacher training focused on professional noticing. These authors cite four different studies as examples of what this training can look like: Fernández, Llinares, and Valls (2013), McDuffie et al. (2013), Schack et al. (2013), and Star and Strickland (2008). A review of these four articles showed significant differences in how this training was approached. Two of the studies had participants work independent of each other (Fernández et al., 2013; Star & Strickland, 2008); one had a mixture of independent skill development followed by discussion around professional noticing (Schack et al., 2013); and the last one used exclusively small-group and whole-class discussion (McDuffie et al., 2013). Another way of describing these differences is through a cognitive lens. Using this view, it could be inferred that the approaches of Fernández et al. and Star and Strickland adopted an individual cognitivist stance on the development of professional noticing capacities (Araujo, 1998), whereas McDuffie et al. operated from a social constructivist stance (Pitsoe & Maila, 2012), and Schack et al. used a structure that appeared to merge the two perspectives. It is critical to understand the psychological processes underpinning professional noticing in order to be explicit about how theoretical perspectives might inform the best design of teacher training in this capacity.

Question 3: What Should Teachers—and Researchers—Be Noticing, and What Is the Appropriate Process for Determining This?

In relation to this question, consider the foci of the four chapters discussed in this commentary. Krupa et al.'s analysis focused on secondary mathematics PSTs' journal reflections, which aligned with the three interrelated noticing skills (Jacobs et al., 2010), and were driven by instructions to summarize what the student understood and did not understand about solving linear equations and to describe what they would do next to advance the student's thinking. Teuscher et al. maintained a similar emphasis on journal entries of secondary mathematics PSTs using the prompt: "*Describe observed mathematical thinking where a student was either frustrated or appeared to have misconceptions …*" (p. 37). In comparison, Floro and Bostic's study focused on in-service teachers' professional noticing of student thinking around modeling with mathematics. Males adopted a broader focus of upper grade mathematics PSTs, investigating what they identified as noteworthy in videos of their peers' teaching.

Comparing the work of Krupa et al., Floro and Bostic, and Teuscher et al. to Males's study, there is a dichotomy between those who limited the purview of professional noticing to students' mathematical thinking and to Males, who extended it to include all aspects of the classroom milieu. Addressing the issue of what components of practice professional noticing should encompass is an important discussion for the field. The greater attention on practices and authentic problem solving outlined in the Standards for Mathematical Practice (Common Core State Standards, Initiative, CSSI, 2011) and the Next Generation Science Standards (NGSS, NGSS for Lead States, 2013) emphasizes the need for teachers to focus on student thinking. However, in light of findings from her study, Males raises the question as to whether a change in context influences what PSTs notice. Is this to say that the scope of professional noticing needs to be adaptable to the different contexts in which it is studied? If so, this would create a great challenge to the field in terms of transferring research insights across different contexts.

It seems likely that, in order to address the issues raised in the last two paragraphs, it will be necessary for those working in the field of professional noticing to create a theoretical model of this construct in the same manner that Magnusson, Krajcik, and Borko (1999) did in association with *pedagogical content knowledge* (PCK). Such a model would help all who wish to use this construct both in research and in teacher preparation to better conceptualize what it is, and to understand how it is related to other constructs—including PCK. In doing so, it also would seem prudent to ensure that teachers' voices are part of the conversations around what professional noticing entails, to what aspects of classroom practice it should be applied, and discussion around what theoretical model best captures our understanding of professional noticing.

Question 4: What Methods and Data Will Allow the Field of Professional Noticing to Push Itself Forward and Answer [Some of] the Questions Posed in This Commentary?

Not surprisingly, there was noticeable variation in the research approaches presented in the four studies in this section. While all of the researchers coded data, the source of the data was varied: interview transcripts (Floro & Bostic), assessment responses (Krupa et al.), journal responses (Teuscher et al.), and video feedback (Males). The majority of the coding was based on a priori categories (Krupa et al.; Teuscher et al.; Males), although Teuscher et al. generated subcodes after an initial analysis of the data, and Floro and Bostic employed emergent coding. The codes themselves ranged from very broad categories—classroom environment, classroom management, tasks, mathematical content, and communication in Males's study—to the very specific categories of Floro and Bostic's investigation (use mathematical models appropriate for the focus of the lesson; encourage student use of developmentally and content-appropriate mathematical models; remind students that a mathematical model used to represent a problem's solution is a work in progress, and may be revised as needed). Only one of the studies was purely qualitative (Floro and Bostic). The other three investigations quantified the data in some way in order to make comparisons before and after an intervention (Krupa et al.), across PSTs with different research experiences around student mathematical thinking (Teuscher et al.), or to indicated changes in PSTs' noticing across two semesters (Males).

Although much was learned from these methodological approaches, we would like to consider how other ways of exploring the data might extend our insights into professional noticing. The use of subcodes by Teuscher et al. suggests one way it is possible to come to a deeper, more nuanced, understanding of the processes involved in professional noticing. Levin and Richards (2011) have expanded this idea, developing levels of action for each of the components. If the validity of such levels can be demonstrated empirically in terms of describing differences in the way individuals engage in professional noticing, they might assist the effort of more fully articulating what this construct represents and the diversity of ways it is employed across contexts. Related to this, the analysis by Teuscher et al. was unique in that they examined relationships between the codes for the different parts of the process; this led them to identify four “types” of noticing: General observation and general interpretation, student mathematical thinking, student mathematical thinking and general interpretation, and student mathematical thinking and root interpretation. Further use of this approach would have two beneficial outcomes: (1) It would provide data that could support efforts at uncovering the cognitive mechanisms of professional noticing and (2) it would allow researchers to more thoroughly describe how individuals engage in the practice of professional noticing.

In discussing the interviews they analyzed in their study, Floro and Bostic note, “The goal of the interview…is to make sense of teachers noticing moments through their reflection on unique instructional moments” (p. 79). This statement highlights the ultimate goal of the work being done in the field of professional noticing: *making sense* of the ways teachers engage in this practice so as to better support their use of it to improve instruction. Given this goal, it is necessary to consider what might be missing from the approaches used by the researchers in this section pursuant to achieving it. One approach that was not utilized by any of the authors or in any of the research in this area that we have been able to locate, is that of *phenomenography* (Dall’Alba & Hasselgren, 1996). This approach has proven effective in coming to understand other phenomenon in education (e.g., Åkerlind, 2008), and it is likely that it would have value in making sense of professional noticing. Larsson and Holmström (2007) explain that,

> phenomenography is the study of how people experience, understand or conceive of a phenomenon in the world around us. The investigation is not directed at the phenomenon as such, but at the variation in people’s ways of understanding the phenomenon (p. 56).

From this description, it is apparent that adopting a phenomenographic stance would incorporate teachers’ voices in models of professional noticing, and produce more holistic descriptions of what this practice entails. Further, researchers would likely better understand *how* teachers see the processes involved in professional noticing, how they think about engaging in those processes, and look at how changes in teachers’ actions around practice are linked to changes in students’ thinking and learning.

Concluding Remarks

The authors contributing to the chapters in this section provided the field of professional noticing valuable insights across the contexts in which their four studies took place. For example, Males described how PSTs in her study tended to focus on *teacher actions* rather than on *student actions* as they analyzed peer-teaching videos. She leaves us to ponder whether different contexts may offer different opportunities for the kinds of events that teachers notice. This underscores the effect context may have on what teachers notice in the classroom.

In a sense, the studies by Krupa et al. and Teuscher et al. offer comparative interventions for developing PSTs' capacity in professional noticing of mathematical thinking. Krupa et al. employed the use of a one-on-one student interview on mathematical thinking to build PSTs' capacity in this area, whereas Teuscher et al. explored how long-term video analysis of student mathematical thinking affected PSTs' capacity for professional noticing in their own instruction. Teuscher et al. asserted that long-term video analysis had significant influence on PSTs' professional noticing abilities in real time. In contrast, PSTs in Krupa et al.'s study demonstrated growth in attending and interpreting, but showed little change in responding to student thinking. These findings suggest a difference in the cognitive demands of the three components of professional noticing, as well as a need for a different approach for developing PSTs' capacity in the third component—deciding. Outcomes also indicate more remains to be learned about a holistic set of experiences that can enhance PSTs' capacity to engage in professional noticing.

Floro and Bostic's study explored in-service teachers' capacity in professional noticing. Two themes emerged from their investigation: teachers' abilities to notice students' struggles with structure within mathematical tasks, and their abilities to notice student struggles translating between representations while problem solving. These findings suggest that context can affect what teachers' notice. What remains to be explored is whether this important outcome was a function of the lesson content, the training of these teachers, the teachers' particular classroom experiences or other factors—and then to determine how to use that in preparing future teachers.

The set of studies found in this section present those currently working in and those coming to the field of professional noticing with a good foundation for exploring this fertile terrain. In order to ensure the most productive journey into the future of this field, it seemed valuable to map the territory by not only describing what these researchers *have done*, but also to give significant attention to what *needs to be done*. In this sense, the broad questions posed throughout this commentary are intended to function like Hilbert's questions posed to the mathematical community at the beginning of the twentieth century: as a roadmap toward a deeper understanding of professional noticing and how we can use it to improve teaching and learning.

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From a Framework to a Lens: Learning to Notice Student Mathematical Thinking

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Abstract Teaching is a complex endeavor that necessarily requires teachers to attend to some activities and ignore others. This case study focuses on prospective teachers' learning to notice student mathematical thinking. We frame our view of noticing with the professional noticing framework (Jacobs, Lamb, & Philipp, in Journal for Research in Mathematics Education 41:169–202, 2010), and our view of student mathematical thinking with the MOST analytical framework (Leatham, Peterson, Stockero, & Van Zoest, in Journal for Research in Mathematics Education 46:88–124, 2015). We share evidence that a research experience that focused prospective teachers in a sustained, intense experience focused on articulating student mathematical thinking through focused video analysis influenced their ability to notice in-the-moment student mathematical thinking during their student teaching experience.

Keywords Student mathematical thinking · Video analysis · Mathematics student teaching · Learning to teach · Field experience

Teaching is a complex endeavor that necessarily requires teachers to attend to some activities and ignore others. As Sherin and Star (2011) observed, "a teacher is bombarded with a blooming, buzzing confusion of sensory data" (p. 69) that they must sift through to make in-the-moment decisions that will support student learning. Some expert teachers have developed the "ability to monitor the complex, chaotic environment of a classroom and home [sic] in on key features relevant to monitoring student understanding" (Miller, 2011, p. 51). Many teachers, however, tend to become overwhelmed by all the sensory data and to resort to cognitive

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tunneling, wherein they narrow their attention when performing complex tasks (Miller, 2011). Narrowing can be productive, however, if it allows teachers to disregard elements of classroom complexity that do not contribute to or that distract from productive teaching. An important objective of mathematics teacher education is to assist prospective teachers in learning how to do an appropriate amount of narrowing—how to sift through the "blooming, buzzing confusion" in a classroom and notice important aspects of teaching that support student learning.

Efforts to improve the teaching and learning of mathematics have emphasized the need for students to be active participants in a joint endeavor to construct mathematical meaning (e.g., National Council of Teachers of Mathematics, 1991, 2000, 2014). Such an approach to teaching requires teachers to analyze the student mathematical thinking that is elicited so they can use that thinking to help facilitate student learning. It is thus no surprise that a body of research related to teacher noticing of various aspects of students' mathematical thinking has recently emerged (e.g., Jacobs, Lamb, & Philipp, 2010). Studies have demonstrated that such noticing can be learned (Star, Lynch, & Perova, 2011; Stockero, 2014), but it does not come naturally (Star & Strickland, 2008), and prospective teachers experience significant difficulties in learning this skill (Peterson & Leatham, 2009; Stockero, 2008). Furthermore, there is a need for more evidence of the influence of preservice teachers' learning experiences (Cochran-Smith, 2005). This chapter reports the results of a post hoc analysis of data that seemed likely to provide such evidence—evidence of a connection between noticing skills that were developed prior to student teaching and the tendency to apply those noticing skills during student teaching.

The fortuitous circumstance was as follows: Four student teachers who began their student teaching experience having taken the same coursework (in some cases with the same instructors) were asked to identify interesting student mathematical thinking during student teaching. In addition to their regular coursework, two of these student teachers had engaged in a prolonged research experience identifying student mathematical thinking in classroom videos. In this case study, we share evidence that this research experience influenced their ability to notice in-the-moment student mathematical thinking during their student teaching experience. In essence we present an existence proof that preservice teachers' preparation experiences before student teaching can have an impact on noticing during student teaching.

Theoretical Framework

This case study focused on prospective teachers' learning to notice student mathematical thinking. We frame our view of noticing with the professional noticing framework (Jacobs et al., 2010), and our view of student mathematics thinking with the MOST analytical framework (Leatham, Peterson, Stockero, & Van Zoest, 2015).

Jacobs et al. (2010) developed a construct for a specialized teacher noticing—*professional noticing of children's mathematical thinking*. The authors narrowed their focus to professional noticing of elementary children's mathematical thinking, which they conceptualized "as a set of three interrelated skills: *attending* to children's strategies, *interpreting* children's understandings, and *deciding* how to respond on the bases of children's understanding" (Jacobs et al., 2010, p. 172, *emphasis added*). *Attending* was described as the extent to which teachers attend to the mathematical details in children's strategies. For this study, we characterized attending as the extent to which student teachers attended to the details of "students' mathematical ideas that surface during instruction" (Stockero, 2014, p. 241). Jacobs et al. (2010) described *interpreting* children's understanding as the extent to which teachers connected children's strategies with research on children's understanding. We took a more liberal approach to interpretation, looking instead for evidence of student teachers trying to make sense of the student mathematical thinking to which they had attended. Finally, Jacobs et al. (2010) described *deciding* as the degree to which teachers' responses were related to the children's thinking and to research on children's understanding of the mathematical concept. We use Jacobs et al.'s three interrelated activities (attending to, interpreting, and deciding how to respond to student thinking—which we refer to from here on as "responding") to examine similarities and differences between two sets of student teachers' abilities to notice student mathematical thinking in a real-time setting early in their student teaching experience.

We view student mathematical thinking as conceptualized in the MOST Analytic Framework (Leatham et al., 2015), a framework designed to facilitate research on productive use of student mathematical thinking during instruction. A MOST—"Mathematically Significant Pedagogical Opportunity to Build on Student Thinking" (Leatham et al., 2015, p. 91)—is an instance of student mathematical thinking that, if made the object of classroom discussion, would help the mathematical understanding of the students in the class move forward. When one applies the MOST analytic framework to video analysis of classroom mathematics discourse, the framework requires that one first "observe some student action that provides sufficient evidence to make reasonable inferences about" (p. 92) what a student is thinking mathematically. The framework then calls for the articulation of that inference, wherein one records the observed student action "as a complete sentence and clarifying, but not correcting, the student's language" (Van Zoest, Merrill, Leatham, Peterson, & Stockero, 2014, p. 8). In articulating this inference, one incorporates references to ideas that were part of the classroom dialogue, such as the teacher's questions that elicited the student thinking. Thus, for this study we viewed *student mathematical thinking* as the articulation of what a student's words or actions meant in the context of the classroom dialogue.

Literature Review

While the benefits of eliciting and using student thinking during instruction are well documented (e.g., Ball, 2001; Leinhardt & Steele, 2005; Schoenfeld, 1998), researchers continually find that teachers, including experts, struggle to do so in ways that support or extend student thinking in their classrooms (e.g., Brodie, 2011; Franke et al., 2009; Peterson & Leatham, 2009). Ball (2001) and Schifter (2001) provided several reasons why eliciting and using student thinking may be difficult: (1) student thinking is not always articulated clearly, (2) teachers must decenter to make sense of how students are thinking about situations, (3) teachers need to identify how students are thinking about the concepts in order to support or extend their thinking, and (4) teachers tend to focus on pedagogical aspects of teaching rather than on student mathematical thinking. In addition, Leatham et al. (2015) suggested that teachers may struggle to capitalize on student mathematical thinking because of the "complexity of recognizing and interpreting" (p. 89) student thinking. Thus, one way to understand barriers to eliciting and using student mathematical thinking is to look at these barriers through the lens of noticing.

Researchers have often used selected video clips to assist in developing teachers' abilities to notice specific elements of teaching (e.g., Kersting, Givvin, Sotelo, & Stigler, 2010; Santagata, Zannoni, & Stigler, 2007; Sherin & Han, 2004; Sherin & van Es, 2005). Early results of such research found that teachers, especially novice teachers, tended to notice a disparate collection of things (e.g., general teacher and student actions, classroom management, posters on the wall) (Sherin & van Es, 2005; Star & Strickland, 2008; Star et al., 2011), rather than important pedagogical moves or student mathematical thinking. To address these shortcomings, researchers have explored the use of guidelines, questions, or protocols designed to focus teachers' attention while watching video clips. Results from such studies have found that teachers, including prospective teachers, are more attentive to specific aspects of teaching related to the protocols guiding their noticing (Santagata & Angelici, 2010; Sherin & van Es, 2005; Star & Strickland, 2008). In studies such as these, however, researchers seldom make explicit the nature and scope of the learning activities that teachers engage with in order to better understand the aspect of instruction that is emphasized by the protocols. These learning activities seem to encompass broad aspects of teaching practice, rather than focus on sustained, intense experiences that are directly related to the specific aspect researchers want teachers to notice. Our study contributes to this literature by analyzing a case of student teachers who were engaged in a sustained experience focused on articulating student mathematical thinking and how that experience influenced their noticing during student teaching.

Others (e.g., Stockero, 2008, 2014; Van Zoest, Stockero, & Taylor, 2012) have begun this focused work . Stockero (2014) engaged prospective secondary mathematics teachers during their field placement course in a sustained process of learning to notice using a framework for Mathematically Important Moments (MIMs). This experience required prospective teachers to work collaboratively with the researcher to modify and revise their own understanding of MIMs as they applied the framework repeatedly over an extended period of time. The researcher facilitated this learning process by pushing the prospective teachers to identify the importance of the mathematics in the instances they had selected each week. Approximately midway through the semester, the framework became the explicit coding window that the prospective teachers continued to use as they identified instances of MIMs. Results indicated that prospective teachers' shifted their noticing from teacher actions and explanations to students' thinking, questions, and comments related to the mathematics being studied. Our work provides further evidence of the potential impact on noticing of sustained application of a framework. Our work differs in that our study examines the use of a smaller framework—the student mathematical thinking criterion of the MOST Analytic Framework (Leatham et al., 2015)—over a longer period of time. Furthermore, although the initial experience of our student teachers is similar to those in Stockero's study, in that they applied a framework to the analysis of videos of lessons, the latter experience wherein we measure their noticing skills was based on real-time observations, where student teachers reported the details of their noticing without the ability to replay video in order to aid their analysis.

Methods

The Brigham Young University (BYU) undergraduate secondary mathematics education (ME) program is a large program with approximately 185 enrolled pre-service secondary teachers (PSTs) and about 40 graduates per year. Program graduates qualify for a Utah provisional secondary (grades 7–12) teaching certification. The BYU ME program differs from many U.S. ME programs. Although PSTs enroll in the typical mathematics courses (41 semester credit hours) that other universities require, they take five core ME courses (16 semester credit hours) prior to student teaching, all from faculty in an ME department that is housed in a College of Physical and Mathematical Sciences. The five core ME courses are grounded in PSTs gaining both mathematical and pedagogical knowledge for teaching. Each course concentrates on specific aspects of mathematics (e.g., ratios, fractions, exponential functions) and pedagogy (e.g., learning mathematics, task design, assessment) and is purposefully designed to contribute to an overall program emphasis on the importance of eliciting and using student mathematical thinking during instruction. Across the courses, the program emphasizes a general four-phase structure to task-based mathematics lessons: launching student inquiry, supporting productive student exploration of the task, facilitating discourse and public performances, and unpacking and analyzing students' mathematics. That said, no framework for the professional noticing or using of student mathematical thinking is formally introduced or studied. The four participants in this study had participated in the ME program at roughly the same time and thus had very similar course experiences of this nature.

Beyond this common program experience, all four of the student teachers that make up this case study had elected to participate in an undergraduate research experience in mathematics education prior to student teaching. Two student teachers (referred to hereafter as STs) had worked on *Project Pathways*, a National Science Foundation (NSF) Math and Science Partnership project (No. EHR-0412537). Their research experience involved analyzing large data sets to determine students' understanding of different mathematical concepts, was largely statistical in nature, and was directed by the first author. The other two student teachers had worked on the NSF-funded project *Leveraging MOSTs* (Nos 1220141, 1220357, and 1220148) under the direction of the second and third authors. Their research experience involved using the student mathematical thinking criterion of the MOST Analytic Framework (Leatham et al., 2015) to analyze video of secondary mathematics lessons. This analysis required them to watch secondary mathematics classroom videos and identify all instances of student thinking during whole-class discussion that were "potentially mathematical." As each instance was marked in the video analysis software—Studiocode (Sportstec, 2013)—they then articulated the student mathematics according to the project coding guidebook (and as described previously in the theoretical framework). Because this focused video analysis is the defining feature of this case study, we refer to these student teachers as STVAs, where VA stands for *video analysis*. During their time working as research assistants, the STVAs articulated hundreds of instances of student mathematical thinking. The training for this work was ongoing and grounded in refining their understanding of the framework through practice and reconciliation. They coded initially on their own, then each week the individual coding was combined, compared, and reconciled with each other (facilitated by a graduate research assistant). Any instances that could not be reconciled among the three of them were then discussed with the second and third authors. Meetings of this nature took place regularly over the course of about a year and a half. We feel it is safe to say that the experience immersed these STVAs in this particular framework for viewing student mathematical thinking.

Because the BYU student teaching structure is atypical, we describe it briefly to make more clear the context of the data. (See Leatham & Peterson, 2010 for a more detailed description of the structure). Student teachers are placed in pairs with a cooperating teacher. The STVAs were paired with each other and the two STs worked with a different cooperating teacher. The four student teachers worked together as a cluster with the first author as their university supervisor. During the first five weeks of student teaching, each student teacher taught once a week and was observed by the other student teachers in their cluster as well as their cooperating teacher and university supervisor. In conjunction with this periodic teaching in the early weeks, the student teachers do a significant number of focused observations and reflective activities. The primary goal of the overall student teaching structure is to minimize a focus on classroom management and survival and to maximize a focus on the practice of eliciting and using student mathematical thinking (Leatham & Peterson, 2010).

One of the reflective activities the student teachers completed was a daily journal of what they did and noticed each day. As part of the journal, they were asked to respond to the following prompt:

> Describe observed mathematical thinking where a student was either frustrated or appeared to have misconceptions. If you were to work with this student, what questions would you ask and when would you ask them? If you were to use this student's thinking as part of a class discussion, how would you use it?

Student teachers' responses to this prompt give insight into the progress they were making with respect to their abilities to notice student mathematical thinking.[1] The student teachers were asked to complete journal entries every day for the first five weeks of student teaching. Had the student teachers written a single response for each day's prompt, they would have logged 23 responses each during these five weeks. However, because the student teachers periodically either did not answer this prompt or provided multiple distinct answers on the same day, the number of responses that each student teacher submitted varied (16, 22, 23, and 31, for a total of 92 journal responses).

We conducted this post hoc analysis because, based on initial observations, the substance of the journal entries for these four student teachers seemed to vary drastically. We employed general qualitative methods (Creswell, 1998) to code, refine, and recode the data. We began with somewhat broad codes that paralleled the three interrelated skills of professional noticing (Jacob et al., 2010): (1) identifying and describing (attending to) student mathematical thinking, (2) interpreting that thinking, and (3) describing how they would respond were they given the opportunity to do so (deciding). Having blinded the data, the first author and a research assistant tested out the codes on the entries from one participant's journal. The second and third authors then coded the same entries and we came together to reconcile the coding and, through this process, refine our definitions for these codes, creating sub-codes to capture nuanced differences within these broader codes.

Having refined our codes through analyzing one student teacher's journal entries, the first author and a research assistant coded the journal entries from the remaining three student teachers. To assess the reliability of this coding, the second and third authors each coded a different subset of 12 journal entries (four per remaining student teacher) and the results of their codes were compared to the first author's coding. The agreement between coders ranged from 83 to 100% for each code. Once the data were coded, and in order to facilitate comparison, we calculated a total percentage of journal entries that were coded in each of the categories, discussed in more detail in the following section, for individual student teachers and by groups of student teachers (STVAs and ST).

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[1]Although there were other parts of the journal, only responses to this prompt were used as data for this study.

Results and Discussion

We organize the results by presenting the percentage of journal entries receiving a particular code for individual student teachers and by sharing typical journal excerpts that demonstrate the different subcategories across the three distinct professional noticing skills: attending, interpreting, and responding. We then discuss the results using the interrelated nature of these skills to differentiate among four types of noticing that were present in our data and specific to the groups of student teachers (STs and STVAs).

Attending

Attending to student mathematical thinking was an explicit part of the prompt in the journal entry; therefore, if student teachers answered the prompt (i.e., referenced student mathematical thinking in any way) we saw this answer as evidence of them attending to student mathematical thinking. As we analyzed the entries, however, two subcategories of attending emerged: *general observations* and *student mathematical thinking*. The primary distinction between the two subcategories is that, in the latter, there was sufficient evidence in the entry for us to infer the student mathematics of the instance, as per the MOST Analytic Framework (Leatham et al., 2015). Table 1 displays the percentages of the four student teachers' journal entries that were coded as attending to student mathematical thinking across the two subcategories.

Table 1
Percentage of student teachers' journal entries attending to student mathematical thinking

	ST1 (n = 16) (%)	ST2 (n = 31) (%)	Total (n = 47) (%)	STVA1 (n = 23) (%)	STVA2 (n = 22) (%)	Total (n = 45) (%)
Attending			100			100
General observation	19	87	64	4	5	4
Student mathematical thinking	81	13	36	96	95	96

General observation. The following journal excerpt is typical of a *general observation* of student mathematical thinking: "Some students were confused about how to calculate the period of a trigonometric function" (ST2). In this excerpt, ST2 describes a general observation of the mathematics with which students are confused, but there is no specific student mathematical thinking articulated. As shown in Table 1, although general observations such as this were made by all four student teachers, this type of response was typical of ST2, for whom the vast majority of her journal entries reported only general observations of student mathematical thinking. The other three student teachers, particularly the STVAs, made general observations far less frequently.

Student mathematical thinking. The following journal excerpt is typical of that which received the *student mathematical thinking* sub-code: “Some of the students wanted to know how [the teacher] knew whether to use a sine or cosine equation for the different modeling problems.” (ST1). Here ST1 reports the student mathematical thinking in enough detail that we can infer what the student may have said, perhaps something like, “How do we know which trig function to use when we solve these problems?” As shown in Table 1, student mathematical thinking was reported by all four student teachers; however, this type of response was quite common for ST1, and predominates for STVA1 and STVA2.

Interpreting

Although interpreting student mathematical thinking was not an explicit part of the journal prompt, we found that a majority of the entries nevertheless included some evidence of interpretation. Again, analysis of the entries revealed two subcategories of interpretation: *general interpretations* and *root interpretations*. General interpretations tended to be broad generalizations about students being confused or not understanding a particular topic or problem. By contrast, root interpretations attempted to identify possible reasons behind the student mathematical thinking or what the student’s thinking might mean in relationship to their understanding of the mathematics. Table 2 displays the percentages of the four student teachers’ journal entries that were coded as interpreting student mathematics across the two subcategories.

Table 2
Percentage of student teachers’ journal entries that interpreted student mathematical thinking

	ST1 (n = 16) (%)	ST2 (n = 31) (%)	Total (n = 47) (%)	STVA1 (n = 23) (%)	STVA2 (n = 22) (%)	Total (n = 45) (%)
Interpreting			81			96
General interpretation	56	90	79	57	68	62
Root interpretation	6	0	2	35	32	33

General interpretation. The following journal excerpt is typical of a *general interpretation* of student mathematical thinking: “One student in B8 was particularly struggling with radians. She wasn’t sure how to convert between degrees and radians and wasn’t sure how to find area/perimeter of circles” (ST1). In this excerpt, ST1 provides a general interpretation that the student was struggling with some aspect of radians, but provides little evidence of how she made sense of the student mathematical thinking. This general interpretation was found in the majority of the four student teachers' journal entries; in contrast with the other three student teachers, ST2 provided this type of interpretation in 90% of her journal entries.

Root interpretation. The following journal excerpt is typical of those that provided a *root interpretation* of the student mathematical thinking:

> The question on the board was, "Find the probability of rolling 2 dice and getting a 5 and an even number." I heard a student say to his neighbor, "It's 3/36 because there are 36 total possibilities and 3 ways to get a 5 and an even number, since the 5 comes first." *This student interpreted the problem as asking for the probability of the first die being a 5 and the second die being an even number*. (STVA1, emphasis added)

In this example, STVA1 provides a root interpretation (in italic) of the student mathematical thinking—an inference about how the student may have interpreted the problem in order to result in making such a statement. This type of journal entry occurred rarely for the two STs; by contrast, the two STVAs included a root interpretation in a third of their journal entries.

Responding

The intention of the second and third questions in the journal prompt (*If you were to work with this student, what questions would you ask and when would you ask them? If you were to use this student's thinking as part of a class discussion, how would you use it?*) was to encourage the student teachers to think about how they might respond to the student mathematical thinking had they been teaching—to engage them in the "deciding" aspect of noticing. Three subcategories of responses emerged from our analysis: *no clear connection, elaborated*, and *facilitated*. If the student teacher's response was not related or connected to the identified student mathematical thinking, then it was coded as *no clear connection*. The next two subcategories within responding were based on variations in the nature of the teacher move. If the response focused on how the teacher would discuss or elaborate on the mathematics in order to address the student struggle or misconception, the response was coded as *elaborated*. On the other hand, if the response provided evidence that the student teacher intended to facilitate further student engagement in making sense of the mathematics in order to address the student's frustration or misconception, the response was coded as *facilitating*. Table 3 displays the percentages of the four student teachers' journal entries that were coded as responding across the three subcategories.[2]

[2]We note that we were very liberal in our coding. If student teachers mentioned anything that seemed to indicate an intent to facilitate student thinking it was coded as such. We also acknowledge that analyzing journal entries does not allow us to access the full interaction the student teachers may have envisioned.

Table 3
Percentage of student teachers' journal entries that responded to student mathematical thinking

	ST1 (n = 16) (%)	ST2 (n = 31) (%)	Total (n = 47) (%)	STVA1 (n = 23) (%)	STVA2 (n = 22) (%)	Total (n = 45) (%)
Responding			77			98
No clear connection	0	6	4	4	5	5
Elaborated	25	29	28	13	32	22
Facilitated	75	29	45	83	59	71

No clear connection. The following journal excerpt is typical of those that provided a response that was not connected to student mathematical thinking:

> I thought that many of the students would be able to graph the tangent function, especially because some of them had seen the function before and could describe its undefined properties. They all had the points on the graph, but they just sat there, and I wasn't sure what was wrong. *I think what would have been more helpful was to give them less points and then afterwards discuss with them which part is actually one period of the parent function*. (ST2, emphasis added)

ST2 makes a suggestion as to how she would respond (in italic), but this decision was not a response to student thinking. Rather, she described what she could do in the future in order to avoid or prevent such student thinking. This type of response was rarely given, but three student teachers (ST2, STVA1, STVA2) provided at least one response in this subcategory.

Elaborated. The following journal excerpt is typical of those that provided a response that elaborated on student mathematical thinking:

> I would simply remind him to check each step that he did. When he would get to the one that involved negatives I would ask him, "What do we get when a negative times a negative is positive?" He would immediately get the answer and correctly finish out the problem. (STVA2)

In this example, STVA2 provides a response that has her telling or reminding the student of a rule. Although this response is clearly related to the observed student thinking, the cognitive work in the interaction remained with the student teacher. This type of response was found across the four student teachers' journal entries, ranging from 13 to 32%.

Facilitated. The following journal excerpt is typical of those that provided a response through which the student teacher intended to facilitate student mathematical thinking:

> Some questions I could have asked this student are, "What is the question asking us to find? What do our graphs illustrate? How do the painters' rates compare in both graphs? Why are they the same?" I would definitely have students discuss this frustration in pairs before discussing it as a class because I definitely see this point as something important for interpreting graphs correctly. (STVA1)

In this example, STVA1 proposes asking the student questions to facilitate further student engagement in making sense of the mathematics. This response is related to the observed student thinking and the cognitive workload is passed to the student rather than remaining with the student teacher. This type of response was found across the four student teachers' journal entries, but varied from 29 to 83%. When the student teachers' hypothetical responses were characterized by these three subcategories we see some variation, but no clear distinction between the two groups.

Summary

Table 4 summarizes the percentage of total journal entries by the two groups of student teachers (STs and STVAs) across all noticing skills and subcategories. These data provide initial evidence of a difference in noticing between the STs and STVAs. For example, while both groups of student teachers attended to student mathematical thinking, STs typically reported a general observation (64%); whereas, the STVAs typically reported student mathematical thinking (96%). Both groups of student teachers also seemed to interpret student mathematical thinking, but the STVAs provided a root interpretation 33% of the time, while the STs provided a root interpretation only once across their combined journal entries. We also note that the STVAs responded to student mathematical thinking more often than the STs and that those responses were more likely to engage students in mathematical reasoning.

Table 4
Percentage of journal entries coded in each skill and subcategory for STs and STVAs

Skills and Subcategories	STs (N = 47) (%)	STVAs (N = 45) (%)
Attending	100	100
General observation	64	4
Student mathematical thinking	36	96
Interpreting	81	96
General interpretation	79	62
Root interpretation	2	33
Responding	77	98
No clear connection	4	5
Elaborated	28	22
Facilitated	45	71

Types of Noticing

As Jacobs et al. (2010) suggested, the skills of attending, interpreting, and responding are interrelated skills. Therefore, although we have discussed these skills in isolation, we realize that these skills are nested and should be analyzed as a whole. We now present four types of noticing student mathematical thinking, as characterized by varying combinations of the attending and interpreting subcategories. (We omit the responding subcategories and associated portions of the entries in these types because we did not find qualitative differences among the four student teachers in the ways they responded to student thinking when viewed in connection with the other skills. That is, the responding subcategories did not help to further delineate the types of noticing we identified.). These four types of noticing characterize the vast majority of the journal entries.

General observation and general interpretation. A first type of noticing is *a general observation* and a *general interpretation* of student mathematical thinking. This type of journal entry was much more typical for ST2 (87% of entries) than for the other three (19, 4, and 0%). You will notice that these percentages are exactly the same as for a general observation only. This is because it is impossible to provide some type of general observation without having done some general interpreting. The following journal excerpt is typical of this type of noticing:

> We noticed while grading the test corrections that students had not yet figured out the main concepts on the test. ***For example, they did not understand how to use the compound interest formula when interest is compounded more than annually. Also, they did not know how to use a rate given in the problem or the differences between compounding discretely versus continuously. The major problem that I saw was a lack of understand [sic] of exponential decay—that the exponent should be negative***. (ST2, emphasis added)

ST2's journal entry provides a general observation and interpretation of the student mathematical thinking (in bold, italic[3]). We are not given enough information to infer what the students were doing incorrectly on their tests as it related to the compound interest formula or exponential decay functions.

Student mathematical thinking. A second type of journal entry included only *student mathematical thinking*. (As mentioned before, the journal prompt did not request an interpretation, but interpretations were given in the majority of the entries.) This type of journal entry was more typical for ST1 (38%) than the other three (10, 4 and 0%). The following journal excerpt is representative of this type of noticing:

> *Some of the students wanted to know how [the teacher] knew whether to use a sine or cosine equation for the different modeling problems*. Because we were short on time, I think she explained that by the graph we created, we compared it to our parent functions and determined which graph it was most similar to… (ST1, emphasis added)

[3]For this section, the parts of the excerpt related to attending will be highlighted by using italic and the parts of the excerpt related to interpreting will be highlighted by using bold.

ST1's journal entry provides enough information to infer the student mathematical thinking (in italic), and then goes on to explain how the teacher responded, yet ST1 never interprets the student thinking.

Student mathematical thinking and general interpretation. A third type of journal entry included the *student mathematical thinking* and a *general interpretation*. These journal entries were more typical for the STVAs (52 and 68%) than for the STs (38 and 3%). The following journal excerpt is representative of this type of noticing, "**One student in B8 was particularly struggling with radians**. *She wasn't sure how to convert between degrees and radians* and wasn't sure how to find area/perimeter of circles" (ST1, emphasis added). ST1 provides the student mathematical thinking (in italic) and a general interpretation (in bold) of the thinking. It is interesting to note that ST1's interpretation was that the student was struggling with radians, yet the student mathematical thinking involved converting between degrees and radians and provides no evidence of a struggle particular to radians.

Student mathematical thinking and root interpretation. A fourth type of journal entry included *student mathematical thinking* and a *root interpretation of the student thinking*. These entries were somewhat rare, occurring in only 15 instances across the 92 journal entries. That said, the STVAs' entries accounted for 14 of the 15 entries. The following excerpt is typical of this type of noticing:

> *A student (as well as many others) believed that if a table grew by a constant growth pattern (slope), it was proportional*. This is a really interesting misconception to me because I know that it's also common. **It's hard when every proportional relationship is linear, which means that it has that constant growth pattern, or slope. So when we put "constant growth pattern" under our list of characteristics of proportional relationships, but then explain that it's not always visible in our table AND that you must have the other 3 relationships we were talking about, I can see why this would be so difficult for kids to understand**… I feel like the difference between slope and equivalent ratios is going to be a big misconception many students are going to have. (STVA1 emphasis added)

In this example, STVA1 identified the student mathematical thinking (in italic) along with a root interpretation (in bold) of why the students may have thought that if a table of values grew by a constant growth pattern it was proportional.

Summary. Table 5 summarizes the percentage of total journal entries by the two groups of student teachers (STs and STVAs) across the four types of noticing. ST2 tended to attend to and interpret student mathematical thinking at the general level —the majority (87%) of her journal entries was coded as this type of noticing. ST1 tended to attend and interpret by providing some detail about the student mathematical thinking and then offering a general interpretation for about half of those entries. The two STVAs tended to attend and interpret by providing some detail about the student mathematical thinking and then offering one of the two kinds of interpretation—about a third of these interpretations were root interpretations.

Table 5
Percentage of journal entries coded in each type of noticing for STs and STVAs

	ST1 (n = 16) (%)	ST2 (n = 31) (%)	**Total** (n = 47) (%)	STVA1 (n = 23) (%)	STVA2 (n = 22) (%)	**Total** (n = 45) (%)
General observation and general interpretation	19	87	64	4	0	2
Student mathematical thinking	38	10	19	9	0	4
Student mathematical thinking and general interpretation	38	3	15	52	68	60
Student mathematical thinking and root interpretation	6	0	2	35	27	31

STVAs identified and articulated specific student mathematical thinking just under three times as often as the STs. In most cases, the STVAs articulated this thinking and included an interpretation of it. By contrast, the STs only articulated the student mathematical thinking in just over a third of the journal entries and only about half of those entries included an interpretation. Although the STVAs provided a root interpretation of student mathematical thinking about a third of the time, the STs did so only one time. The STVAs provided much more evidence of their abilities to notice details of student mathematical thinking.

Conclusion and Implications

The results of this case study provide evidence to support the claim that PSTs' engagement in focused video analysis had a significant influence on their professional noticing of student mathematical thinking in *real time*. These journal entries were not based on watching a video of the lesson and going back through the video multiple times to capture the student mathematical thinking. Rather, these written reflections were based on real-time observations. The STVAs seemed to notice student mathematical thinking—amidst all the "blooming, buzzing confusion" (Sherin & Star, 2011, p. 69) in the classroom—at a different level than their ST counterparts. One possible reason for this difference in noticing is the STVAs' focused video analysis of identifying and articulating student mathematical thinking, particularly given the close relationship between the nature of that experience and the foundational noticing skill of attending to student mathematical thinking. The STVAs were much more likely to attend to student mathematical thinking at a deeper level, and that attention seems to be critical to providing the detail necessary for deeper levels of interpreting.

To provide further evidence for this possible connection, the two STVAs were interviewed and asked whether working on the research project had impacted their teaching. This excerpt captures the essence of both STVAs' responses to this question:

> In the moment when I am teaching I find myself mentally coding. Or when I am observing another teacher, like my student teaching partner, I will be coding her lessons in my head. It's definitely different when I can't rewind and analyze over and over, but in the moment I try and think about what the students are saying mathematically, what I need to be asking of them in order to better understand what they are saying, and what moments would be determined as MOSTs and how I can really capitalize on those experiences and how I can further my goals of the lessons. (Interview STVA 2, 2015)

Statements such as this suggest that the STVAs were explicitly aware of the influence the focused video analysis had on them, because they realized they were asking themselves questions while teaching that were similar to those they had used while coding videos for the research project. In essence, the STVAs had become attuned to the practice of identifying and articulating student mathematical thinking, as evidenced by the fact that 96% of their journal entries attended at the level of articulating student mathematical thinking.

Although we are enthusiastic about these results, we are aware that many may ask questions such as the following: How feasible is it for all PSTs to participate in a research experience that includes focused video analysis, particularly one as extensive as the one the STVAs experienced? What aspects of focused video analysis (e.g., nature of framework, quantity of experience, the medium of video) are necessary to gain results such as these? In what ways might these necessary aspects be integrated into ME coursework? Although we do not as yet have answers to these questions, we see the results of this case study as compelling evidence to pursue answers. We are convinced that it is worth trying to incorporate focused video analysis into coursework to enhance PSTs' learning of the skills necessary for professional noticing as we attempt to gather greater evidence of its impact.

For example, PSTs could be taught specific, focused framework around particular aspects of effective mathematics instruction and then engage in research-like activities. Having coded video excerpts individually, PSTs would then compare those identified instances with those of other PSTs in order to come to some reconciled agreement about the instances and their interpretation of the framework. The group's agreed-upon instances could then be compared with those of other groups in whole-class discussion in order to refine the understanding of the framework (and the underlying practice) for all members of the class. The process of reconciling instances and interpretation of the framework provides opportunities for PSTs to justify their thinking among the group members or between groups and is similar to the activity in which the STVAs participated as research assistants. Thus we believe this reconciling and refining that an understanding of the framework is a critical aspect of turning that framework into a lens through which they will tend to view teaching practices.

Finally, although all five core mathematics education courses at BYU emphasized the value of instruction that focuses on eliciting and building on student mathematical thinking, our two STs only identified student mathematical thinking in 36% of their journal entries. This result suggests that our PSTs' participation in coursework focused on attending to student mathematical thinking throughout their program may need to be improved, possibly because their abilities to engage in the foundational noticing skill of attending (in detail) to student mathematical thinking is limited. In spite of the fact that we only had four student teachers' journal entries upon which to base our conclusions in this study, the evidence suggests that the focused video analysis experience had a positive influence on the STVAs' ability to notice student mathematical thinking. Mathematics educators should seriously consider designing courses that engage PSTs in experiences similar to those of our STVAs.

This case study demonstrates that student teachers, despite being novices, are capable of decentering and noticing student mathematical thinking. Such results likely require significant time to grapple with important frameworks for making sense of complex mathematics teaching practices. When given these opportunities, however, it is possible for such frameworks to be transformed into lenses through which these teachers tend to "naturally" view their teaching. We see such possibilities as extremely promising in helping us as mathematics teacher educators have a truly meaningful impact on our future teachers.

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Investigating Secondary Preservice Teacher Noticing of Students' Mathematical Thinking

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Abstract Based on promising work conducted with practicing and preservice teachers at the elementary level to scaffold teacher noticing, we propose that secondary preservice teachers (PSTs) can similarly be supported in their development of professional noticing. Through the lens of research results obtained by studying the effects of a curricular module designed to develop secondary mathematics PSTs' noticing, we discuss aspects of teacher noticing constructs at the elementary level that are applicable to secondary and aspects that require modification for transferability. Further, we describe the impact of the curricular module, including a task-based clinical interview, on secondary PSTs' ability to attend, interpret, and respond to student thinking.

Keywords Noticing · Assessing noticing · Interview assignment · Secondary methods

In order to effectively implement student-centered instruction that fosters mathematical proficiency, teachers are required to not only recognize and interpret student thinking but also to incorporate students' current understandings as a basis

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for instruction (Franke, Kazemi, & Battey, 2007; NRC, 2001). Research shows that elementary teachers' abilities to elicit and build upon student thinking during mathematics instruction is positively associated with students' mathematics achievement (Bobis et al. 2005; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). While the empirical link is less established at the secondary level, a strong theoretical foundation supports the claim that by focusing on student thinking, mathematical understanding will increase (Brown & Cocking, 2000; Walshaw & Anthony, 2008). As evidenced by the dominance of traditional methods of instruction and teacher-centered patterns of discourse, this is not standard practice, especially at the secondary level (Cazden, 2001; Hiebert et al., 2003; National Mathematics Advisory Panel, 2008). Therefore, a need exists to build secondary mathematics teachers' ability to make sense of student thinking.

The role of student thinking in guiding instructional decisions is gaining in prominence among researchers of professional noticing. Sherin, Jacobs, and Phillip (2011), articulate five core questions for researchers in the emerging field of noticing, including: (1) Is teacher noticing trainable?, and (2) How can researchers most productively study teacher noticing? In this chapter, we contribute additional evidence towards answering these two open questions by discussing if a curricular module, centered on a task-based clinical interview assignment, can foster preservice teachers' (PSTs') noticing of student thinking as a first step toward student-centered instruction. In addition, we characterize how researchers can assess secondary PSTs' noticing by building upon prior efforts with elementary preservice and in-service teachers.

Theoretical Framework

Teacher Noticing

Noticing is a common word in the English language, but to understand what is meant by *teacher noticing*, we return to some of the origins of noticing as a construct. Goodwin (1994) defines professional vision as consisting "of socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606). He elaborates on three work practices: coding, highlighting, and producing material representations, through which professional vision is both evidenced and developed. The coding scheme establishes an orientation that focuses or filters one's perceptions toward the work practices and accepted ways of knowing particular to a professional group. Highlighting refers to ways in which features relevant to the work are made visible. Similar to Mason's (2002) notion of marking or calling out, highlighting through verbal, visual or physical means serves to shape one's perceptions by making explicit the features of the event to which one is expected to attend while moving other features to the background. The production and articulation of representations refers to ways in which written and verbal communication, human interactions, and tools are used to build representations central to disciplinary work. In a classroom full of students, the skills a teacher would use to engage in coding, highlighting, and producing representations are likely very different from the skills needed in other social contexts.

Mason (2002) stated, “every act of teaching depends on noticing: noticing what children are doing, how they respond, evaluating what is being said or done against expectation and criteria, and considering what might be said or done next” (p. 7). He introduced the notion of *awareness* to characterize the ability of noticing as a consequence of structuring teachers’ attention about relevant teaching events. For mathematics instruction, this awareness is important so that teachers can identify significant mathematical ideas that students present and make judgments about their next instructional steps based on the students’ thinking and understanding.

Our use of noticing in this chapter draws upon a conceptualization of *professional noticing of student thinking* (Jacobs, Lamb, & Philipp, 2010) comprised of the three interrelated skills of “attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings” (p. 172). Teachers must notice (attend to) the specific mathematical ideas evident in students’ written work or verbal responses in order to make sense of (interpret) that thinking. Interpretations of students’ thinking can then be used to inform teachers’ next steps (respond).

Is Teacher Noticing Trainable?

Research has shown that elementary PSTs can develop professional noticing skills through use of video and guided reflection (McDuffie et al., 2013; Schack et al., 2013; Star & Strickland, 2008). McDuffie and colleagues demonstrated how repeated cycles of video viewings enabled elementary PSTs to attend to and interpret students’ multiple knowledge bases and reflect upon equitable teaching practices. Schack and colleagues reported similar success following an intervention to advance elementary PSTs’ ability to notice student thinking utilizing video-recorded clinical interviews followed by actual diagnostic interviews. PSTs demonstrated growth across all three components of professional noticing, although no PST achieved the highest level of sophistication for either interpreting student thinking or deciding how to respond.

In their research at the elementary level, Fernández, Llinares, and Valls (2013) characterized how PSTs attended to and interpreted students’ mathematical thinking when analyzing students’ written work on proportional reasoning tasks. They identified a four-level developmental sequence to describe the advancement of PSTs’ noticing abilities in the specific mathematical domain of multiplicative thinking. Trajectories like the ones created by Fernández and colleagues can help mathematics teacher educators assess noticing.

Improvements in noticing have also been achieved through professional development (PD) efforts (Goldsmith & Seago, 2011; Jacobs et al., 2010; Jacobs, Lamb, Philipp, & Schappelle, 2011; van Es, 2011; van Es & Sherin, 2008). During a Cognitively Guided Instruction workshop, Jacobs and colleagues (2010, 2011) documented how expertise in professional noticing grew over time. Similarly, van Es and Sherin (2008) detailed the development of teachers' selective attention to and knowledge-based reasoning around student thinking through participation in researcher-facilitated video clubs. Finally, Goldsmith and Seago (2011) increased opportunities for secondary grade teachers to notice student thinking through use of classroom artifacts, mostly videos or student work samples, in order to link instructional choices with evidence. This research with practicing teachers demonstrates how repeated opportunities to engage in collaborative noticing, guided by analytic frameworks and specific prompts for discussion, can direct teachers' attention to mathematically important aspects of student thinking.

In summary, research, primarily at the elementary level, shows promise for developing both practicing and PSTs' noticing through well-sequenced, scaffolding activities (Jacobs et al., 2010; McDuffie et al., 2013; Schack et al., 2013). However, scant research has been conducted on teacher noticing at the secondary level. Therefore, efforts that explore the viability of leveraging research-based constructs from the elementary level to support secondary PSTs' ability to notice student thinking are needed. To this end, we designed an intervention that incorporates scaffolding activities utilized successfully at the elementary level, such as viewing and reflecting upon videos of clinical interviews, to be used in a secondary methods course in order to foster PSTs' professional noticing of student thinking.

Productively Assessing Noticing

In order to gauge the viability of the intervention, we needed a way to assess the impact of the module on participants' noticing. In particular, we were interested in observing changes in the noticing demonstrated by PSTs (e.g., affect of students, students thinking processes, etc.) as well as changes specific to the three components of professional noticing of children's thinking as defined by Jacobs et al. (2010). Thus, we drew upon prior work of similar noticing interventions. Jacobs et al. (2010) administered two assessments based upon classroom artifacts to determine if differences existed in professional noticing across groups of elementary teachers. Teachers with varying levels of teaching and professional development experience watched a 9-minute video of a lesson and received samples of student written work. Based upon the video and work samples, teachers wrote responses to prompts specific to the three interrelated skills of: attending, interpreting, and deciding how to respond. To analyze the open-ended responses, researchers used a two-point coding scale for attending (evidence or lack of evidence), and a three-point scale for interpreting and deciding how to respond (robust, limited, or lack of evidence).

In addition, pre- and post-assessments (Goldsmith & Seago, 2011; Schack et al., 2013; Star, Lynch, & Perova, 2011; Star & Strickland, 2008) and before- and after-writing assignments (Sherin & van Es, 2005) have been used to measure changes in individual PST's noticing. In a secondary methods course, Star and colleagues administered pre- and post-assessments in which PSTs responded to numerous questions for each of five observational categories: classroom environment, classroom management, tasks, mathematical content, and communication after viewing an approximately 50-minute video of classroom instruction. It is important to note that the main focus was on noticing of instruction and not on student thinking. The research team designed the questions on the assessments to have correct answers, and the PSTs achieved statistically significant gains in their ability to notice by the completion of the class. Conversely, Sherin and van Es (2005) collected and analyzed qualitative data to report changes in what PSTs noticed in student thinking as a result of video club sessions. Changes in noticing were evident in PSTs' narrative essays that were written after analyzing video-recorded lessons of their own instruction.

Goldsmith and Seago (2011) administered a pre-/post-artifact analysis to document shifts in secondary teachers' noticing. The teachers participated in one of two professional development programs that made deliberate use of practice-based artifacts to support teachers' use of student evidence in instructional decision-making. The assessments included teacher responses to a 5-minute video clip of 6th grade students presenting solutions to a linear function task and three associated samples of written student work. The scoring rubric attended to the tone and topic of the responses to assess the extent to which responses emphasized student understanding and potential versus student deficits, and incorporated evidence to support claims. In addition to the pre-/post-artifact analysis, the researchers accessed recordings of the professional development discussions to document qualitative shifts in teachers' ability to use evidence to support their interpretations, notice the potential in student thinking, and attend to the mathematics in more detail.

Embedding the three prompt framework (Jacobs et al., 2010) within a video-based pre- and post-assessment model, Schack and colleagues (2013) measured elementary PSTs' ability to notice student thinking by having them watch a short video of a student interview and respond to prompts similar to those used by Jacobs et al. Their intervention was grounded in each of the three stages of the professional noticing framework in the context of learning progressions from the Stages of Early Arithmetic Learning (SEAL) and consisted of a module of in-class sessions and an out-of-class assignment that involved conducting a task-based interview. Researchers coded PSTs' responses utilizing a four-part ranking for the attending category (inaccurate, limited, salient, and elaborate) and a three-part ranking for interpreting and deciding. These methods have enabled researchers to discriminate among different levels of noticing and make general claims about the effects of a variety of interventions. However, these coding schemes are often specific to particular content areas, such as early number (Schack et al., 2013) and proportional thinking (Fernández et al., 2013).

Methodology

Drawing on Jacobs et al.'s (2010) noticing framework, we designed a study to determine whether a curricular module supported shifts in secondary preservice teachers' attention to and interpretations of student thinking. We posed the following research question: *What impact does a structured curricular module have on PSTs' noticing of student thinking?* By addressing this question, we contribute evidence towards the open issues of whether noticing is a teachable skill and how researchers can study teacher noticing specific to secondary mathematics education. An overview of the instructional aspects of the curricular module will be provided; however, for a detailed description of the interview assignment, readings, and associated class discussions, see Lesseig, Casey, Monson, Krupa, and Huey (2016).

Curricular Module

The curricular module consists of pre- and post-assessments, readings, in-class discussions, a structured interview with a secondary student, and a written reflection (Table 1).

Table 1
Interview module activities and timeline

Week number	Module activities
1	• Pre-Interview Video Assessment • Solving Equations Protocol problems and discussion • Interview Assignment introduced • Questioning article and class discussion
2	• Students complete Interview Assignment (outside of class)
3	• Interview Assignment due • Debrief assignment experiences • Post-Interview Video Assessment

PSTs first read the article, *Questioning our Patterns of Questioning* by Herbel-Eisenmann and Breyfogle (2005), to prepare for the type of question-posing needed during the interview assignment. A class discussion followed to further compare how a funneling questioning pattern, where the teacher asks a sequence of questions that guide or "funnel" the students to think as the teacher would in order to reach a desired goal, differs from a focusing questioning pattern where the teacher asks questions that elicit and build upon presented student thinking. An emphasis was placed upon the use of focusing questions during the interview assignment. The interview assignment, adapted from Huntley, Marcus, Kahan, and Miller (2007), required that each PST interview a secondary student and follow a specific protocol (Appendix A). The protocol outlines students' strategies for solving linear equations in the cases of one unique solution, an infinite number of solutions, and no solution. We chose linear equations because solving linear equations in one variable using multiple representations is a key topic in the Common Core State Standards for Mathematics (CCSSI, 2010) and is difficult for many students to learn (Kieran, 1992). We also know that students have a difficult time making connections between graphic and symbolic representations of solutions (Huntley et al., 2007; Knuth, 2000).

After conducting the interview, the PSTs rated their student's problem-solving, versatility, and adaptability levels by analyzing the student's verbal and written responses according to rubrics provided in the assignment, mirroring the analysis performed by Huntley et al. (2007). The PSTs then wrote a reflection paper to communicate the results of the student interview. In their reflections, PSTs were asked to document what they noticed about the student's thinking in relation to the three interrelated noticing skills (Jacobs et al., 2010) by summarizing what the student understood and did not understand about solving linear equations and describing what they would do next to advance the student's thinking.

To document the PSTs' noticing of student thinking before and after the instructional components, we administered pre- and post-assessments utilizing video recordings of a task-based interview with a student similar to the design of Schack et al. (2013), and PSTs responded to targeted noticing prompts after viewing purposefully selected videos of student thinking. In the approximately 5-minute videos, PSTs watched a student solve quadratic function tasks. Specifically, the tasks presented to the student were: (1) Solve for x: $x^2 - 4x + 4 = 0$, (2) Could you solve that another way? (3) Solve for x: $x^2 - 2x + 3 = 0$, and (4) When you look at the problems, what would the graph of this (interviewer points to task 1 and then 2) look like? The interviewer encouraged the student to elaborate on written work in an effort to reveal understandings and probe for connections between symbolic and graphical representations. The questions posed in the video interviews were similar in nature to the interview assignment, but the tasks were noticeably different due to the quadratic nature.

The pre- and post-assessments required the PSTs to watch the video, then answer prompts designed to elicit what they noticed. After viewing the video one time, PSTs responded to the prompt: "As you watch this video, what do you notice?" in order to gauge their noticing in general, similar to van Es and Sherin (2008), as well as their attention to the student's mathematical thinking. Then, the entire video was replayed a second time and PSTs responded to the prompts: "How would you describe what this student understands?," and "Describe some ways you might respond to this student and explain why you chose those responses," in order to assess the three nested components of mathematical noticing presented by Jacobs et al. (2010). Each question was intentionally designed; however, it is important to note that all three responses to the prompts were considered when seeking evidence specific to the components of noticing. The first prompt provided information about what PSTs noticed in general, as well as how PSTs attended to student strategies (*attending*); the second prompt assessed the extent to which PSTs were able to make sense of student understanding (*interpreting*); and the third captured PSTs' ability to respond based on this understanding (*responding*).

Participants

The participants for this study were 32 secondary mathematics PSTs, 26 undergraduates and 6 graduates, enrolled in secondary mathematics methods courses at three United States universities. At each university, students were only required to take this one mathematics methods course. Though many of the students had completed up to 60 h of practicum experience, few had analyzed students' mathematical thinking prior to the course.

Data Sources

The primary data sources utilized in this paper are PSTs' pre-post-assessment responses. Additional data included field notes from class sessions in which the module was implemented and PSTs' interview reflection papers, which complemented the pre-post data.

Analysis

Pre-post-assessment data. Two researchers independently categorized PSTs' demonstrated abilities to attend, interpret, and respond to student thinking on the pre- and post-assessments utilizing a coding scheme adapted from Jacobs et al. (2010). Each of the three noticing components, attending, interpreting, and responding, was ranked as either: no evidence (0), limited (1), and emerging ability (2). See Appendices B, C, and D for the full rubrics with illustrative examples. We chose the term "emerging" to categorize the highest level of response for this study because we felt none of the PSTs were able to provide responses to student thinking at an expert level, not surprising at this point in their professional career. Our definition of each rank was informed by our impressions after reading all of the responses coupled with characteristics of expert noticing depicted in the literature. Initially we double-coded a subset of responses to ensure rankings were applied consistently over time and across multiple researchers, and we maintained a codebook with clear operational definitions and examples (Miles & Huberman, 1994). Once we reached agreement on the rankings for the subset, we continued with an iterative process of independently double-coding subsets of responses and meeting frequently to compare rankings. In order to reach consensus on rankings, we referred to the codebook and discussed on any discrepancies until agreement was reached.

We determined the *attending* rank from the PSTs' description of the methods or procedures the interviewee demonstrated. Sample responses for each level of *attending* are provided in Appendix B. Similarly, to determine a score for *interpreting* (Appendix C), we looked qualitatively at the extent to which PSTs detailed the mathematics that the student did or did not understand. Indicators included whether the PST named a specific math idea or relationship, mentioned both strengths and weaknesses, differentiated between procedural and conceptual understanding, or noted specific limitations in students' thinking. Finally, rankings for ability to *respond* (Appendix D) to student thinking were based on the extent to which PSTs' responses connected student understandings to subsequent steps.

To further characterize what PSTs noticed, segments of written responses on the pre- and post-assessments were color-coded and annotated to denote references to: (1) mathematical concepts, (2) interviewer questions, (3) the disposition of the interviewed student, and (4) specific connections to graphing or checking solutions. The open-ended nature of the prompts allowed us to assess what PSTs are naturally attuned to when they observe students working on mathematics. Thus, some PSTs noticed student dispositions, including the comfort level, confidence, or nervousness of the student. We chose the categories initially based on our research questions and found later that they captured the main themes evident in the responses. In other words, the coding was both inductive and a priori. Analysis within and across color-coded text provided frequency counts and revealed similarities and differences in PSTs' noticing. Together, these analyses provided both quantitative and qualitative means of describing changes in PSTs' noticing.

Findings

Impact of Assignment on PSTs' Noticing

The PSTs provided evidence of increased *attending* to and *interpreting* students' mathematical thinking on the post-assessment, but no change in *responding*. From the pre- to post-assessment, 12 (38%) PSTs exhibited positive gains in *attending* to students' mathematical ideas with slightly fewer PSTs, 8 (25%), demonstrating gains associated with *interpreting*, and no notable change in PSTs' *responding* to students. For *attending* and *interpreting*, the dominant trends in the pre- and post-assessments responses are presented coupled with a discussion of how responses qualitatively changed. This is followed by an integrated view of changes across *attending* and *interpreting*. Responding appropriately was not a focus of the curricular module and no changes were observed from pre- to post-assessment; however, we present the results as a potential baseline indicator for future efforts. Finally, we present results of the categories of responses evident in students' written work from the pre- to post-assessments.

Dominant Trends in Attending

Of the three elements of noticing, *attending* improved the most. Table 2 reports the individual (n = 32) pre- and post-assessment changes for *attending* and shows that 38% (n = 12) of PSTs demonstrated improved responses. Fifty-three percent (n = 17) remained unchanged from pre- to post-assessment, and 9% (n = 3) had lower scores on the post-assessment.

Table 2
Attending responses on the pre- and post-assessments

Assessment score	Attending		
Post-	Pre-		
	No evidence	Limited	Emerging ability
No evidence	9 (28%)	0 (0%)	0 (0%)
Limited	6 (19%)	5 (16%)	3 (9%)
Emerging	3 (9%)	3 (9%)	3 (9%)

Half of the 18 PSTs demonstrating *no evidence* of *attending* to students' mathematical ideas on the pre-assessment provided responses at a higher level on the post-assessment. Such an improvement is illustrated in Nina's responses below. Note how on the post-assessment the focus shifted away from the student's disposition toward their mathematical work.

> **Pre**: The student is at first unsure of their problem. Then they start into it. She seems nervous and does require a couple prompts. The interviewer actually prompts her on her answers and the student corrects herself. She has learned the basics of factoring but is not confident. She did not learn enough to understand negative square roots. This is the grade math. She is much more advanced then the students I encounter who took/came out of the everyday math program. (No evidence)

> **Post**: Student factored and checked her factoring on first problem. Second problem couldn't factor, she checked using foil. So she used quadratic equation. She made one small error but fixed it. Graphing: She saw a way to generalize about graphing [the] first problem. Error in quadratic equation. She did not know how to handle graphing complex numbers. I guess it would be the plotting of the intercept. (Limited)

Typical of other *no evidence* responses, Nina does not attend to mathematical details in the pre-assessment but instead comments on the student's confidence and the interaction with the interviewer. However, in her post-response, Nina provides details of the student's method and notes small errors the student made while solving.

Of those PSTs demonstrating *limited* evidence on the pre-assessment, 38% advanced to *emerging ability* on the post-assessment. We believe this is good for PSTs in their methods course and shows growth following the intervention. Curtis' pre-post-responses serve as a second example of ways PSTs advanced in *attending* specific to gathering evidence before drawing conclusions or interpretations of what students understand and know.

> **Pre**: (1) She is not sure of the zero property. (2) Doesn't have an understanding of what the graphs look like (ex.) shape and how the solution set would affects it. (3) Not sure about how imaginary numbers work with square roots. (4) Doesn't understand when you can cancel out terms. (Limited)

> **Post**: (1) The first thing she does with each one is trying to factor using (x) (x). (2) After factoring she checks her answer to see if it is correct. If it doesn't work then she uses the quadratic formula. (3) She didn't divide the whole numerator by 2a. (4) Graphing was ok, she had some idea. (Emerging ability)

In his pre-assessment response, Curtis jumped to interpretation, commenting on what the student may or may not understand. While this response is focused on mathematics, rather than affect, the statements are broad generalizations not necessarily grounded in details of the student's mathematical thinking. In contrast, Curtis' post-assessment response includes a step-by-step description of the student's strategy.

Qualitative changes in attending. In addition to PSTs providing more evidence of attending to students' mathematical thinking, the type of attending qualitatively shifted. During the pre-assessment, most (88%, $n = 28$) of the PSTs attended to some mathematical aspect of the interviewee's thinking. On the post-assessment however, PSTs went beyond simply noting mathematical concepts and procedures to include more references to sense-making activities, specifically graphical interpretations and checking work, which were emphasized in the interview assignment. The number of PSTs attending to at least one of these sense-making aspects increased from 28% ($n = 9$) on pre-assessment responses to 72% ($n = 23$) on the post-assessment. Below are Fran's pre- and post-assessment responses to the first prompt. Instances of *attending* to graphical representations and checking work are underlined.

> **Pre**: I notice that the student used different methods to come to a solution when working on these tasks. She started task one by organizing her thinking about the problem into a table with the factors for "c" and their sum. She quickly realized that she could factor this problem and seemed to easily come to the correct answer. She was also aware of how to use the quadratic formula on the third task, but it took the interviewer prompting her through questions to realize the problem had no real solutions. She also seemed to be unsure about how to draw a graph of task one. (Emerging ability)

> **Post**: I notice the student used the FOIL method to check her work, and by doing this the second time, she realized she could not factor and then turned to using the quadratic formula (another method of symbolic manipulations). When prompted to solve the problems another way, she was able to talk about the concept of graphing in terms of "finding and graphing the zeroes" for the first equation, and she was able to somewhat sketch a graph as a result. But for the second equation, she seemed stuck and unable to visualize what the graph would look like. She seemed to be unable to recognize that the two solutions she had obtained symbolically were not real solutions. (Emerging ability)

Both responses contained significant mathematical details and were scored as *emerging ability*. However, in her post-response, Fran not only noted the result of the student checking her work, but elaborated more fully on the student's attempts to make sense of the problem graphically.

Dominant Trends in Interpreting

For the second component of teacher noticing, *interpreting*, most PSTs exhibited a *limited or emerging ability* both before and after the assignment as shown in Table 3. A slight trend in improvement is present with 25% (n = 8) participating teachers providing a more advanced response on the post-assessment. Predominantly, PSTs remained unchanged in their level of response from pre- to post-assessment (63%, n = 20). All but 9% (n = 3), demonstrated some ability to *interpret* student thinking on the post-assessment.

Table 3
Interpreting responses on the pre- and post-assessments

Assessment score	Interpreting		
Post-	Pre-		
	No evidence	Limited	Emerging ability
No evidence	1 (3%)	1 (3%)	1 (3%)
Limited	3 (9%)	13 (41%)	2 (6%)
Emerging	1 (3%)	4 (12%)	6 (19%)

Renee's responses below illustrate a typical improvement from *limited* to *emerging ability* with increased specificity of observations and connections to evidence accompanying interpretations of the student's understandings.

> **Pre**: She understands the process of factoring and the quadratic formula. She may not understand that zero is not the same answer as no solution. (Limited)
>
> **Post**: She understood that if you cannot factor, you can go to the quadratic formula. She understands some graphical representations of figures and how those come about. She understands how to use the quadratic formula, but not that a negative under the square root yields the answer of no solution, because it involves imaginary numbers. (Emerging ability)

While Renee lists mathematical ideas the student may understand, or still needs to understand in both responses, her post-response is more specific. Instead of merely naming the quadratic formula, she notes that the student understands how to use the formula, but may not completely understand what the results indicate. Renee also made direct reference to graphing, although her interpretation of exactly what the student understands about graphical representations is unclear.

Qualitative changes in interpreting. Similar to *attending* to students' thinking, qualitative changes in the *interpreting* responses were evident from the pre- to post-assessments, but to a lesser degree. The number of PSTs' responses that included connections to graphical representations or checking work increased from 28% (n = 9) on the pre-assessment to 44% (n = 14) on the post-assessment. Again, these aspects were emphasized in the module, building upon the Huntley et al. (2007) article, as important elements in gauging algebraic proficiency.

Changes Across Attending and Interpreting

In contrast to Jacobs et al. (2010) and Schack et al. (2013), we ranked PSTs' responses for *attending* and *interpreting* independently and did not discriminate between interpretations based on explicit, detailed accounting of student work versus those perhaps based on more limited or instinctual noticing of the student strategies. PSTs were able to demonstrate interpreting even if they had not provided specific details for attending in their response to the first prompt, as long as the interpretation was consistent with what happened in the video. This allowed us to investigate the PSTs' ability to integrate the component parts of noticing. On the pre-assessment, 91% (n = 29) of the PSTs provided equivalent or higher ranked responses for *interpreting* in comparison to *attending*. This trend continued into the post-assessment with 94% (n = 30) of the PSTs providing equivalent or higher ranked responses for *interpreting* in comparison to *attending* (Table 4). Rarely on the pre- or post-assessments did PSTs provide more robust responses for *attending* than *interpreting*.

Table 4
Comparison of attending and interpreting responses on the pre- and post-assessment

	Pre-assessment score attending					Post-assessment score attending		
Pre-assessment score interpreting		No evidence	Limited	Emerging ability	Post-assessment score interpreting	No evidence	Limited	Emerging ability
	No evidence	4 (12%)	1 (3%)	0 (0%)		2 (6%)	1 (3%)	0 (0%)
	Limited	13 (41%)	3 (9%)	2 (6%)		7 (22%)	10 (31%)	1 (3%)
	Emerging ability	1 (3%)	4 (12%)	4 (12%)		0 (0%)	3 (9%)	8 (25%)

Our data indicates that PSTs scored higher on the component skill of *interpreting* than *attending*; perhaps indicating it was more natural for PSTs to begin to consider implications for what students understood, as they believe this is what is expected of teachers. The interview assignment created an awareness of the need to base interpretations on evidence of student's mathematical thinking. On the post-assessment results, the *interpreting* and *attending* rankings were more closely aligned, indicating that PSTs were basing their interpretations on the evidence at hand in a deliberate manner. However, 31% (n = 10) of PSTs still scored higher on *interpreting* than *attending* on the post-assessment. This result is observed in the qualitative analysis of the PSTs' responses to question one, "what do you notice," which often contained statements prefaced with "the student knew…" or "the student understood…." Granted by limiting our analysis to written evidence, we cannot know with certainty what PSTs noticed as a basis for *interpreting* student thinking, we only know what PSTs documented about their noticing. However, jumping to interpretation of student understanding before fully unpacking the mathematics in a response can be problematic and is something worth further investigation. As noted by Jacobs et al. (2011), "strategy details provide a window into a child's understandings and should form the basis for teachers' decisions about how to respond" (p. 109).

Dominant Trends in Responding

Of the three elements of noticing, *responding* results largely remained the same with 59% (n = 19) of PSTs remaining at initial levels and predominantly at the *limited* rank. Table 5 shows that 22% (n = 7) of PSTs provided less sophisticated responses on the post-assessment, and 19% (n = 6) provided more advanced responses.

Table 5
Responding responses on the pre- and post-assessments

	Responding		
Post-	Pre-		
	No evidence	Limited	Emerging ability
No evidence	3 (9%)	5 (16%)	0 (0%)
Limited	4 (13%)	15 (47%)	2 (6%)
Emerging	0 (0%)	2 (6%)	1 (3%)

The curricular module employed as our intervention did little to equip PSTs with ways of responding to student thinking, so this result is not surprising. We share these results as a potential baseline for future research endeavors. In that vein, we provide Curtis' pre- and post-responses to the third prompt, which demonstrate an improvement in *responding*.

> **Pre**: I would go over graphing equations so she knows how they would look and what critical points are. Also go over what happens when there is a negative square roots. Improving on those two things will help out a lot. (Limited)

> **Post**: (1) I would ask her if the—b term in the quadratic formula is divided by 2a. It would get her to think about if she remembers the formula correctly. (2) I would ask her what it means to take the square root of a negative number. See if she can recall imaginary numbers. (Emerging ability)

In the post-assessment response, Curtis connects his next instructional moves to evidence provided by the student and expresses a desire to connect to her thinking and understandings, a change from the generality of his pre-assessment response. Below is a pair of responses from Willie, which remain at the "limited" ranking.

> **Pre**: I would explain what a parabola was and what the factors that we get mean. Also talk about proper canceling of terms. (Limited)

> **Post**: I would point out why negative roots don't work and help her understand when there is no answer for what x equals. (Limited)

Willie's responses show perhaps a slight shift toward what the student was thinking, but not in a way that values the students' thinking. His response highlights errors made by the student versus building from her understanding toward a productive approach.

Themes Evident in PSTs' Prompt Responses

Recall, we also coded the pre- and post-assessments based on the following four themes: mathematical concepts, interviewer questions, disposition of the interviewed student, and specific connections to graphing or checking solutions (items highlighted throughout the assignment). Table 6 shows the percent change of instances, a PST could have zero, one, or multiple instances within a response to a prompt on either the pre- or post-assessment of each theme.

Table 6
Percent change of instances of themes from pre- to post-assessment

	Mathematical concepts	Connections to graphing or checking solutions	Interviewer questions	Disposition of the interviewed student
Percent change attending	11%	156%	13%	−82%
Percent change interpreting	7%	56%	0%	−100%
Percent change responding	50%	25%	−50%	NA[a]

[a]*Note* No PST made remarks about student dispositions when answering responding prompt

For the *attending* and *interpreting* prompts, remarks about the student's disposition decreased in PSTs' responses from the pre- or post-assessment and were not evidenced at all on the *responding* prompt. While a reduction in responses focused on student disposition that occurred from pre- to post-assessment, instances of connections to key facets of algebraic proficiency, as described by Huntley et al. (2007) and elaborated upon via readings and discussions in the curricular module, increased. Further, the number of times each PST mentioned checking work or graphical representations increased in frequency.

Though there were no noticeable changes in the quality of results for *responding*, the global trends discussed above carried through to this category as well with a 50% increase in instances of discussing mathematical concepts in the *responding* prompt on the post-assessment, a 25% increase in their instances of connections to key facets of algebraic proficiency coupled with a 50% decrease in responses that referred to the interviewer questions.

Discussion and Implications

Though asking PSTs to interview students is not necessarily a novel assignment, its use with secondary PSTs is not well documented and little is known about what secondary PSTs learn through such experiences. Our findings indicate that this curricular module, centered upon a student interview, advanced PSTs' noticing of student thinking. This finding is especially critical for secondary education students whose preparation largely revolves around acquisition of content knowledge. Below, we discuss similarities and differences between our results and the work conducted at the elementary level and the transferability of noticing work completed at the elementary level for secondary PSTs.

We observed gains in PSTs' ability to *attend* to and *interpret* student thinking, and no change in their demonstrated ability to *respond* to students. These findings are consistent with previous research on noticing (e.g., Jacobs et al., 2010; Schack et al., 2013). That is, while expertise in both *attending* and *interpreting* can be advanced through repeated opportunities to consider student thinking, *responding* to student thinking, the most demanding of the three components, does not necessarily increase with more experience but requires more focused coursework and professional development experiences. Though we cannot claim that the curricular module alone led to these shifts in PST noticing, this research points to the benefits of increasing PSTs' awareness of and attention to student thinking through intentionally sequenced coursework. Specifically, we designed the module activities to include viewing videos of task-based interviews, course readings, interviewing a student, and structured reflection.

Our analysis of PSTs' responses revealed several themes regarding the noticing of secondary PSTs. First, PSTs tended to focus on student errors and to describe their instructional responses in terms of "fixing" student mistakes. Second, although *attending* and *interpreting* improved, PSTs' descriptions of student strategies lacked specific mathematical language and tended to be vague, suggesting that PSTs may lack the language skills and mental structures necessary to effectively communicate what they noticed. Consistent with this result, we found a third trend of rates for *interpreting* being higher than those for *attending*, though this was more pronounced in the pre-assessment, and that PSTs made broad generalizations when describing student understandings. These findings suggest that our secondary PSTs might be prone to engage in what Mason (2002) would describe as unproductive noticing based on minimal evidence. In future efforts, we advocate the need to be more explicit with PSTs about the nested components of noticing by requiring PSTs to support interpretations with observations and evidence of student's mathematical thinking. In addition, we recommend requiring PSTs to consider and comment upon the strength of evidence utilized to generate interpretations.

These themes and results raise a number of issues that are unique to the preparation of secondary versus elementary mathematics PSTs. Secondary programs often have a strong content focus in mathematics coursework with fewer experiences tied to child development, learning, and thinking, making the need for developing PSTs' abilities to notice student thinking all the more important in methods courses. Across this work, we see a clear emphasis on the importance of grounding interpretations of student thinking in specific verbal or written evidence in relation to significant mathematical understandings. Key indicators of advanced noticing include: specificity in recalling details; supporting statements with evidence; moving beyond description or evaluation; offering alternative explanations; and moving beyond correctness or student errors when assessing understanding (Fernández et al., 2013; Goldsmith & Seago, 2011; Jacobs et al., 2010; McDuffie et al., 2013; van Es & Sherin, 2008). These are indicators that can and should be incorporated into secondary methods coursework.

The research reported in this chapter provides a useful contribution to mathematics educators by building upon research on professional noticing of student thinking conducted at the elementary level and adapting similar coding schemes and analyses to measure secondary PSTs' ability to *attend* to, *interpret*, and *respond* to student thinking. While we found the framework to be viable, a number of other factors, discussed above, are unique to secondary PSTs. This research also highlights areas of our curricular module that we can improve and exposes additional needed supports to further foster secondary PSTs' professional noticing.

In order to develop PSTs' ability to connect interpretations to evidence, secondary instructors require additional supports in the form of ecologically valid artifacts from classrooms, such as to video clips, sample student work, and other artifacts of student thinking (Ball & Cohen, 1999; Wilson & Berne, 1999). While we recognize there are some such resources available, in order to create an instructional module that supports secondary PSTs' noticing of student thinking, additional materials and resources must be created and cataloged in a way that is grounded in research. Artifacts that document secondary students' thinking should include multiple representations or avenues for solving tasks and highlight common misconceptions. Such resources are not easily accessible at the secondary level and would give teachers, including PSTs, support in developing lessons and responding to thinking on specific topics.

In addition, common language and research-based learning progressions to describe the mathematics at the secondary level is lacking, whereas at the elementary level, learning progressions, and common terminology exists. Developments such as Cognitively Guided Instruction and Stages of Early Arithmetic Learning at the elementary level provide frameworks for listening to children's mathematical thinking and ways of using that knowledge to inform the next steps in instruction. Schack et al. (2013) and Jacobs et al. (2010) utilize these trajectories in their coursework and professional development programs. Similar research is not available in the same form at the secondary level, posing additional challenges for secondary methods instructors in supporting PSTs' knowledge of student strategies and research-based instructional responses.

Further, developmental trajectories for advancing noticing abilities have been created for some content areas at the elementary level (Fernández et al., 2013), yet do not exist at the secondary level. The trajectory created by Fernández and colleagues is specific to a particular mathematical domain and is not transferable to work outside of proportional reasoning tasks. Similar to this work, our characterization of how PSTs' noticing changed is specific to the mathematical content of solving linear tasks. The student-learning research upon which the interview assignment is based describes a web of relationships and understandings that students must possess and have facility with in order to have a robust understanding of solving linear equations, rather than a sequential learning trajectory (Huntley et al., 2007). More research that describes the foundational ideas and processes in high school mathematics and the relationships between them is needed to further support PSTs' development of noticing. In addition, Huntley and colleagues provided a well-defined learning goal, common to all students. It is important that PSTs should have learning goals, which connect back to these foundational ideas.

It is clear that the module provides an initial step towards developing PSTs' ability to professionally notice student thinking. Our current work expands the module to include assignments with a targeted focus on improving PSTs' ability to respond. In addition, an increased emphasis on linking interpretations to evidence or suppressing interpretations until evidence is found is a pressing need, so that we do not inadvertently reward unproductive noticing based on minimal evidence. From our findings, it appears that secondary PSTs are predisposed to identify and judge areas of weakness in student thinking, which is possibly the cultural view of what a mathematics teacher does in the United States.

Recall, Mason (2002) stated, "every act of teaching depends on noticing" (p. 7). We are working toward developing the noticing discipline by supporting PSTs' attention to this deliberate practice and highlighting that attending to students' mathematical thinking is an inherent expectation and critical aspect of designing instruction. We have confirmed evidence from others that teacher noticing can be trainable, in this case with secondary PSTs, and have provided a glimpse into how the effectiveness of a noticing intervention might be assessed.

Appendix A: Solving Equations Interview Protocol

Interviewer: *I am going to ask you some questions about solving equations. I am very interested in how you come up with your answers, so it is important for you to tell me what you are thinking as you are working. The interview will not be graded, so you don't have to worry about wrong answers.*

Please solve each equation for x.

Give the student the following problems and allow students ample time to work.

Problem A: $2x + 3 = 5x - 9$
Problem B: $2(3x + 4) = 6x + 8$

Problem C: $2(3x + 4) = 6x - 5$

Feel free to ask students *Why did you do that?* Or *What were you thinking when you did that?*

Wait for the students to respond. Using the coding rubric, record the response and strategies used. If students used only one strategy on any problem then continue to probe 1.

- Probe 1: *Could you have solved these in another way? If so, show me how. If not, why*?

Wait for the students to respond. Record the response and the strategies used. If the students did not use graphing then continue to Probe 2.

- Probe 2: *What would be the relationship between the graphs of each side of the equation*?

You may continue to ask questions but only for clarification of students' thinking and understanding. Do not ask questions that will lead your student to the answers as you are searching for what the students know and are able to do without help.

Interviewer: *Thank you for participating in this interview.*

Appendix B: Attending Category Exemplars

Emerging ability. Written responses attend to significant mathematical ideas, describe strategies employed by the student in a coherent and comprehensive manner, and refrain from evaluative comments.

Example:

> Her first instinct is to solve algebraically and she checks her answers using the FOIL method. After she factors and checks her answer using the FOIL method, she realizes that it does not work and uses the quadratic formula. She loses her ± towards the end of her work on the second problem. Uses the term "zeroes" and explains that the first problem would have two "zeroes" meaning "it crosses the x-axis twice." Defines the term "zero" in terms of what it would look like on a graph rather than what it actually meant graphically. She states the first problem will have two zeroes but her graph touches the x-axis at one point, $x = -2$? [Lynn, post]

Scoring Rationale: While the response does not address all the mathematics present in the video, the PST is able to systematically describe steps that the student used to solve the problem. In addition, the PST recalls specific statements that the student made during this process and highlights a potential source of confusion for the student, seeking two zeroes but only locating one x-intercept.

Limited. Responses attend to elements of the student's mathematical thinking and/or solution strategy, but with less detail and structure. These responses often entail disjointed observations.

Example:

Student jumps straight to factoring but is also able to use quadratic formula. Student seems to understand that two repeated answers means the graph touches the x-axis to make a double zero. I'm surprised to see the student checking her own work. [Max, post]

Scoring Rationale: The PST notes several specific mathematical terms (i.e., factoring, quadratic formula, double zero) but provides few details of the process the student engaged into arrive at a solution. The PST offers an inferential statement about what the student understands without citing evidence from the video.

No evidence. Responses attend to non-mathematical aspects of the interview (i.e., student confidence or demeanor, interviewer questions) or make general observations with no mathematical details.

Example:

The girl seems shy and timid which is why she is quiet and doesn't speak clearly. She knows for the most part the material, however, she still gets confused on some things. Now that I think about it, I'm not sure if she's confused or trying to rush and messes up cause she's nervous in front of camera. [Nicolas, pre]

Scoring Rationale: Response focuses on student disposition and does not mention any specific mathematical ideas.

Appendix C: Interpreting Category Exemplars

Emerging abilities. *Written responses need to incorporate both strengths and weaknesses of the student's thinking and connect interpretations to specific mathematics ideas and relationships evidenced in the video.*

Example:

The student understands how to solve for *x* by factoring or using the quadratic formula, and knew how many zeros a equation could have. However, she wasn't able to graph the second problem with the negative square root. I believe she doesn't know what it means to have a "no real" solution. [Tremaine, post]

Scoring Rationale: Although not a comprehensive account of all the mathematical ideas, the response addresses both what the student understands (how to solve by factoring or quadratic formula) and may not yet understand (meaning of no real solution). Importantly, this assessment is based on a noticing that the student was unable to graph the second problem.

Limited. Responses interpret elements of the student's mathematical thinking and/or solution strategy based on the evidence provided, but are not in a connected or comprehensive manner. Responses often include over generalizations without direct connections to the data within the video-recorded interview or focus upon only one aspect.

Example:

I would say the student understands how to solve quadratics through graphing, foiling, and the quadratic formula, but doesn't have a good grasp on imaginary numbers. [Bob, post]

Scoring Rationale: Response identifies strengths and weaknesses in student understanding. However, both are limited to a list of general mathematical topics without any elaboration or supporting evidence.

No evidence. The students' thinking is interpreted in a non-mathematical manner, incorrectly, or too vaguely to be meaningful.

Example:

I think she focuses on the equation in the beginning equaling zero that she keeps thinking that the final answer is $x = 0$. She does that for both problems. I hear noise in the background that could cause her to lose focus. [Nicholas, pre]

Scoring Rationale: The PST tries to rationalize the student's response (i.e., confuses equation equaling zero with final answer or distracted by noise) rather than interpreting the mathematical thinking. No specific mathematical ideas are mentioned that the student either does or does not understand.

Appendix D: Responding Category Exemplars

Emerging abilities. Written responses need to offer questions to further probe or extend student thinking based on what the student understood. Responses could include additional tasks, new representations, questions, or instructions to confront a misconception in student understanding evidenced in the video. It is important that responses include a rationale for furthering the students' thinking and is connected to how the PST answered the previous prompt about what the student understands.

Example:

I would spend more time exploring graphs and equations. She has clearly spent time on factoring to solve quadratics but she doesn't understand what that sol[utio]n means. I would also talk about when and why we can cancel things out of fractions. Then I would talk about eq[uatio]ns with no real sol[utio]ns, what that means, and how zero is a solution. In all of these difficulties, she lacks understanding why things are done, whatever I did as her teacher, I would want to spend time explaining the reasoning for it. [Sandra, pre]

Scoring Rationale: Multiple next steps are provided that connect clearly to what the student seemed to understand and not understand. The response also stresses the importance of knowing why a procedure works instead of relying upon memorized steps for the students to take to get the correct answer.

Limited. Responses are connected to the PSTs' response given in the previous prompt regarding what the student understands and no rationale for next steps is given, or a rationale for next steps is given, but the response is not connected to what the student understands.

Example:

I would go over graphing equations so she knows how they would look and what critical points are. Also, go over what happens when there is a negative square roots. Improving on those two things will help out a lot. [Curtis, pre]

Scoring Rationale: The response makes connections between the next steps and what the student understands and areas of challenge. However, it is unclear what "going over" means and rationale for these next steps is not well documented.

No evidence. There is neither a connection to the mathematics the student understands nor a rationale for the suggested next steps. These responses are generally vague and discuss larger concepts without identifying specific next steps based on the student's thinking. Responses may also list topics the PST might cover next, but remain unclear on how topics would be addressed and why.

Example:

I'd point to decisions she made, ask her why she made them, ask what we learned in class and if there was perhaps another way to solve that. Help her remember basic rules and ask her why those are. [Nora, pre]

Scoring Rationale: The response is vague with no specific mathematics connection to the student's thinking and no meaningful rationale for next steps.

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A Case Study of Middle School Teachers' Noticing During Modeling with Mathematics Tasks

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Abstract Schoenfeld (Mathematics teacher noticing: Seeing through teachers' eyes. Routledge, New York, pp. 223–238, 2011) wondered about the transferability of teacher noticing across contexts (e.g., different grade levels and task types). This chapter focuses on middle school teachers' noticing during instruction that promoted modeling with mathematics, which is one of eight Standards for Mathematical Practice (SMPs) found in the Common Core State Standards (CCSSI in Common Core State Standards for Mathematics. Author, Washington, DC, 2010). A case study approach was used to explore middle school teachers' noticing during instruction promoting modeling with mathematics. This study focuses on two middle school teachers who enacted modeling-focused lessons. Lessons, videos, and interview data were analyzed using inductive analysis (Hatch in Doing qualitative research in education settings. State University of New York Press, Albany, NY, 2002). We drew two impressions from the data. The first was that teachers' noticing focused on fostering students' use of multiple representations. The second result was that teachers' noticing was framed in ways to assist with making sense of a modeling task or its solution. We connect these results to transferability of teaching noticing, specifically to instruction promoting modeling with mathematics.

Keywords Middle school · Problem-solving · Standards for mathematical practice · Common core state standards-Mathematics · Representations

Teachers manage a number of instructional elements everyday including mathematical tasks, mathematical discourse and interactions during those tasks, and the learning environment (National Council of Teachers of Mathematics [NCTM], 2007). They typically are making choices to attend to certain instructional moments, interpreting those moments, and deciding how to proceed. These

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formative assessments are “the process of gathering evidence about a student’s knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes” (NCTM, 1995, p. 3). Formative assessment supports learning by allowing teachers opportunities to gauge the degree to which students are meeting desired instructional goals and make adjustments (Clark, 2012; William, 2007). Evidence from research on teachers’ assessment suggests that teachers ought to pay close attention to students’ understanding, or lack thereof, so they can best attend to the students’ learning needs and support learning outcomes (Clark, 2012; William, 2007).

Mathematics teachers gather much evidence during instruction; however, the process of what teachers attend to, how they interpret that information, and what they decide to do have not been clear (Jacobs, Lamb, Philipp, 2010; Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder, 2013). This process is *teacher noticing*, or put another way “the processes through which teachers manage the ‘blooming, buzzing confusion of sensory data’ with which they are faced,” (Sherin, Jacobs, & Philipp, 2011, p. 5). For example, Jacobs and colleagues (2010) investigated teachers’ noticing of children’s mathematical thinking as an aim to explore ways in which they might respond to a child’s just-in-time thinking. A cross-sectional study of 131 prospective and practicing teachers, all who had differing amounts of teaching experience, was observed. The researchers concluded that there were various forms of teacher noticing, depending on teaching experience. Teaching experience is one of many variables that influence teachers’ noticing; others include lesson objectives and grade levels of the teacher (Schack et al., 2013; Sherin et al., 2011; Thomas, Eisenhardt, Fisher, Schack, Tassell, & Yoder, 2015). The focus of this chapter is to explore teacher noticing in a particular context and add to the emergent foundation of knowledge in this area. Ultimately, mathematics education researchers may be able to unpack instructional decisions better through such a focus (Schack et al., 2013; Sherin & Star, 2011; Thomas et al., 2015). We frame our work around mathematics instruction in the Common Core era, specifically focusing on teachers’ promotion of the Standards for Mathematical Practice (SMPs; Common Core State Standards Initiative [CCSSI, 2010]).

Related Literature

Standard for Mathematical Practice: Modeling with Mathematics

The SMPs describe a set of mathematical behaviors and habits for students to experience (Table 1).

Table 1
Standards for mathematical practice

Standard for mathematical practice #	Title
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for regularity in repeated reasoning

Teachers in states that have adopted the SMPs are expected to read and understand them, then design instruction promoting them. The SMPs and content standards serve as the expectations of what students should learn and do while engaged in K-12 classroom mathematics teaching (CCSSI, 2010). Modeling with mathematics, the fourth SMP, states

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace… They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. (CCSSI, 2010, p. 7)

Instruction promoting modeling with mathematics can "engage [problem solvers] in a process of interpreting mathematical situations" (Zawojewski, 2010, p. 238). Thus, SMP 4 is operationalized as requiring students to apply real-world knowledge, make assumptions and approximations, and continuously evaluate the reasonableness of a result (Bostic, 2015). Teachers promoting modeling with mathematics (SMP 4) are expected to use mathematical representations appropriate for a lesson, encourage students to use a variety of developmentally and mathematically appropriate models while problem-solving, and continuously remind students to revise their models (Fennell, Kobett, & Wray, 2013). Students' ability to strategically employ multiple representations (e.g., written symbols such as variables, expressions, and equations; tables; diagrams and pictures, concrete manipulatives, and verbal language) during problem-solving is linked to their problem-solving performance (Yee & Bostic, 2014). Broadly speaking, past literature has framed representations as symbolic (i.e., written symbols) and nonsymbolic (i.e., all others) (see Yee & Bostic, 2014 for a review). Translating between representations (e.g., verbal language to a variable) is embodied within this notion because effective and efficient problem-solving often requires navigating between various representations (Yee & Bostic, 2014). It is critical that teachers encourage students to develop flexibility with a variety of representations during problem-solving, which includes tasks promoting modeling with mathematics. The present research provides some insight into what teachers notice during instruction promoting modeling with mathematics and how their instructional decisions aim to benefit students' learning.

Situating the Study in Teacher Noticing

Teacher noticing is a means for teachers to engage in formative assessment practices because "teachers must recognize students' thinking…as it happens and make…instructional choices in response to what they notice" (Luna, Russ, & Colestock, 2009, p. 1). Erickson (2011) argues that teachers tend to engage in noticing as a means to make decisions to benefit students' learning and/or instruction. Teacher noticing includes two key processes: "attending to particular events…[and] making sense of events in an instructional setting" (Sherin et al., 2011, p. 5). Sense making for our study includes interpreting and deciding on a response (Sherin et al., 2011, p. 5).

We approach teacher noticing as a two-stage process: Attending encompasses the first stage then interpreting and deciding characterize the second stage. *Attending* is when the teacher gathers evidence of a student's thinking as it happens in the moment (Jacobs et al., 2010; Schack et al., 2013; Thomas et al., 2015). This may include how a child might behave or how he/she uses specific tools for an activity. Attending leads to sense making, which includes interpreting and deciding. *Interpreting* is when a teacher examines the gathered evidence from the attending phase and "coordinat[es] the observed actions with what is known about…development in a particular area" (Thomas et al., 2015, p. 296). The *deciding* phase is when a teacher collects (considers) the information/observations gathered during the earlier phases and makes an informed decision on how to act (Jacobs et al., 2010; Schack et al., 2013; Thomas et al., 2015). There are multiple decisions a teacher could make using the evidence. For instance, imagine a teacher noticing a small group of students problem-solving and expressing difficulty. The teacher's noticing may lead him/her to re-teach the material in a different manner. Or, the teacher may dismiss the students' difficulties and move on with the lesson. An observer of this situation may wonder what the teacher attended to and how they interpreted the data to make a decision on how to proceed. Moreover, it is uncertain whether teacher noticing might be similar or different across various contexts in the Common Core era. We drew upon this uncertainty as a way to build the foundation of teacher noticing literature.

Schoenfeld (2011) summarizes and pushes the field of teacher noticing forward with a couple thoughts and wonderings. First, teachers need robust pedagogical, mathematical, and mathematics pedagogy knowledge to teach students in the Common Core era because the content and practice standards are not necessarily easy to discern upon inspection (Bostic & Matney, 2014). Second, and germane to this chapter, what does teachers' noticing look like in various contexts? The field of teacher noticing must begin to investigate the transferability of teacher noticing across contexts (e.g., different grade levels, task types, instructional foci; Schoenfeld, 2011). To date, no published study has explored teacher noticing through the lens of instruction promoting the SMPs, much less one or more SMPs. Research on instruction promoting modeling suggests that it is unique from non-modeling instruction (Lesh & Zawojewski, 2007). Moreover, instruction promoting the fourth SMP (modeling with mathematics) tends to appear different from instruction highlighting other SMPs (Bostic, 2015). The present study explores middle school teachers' instruction to better understand teachers' noticing within this context, and respond to Schoenfeld's wondering.

Synthesis

Drawing on a teacher noticing framework, this study takes up Schoenfeld's (2011) wondering about teacher noticing within various contexts (e.g., types of tasks and grade levels). The purpose of this study was to examine how middle school mathematics teachers engage in teacher noticing during instruction that supports modeling with mathematics. Our research question is: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics?

Method

Methodology

The methodology used for this research is a case study, which "investigates a temporary phenomenon in its real-world context" (Yin, 2014, p. 237). It involves analyzing data from one or more cases to confirm a specific phenomenon happening within those cases (Yin, 2014). This chosen method of research is appropriate as case studies allow researchers the ability to explore a novel phenomenon.

Participants

One male and one female teacher are the focus of the present study. They are identified by pseudonyms, Mr. Brown and Mrs. Zelda. Mr. Brown and Mrs. Zelda were purposefully selected from the larger sample of 38 teachers because they successfully enacted tasks promoting SMP 4 (i.e., modeling with mathematics). Initially, we considered data for this study from 38 middle school teachers (i.e., grades six–eight) who volunteered to take part in yearlong professional development (PD) in Ohio. The aim of the PD was to foster sense making of the Common Core State Standards for Mathematics, particularly the Standards for Mathematical Practice (SMPs).

Mr. Brown was a seventh-grade teacher with 10 years teaching experience. He held a Masters in Education and self-identified himself as Caucasian. Mr. Brown's district is suburban with a low student poverty rate (Ohio Department of Education, 2015). Mrs. Zelda was a seventh-grade teacher with 19 years teaching experience. She earned her Masters in Education and also self-identified herself as Caucasian. Mrs. Zelda's district is a small town with a high student poverty rate (Ohio Department of Education, 2015).

Data Collection

A goal of this study was to closely examine the noticing of middle school teachers who promoted modeling with mathematics during their instruction. To meet that goal, there were two parts to data collection: video and lessons followed by participant interviews.

Videos and lessons. Teachers submitted lesson plans and videos of instruction after experiencing the PD. A team of mathematics education researchers reviewed the lessons for intended foci and later coded videos using a SMP look-for protocol (Fennell et al., 2013). The team examined the lessons and sorted them based on teachers' stated goals and the SMPs they intended to address during instruction. The SMP look-for protocol suggests observable behaviors that a teacher might enact during instruction. There are three statements specific to teachers' promotion of SMP 4: Modeling with mathematics: (1) Use mathematical models appropriate for the focus of the lesson; (2) Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids); (3) Remind students that a mathematical model used to represent a problem's solution is a work in progress, and may be revised as needed (Fennell et al., 2013, p. 12). A randomly selected sample of 20% of the data collected from the PD was coded independently by members of the research team to determine interrater agreement. They agreed 96% of the time, which exceeds the minimum threshold (90%; Ary, Cheser-Jacobs, Sorenson, & Razavieh, 2009).

It was minimally sufficient to say teachers' instruction promoted modeling with mathematics if there was evidence for one of the three statements. Because the nature of this study is geared toward teacher noticing practices within modeling with mathematics instruction, we examined only those who displayed more than minimally sufficient evidence. That is, this purposeful sample is composed of teachers who had two or more indicators for SMP 4 from the SMPs look-for protocol. Mr. Brown and Mrs. Zelda's lessons and videos demonstrated (a) an intentional focus on modeling with mathematics and (b) evidence for at least two of the three statements for modeling with mathematics. These two teachers were then sent requests for follow-up interviews regarding their noticing.

Interviews. The goal of the interview, much like in past teacher noticing research (e.g., Sherin, Russ, & Colestock, 2011) is to make sense of teachers noticing moments through their reflection on unique instructional moments. Each interview lasted approximately 60 minutes and was conducted at the participant's school. The teacher watched his/her instruction on a laptop computer while being filmed by a video recorder. A video camera was placed so that it captured teachers' verbal statements and nonverbal cues during the interview. The interviewer paused the video of the teacher on the laptop for one of two occasions. The first occasion was when a teacher asked the interviewer to stop the video to discuss a specific teacher noticing moment. The second occasion was when the interviewer observed the teacher attending to a unique instructional event during the video of the lesson. For example, the video was paused often when the teacher stepped toward a student who shared a misconception. After a pause for either occasion, the interviewer posed a series of questions with regards to the teacher noticing process of a specific event. The first question was: What made you attend to this specific event? The second prompt was: Describe your thought process during the student–teacher interaction. The third question was: What did you interpret from this event? The fourth and final question was: What did you decide to do after you interpreted what was going on with the situation? Participants were encouraged to share any remaining thoughts at the end of the interview.

Data Analysis

The focus of the analysis was on data collected during the interview. Mr. Brown and Mrs. Zelda's interviews were analyzed using thematic analysis (Hatch, 2002). The goal of thematic analysis is to generate plausible themes based on a plethora of evidence and paucity of counter examples (Hatch, 2002). An analytical approach such as thematic analysis is appropriate in case study work because it allows the researcher to describe and explore a phenomenon of interest (Yin, 2014).

The data analysis was performed in seven steps. First, each interview was viewed in its entirety. During the second step, memos were made when teachers asked to pause the video to discuss their noticing during specific situations. Third, notes were made of specific statements spoken by the teacher during the teaching to find general impressions across the participants. Fourth, general impressions that were common between Mr. Brown and Mrs. Zelda were collapsed into initial, tentative themes. Fifth, evidence supporting (or not supporting) was sought within the interviews, videos of teachers' instruction, and the lessons. Sixth, the interviews were watched a second time to explore the degree to which impressions matched the data. Finally, impressions were synthesized to become themes describing middle grades mathematics teachers' noticing during tasks that promote modeling with mathematics.

Results

We present two themes arising from the data to answer the question: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics? The first theme was that Mr. Brown and Mrs. Zelda noticed students' struggles with structure found within the tasks. The second theme was that teachers in this case study noticed students' engagement (and struggles with) in translating between representations while problem-solving.

Theme 1: Structure in Mathematics

Each teacher who conducted a modeling with mathematics task shared how students struggled with the inherent mathematical structure within the assigned problem. The Standard of Mathematical Practice (SMP) #7 (CCSSI, 2010) suggests that students should look for patterns and specific structure within a problem. The SMP look-for protocol states three aspects indicative of teachers fostering this SMP.

> (a) Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains. (b) Recognize that the quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson. (c) Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways (e.g. $76 = (7 \times 10) + 6$); discussing types of quadrilaterals, etc. (Fennell et al., 2013, p. 13)

Mr. Brown and Mrs. Zelda engaged students in exploring mathematics to deepen their understanding and promoted modeling with mathematics.

Mr. Brown implemented a modeling with mathematics task focusing on using the slope-intercept equation to understand payment for various jobs. The context of his mathematical task was that an individual made $10 per hour for their job and received a signing bonus of $20 (Figure 1). Students determined how much money the individual earned given a number of hours (i.e., represented as x in the equation). The goal was to create a suitable mathematical model for any number of hours and perhaps, be able to transfer this model to situations with different hourly rates and signing bonuses. Students selected and used input/output tables to record data. Most plotted their data from the table onto a coordinate plane. Students sought to analyze the relationship between hours worked and money earned through coordinated efforts with multiple representations.

Description: You are getting your first job delivering papers. The job requires you to deliver papers in town according to the addresses on the list. For performing this job, you will be receiving $10 per hour. Because you have chosen to accept this job, you are receiving a one-time, $20 signing bonus. Your job is to figure out how much money you would make when working "x" amount of hours. Fill in the table below to display your findings. After you fill out the table, graph your findings accordingly.

Input (X)	**Output (Y)**	What is the equation for this task? How much money do you have when you start?

Figure 1. A portion of the task shared during Mr. Brown's seventh-grade instruction.

The first situation that arose within the lesson where Mr. Brown attended to a student's thinking was approximately 14 minutes into the lesson. One student, instead of leaving the equation in the form y = 10x + 20, added 10x + 20 to make 30x. The misconception that the student had was to collect unlike terms, therefore misunderstanding a key mathematical property: collecting like terms to simplify expressions. Mr. Brown asked to pause the video and talk about attending to students' work on the task. Just a moment before he asked to pause the video, Mr. Brown approached a student and asked him about manipulating an algebraic expression. Mr. Brown said that, "…what this student was doing was he was misunderstanding the equation. What I noticed he was doing was adding 10x + 20 to get 30x thinking he was going to get the same answer as 10x +20." He interpreted that this student and others seemed to misunderstand how to collect terms within an expression. To that end, he decided to manage this misconception by intervening.

> I [Mr. Brown] had to intervene and point out to the student that you can't add unlike terms. So I had to tell him you have to keep it in 10x +20 form. I showed him the difference if I were to keep his answer of 30x versus keeping it as 10x +20. We plugged in values for x and got different results, thus coming to the conclusion that when you add 10x + 20 to equal 30x, it changes the whole problem.

This instance of Mr. Brown's noticing during the task was consistent throughout the interview. He felt that students ought to show flexibility with the mathematical structure found within mathematical models. Mrs. Zelda's lesson coincides with students not understanding the structure within problems shared during a lesson.

Mrs. Zelda conducted a lesson promoting modeling with mathematics that involved students creating equations for a bike tour that includes profit, revenue, and expenses. Bike tours are a common business for a nearby vacation spot where many students' families work and visit. Figure 2 shows the table and description students were given.

Description: Rider Inc. is a business that rents out bikes and camp space. They have some data that shows them the dollar value of a single customer, all the way to 3 customers. The manager has asked you to construct an equation that works for "x" amount of customers. Please finish the table for up to 6 customers, then construct an individual equation for each: profit, revenue, and total expenses that would work for all amounts of customers.

Customers	Revenue	Bike Rental	Food and Camp Costs	Total Expenses	Profit
1	$350	$30	$125		
2	$700	$60	$250		
3	$1050	$90	$375		

Figure 2. The table and description used for Mrs. Zelda's task in her lesson.

Mrs. Zelda pointed out a particular situation within her lesson while watching the video of her lesson. About halfway through the lesson she recalled students started to ask more questions when asked to construct a general equation for profit, total expenses, and revenue. She shared that year after year, students tended to struggle with this because they had a hard time thinking abstractly and generally about equations, as opposed to concrete thinking. Mrs. Zelda paused the video to discuss when she saw students struggling to construct equations from the table. During that moment, she chose to guide students toward constructing an equation, that is, connecting representations (i.e., tables and equations) using mathematical operators. She claimed the students knew how to calculate the total expenses and profit but they had difficulty constructing an equation for the situation. She clarified this further during the interview through a role-play. She role-played the teacher and students' mathematical actions and statements.

Mrs. Zelda: A lot of times especially with linear equations, when they are making equations or putting a situation into an abstract equation, there sometimes is a disconnect. They know what to do, but they can't put letters and numbers and operations together. So I always pull back. … *[Begins role-play]*

Mrs. Zelda: [Teacher]: So if there is one customer and they are going to an amusement park, how much are they going to pay? $40 bucks. What if they bring their boyfriend or girlfriend?'

Mrs. Zelda: [Student] $80 bucks.

Teacher: Ok what if they bring, boyfriend and two other friends? Well then they figure it out. What did you just do?

Student: Well I multiplied.

Teacher: What did you multiply?

Student: I multiplied 40 times however many people were going.

Teacher: Ok, so how can I write that so if I want 120 people going with me?

Student: Well I multiply 40 times 120.

Teacher: What number stayed the same? What number changed? The number that changed, give me a letter that works with that. Whether it's friends, customers, or jellybeans, or whatever. *[Ends role-play]*

Mrs. Zelda: Then they [students] are like "oh." So then what does that tell you when I take 40 times the number of people? The cost of the admission. So sometimes you have to break it down and then go for simple, and then you can get to where they are getting now.

This role-play is evidence that Mrs. Zelda noticed an issue and decided to guide students in their thinking on how to construct an equation. Equations have a unique yet specific mathematical structure that her students seemed to misunderstand. She employed a simple, relatable experience when talking with students about the price to go to an amusement park. Then she built the problem into more complex situations when she saw more students expressing understanding, eventually constructing an equation with a coefficient and variable. She noticed that students were able to carry out procedures to solve the problem but had difficulty expressing the problem situation as an equation using variables. That is, students struggled to move from the situation embedded within the task to creating a table and ultimately to an equation that characterized the structure inherent within the problem.

In conclusion, Mrs. Zelda attended to students struggling with abstracting an equation for the whole problem and constructing the formula using variables. She interpreted that students knew how to find the different values for the table, but could not provide the general equation for profit, revenue, or total expenses. Mrs. Zelda decided to provide her students with a simpler example of going to an amusement park and bringing friends and/or family. This decision guided students to conceiving how they might construct a formula for the particular problem within the lesson. In Mr. Brown's and Mrs. Zelda's cases, it is clear from these teachers' voices that looking for and making use of structure within tasks addressing SMP 4 is important.

Theme 2: Translating Between Representations

A second theme drawn from the interviews was that teachers attended to students' facility translating between representations. As shared in the previous theme, Mr. Brown and Mrs. Zelda attended to situations when students demonstrated difficulty connecting the meaning of situations to terms within equations. They used multiple representations and aimed to assist students to problem-solving using one representation (e.g., graph or table) then translate to another representation (i.e., expression or equation). Concomitantly, teachers felt the need to foster students' sense making of mathematical structure through various ways. Mr. Brown explicitly told and showed students the correct way to manipulate one mathematical structure while Mrs. Zelda scaffolded students' thinking through different examples that might foster greater connections and correct the misconception. Thus, there are natural connections between mathematical structure and representation usage.

Mathematical language (i.e., verbal representations) was a key part of students' struggles with translating between representations and is explored further.

In Mr. Brown's and Mrs. Zelda's lessons, students misunderstood language within the problem or they did not know what certain words meant. For example, Mrs. Zelda shared several thoughts related to the importance of mathematical language during tasks addressing modeling with mathematics. A focus of Mrs. Zelda's lesson was to determine values for variables in an equation using a table. She noticed through some of her questioning that students expressed confused facial gestures when talking about words such as revenue, expenses, and profit. She shared the following during her interview while viewing her lesson and discussing one student–teacher interaction.

> I think they didn't understand the definition of revenue. Therefore with taking the information they had, I guided them through that, and showing them 'this [points to the problem then moves her finger to a specific term] is revenue'. I probably should have said 'What is revenue?' so the definition and vocabulary was there. So I think that's what it was. They didn't get what revenue was. They were looking at profit. They don't know what profit is. They've heard about it, but they don't know what it is, along with expenses and revenue. So it was just a quick lesson to show linear relationships but then they [textbook?] were throwing all this other jargon and vocabulary in. So it sounded like they [students] just didn't understand the vocabulary.

Mrs. Zelda attended to students who seemed confused about the problem and were not progressing in their problem solving. She shared that her interpretation was that they consistently did not understand the problem's language, thus they were uncertain of the problem's goal. This led to her decision to guide students to better understand the problem's language so they might translate verbal representations into symbolic forms (e.g., variables and equations). This happened several times during the lesson and Mrs. Zelda commented on it frequently during the interview. Mr. Brown shared a similar sentiment during his interview when watching his interactions with students.

Mr. Brown expressed that he also tended to focus on mathematical language as a noticing during tasks promoting modeling with mathematics.

> I [Mr. Brown] tend to notice that students have a hard time understanding the variables and constants [in equations]. Like when I use this equation in a story problem context, I find students have a hard time understanding 'mx' means slope times a number. Then they sometimes forget to add the constant at the end, which is what happened in this example. So really the students have trouble understanding how to read the problem.

To summarize, he attended to a student who had a question about completing the problem because of mathematical language written in symbolic terms. He noticed that this student was not adding the constant to each input. Next, he interpreted that this student was confused about what a constant is and what to do with it. He also saw that other students were making the same mistake. After interpreting that students did not know what to do with the constant, or know what it meant within the context of the equation, Mr. Brown decided to address the whole class and remind students to add the constant to each input. Students needed assistance making sense of the mathematical language, as written in abstract terms (i.e., symbolic representation as a variable) rather than in Mrs. Zelda's case that was represented as words (i.e., nonsymbolic representation as a word). This is a struggle for the teacher and students related to representational translations.

Further evidence of Mr. Brown's noticing students' language (verbal representation) came from his response when asked what he finds himself attending to most often when enacting tasks aimed at fostering modeling with mathematics.

> I [Mr. Brown] often find myself having to go over [the whole problem] with the whole class and explain what the problem is asking …. Students tend to forget how to *read* [emphasis added] the problem, so if I find that multiple students aren't getting it, chances are a majority aren't getting it, which means I should address the class as a whole [about the words in problem].

Mr. Brown shared he had attended to enough instructional situations to make sense of the feeling when students may not understand the words within a task associated with SMP 4. Taking instructional time to assist students with making sense of words (i.e., verbal representations) assuredly helped students' understanding of the situational context in the problem, which allowed them to create more appropriate mathematical models and ultimately, generate viable mathematical models (solutions). A related situation arose during Mrs. Zelda's interview.

Mrs. Zelda attended to a frustrated student during her instruction. The student was perplexed by the task and interjected his question while Mrs. Zelda was speaking. Mrs. Zelda shared that her experience was that if this particular student was confused then it was typical for others to feel similarly. Mrs. Zelda's response during the interview highlights her pathway through the noticing framework:

> …he's [the student] forcing me to re-direct and back up and say 'let's look at the revenue, and the expense, and the profit to find a pattern. What's going on?' Then he [the student] took it to "well now we have to look at the pattern." [Mrs. Zelda asks] 'How did you get that pattern? What do you do? Ok do it.' So he, in his mind, was kind of thinking through the process out loud. Which sometimes they have to do because sometimes when going around I see the blank look of "I don't get it." [Mrs. Zelda] 'Ok what don't you get? You have to look at it. What do you know? How do you get the numbers, go back and forth. Then show me what you do next.' And usually if they *verbalize it out loud* emphasis added], it's kind of like a certification that I [the student(s)] am doing it right. And that's what I find a lot of times is they [the students] just need to verbally say it out loud…

Here again, students struggled but sharing ideas aloud and making sense of the words with teacher assistance supported students' problem solving. Mrs. Zelda attended to one student's difficulty with the language in the task so they might translate this word (verbal representation) into a variable (symbolic representation). She perceived this student as a voice for the class hence interpreting that multiple students were also confused. She decided to encourage him and others to use their own words and verbally problem solve. That is, use verbal language as a means to translate between various representations (e.g., symbols and tables) in the task and their problem-solving strategies. It is clear that Mrs. Zelda noticed the representations embedded in tasks promoting modeling with mathematics, especially her students' language. This assisted her students to connect the language in the problem, the words students chose to explain their thinking and/or questions during problem solving, and their symbolic representations.

Discussion

The goal of this research was to identify what middle school teachers notice within instruction promoting modeling with mathematics. Both Mr. Brown's and Mrs. Zelda's instructions were analyzed using a teacher noticing framework to answer the question: What do middle school teachers notice during mathematics instruction that promotes modeling with mathematics?

It was evident that instruction promoting modeling (SMP 4) is qualitatively correlated with a focus on mathematical structure (SMP 7). SMP 7 (Look for and make use of structure) states that students "can step back for an overview and shift perspective" (CCSSI, 2010, p. 8). Relatedly, students engaged in this standard are expected to shift their representational thinking while doing mathematics. They attended to students' struggles with translating between representations, particularly moving from mathematical language embedded in the problem (verbal representation) to symbolic forms. If students do not understand the language embedded within modeling with mathematics tasks then they may not necessarily understand the problem much less be able to solve it (Yee & Bostic, 2014). Tasks fostering modeling with mathematics include several cognitive facets including reading text and other mathematical representations, connecting those representations, and drawing upon them during further problem-solving (Bostic, 2015). It is no surprise that Mr. Brown's and Mrs. Zelda's noticing was focused on ways to encourage students' flexibility with representations during modeling with mathematics instruction as a means to help students arrive at a reasonable result. Instruction promoting modeling with mathematics appears to have a unique facet not raised in prior teacher noticing literature: Mr. Brown and Mrs. Zelda notice how students engage with mathematical structure and translate between representations.

These findings supplement the burgeoning research on teacher noticing with evidence within a specific context. Schoenfeld (2011) called for teacher noticing to address specific contexts, specifically, teacher noticing across instructional contexts fostering modeling with mathematics. We considered numerous variables including years of experience (ten or more), grade levels (middle school), type of task (promoting SMP 4), education completed (M.Ed), and district-level differences (suburban with low poverty rate compared to rural with high poverty rate) and were able to draw out themes across the two cases. Seasoned teachers who are knowledgeable about instruction promoting modeling with mathematics (and other SMPs) are focused on supporting students to look for structure within these tasks and translating between representations. Drawing across these two cases (but not generalizing to the greater population), we conjecture that there are similarities across contexts focused on modeling with mathematics.

Limitations and Future Research

There were a couple limitations to this study. One limitation was that this case study purposefully focused on two teachers' instruction hence results cannot be generalized to the greater population of middle school teachers. Further studies might examine the noticing patterns of more middle school teachers during modeling with mathematics instruction. Given that Mr. Brown and Mrs. Zelda were knowledgeable of the SMPs and instruction promoting them, this begs the question: How does middle school teachers' noticing during modeling with mathematics tasks develop as they gain greater confidence enacting such problems? What differences exist between middle school teachers' noticing during modeling with mathematics instruction and non-modeling with mathematics instruction? Future research may respond to questions like these that build upon this case study. Relatedly we wonder: What do similar (e.g., years of teaching experience and education completed) elementary and high school teachers notice during modeling with mathematics instruction?

A second limitation was that not all the recordings of the two teachers' lessons captured every student interaction. It is possible there may have been situations that were not visible on camera or recalled by Mr. Brown or Mrs. Zelda. Future researchers might consider placing multiple cameras around the room and asking students and teachers to wear microphones to record every student–teacher interaction. Capturing more interactions may allow for deeper exploration into what middle school teachers notice during instruction promoting modeling with mathematics.

Final Thoughts

The goal of this case study was to closely examine two middle school teachers' instruction to understand what they notice during mathematics instruction that promotes modeling with mathematics. Mr. Brown and Mrs. Zelda attended to instructional events most closely associated with mathematical structure and translating between representations. Such a focus is needed during instruction in the Common Core era that includes standards describing mathematical behaviors not typically found in many previous state-level standards.

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Using Video of Peer Teaching to Examine Grades 6–12 Preservice Teachers' Noticing

Lorraine M. Males

Abstract In this chapter, I describe what features secondary (6–12) mathematics preservice teachers (PSTs) identified as noteworthy in lessons taught by their peers in the context of a methods course. PSTs planned, taught, and reflected on at least two lessons from middle and high school reform-oriented materials taught to their peers across two semesters. Lessons were recorded and uploaded to VoiceThread, a web-based application that enables users to comment on video and these comments served as data for this study. Results indicated that across the two semesters, PSTs most frequently identified aspects related to communication, mathematics content, and classroom management with less evidence of attention to classroom environment and tasks. Although PSTs more often noted teacher talk or actions, rather than student talk or actions, the percentage of comments related to student talk or actions increased in the second semester. These results are significant in that they illustrate that PSTs can identify noteworthy features of classroom instruction and this assignment served as an opportunity for PSTs to do this.

Keywords Noticing · Secondary prospective teachers · Teacher education · Peer teaching

According to Mason (2002), noticing is something that we do all the time, but in a profession "we are sensitised to notice certain things" (p. xi). This act of noticing is not new. Research has indicated that experienced teachers do, in fact, notice or have the ability to attend to and interpret classroom situations (Berliner, 1994). However, this ability is often perceived as developing over time as it requires extended opportunities to focus on aspects of practice and make connections between teaching and learning (Amador, 2016; Jacobs, Lamb, Philipp & Schappelle, 2011; van Es, 2011, Van Es & Sherin, 2002). I argue, as do others (Star & Strickland 2008; Van Es & Sherin, 2002) that preservice teachers (PSTs) who lack the experiences that more veteran teachers possess, can learn to notice.

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Furthermore, since one of our primary responsibilities as mathematics teacher educators is to facilitate the development of our preservice teachers (PSTs) as professionals and noticing is a key aspect of becoming a professional, we cannot ignore noticing. This paper describes a study that examined what secondary (6–12) mathematics PSTs identified as noteworthy when watching video of their peers teaching and provides implications for teacher education and research.

Preservice Teachers' Abilities to Notice

Early research on noticing suggested that teachers' classroom experience is related to their ability to notice (Berliner et al., 1988), something that, unfortunately, PSTs do not have. However, more recent research shows that despite PSTs lack of experience, they do have the ability to notice and that this ability can be cultivated.

One promising activity that has been found to improve teachers', including PSTs', abilities to notice is watching and discussing video of practice (Sherin & Han, 2004; Star & Strickland, 2008; van Es & Sherin, 2002). Video has been shown to be a useful tool in helping to focus PSTs' attention on aspects of teaching and learning mathematics (Star & Strickland, 2008). Rather than use videos as a way to give PSTs models of expert teaching or analysis, researchers have begun to use videos as contexts for providing opportunities for PSTs to examine and analyze classroom practice. Star and Strickland (2008) used video to examine whether a course with a focus on observation of practice, improved PSTs' noticing. PSTs were asked to complete pre- and post-written instruments designed to investigate what participants attended to after watching a video of an 8th grade mathematics classroom from the US Public Release TIMSS videos. They found that PSTs improved in their abilities to notice classroom features, such as the classroom environment, mathematical content, tasks, and communication. In a replication study (Star, Lynch, & Perova, 2011) however, results indicated that PSTs similarly improved in noticing classroom environment and communication, but did not improve in noticing tasks or mathematical content.

Other work with video has included the use of video clubs where teachers watch their own video and/or that of their colleagues. Through this work, researchers have found that teachers', both inservice and preservice, noticing abilities can shift from the general noticing of sequences of events to a more focused noticing of particular moments (Sherin & Han, 2004; van Es & Sherin, 2002, 2011). For example, van Es and Sherin (2002) found that PSTs' noticing was supported by the use of a Video Analysis Support Tool. This tool asked PSTs who were engaged in an internship in schools to analyze video from their own classrooms, allowing them to draw on their knowledge of their particular context. van Es and Sherin found, like others, that by using this tool, PSTs were supported in analyzing their practice via call-outs (Frederiksen, Sipusic, Sherin, & Wolfe, 1998) rather than chronological descriptions of events. In addition, PSTs used evidence to support the importance of these call-outs. Finally, they found that PSTs were more interpretative. For example, PSTs using the tool explained what students meant when they analyzed student thinking or how a teacher move impacted student understanding. In addition, these studies indicated that teachers', both inservice and preservice, observations shifted from discussing what the teacher was doing to what students were doing or saying.

Furthermore, Star and Strickland (2008) argue that a PST's ability to learn from their teaching is dependent on their ability to notice and Sherin and van Es (2005) argue that improving PSTs' abilities to notice should be an explicit focus of teacher preparation.

Teacher Noticing

Teaching is a complex profession that requires those who assume the role of teacher be able to make decisions by attending to and making sense of their environment, core activities linked to what it means to notice. The roots of noticing are not unique to the study of teaching. Although different professions may require different skills, each has what it calls an expert and according to Miller (2011), researchers across domains have been concerned with the development of models for what is involved in "expert looking" and how this expert looking is developed. Where teaching is concerned, Miller suggests that expert looking is dependent on "Situational Awareness." Situational awareness involves the perception of elements in an environment, comprehending the meaning of these elements, and projecting their status in the future (Endsley, 2000). Perception, however, is selective (Goodwin, 1994), meaning that we perceive only a subset of what is available for us to see. Due to the selective nature of perception, Goodwin calls the act of seeing and understanding that is socially organized in order for particular groups to understand events as "Professional Vision" (Goodwin, 1994, p. 606). As a teacher, one must develop this professional vision in order to focus on the aspects of classroom practice that enable them to do the work of teaching.

In this paper, I use van Es and Sherin's (2002) definition of noticing that proposes three key aspects of noticing: (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions. I chose to use this definition due to its broad focus on classroom situations and its prior use in studies of preservice secondary teacher noticing. Although all three aspects are critical, like Star and Strickland (2008), I focused primarily on the first aspect, as identifying what is important or noteworthy is the first step in developing ones' capacity to notice. This aspect of noticing has garnered attention from other researchers who use terms such as making call-outs (Frederiksen et al., 1998), highlighting (Goodwin, 1994) or attending (Jacobs, Lamb, and Philipp, 2010). This aspect of noticing, particularly for preservice teachers, is foundational. Without identifying what is important one cannot make connections between theory and practice, nor can they use these connections to reason or make decisions.

My goal for this study was, like Star and Strickland (2008), to contribute to this existing literature by engaging PSTs in a noticing activity within the context of a secondary mathematics teaching methods course. However, this study also draws on the video club research and differs from Star and Strickland because videos of PSTs teaching served as the central object of observation. In this study I address the question: What do PSTs identify as noteworthy when watching videos of their peers teaching? Using the framework developed by Star and Strickland, I describe the classroom features that PSTs identified as noteworthy, evidenced by their explicit comments on videos of their peers teaching in a microteaching setting.

Methods

Context and Participants

This study took place in the context of a secondary (6–12) mathematics teaching methods sequence at a large mid-western university. This sequence consisted of two teaching methods courses for which I was the instructor. The courses broadly focused on issues of mathematical thinking and learning with a focus on access and equity, lesson and unit planning, working with curriculum materials, and classroom discourse (i.e., interaction patterns, questioning, discourse moves), with the latter two topics being focused on more in the second semester than in the first. Participants (n = 21) included 15 undergraduates enrolled in a 4-year program and six masters + certification students enrolled in a 14-month program. Undergraduates typically take these two courses during their junior and senior year before completing a semester of student teaching. Graduate students take the first methods course in the summer, their first semester of enrollment, and then join the undergraduates in the second course in the fall semester. While enrolled in the second methods course all students were also enrolled in a practicum course, which involved attending a local middle or high school each day for approximately two hours and completing observation, analysis, interview, and teaching assignments.

Data Collection

The data for this study were collected in each of the two methods courses from a microteaching assignment. PSTs completed this assignment each semester by planning, teaching, and reflecting on one lesson from a reform-oriented curriculum series. Reform-oriented curriculum materials were chosen in order to expose PSTs to materials that looked quite different from the materials they had experience with.

In the first semester, lessons were chosen from the Grade 7 Stretching and Shrinking unit from The Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). In the second semester, lessons from The Interactive Mathematics Program's Year 3 High Dive unit and Year 4 How Much? How Fast? unit (Fendel, Resek, Alper, & Fraser, 2008) were used since together these units addressed the same unit problem. Rather than use an assortment of lessons from an assortment of materials, I purposefully chose to use lessons from one unit so that, in addition to practicing planning, teaching, and reflecting on a lesson, PSTs could also experience the unfolding of a sequence of mathematical ideas in a unit as a student might. These units were chosen to provide students with content that was relevant and spanned the types of courses they may teach, including geometry, algebra, and trigonometry. I specifically chose units from The Interactive Mathematics Program that were more difficult and potentially less familiar in order to provide students with as authentic an experience as possible while still using high school curriculum materials.

PSTs were asked to plan the lesson using the curriculum materials and submit a written copy of a modified version of the Thinking Through a Lesson Protocol (Smith, Bill, & Hughes, 2008). Components of this plan can be seen in Figure 1.

- the topic
- Standards addressed
- objective(s)
- assessment(s)
- materials needed
- a plan for all activities in each of the lesson phases (i.e., Launch, Explore, Summarize)
- a summary statement for the lesson
- homework assignment
- anticipated solutions to the tasks that students would complete in class and at home

Figure 1. Components of the lesson plan.

Within the plan of activities for each of the lesson phases, PSTs were asked to include details for (a) what and how students were working (e.g., how students were configured, what work they were doing, how they were recording their work), (b) anticipated student thinking and questions, and (c) teacher moves. PSTs were also asked to submit all anticipated solutions to the tasks that students would complete during class and for homework. Using this plan, each PST enacted a 30 min lesson at some point within the semester, with their peers participating in the lesson as middle or high school students and providing brief written and verbal feedback at the conclusion of the lesson. All lessons were recorded and uploaded to VoiceThread (VoiceThread LLC, 2013), a web-based application that enables users to upload, among other formats, video, and invite others to record commentary on the video. In addition, to the immediate feedback provided at the conclusion of lessons, in each semester PSTs were assigned to provide detailed feedback (at least four comments) on two lesson videos on VoiceThread via text, audio, or video. PSTs were asked to provide feedback that was intended to help the teacher think more deeply and reflect on particular aspects of their lesson and were required to use, in each of their comments, the phrases “I notice…” and “I wonder…”. Before embarking on this activity, PSTs practiced providing this kind of feedback on a video from my first year of teaching.

This assignment was completed by each PST in both of the methods courses. Therefore, over the course of the two methods classes, which occurred over two semesters, each PST planned and taught two lessons and provided detailed feedback on lesson videos four times. This resulted in a total of 41 lessons (since one PST dropped the second methods course and did not teach a lesson) and 716 detailed comments. These comments were used as data to identify what PSTs described as noteworthy in the lessons of their peers and to characterize these observations. Figure 2 illustrates the timeline for data collection.

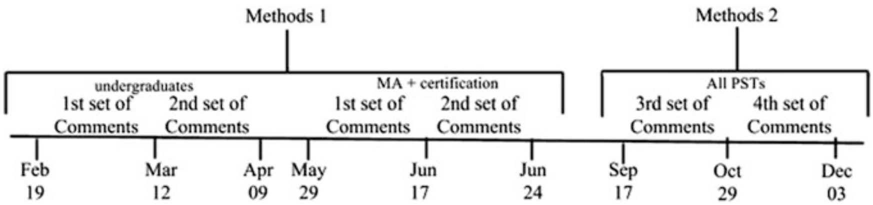

Figure 2. Timeline of data collection.

Data Analysis

Star and Strickland’s (2008) five observation categories were used as a coding scheme to describe what PSTs identified as noteworthy about their peers’ lessons, as evidenced by their comments on lesson videos. It is important to note that PSTs may not have left a comment for every aspect that they identified as noteworthy, but for the purposes of this paper I can only report on what PSTs explicitly commented on. These categories included: classroom environment, classroom management, tasks, mathematical content, and communication. See Table 1 for a description of each of these categories [taken directly from Star and Strickland (2008)] and for sample comments in each category.

Table 1
Observation categories used to code comments

Category	Description	Sample comment
Classroom environment	Includes physical setting such as desk arrangements, materials and equipment available and utilized, demographics of students and teacher, class size, grade level, and course title	I noticed that you used the board to demonstrate how to draw the figure, I wonder if the use of a projector of some sort would have helped demonstrate the activity

(continued)

Table 1 (continued)

Category	Description	Sample comment
Classroom management	Includes the ways the teacher deals with disruptive events, pace changes, procedures for calling on students or handling homework, and the teacher's physical presence (e.g., patterns of moving around the classroom, strategies for maintaining visibility, tone, and volume of voice)	I noticed that at times you would start speaking on a certain idea but then would start talking quietly and it was hard to hear what you were saying. I wonder if you spoke a little more clearly if it would be easier for students to hear what you are saying
Tasks	Refers more generally to activities students do in the class period (e.g., warm-ups, worksheets, taking notes, presentations, passing out papers) or future activities such as homework or upcoming quizzes	I notice here you started the class with a warm-up—this seems to be a good idea. I wonder if a slightly shorter warm-up would have left more time for the part about triangles, though
Mathematical content	Includes representation of the mathematics (graphs, equations, tables, models), examples used, and problems posed	I noticed you went from a unreduced scale factor to percentages and then to a reduced scale factor. I wondered if maybe just going from the unreduced to the reduced would have been a little bit clearer to the students.
Communication	Refers to student-to-student as well as teacher-to-student talk and includes questions posed, answers or suggestions offered, and word choice	I noticed (@1:10) a student asked you what you meant by "divide" and in your explanation you used words like "this" to refer to things like triangles and I wonder if it would be better to use more descriptive and explicative words in referencing things in your explanations

All data was collected in a spreadsheet to organize coding. A tab was created for each PST that included their comments on each of the four lesson videos they were assigned. Each text comment was directly copied from VoiceThread and placed into a spreadsheet. Any comments that were left via video or audio were transcribed and placed into the same spreadsheet. Two researchers (the author and a mathematics education graduate student) used consensus coding (Orwin & Vevea, 2009) to code each comment. Each researcher first applied the coding scheme to each comment separately and then met to discuss and come to consensus. Although there were few disagreements, some did occur. It was the case that many of these disagreements were the result of one coder overlooking something in the comment or in the definition of the coding category. For example, when coding one comment dealing with the way a PST called on students using popsicle sticks, one coder coded this as communication, whereas one coded this as classroom management. During the discussion both coders read through the definitions of each category and decided that based on the definitions that this should be coded as classroom management since the definition of this category includes “procedures for calling on students.” In some instances coming to consensus involved agreeing that a particular comment should be coded in two categories as it addressed pieces of the definitions of more than one category. Coders were open to adding categories during analysis and discussion, but the analysis did not lead to additional categories as the Star and Strickland (2008) categories adequately described the data. A smaller subset of the data (20% of comments across different PSTs) was analyzed 5 months after initial coding by the author to search for disconfirming evidence. This analysis did not result in the changing of codes, the characterization of the coding categories, nor did it result in the addition of any categories.

During initial coding, we began to notice that comments either focused primarily on the teacher (e.g., teacher moves, utterances, mannerisms) or students (e.g., utterances, perceived confusion). In order to capture these differences and examine patterns, we decided to also code each comment for its primary focus, student, or teacher. We returned to comments we had already coded to capture this.

Results

In this section, I present the results from my analysis. I begin by first describing the quantity of comments left by PSTs. I then follow this by describing the focus of these comments, first addressing what PSTs noted with respect to the teacher and to students and then to the observation categories. However, before turning to the results, I think it is important to recognize that the comments left by PSTs do not necessarily provide a complete picture of what they found noteworthy. By this, I mean that because a PST left four comments (the minimum required) on a peer’s lesson video, does not necessarily mean that they only found four aspects of the lesson noteworthy. As with any assignment, it is just that, an assignment. Nevertheless, the comments left by PSTs provide a glimpse into aspects that they did indeed find noteworthy enough to comment on. Another important point to mention is a potential limitation of the VoiceThread environment. The second PST to comment on the lesson video was able to see and/or hear the first PST’s comments. This may have impacted what the second PST commented on in two ways. First, the second PST to comment may have been influenced by the first PST’s comments and may have been apt to comment on the same aspect of the lesson. Second, the second PST to comment may have felt the need to comment on something different than the first so as not to “repeat” comments, possibly forcing this second PST to find aspects of the lesson noteworthy that they may not have if the comment from the first PST were not already visible on the video. In a second pass through the data I found that the former was not evident; the second PST did not comment on the same aspects that the first PST did. However, it is unclear if the second PST was forced to comment on different aspects given the comments that were already visible to them. I have no way to know if the first PST's comments influenced the comments provided by the second PST.

Quantity of Comments

Although a crude measure, I begin the results by first describing the number of comments left by PSTs. While quantity does not provide insight into *what* PSTs found noteworthy, the number of comments left nonetheless may provide a glimpse into *how much* PSTs found noteworthy. Across both semesters PSTs left a total of 716 comments (354 in the first semester and 362 in the second). Table 2 includes the total number of comments left on each lesson video.

Table 2
Total number of comments left on lessons

Total number of comments left per lesson	Number of lessons
0–8	4
9–16	20
17–24	10
>24	7

Of the 41 lessons four lessons (10%) received eight comments, meaning that each commenter left the minimum number of comments required for the assignment. Seven lessons (17%) received more than 24 comments, with most lessons receiving 9–24 comments. Of the 82 sets of comments only 17 sets (9%) included the minimum number of comments. This means that 91% of comment sets included more than four comments, indicating that many PSTs may not have only been using the four-comment requirement to guide their viewing of the video. In the next section, I discuss the focus of the comments.

Focus of Comments

I begin with describing what teachers found noteworthy with respect to teacher actions and student actions across the two semesters. I follow this with an overview of the percentages of comments in each category across the two semesters, pointing out differences between comments left in each semester, when possible. I then provide more qualitative descriptions of the comments in the most coded categories and include representative excerpts directly from PSTs. All PST names are pseudonyms.

Attention to teachers and students. Not surprisingly, it was clear when we began our analysis that PSTs' comments focused on teacher actions. However, it was also true that PSTs often commented on students. Hence, we decided to code comments as Teacher if they primarily focused on the teacher without any reference to students and Student if they referred to students in any way, including interactions between the teacher and a student or group of students or actions of or interactions between students. Table 3 provides the percentages of comments that focused primarily on the teacher and the percentage of comments that focused primarily on students by semester.

In both semesters more comments focused primarily on the teacher. However, there was a shift in Semester 2. Although more comments were still primarily

Table 3
Percentages of comments focused on teacher and students by semester

Focus	Semester 1	Semester 2
Teacher	88	69
Students	12	31

focused on the teacher, 31% of all comments (compared to only 12% in Semester 1) focused primarily on students. Most of these comments indicated that PSTs were noting interactions between teachers and students rather than interactions between students. This was most noted by PSTs when there was a sense that students might not understand. For example, comments such as this one from Katherine were most common, "At 4:40 I noticed that Becky was concerned that her group members were getting different answers for Harry's height and I wonder if this would have been a good opportunity to address this issue with the class and explain that this particular problem is going to have more than one answer and why that is." Although the PST is providing a suggestion to the teacher the primary focus of the comment was a student's concern about their group's answer.

Observation categories. Results indicated that PSTs' comments clustered around three of the observation categories: communication, classroom management, and mathematical content. Table 4 provides the percentages of comments in each of the categories further delineated by the total number in each category focused on Teacher and Students.

Table 4
Percentages of comments and total number focused on teacher and students

Observation category	Percentage of comments	Number of comments within category that focused on teacher	Number of comments within category that focused on students
Classroom environment	1	7	0
Classroom management	24	159	10
Tasks	9	55	7
Mathematical content	25	161	18
Communication	42	278	21

Across both semesters PSTs commented most often on communication, accounting for 42% of codes, with mathematical content and classroom management accounting for 25 and 24%, respectively.

Only 9% of comments addressed Tasks and only 1% addressed Classroom environment. The lack of comments addressing Classroom environment is likely a consequence of the context of the assignment. Classroom environment was defined as comments that addressed the physical setting including desk arrangements, materials, and equipment, and aspects related to the demographics, class size, and grade level. Since this was an assignment within a methods course and the PSTs who were commenting were members of the course, no comments addressed class demographics, size, or grade level. In addition, no comments addressed the room arrangement. For the assignment PSTs were expected to have students work in groups during the explore phase of the lesson and in our methods classroom tables were already arranged in groups of three or four students. PSTs rarely rearranged the tables or student seats and this likely contributed to the lack of comments focused on this aspect of the lessons. All of the comments that were coded in this category addressed equipment or materials (e.g., use of projector, iPad).

Tasks referred to the general activities students do in class or future activities such as homework or upcoming quizzes. The attention to Tasks might also be influenced by the context of the assignment. In the assignment, PSTs were asked to use the provided curriculum materials to plan the lesson and most followed the suggestions in the teachers' guide. Therefore, when commenting, PSTs may not have been attuned to noticing aspects related to the general activities of the class as PSTs may have interpreted the activities as provided for the teacher and students. There was a slight increase in the attention to Tasks in the second semester (up from 8 to 10%). PSTs commented more often on activities such as warm-ups and worksheets. It is important to note that the curriculum materials that PSTs were asked to use as a resource in the second semester differed from the first. Although this may be a contributing factor it was also the case that when teaching their second lesson PSTs added more aspects to the lesson, like creating their own worksheet or incorporating a warm-up activity (something that they observed in their practicum setting during the second semester).

Across the two semesters the most common observation categories remain the same, but there was a shift in the distribution of these categories across comments. Table 5 illustrates the total number of comments with the accompanying percentages in parentheses for the three most common categories for each semester.

Table 5
Frequencies and percentages of comments addressing classroom management, mathematical content, and communication by semester

Category	Semester 1	Semester 2
Classroom management	121 (31)	70 (17)
Mathematical content	108 (27)	93 (23)
Communication	132 (33)	199 (50)

Communication was the most noted observation category across both semesters, however, it accounts for a much larger portion of comments in the second semester than in the first. Half of all comments left in the second semester were related to communication while communication only accounted for 33% in the first semester. Classroom management was the second most noted category in the first semester, but attention to this decreased in the second semester as did attention to mathematical content, which was the second most common category in Semester 2. In the next Section, I describe the comments in each of these categories.

Communication. Comments categorized as Communication referred to those that focused on discourse including student-to-student and student-to-teacher talk. Other than implicit comments that alluded to the fact that PSTs recognized that students were talking with each other (e.g., "group discussion," "explain group's process") there were no comments that addressed student-to-student talk. Granted, this could be due to the fact that it was hard to hear conversations between students on the video, but is also likely that PSTs attended more to the teacher than to students. Instead, all comments focused on student–teacher talk and included issues of engagement, purpose, and clarity. For example, many comments described general interactions between teachers and students and questioned whether these served to open up or close students' engagement with the ideas, such as the following comments from James and Jeff,

> James: I noticed that you asked Michaela what their process was and then almost "answered" it by giving different ways, and I wonder if you could let the student explain some, and then if they need help or guidance, then give them ideas.

> Jeff: I noticed when you were explaining the scale factor it was a one-sided discussion where the students just watched you explain it. I wonder if this was because you felt pressed for time or if you meant to include students' thoughts in the exploration…

Other comments focused specifically on word use to get at purpose and clarity, such as the comments below from Heidi and Sandra,

> Heidi: At 2:54 I noticed you were walking around asking groups what angles they used. I wonder, what was the purpose of this question? Would maybe a more beneficial or insightful question be, "how did you choose your angles?"

> Sandra: I noticed that you let the students just shout out 6 2 then 6 3 and wondered if you felt it would be necessary to ask for the x value and y value to express clarity"

These comments indicated that Sandra and Heidi noted specific words that were used (or not) by teachers when questioning students or listening to their responses.

Finally, another frequently mentioned aspect of communication involved the use of wait time. These comments often indicated that PSTs noted when wait time was not used, but might be useful. Many instances proposed wait time be used after a teacher asked if students had any questions, like the comment below from Valerie, or when asking another question that required students to apply their reasoning, like the comment from Mindy.

> Valerie: I noticed a couple of times you asked if everyone understood or if anyone had any questions but didn't give much time for students to answer. I wonder if some students might have had questions but weren't able to ask them because you moved on too quickly.

> Mindy: I noticed after asking, "Does everyone agree with that?" you moved on. I wonder if it would be good to try using wait time, you don't have to but it's just a thought.

Mathematical content. Mathematical content was noted in at least 20% of comments in both semesters, with a slight decrease from the first to the second semester. Across the two semesters the comments that PSTs left related to the mathematics could be grouped into five categories: definitions (45%), representations (27%), examples (20%), notation (5%), and tools (4%).

Definitions. By far, definitions accounted for the largest focus of mathematical content comments. 45% of all mathematical content comments addressed definitions. These comments came in two forms. Comments regarding a lack of defining and comments that addressed how a teacher defined.

Comments regarding the lack of definitions included varying levels of specificity with some merely questioning the potential usefulness of defining, such as Linda's comment, "I noticed that you used the word transversal. I wonder if [it] would have been helpful to define it for the class," whereas others indicated more reasoning as to why the definition might be needed, such as Pam's comment:

> Pam: I noticed that you used the words "defend their answer" and I'm wondering if everyone would know what it meant when you said "defend". I'm wondering if you could have used the language "explain how…" and then used "defend", it would maybe be better understood by high school students.

The most common type of comment addressing definitions was related to how a teacher provided a definition and the ways in which students engaged with the definition, such as the following comments from Helen, Brenda, and Diana:

> Helen: I noticed that you gave the definition and factors for polar coordinates and then just moved on, and I wonder if you could have maybe opened it up for a little bit of discussion or asked if there were any questions in case students had any.

> Brenda: I noticed that you asked what the class thought of the word identity rather than just presenting a definition. I wonder if that helps them remember the definition.

> Diana: I noticed you presented a true identity, I wonder if it would be beneficial to have students test that identity or have students at first try to write their own identity equations after just hearing the definition.

These comments indicated that Helen, Brenda, and Diana noted aspects related to the teacher's choice as to how to provide a definition and how this may impact students, with Helen wondering if the teacher moved on too quickly without discussion and Brenda and Diana considering the impact of asking students to engage with the definition in some way before and after providing a formal definition.

Representations. Across the two semesters the next most common aspect of mathematics content addressed in comments was representations, which accounted for 27%. These comments tended to be more common in the second semester than in the first and the focus was on connecting representations, such as the following comments from Frank and Todd:

> Frank: I noticed how you wanted to put what Diana was saying in words! It is a great way to move from a geometrical representation to something more concrete! It reminded me of the Border Problem.

> Todd: I notice at this point you describe the sine function as being used to find the height of the triangle. I wonder if it would have been beneficial to review the unit circle representation of trigonometric functions at this time—it might lead to an opportunity for explaining why the expression is multiplied by the radius—in terms of similar circles/triangles."

Examples. Finally, across both semesters, 20% of all content comments addressed the teachers' use of examples. Typical comments with regard to examples included PSTs pointing out that additional examples, or different examples (e.g., not the one presented in the textbook) might be useful, such as the comments below from Pam and Frank:

> Pam: I noticed how you explained how to write height to width in terms of a fraction. I am wondering if you could have given a few other examples in order to check the student's understanding.

> Frank: I noticed you stuck with the example in the book and I'm wondering if you maybe presented your own example instead of the book you could have portrayed that the transition from small to big, big to small makes a pretty large difference!

Other comments got at how the teacher engaged (or did not engage) students with the example, such as the following comments from Linda and Heidi:

> Linda: At 2:30- I noticed you did all the counting and work in your example. I wonder if it would have been an easy way to engage the students to have them count the height and width of the shape.

> Heidi: When you introduced the sun casting a shadow example, you told the students where the hypotenuse came into play, I wonder if the students could have come to this conclusion themselves after thinking about it for a little while if you asked them, would this have been helpful for them to think through or not?

Finally, some comments called into question specific aspects of an example such as the comments below from Becky and Donna:

> Becky: I noticed that the two smaller sides of these figures adds up to exactly the length of the larger side, which means they are not actually triangles. I wonder if different values could have been used or if this could end up confusing students when working on future problems.

> Donna: I noticed in the first example that the numbers on the triangles were possibly a bit unwieldy for a seventh-grader. Maybe choosing two prime numbers and a non-prime for the lengths of the sides would have been advisable to avoid the need for reduction. If practice with reduction was desired, perhaps it could be by small, whole number factors.

Becky's comment indicated that she noted the accuracy of the example and found it to be mathematically incorrect whereas Donna noted the choice of numbers pointing out that the number choices may present potential difficulties for students.

Classroom management. The classroom management observation category was defined by attention to the way teacher's deals with disruptive events, pace changes, procedures for calling on students or handling homework, and aspects of presence, such as how the teacher moved around the classroom, stayed visible, and the tone and volume of the teacher's voice. The context of the assignment likely had a great impact on what PSTs found noteworthy related to classroom management. Since these lessons were taught in a methods course disruptive events were virtually nonexistent and so PSTs did not have the opportunity to notice such aspects. That said, other aspects of classroom management were noted frequently by PSTs, with the most common being issues of time management or pacing and teacher position and voice.

Comments related to time management usually addressed the ways in which teachers made use of time, such as issues related to running out of time or pacing such as the following comments by Walter and Diana,

> Walter: I noticed that we ran out of time toward the end of the lesson. I wonder if we maybe could have gone over the last problem or two as a class instead of working in groups again.

> Diana: I notice here you started the class with a warm-up—this seems to be a good idea. I wonder if a slightly shorter warm-up would have left more time for the part about triangles, though.

In addition, issues of position and movement around the room were common, such as the following comments from Eric and Helen who commented on teacher position during whole-class and small-group work

> Eric: I really liked the verbal aspect of your lesson, but I noticed that your body posture is often oriented toward the whiteboard. It might be easier to overcome this by making the students do a lot of the work you are trying to write down. This way you are having more of a two-way conversation, instead of just talking to the class?

> Helen: I noticed that you kind of start walking away from the table while you are still answering the question and the student is still working on it, and I wonder if this would discourage the student from asking further questions if they had any more.

Finally, there were comments related to voice, all of which addressed volume, such as the comment from Aaron, "I noticed that you were very soft spoken in small groups and I liked that however I wonder if you could turn up the volume since it might be hard to hear you."

Discussion

The purpose of this study was to examine secondary (6–12) PSTs' noticing. Specifically, I aimed to describe what classroom features PSTs identified as noteworthy when viewing video of their peers teaching, as evidenced by comments they left on their peers' lesson videos. Results indicated that across the two semesters PSTs were enrolled in a secondary mathematics methods course sequence they most frequently noted communication, mathematics content, and classroom management with less evidence of attention to classroom environment and tasks.

Although prior research indicates that PSTs are not necessarily perceptive in identifying salient features of mathematics lessons, like other studies, this study illuminated that PSTs can, in fact, identify noteworthy aspects of classroom situations. Similar to Star and Strickland (2008) and Star et al. (2011) who found that PSTs could identify aspects of classroom situations from a full-length video of a mathematics lesson, I found that PSTs could identify noteworthy aspects of classroom instruction in videos of lessons taught by their peers. While both contexts provided opportunities for PSTs to observe salient features of classrooms, it seems that these different contexts (teaching in a secondary classroom versus peer teaching in a methods course) may have provided different opportunities. I return to this point in the next section.

In addition, perhaps unsurprisingly, results showed that when PSTs viewed lesson videos they noted teacher talk and/or actions more than student talk and/or actions. While this might not be surprising, as this was in the context of a methods course where PSTs are learning to teach, there was a significant amount of student talk throughout each lesson, both in the context of whole-class discussion and small group work. Albeit the students were themselves PSTs, not secondary mathematics students. Nonetheless, these PSTs were engaging in the mathematics of the lesson and responded to and questioned other students and the teacher as students would in a secondary mathematics class. Although this focus on the teacher was evident in both semesters, there was a shift in the distribution of teacher- versus student-focused comments with student-focused comments increasing from 12% in the first semester to 31% of comments in the second semester. This shift indicates that PSTs' comments did more often involve student talk and/or actions in the second semester compared to the first semester, indicating that PSTs were able to shift their focus to students rather than the teacher. There were also some differences in terms of what was identified across the two semesters. Although the most frequently noted observation category was communication in both semesters a much larger proportion of attention was devoted to this in the second semester (50%) than in the first (33%). This could be explained by the content of the methods courses. A large portion of the second semester was devoted to the study of classroom discourse and included reading about and analyzing interaction patterns, questioning, and discourse moves. This likely contributed to students' explicit attention to communication. Similarly there were changes, albeit much smaller, in attention to both mathematics content and classroom management. Both mathematics content and classroom management were noted less frequently in the second semester than in the first. One hypothesis for the decrease in attention to mathematics content could be that the mathematics content in the second semester, which involved trigonometry and physics, was harder than that in the first semester, which involved similarity and congruence. It became clear throughout the second semester that more PSTs were struggling with the mathematical content and this may have impacted their attention to the mathematics or at least their willingness to comment on the mathematics. It is unclear why the attention to management decreased so much, but this could be due to the fact that students were required to leave four comments and that PSTs' were noting more about the ways in which teachers and students communicated, thus decreasing the number of comments that addressed classroom management. Finally, as mentioned earlier, it was also the case that there was more attention to Tasks, although still not nearly as much as the other three categories, in the second semester than in the first. This is likely due to PSTs making the lesson more their own in the second semester and because in the second semester PSTs were enrolled simultaneously in a practicum experience in secondary schools. In the second semester, PSTs incorporated tasks that they saw enacted in their placement classrooms like warm-ups and this may have provided more opportunities for PSTs to attend to tasks.

Using Peer Video as a Context for Noticing

For this study, the context within which PSTs were asked to notice was the lessons of their peers, lessons taught within a secondary mathematics methods course. Like other studies, this study indicated that PSTs could, in fact, notice classroom features. However, this raises the question as to whether the change in context influences what PSTs noted. Comparing the results of this study with the results of the studies by Star and colleagues (2008; 2011) provides a glimpse into the potential differences. Whereas Star and colleagues saw greater attention to classroom environment and management, PSTs in my study identified classroom management as noteworthy, but also noted quite frequently communication and mathematics content. It might not seem surprising that the PSTs in my study focused less on classroom environment and management. First, it may be that PSTs did not make note of the classroom environment as they would have had they not been viewing a video of a class that they themselves were a member of. As the saying goes, it is hard to see the forest through the trees. However, this diminished focus on the classroom environment may have allowed students to pay attention to other aspects of the lesson that may be hard to pay attention to in a secondary classroom. Being so close to the situation sometimes limits a person's perspective. Second, it is likely that classroom management was not as much of a focus because there were less management issues to note. Not that management revolves solely around disruptive situations, but this is part of classroom management and there were few, if any, disruptive events, unlike in the videos used by Star et al. (2011) where PSTs noticed the teacher maintaining control of the classroom. In addition, other management features such as taking attendance were not relevant in the methods context. While my PSTs' attention to mathematics content was modest, their attention to communication was quite frequent. This attention to communication may have been more possible due to their more limited opportunities to attend to classroom management issues. Although these differences are not surprising, they highlight how different contexts may provide different opportunities for PSTs to notice classroom features.

Implications for Teacher Education and Research

As emphasized by Sherin and van Es (2005), improving PSTs' abilities to notice should be an explicit focus of teacher preparation. My study is an example of just one assignment that could help teacher educators examine the noticing capabilities of their PSTs. However, we cannot just stop there. Noticing our PSTs' noticing is not enough. We need to provide them with opportunities to improve their noticing capabilities and not only identify classroom features (as I focused on in my study), but interpret these features and decide how to respond. We also need to provide a variety of contexts within which to notice. As discussed above, my study likely differed from that of Star and colleagues (2008; 2011) because there were different opportunities for noticing in an 8th grade math lesson taught by an experienced teacher than in a secondary mathematics lesson taught by a PST to his or her peers. Providing opportunities within multiple contexts could help to highlight classroom features in different ways.

Furthermore, in order to provide PSTs with effective opportunities to develop noticing capabilities, more research is needed. Teacher educators need to understand how different contexts for noticing impact what PSTs can learn to notice. In addition, since teacher educators are tasked with preparing teachers to do more than just notice, understanding how noticing opportunities could be designed to best make use of time and available resources is also necessary. The experience described in this study incorporated opportunities for PSTs to learn to notice in an assignment designed to accomplish other important aspects of teaching, such as providing them with the opportunity to practice planning, teaching, and reflecting on a lesson. It provided evidence that while engaging PSTs with other aspects of teaching, one might also be able to successfully engage students in the process of learning to notice. That said, this assignment may only allow for students to engage in certain aspects of noticing. For example, when PSTs are asked to comment on videos of their peers teaching in the context of a methods course they may be able to focus more intently on aspects such as mathematical content, but not necessarily on aspects, such as classroom management or environment, aspects that are also important in the work of teaching. Therefore, it is important to provide additional opportunities, such as watching video of real secondary classrooms, so that there is an opportunity for PSTs to begin to develop their abilities to notice aspects of classroom instruction that are mostly removed when teaching in the context of a methods course.

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Part II
Examining Student Thinking through Teacher Noticing

Examining Student Thinking Through Teacher Noticing: Commentary

Randolph Philipp, Mike Fredenberg and Casey Hawthorne

Abstract With the growing research base on teacher noticing has come a similar expansion of methodologies used to measure teacher noticing. The six chapters in this section reflect a range of methodologies, and this commentary is organized around three methodological considerations showcased in the chapters: (a) adoption of a conception of teacher noticing, (b) design of data-collection tools, and (c) choice of data-analysis lenses.

Keywords Teacher noticing · Professional noticing · Teachers' knowledge · Preservice teachers · In-service teachers

Classrooms are highly complex environments, and for teachers to create and nurture rich and supportive learning environments for all their students, they must learn to focus their attention among the "blooming, buzzing confusion of sensory data" (Sherin & Star, 2011). One approach teacher educators and professional developers have taken is to decompose the practice of teaching into specific components that might be studied and learned (Grossman et al., 2009), and the practice of noticing has emerged as a growing area of inquiry among researchers in their study of teaching practices (e.g., see Sherin, Jacobs, & Philipp, 2011). This book extends our understandings of teacher noticing, and the authors of the four papers in this section examine *student thinking* through teacher noticing. After addressing one commonly applied noticing framework, we describe contributions from each of the four papers, identifying major questions raised, and finally turn to

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recent work by two of the authors of this commentary (Fredenberg, 2015; Hawthorne, 2016) to consider the knowledge associated with engaging in the practice of noticing of students' mathematical thinking.

Professional Noticing of Students' Mathematical Thinking

Authors of the four papers in this section draw upon frameworks for noticing, mathematical content, and learning, as we mention when discussing each paper, but because the framework *Professional Noticing of Students' Mathematical Thinking* (Jacobs, Lamb, & Philipp, 2010) (hereafter *Professional Noticing*) plays a central role in all four papers, we first describe *Professional Noticing*. *Noticing* is a teaching practice, something one *does*. The construct of *Professional Noticing* is comprised of three practices: *attending* to students' strategies and their mathematical thinking, *interpreting* students' understandings, and *deciding how to respond* on the basis of students' understandings. We highlight two key aspects of this conceptualization. First, the three components are highly interrelated and often occur seemingly simultaneously. For example, when a student responds in a manner that indicates to the teacher an unforeseen conception, the teacher might pose a follow-up question to that student or to other students, and on the basis of additional information, the teacher might modify the lesson. In this example, attending to and interpreting the first student's thinking were virtually inseparable, and the teacher began to formulate a response while interpreting the students' thinking. Furthermore, although these three components of *Professional Noticing* are highly interrelated, for purposes of studying teacher noticing, researchers often isolate the components, an isolation we consider useful for the early development of the construct. The second aspect we highlight relates to the fact that teachers constantly engage in multiple types of noticing. For example, teachers notice whether a small group is working productively or if a student who seems troubled might need medical attention. Although these examples of teacher noticing have clear and direct implications for students' learning, *Professional Noticing* is a particular and explicit focus on the mathematical thinking of students.

The Four Studies

In the chapter by Lee and Choy, they studied preservice teachers from the United States and in-service teachers from Singapore to investigate the role noticing plays in teachers' learning from Lesson Study. They drew upon van Es's (2011) work to consider both what and how teachers notice during two components of a Lesson Study cycle, specifically while planning the lessons and while reviewing and discussing the lessons. They also drew upon the *Professional Noticing* framework (Jacobs et al., 2010) to investigate the extent to which the preservice U.S. teachers and in-service Singaporean teachers attended to, interpreted, and decided how to respond when discussing significant mathematical aspects during lesson-study discussions. Further, they applied a 3-Points framework (Yang & Ricks, 2013) to consider how teachers focus on the mathematical concept or big idea (the *Key Point*), the cognitive obstacle students face when grappling with the main idea (the *Difficult Point*), and the teacher's approach for supporting students while they get at the heart of the lesson (the *Critical Point*). The study showed that during the initial class observations, both groups of teachers found focusing on significant mathematical aspects challenging. For example, the U.S. preservice teachers focused on such nonmathematical issues as management and organization. However, by the final lesson-study discussion, both the preservice and in-service teachers began to notice specific episodes of student thinking. The researchers attributed the increased attention to students' thinking to a concentration during the lesson-study cycle on the 3-Points framework: the preservice and in-service teachers' discussion of the Key Point, the Difficult Point, and the Critical Point. The authors also found that neither the preservice nor the in-service teachers reached the level of noticing such that they engaged in deciding how to respond to students' mathematical thinking. Finally, although they noted that supporting teachers in adopting the 3-Points framework is nontrivial, they concluded that Lesson Study focused on such a framework can be a means to develop noticing expertise.

One noteworthy feature of the study by Lee and Choy is that although they studied two very different groups, U.S. preservice teachers and Singaporean in-service teachers, the results of their study were similar for the two groups. A second noteworthy feature is their infusion of the 3-Points framework, which was designed to focus attention among the participants on the mathematical details. Although such a focus supported the participants in developing noticing skills, none of the teachers engaged in the highest level of *Professional Noticing*. This result provides additional evidence for the challenge of supporting even experienced teachers in learning to respond to students' mathematical thinking.

van den Kieboom, Magiera, and Moyer studied prospective teachers' noticing in the context of one-on-one clinical interviewing taking place as part of a two-course integrated mathematics/field-experience sequence. Unlike the other authors of this section, van den Kieboom et al. situated their study within a well-defined mathematical content domain, the meaning of the equal sign, and they presented a four-category hierarchical framework of student thinking about the equal sign. Their overall goal was to engage prospective teachers in opportunities to rehearse and, subsequently, improve their noticing skills. They found that the prospective teachers' noticing skills improved, with 19 of the 32 prospective teachers showing improvement in attending to and further exploring student thinking about the equal sign; however, the improvement was not statistically significant. They also found that the prospective teachers noticed predominantly the strategies students used to solve a task without focusing on the details of the students' thinking about the equal sign. The authors concluded with two suggestions for improving the focus on the prospective teachers' noticing skills: (a) Use more examples and counterexamples of interviewers attending to and further exploring student thinking about the equal sign, and (b) incorporate “missed opportunities,” whereby prospective teachers watch an interview that might seem similar to one that they conducted and then reflect on how the interviewer might have taken a different direction to explore student thinking concerning the equal sign.

A noteworthy feature of the study by van den Kieboom et al. is their focus on a well-defined mathematical content domain that includes details about students' mathematical thinking, creating opportunities for prospective teachers to grapple with the mathematical details of the students' thinking. We see this approach as holding much promise for supporting the development of professional noticing of students' mathematical thinking.

Amador, Weiland, Hudson, Galindo, and Rogers, drawing upon frameworks of van Es (2011) and Jacobs et al. (2010), carried out a longitudinal study of six prospective elementary school teachers and then focused on one, Mikayla, over three phases: enrollment in a field experience during her junior year (Phase 1), student teaching during her senior year (Phase 2), and her first year of teaching (Phase 3). The authors studied Makayla's noticing in the context of mathematics and science, and a Lesson Study approach was used during Phases 1 and 2 when the six prospective teachers were paired during cycles of Lesson Study. Extensive data were collected, including written lesson plans, videotapes of lessons, field notes and observation, and post-teaching interviews. Two major themes emerged from the study. First, Mikayla emphasized students' mathematics understanding by attending to and interpreting students' thinking in all three phrases, with the greatest changes to her attending and interpreting being measured as the difference between her junior year and senior year. Second, the extent to which Mikayla adapted or modified her teaching in the moment, also grew, with marked changes being measured as the difference between her senior year and her first year of teaching. Also noteworthy, although Makayla's noticing improved in both mathematics and science, her deciding how to respond to students' thinking was evident more in mathematics than in science. Amador et al. theorized that Mikayla may have been limited in her scientific content knowledge vis-à-vis her mathematical content knowledge, accounting for the difference.

Amador et al. followed teachers over 3 years, an ambitious yet powerful means of learning about the development of teacher noticing. Furthermore, by observing Mikayla in two subject areas, the researchers were able to tease out the role that her content understanding played in her deciding how to respond in the moment. In particular, the authors noted that for prospective teachers to respond to content-specific instruction, they must be supported in developing the rich content knowledge needed to do so.

Wells extended the construct of professional noticing of students' mathematical thinking (Jacobs et al., 2010) to incorporate observable gestures, body language, and audible indicators of student thinking, most notably in students' conversations. Data were videotapes of weekly lessons in a fifth-grade class considered to reflect the teacher's normal teaching practices, transcribed with gesture mark-up to indicate the temporal aspect of each gesture. Wells examined common features to which a teacher might attend during group work. Major study results indicate that the manner in which a group engages in conversation is more important than what is said. For example, Wells posited that for a group to progress satisfactorily toward a solution path, the group must first embrace a cooperative demeanor and that an increase in gesture size seemed to indicate progress toward a solution, as did *posture echoing*–group members' adopting a common posture when working and conversing.

By attending to student conversations, including student gestures, to investigate the relationships between group conversations and progress toward a solution strategy, Wells has added another layer to the study of teacher noticing. For example, Wells offered a set of group dynamics that a teacher might find valuable for deciding whether to intervene in classroom group work. In addition, the finding that successful groups appear to immediately establish a supportive conversational atmosphere underscores a key noticeable aspect of group work. We suggest that some of these group dynamics seem to be more easily attended to than others. For instance, a teacher can observe posture echoing from across a classroom, but conversational shifts in a group's discourse requires a more intimate degree of observation. Moreover, noticing initial group dynamics requires a specific focus on each group's opening conversational tones and inflections, and such centered attention might be difficult to achieve across multiple groups. Finally, the results of Wells' work raise for us a question relating to the most efficient use of a teacher's attention: Of the group dynamics that Wells presents, which most contribute to the *Professional Noticing* of students' mathematical thinking?

The Role of Knowledge in Deciding How to Respond to Students' Thinking

The four papers in this section highlight the challenges involved with preparing prospective teachers, and even practicing teachers, to decide how to respond to students' thinking. We are not surprised that this practice is difficult for teachers. Tyminski and colleagues (2014) highlighted the coordinated and integrated manner in which teachers' specialized content knowledge, knowledge of content and students, and knowledge of content and teaching (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) must be held for teachers to engage in deciding how to respond to students' thinking. Perhaps an important issue is understanding not just the type of knowledge needed but also the constellation of knowledge and practice held by teachers and how it supports their in-the-moment decision making. Two recent studies of teacher noticing shed light on this question.

Fredenberg (2015) studied three primary-grade teachers who had more than 13 years of experience teaching mathematics using the principles of Cognitively Guided Instruction (CGI) and more than 6 years of professional development centered on children's mathematical thinking. Fredenberg applied a methodology whereby, in addition to conducting a series of structured clinical interviews and classroom observations, he (politely, with the teacher's preapproval) interrupted immediately after a teacher modified a task for a student and asked the teacher to explain her reasoning for the decision. Combined with semi-structured stimulated-recall interviews, these interruptions enabled Fredenberg to unpack the teachers' knowledge, noticing, and other practices and begin to understand the relationships among these.

Fredenberg (2015) posited that for these teachers the practice of noticing children's mathematical thinking was inextricable from the teaching practices of lesson planning and task design. He argued that when these teachers designed a task, they manufactured within the task architecture frameworks for, first, noticing their respective students' thinking and, second, leveraging their students' thinking to meet specific learning objectives. For example, Fredenberg found that during the task-design process the teachers often anticipated how specific students might react to a problem, and they made precise number choices to provide themselves opportunities for scaffolding moves across the wide range of their students' mathematical knowledge and understandings. Essentially, the teachers appeared to premeditate instructional responses applicable to the various strategies that their students would in all likelihood employ. Fredenberg concluded that for these exemplary teachers, *Professional Noticing* was woven across the domains of lesson planning and lesson enactment, and, hence, for them *Professional Noticing* was not exclusive to classroom teacher-student interactions. On the basis of this finding, we ask: How does *Professional Noticing* become integrated across the practices of exemplary teachers? And what knowledge is required for such integration, or degrees of, to be an attainable outcome of teacher preparation or professional development?

Hawthorne (2016) presented another study of exemplary teachers, but unlike Fredenberg's study in which all three teachers displayed expert noticing, Hawthorne's study showed that although two middle school teachers possessed similar knowledge structures, only one of the two effectively engaged in deciding how to respond on the basis of the students' understandings. Furthermore, the differing degrees of professional noticing correlated with the teachers' respective lesson-planning practices. For example, Jack, who expertly incorporated student thinking into his in-the-moment pedagogical decisions, premeditated his noticing in the lesson-planning process. Jack was deliberate and meticulous in designing his lesson plans, all of which included the nature of the student thinking that he wanted to stimulate and build upon during the lesson. Furthermore, Jack's precise organization of his lesson plans enabled him to anticipate and sequence students' emergent ideas while enacting the lessons. Thus, Jack, like the teachers in Fredenberg's study, actively premeditated his noticing of students' thinking when he proactively attended to specific instances of mathematical concepts and ideas of his students' thinking that he believed would emerge during a lesson.

In contrast, Clara, the second teacher in Hawthorne's study, did not exhibit organization and detail in the lesson-planning process similar to Jack's. Clara did not actively anticipate student thinking when planning a lesson, and, as such, she did not plan instructional strategies to meet specific instances of student thinking.

Consequently, Clara's professional noticing seemed to be much more reactive to student thinking than Jack's, which did not afford her the same opportunities to build on and extend her students' emergent ideas. Particularly noteworthy in Hawthorne's (2016) study is that the two teachers were engaged in the same long-termed professional development, and they both displayed similar mathematical content knowledge of algebraic generalization, the topic they were teaching. Hawthorne argued that the differences in the teachers' noticing could not be explained by their mathematical content knowledge and instead related to the manner in which they anticipated and thought through details of students' mathematical thinking vis-à-vis the generalization process.

Final Comments

In any classroom, one might direct one's attention in seemingly infinite ways, and the study of teacher noticing in general and professional noticing of students' mathematical thinking in particular have helped us understand where teachers place their focus. But understanding *what* teachers do (and do not) notice in mathematics classrooms, as important as it is, leaves those of us charged with preparing new teachers or providing professional development to experienced teachers posing another question: How might we leverage these constructs in our work with prospective or practicing teachers? By focusing the teachers' noticing on students' mathematical thinking, we emphasize this central feature of the mathematics teaching enterprise, and, further, we elevate not just the mathematics and not just the students' thinking, but the important space that lies at the intersection of these critical areas. And in this space we still have much to learn about how the mathematics must be understood for a teacher to effectively engage in professional noticing of students' mathematical thinking or how a focus on students' mathematical thinking leads to teachers' deeper learning of the mathematics. These questions seem to us to be both important and rich, and the papers in this section provide additional examples of how researchers are pursuing the study of student thinking through teacher noticing.

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Mathematical Teacher Noticing: The Key to Learning from Lesson Study

Mi Yeon Lee and Ban Heng Choy

Abstract Lesson Study has been adapted by many countries in support of teachers' learning from their practice. However, learning from Lesson Study does not come naturally and it is unclear how teachers can be supported in such learning. Moreover, lesson preparation, a critical component of mathematics teaching, is still largely under-explored in the study of teacher noticing. This chapter presents an analysis of what and how teachers notice when they make instructional decisions during the planning and reviewing stages of Lesson Study. It compares and contrasts two groups of elementary school teachers: one group of pre-service teachers (PSTs) from the United States, and the other group of in-service teachers (ISTs) from Singapore, in terms of what they see and think about their students' mathematical reasoning during Lesson Study. Using a notion of productive noticing, we provide snapshots of mathematics teacher noticing, which highlights the key role noticing plays in learning from Lesson Study, and offer insights as to how teacher noticing can be supported in the context of lesson planning and reflection.

Keywords Noticing · Lesson Study · Lesson planning · Lesson reflection · Teacher education

Introduction

To teach mathematics effectively, teachers should notice and build on student thinking, adjusting their instruction to support their students' learning (National Council of Teachers of Mathematics [NCTM], 2014). Teaching in this manner is

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both ambitious and challenging, and requires knowledgeable teachers to enact these productive teaching practices (National Research Council, 2005; Smith & Stein, 2011). In light of this, teacher education researchers suggest that meaningful teacher learning occurs when teachers have opportunities to reflect upon their teaching practice and work in professional communities in order to solve instructional problems related to their teaching practice (Ball & Cohen, 1999; Hiebert, Morris, & Glass, 2003). However, participation in these learning communities alone, while deemed necessary, would be insufficient. Instead, it is crucial that teachers develop a common language to discuss issues with regard to teaching and learning (Bryk, 2009; Loughran, 2009).

Mathematics teacher noticing is one such means to improve teaching expertise because how teachers pay attention to and make sense of what happens in their classroom can influence the quality of mathematics teaching (Jacobs, Lamb, & Philipp, 2010; Sherin, Jacobs, & Philipp, 2011). Despite the growing number of research studies on teacher noticing, many of these studies centred on exploring teacher noticing skills displayed when reviewing their teaching videos (Sherin & van Es, 2009; Star & Strickland, 2008; Stockero, 2008), and only a few studies examined teacher noticing during the lesson preparation (Choy, 2014; Santagata, 2011). For example, some researchers provided teachers with another teacher's instruction video and asked them to describe what they notice in the teaching video (Colestock & Sherin, 2009; Kersting, 2008; Star, Lynch, & Perova, 2011), and others asked teachers to retrospectively recall what they were noticing during their own teaching by watching a video from their own classroom (Ainley & Luntley, 2007). In some cases, researchers asked teachers to watch and discuss excerpts of their teaching video with other teachers as a peer group (Sherin & van Es, 2009). As part of a larger study, Choy (2014) explores what teachers notice during the lesson preparation stage of Lesson Study and extends the realm of the study of noticing to lesson planning.

Lesson Study is a collaborative teacher-inquiry professional development approach that emphasizes reflection on practice and students' thinking (Fernandez & Yoshida, 2004; Stigler & Hiebert, 1999), and can be used to effectively develop teachers' expertise and foster meaningful teacher learning. However, learning from Lesson Study does not come naturally (Takahashi & McDougal, 2016). In this regard, Fernandez, Cannon, and Chokshi (2003) highlight three critical lenses*, that of researcher, curriculum developer,* and *student,*needed to learn from the processes of Lesson Study. Adopting these lenses requires teachers to use varying perspectives to focus their attention on mathematically worthwhile aspects (Schifter, 2001) and interpret students' mathematical ideas in order to make instructional decisions productive for enhancing students' reasoning (Jacobs et al., 2010). Thus, we hypothesize that teacher noticing, which consists of observing, analyzing and responding (Sherin et al., 2011), plays a critical role in teachers adopting these lenses. Furthermore, even though it is important to prepare oneself to notice (Mason, 2002), the role of noticing during lesson preparation has been relatively unexplored.

In this chapter, we will examine mathematics teacher noticing during the planning as well as the review stages of Lesson Study by applying these three critical lenses. The key questions that guided our inquiry are as follows:

1. What do teachers notice when they plan and review lessons during Lesson Study?
2. How do teachers notice what they observe during Lesson Study?
3. How can we support teachers to learn from Lesson Study through a focus on noticing?

Theoretical Framework: Learning from Lesson Study

Critical Lenses for Learning from Lesson Study

Setting the different adaptations of Lesson Study implemented by various countries aside, Lesson Study in essence comprises five essential tasks—(1) developing a research theme; (2) working, discussing and anticipating student thinking through mathematics tasks; (3) developing a shared lesson plan; (4) collecting data during observation of research lesson; and (5) conducting a post-lesson discussion (Lewis, Friedkin, Baker, & Perry, 2011). These five tasks can be applied into three phases of a lesson such as lesson planning (Task 1–3), teaching (Task 4), and lesson reviewing (Task 5). Here, we will focus on teachers' discussion during the planning and reviewing phases of Lesson Study.

The potential of Lesson Study to improve teachers' practice (Fernandez & Yoshida, 2004; Lewis, Perry, & Hurd, 2009; Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012) can only be fully realized when teachers learn how to critically examine their lessons (Takahashi & McDougal, 2016). In this regard, Fernandez et al. (2003) provided three critical lenses that can be applied to examine lessons for the purposes of Lesson Study. The first lens is *the researcher lens* that encourages teachers to see themselves as researchers looking into their problems of practice. Putting on this lens requires teachers to develop the appropriate means to investigate their own research questions, and use evidence to explain the success of their intervention before they apply the findings to other similar contexts (Fernandez et al., 2003). The second lens—*the curriculum developer len*s—focuses teachers' attention on how to sequence activities and connect them to students' learning during the lesson. In this aspect, teachers are concerned with orchestrating students' learning both across and within lessons, bearing in mind the developmental progress of students' thinking. Finally, when teachers attempt to anticipate students' possible solutions to main tasks and consider how to use this knowledge to support students' deep understanding of the content, they are beginning to adopt the *student lens*. Adopting these lenses requires teachers to *notice* mathematically meaningful events in the classroom and adapt their instruction based on students' thinking while providing appropriate curricular materials to support students' learning (Schifter, 2001).

What is Mathematics Teacher Noticing?

Mathematics teacher noticing, a form of professional vision (Goodwin, 1994), can be conceptualized in three different ways (Jacobs et al., 2010; Miller, 2011; Sherin & van Es, 2009): noticing as (1) focusing on what teachers attend to; (2) focusing on teachers' interpretation about what they selectively attend to; and (3) combination of three actions such as attending to, interpreting, and responding to student thinking. In this paper, we adopt the third perspective of noticing, which consists of attending to noteworthy events, interpreting these events, and making instructional decisions based on interpretations of the notable events (Jacobs et al., 2010).

To characterize teacher noticing, two main dimensions of teacher noticing are examined: *what teachers notice* and *how teachers notice* (Sherin & van Es, 2009; van Es, 2011). The first dimension describes both *who* (e.g. whole class, student group, individual student, and the teacher) teachers focus on, and *which topics or issues* (e.g. pedagogical strategies, behaviour, mathematical thinking, and classroom climate) they identify. The second dimension captures how teachers analyse what they notice in terms of *analytic stances* (e.g. descriptive, interpretive, and evaluative) and the *depth of analysis* (e.g. whether to provide few details or ground their comments in evidence) when they make their instructional decisions. These two dimensions are also applicable for researchers seeking to examine teacher noticing during the planning, teaching, and reviewing phases of Lesson Study. Even though van Es developed the framework for learning to notice student mathematical thinking, for our study, the use of her framework is extended to investigate what and how teachers notice during the whole Lesson Study processes (see Table 1).

Noticing as a Way to Put on the Three Critical Lenses

It is "wishful thinking" to expect that "something good will happen" just because one gathers "teachers together to talk about practice" (Bryk, 2009, p. 599). As highlighted, it is crucial that teachers adopt the three critical lenses and focus on student reasoning when reflecting on their teaching in order to learn from Lesson Study. However, applying these critical lenses can be very challenging, and requires teachers to focus their attention on noteworthy aspects of their teaching practice. They need to attend to aspects of student thinking from classroom artifacts; student explanations; and discourses, and interpret them using a mathematical perspective before, during, and after a lesson (Goldsmith & Seago, 2013; Jacobs et al., 2010; Schifter, 2001; Smith & Stein, 2011). In many ways, these characteristics of noticing are similar to the notion of *extended noticing*, as proposed by van Es (2011).

Table 1
A framework for learning during Lesson Study

	What teachers notice	How teachers notice
Level 1 *Baseline*	• Attend to irrelevant details that do not have direct impact on student learning • Attend to whole class environment, behaviour, *generic* content and learning and to teacher pedagogy	• Form general impressions of what occurred • Provide descriptive and evaluative comments • Provide little or no evidence to support analysis
Level 2 *Mixed*	• Primarily attend to teacher pedagogy • Begin to attend to *particular* aspects of mathematical concepts and the difficulties associated with them • Begin to attend to particular students' mathematical thinking and behaviours	• Form general impressions and highlight noteworthy events or details • Provide primarily evaluative with some interpretive comments • Begin to refer to specific events and interactions as evidence
Level 3 *Focused*	• Attend to *particular* aspects of mathematics and relate students' confusion to the teaching approaches • Attend to particular students' mathematical thinking	• Provide interpretive comments • Refer to specific students' difficulties, events and interactions as evidence • Elaborate on these specific students' difficulties, events and interactions
Level 4 *Extended*	• Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking	• Provide interpretive comments • Refer to specific events and interactions as evidence • Elaborate on these specific students' difficulties, events, and interactions • Make connections between events and principles of teaching and learning • On the basis of interpretations, propose alternative pedagogical solutions

Note. Adapted from "A Framework for Learning to Notice Students' Thinking" by van Es (2011, p. 139).

Expert teachers, who are highly proficient in this work, can perceive meaningful patterns from what they see, and connect these observations to what they know, to make productive instructional decisions in the midst of a complex classroom environment (Berliner, 2001). These teachers are more sensitive and attuned to task demands and social contexts, and are better able to call upon different but useful strategies to solve their problems in practice (Berliner, 2001; Mason, 2002). This high level of attention is more active and intentional, rather than passive or spontaneous (Erickson, 2011; Mason, 2011; Miller, 2011; Sherin et al., 2011), and constantly seeks to use experience as evidence to form new ideas that can inform future practice (Schön, 1991). Hence, we argue that teachers can hone this specialized seeing, sense-making, and decision making by focusing their noticing on mathematically significant aspects of teaching and learning during the processes of Lesson Study. The three critical lenses put forth by Fernandez et al. (2003) will require teachers to notice specifically the mathematical concept, students' difficulty when learning the concept, and whether their teaching approaches address the difficulty.

The Three-Point Framework

These three areas for focusing noticing are similar to what Yang and Ricks (2013) term as the *Three Points*. They detail how Chinese teachers think about the design of a task in a lesson using three focal points: the *Key Point*, the *Difficult Point*, and the *Critical Point* (p. 54). The Key Point refers to the mathematical concept targeted in the lesson, which is sometimes known as the "Big Idea" (Askew, 2013, p. 6). The Difficult Point is the cognitive obstacle or stumbling block that students face when learning the Key Point. This can refer to persistent errors or common misconceptions that are associated with the concepts being taught. By anticipating students' Difficult Point, teachers begin to adopt the three critical lenses and design lessons targeted at the challenging aspects of learning the concept. The Critical Point is then the "heart of the lesson", which highlights the approach that teachers can use to support students in their efforts to overcome the Difficult Point, in order to learn the Key Point (Yang & Ricks, 2012, p. 43).

As an example, to teach fraction–decimal conversion at Grade 4 (age 10), a teacher may identify the key concept as the fact that common fractions and decimal fractions are different representations of the same number (Key Point); highlight students' confusion in terms of their inability to relate fractions with denominators other than 10 to decimals (Difficult Point), that is, they may put 1/5 as 0.15 because the digits "1" and "5" appeared in 1/5; and the proposed Critical Point is to create tasks where students can relate fractions such as 1/5 to fractions with denominators 10, 100, or 1000. This example illustrates how the Three Points can be used to direct teachers' attention to the relationship between specific aspects of the concept (Key Point and Difficult Point) to the design of the task (Critical Point). However, the ability to describe the details of the Three Points is dependent on a good understanding of mathematics as well as the experience in teaching the subject. Hence, this ability has been used as a distinguishing mark between highly and less proficient teachers in China (Yang & Ricks, 2013).

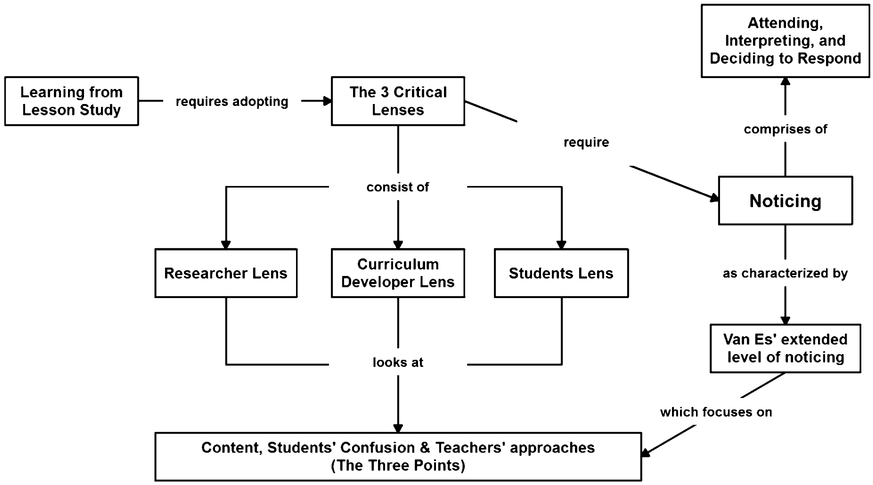

Figure 1. Theoretical framework to learn from Lesson Study.

Together, the Three Points (Yang & Rick, 2013) and van Es' (2011) framework for noticing can provide a useful way to examine what, and how, teachers notice when they proactively adopt the three critical lenses to learn from Lesson Study. In particular, we incorporate the Three Points into van Es′ framework to highlight specifically what teachers notice during Lesson Study. Figure 1 shows the relationships between the different theoretical constructs used in this chapter.

Methodology

Context of the Two Case Studies

Vignettes drawn from two contrasting case studies were explored in this study: pre-service teachers (PSTs) in the United States and in-service teachers (ISTs) in Singapore. These two groups of teachers represent two ends of the teaching experience spectrum with different cultural backgrounds. Even though one may argue that it is unfair to compare PSTs with ISTs from two different countries, we want to highlight that the purpose of this study is *not* to compare them in terms of their noticing expertise. Instead, we want to explore the *common* characteristics of their noticing, which lead to both captured and missed opportunities to learn from the processes of Lesson Study. By selecting these contrasting cases, we believe that the findings have the potential to produce insights about the role of noticing in learning from Lesson Study, particularly when we hypothesized that challenges to noticing productively and the characteristics of more productive noticing might be similar in these seemingly different cases. This replication logic is an important consideration for case study research (Yin, 2003).

The first Lesson Study group consisted of six elementary PSTs (Amy, Christina, Erin, Hera, Jane, and Mary), a facilitator delegated from the university, and a host teacher invited for this study. The PSTs were engaged in six Lesson Study sessions in a U.S. primary school (aged 6–8) through a weekly three-hour field experience. The PSTs were instructed to teach number sense including counting, addition, and subtraction during the field experience. The vignettes, described in this chapter, were developed from episodes, which occurred at the first and last session respectively. The objectives of the first lesson were to count sets of objects up to 60, and to figure out how many pieces would fit into the large shape by allowing students to cover an area of the large shape using smaller pattern blocks. The objective of the last lesson was to help students generate various strategies to add or subtract two numbers.

In the second case, seven ISTs and a school leader participated in six Lesson Study sessions that explored the teaching of fractions for Primary Two students (aged 7–8) in a Singapore elementary school. Four of the teachers have more than 10 years of teaching experience and the others have at least three years. Two of the more experienced teachers—Zelina (25 years) and Hannah (16 years)—are of particular interest in this chapter. The teachers were part of a larger study on teacher noticing conducted by the second author. However, this chapter reports the initial phase of the study, where the ISTs were not introduced to the notions of teacher noticing. The seven teachers, Hannah; Alice; Heather; Heidi; Jacinda; Sherry; and Zelina, worked together to plan a lesson on ordering *Unit Fractions* for Primary Two students. Vignettes, developed from the planning and review sessions of the Lesson Study, are presented and discussed.

Data Collection

Data for both cases were collected and generated through video or voice recordings. The data from U.S. were collected by video-recoding two Lesson Study discussions at the beginning and the end of a mathematics field experience. In the two videos, the same pair of PSTs co-taught the two mathematics lessons and other PSTs observed their lessons. The data from Singapore were collected through voice recordings of the Lesson Study sessions involving all seven ISTs and their school leader, Jaslyn who participated in the discussions. We watched and listened to the video and voice recordings to mark out segments, which reflected similar levels of noticing according to the adapted van Es' framework. These segments were then transcribed for further analysis without editing any ungrammatical or colloquial language.

Data Analysis

For this study, we characterized teachers' noticing in terms of *what* and *how* they notice. By extending van Es' (2011) framework to Lesson Study (see Table 1), we evaluated *how* teachers notice based on six components: (1) whether the statement was general or specific; (2) whether the statement was descriptive, evaluative, or interpretive; (3) whether the statement was based on evidence; (4) whether the statement elaborated on events and interactions; (5) whether the statement made connections between events and principles of teaching and learning; and (6) whether the statement proposed alternative pedagogical solutions. With regard to *what* teachers notice, we coded what they discussed in terms of the Three Points (Yang & Ricks, 2012). To aid our analysis, we developed a matrix (see Table 2) to examine the three processes of noticing (Jacobs et al., 2010) in relation to the *Three Points* (Yang & Ricks, 2012). We then independently completed the matrix by extracting fragments of transcripts from the selected episodes to uncover the emerging themes.

Table 2
An example of a matrix used in analysis

	Attending to	Making sense of	Deciding to
Key Point			
Difficult Point			
Critical Point			

For each selected segment at the respective noticing levels, we analysed *what* a teacher noticed with regard to the Three Points by deconstructing noticing into the three processes: Attending, interpreting, and deciding how to respond. For example, when teachers noticed at the baseline level, they often only noticed whole class environment, behaviour, or teacher pedagogy, which were not directly related to the Three Points. In such cases, we left the matrix blank and rated "missing the 3 points". In cases where teachers' noticing was more focused, we coded what teachers attended, interpreted, or responded to with regard to the Three Points. When there were discrepancies in analysis, we reconciled our differences by intensively discussing them. We then considered two dimensions of noticing (what and how) from the matrices with our notes and assigned the levels of noticing, assigning a lower level of noticing where the two dimensions misalign. Finally, we wrote vignettes illustrative of noticing at that level.

What and How PSTs and ISTs Notice During Lesson Study?

Our findings indicate that both PSTs and ISTs found it challenging to focus on mathematically significant aspects, such as the Three Points, during initial Lesson Study discussions. Of particular interest in this study, we found that both PSTs and ISTs demonstrated a more focused level of noticing when they noticed aspects related to the Three Points. In this section, we present some representative vignettes of what and how the teachers in our study noticed at the different levels of noticing during the Lesson Study discussions at the planning and reviewing stages.

Teacher Noticing During Lesson Planning

PSTs' focusing on aspects less relevant to mathematics. As Takahashi and McDougal (2016) argue, it is critical for teachers to think about the mathematical content and relate the lesson design to the students' thinking. Therefore, without a focus on the Three Points, teachers are unlikely to gain new understanding of mathematics and teaching. We note that the PSTs tend to focus on less relevant issues when discussing the task, especially during the initial Lesson Study session. For example, the PSTs seemed to focus largely on management and organization issues, instead of lesson content or pedagogical strategies, when examining the textbook during the planning stage. In this excerpt, the facilitator asked the PSTs how they could support students in learning to compare the size of two numbers on a number line. Hera began the discussion with the following idea:

1.	Hera	First, we need to think about how to organize students for this activity.
2.	Erin	Are you thinking of teaching number lines per table or per student?
3.	Host teacher	Or just up on the wall or big group.
4.	Jane	It looks kind of like he [the teacher pictured in the textbook] is teaching the whole class.
5.	Mary	Yeah, I think that's his whole class but I think it'd be nice to follow the small group thing. If not, it would be hard to control students.
6.	Jane	Yeah, I was thinking by table or something. So we would have a number strip per table.

As seen from the exchange, the PSTs decided to use the given activity in the textbook without any reflection and focused mainly on logistical issues during lesson planning. They did not consider whether the activities were appropriate or whether they need additional activities to achieve the Key Point. Also, they neither thought about students' possible Difficult Points in learning the Key Point, nor how to help students overcome the difficulties (Critical Point). Furthermore, the PSTs did not attend to students' thinking at all in terms of the Key Point, Difficult Point, and Critical Point during the session, and made generic suggestions about the lesson with little or no justification. For instance, when Jane suggested whole class teaching based on the picture in the textbook, Mary suggested group work because of difficulty of managing students, but without any sound pedagogical rationale. Therefore, PSTs generally did not engage in any analysis of the teaching materials and did not provide any interpretative comments with regard to the choice of strategies. In this respect, PSTs showed the *baseline* level of noticing.

ISTs' baseline noticing of the Three Points. It is possible that the PSTs failed to focus on the Three Points because of their lack of teaching experience, however, focusing on the Three Points can be challenging even for the ISTs. Furthermore, the ISTs may focus on the Three Points superficially without noticing specific details. For example, Hannah, an IST, began the initial discussion by sharing the Lesson Study goals on 'Unit Fractions', and suggested that they sharpen their questioning techniques:

> We have picked fractions as the main cause of concern because of the data that we have collected from last year's P2 [Primary 2] cohort teachers saying that the children are still not good in fractions and particularly the basic skills of ordering fractions… also they are having some problems. Because of the data we have collected from item analysis, we then decided to focus on fractions as our area of concern. And also… we also talked about questioning techniques that we have gone through as a school… how we could actually sharpen our questioning techniques to actually help children to learn fractions…

Although Hannah made reference to the concept and confusion targeted in the lesson, she did not elaborate clearly what she meant. Hannah presented the ordering of unit fractions as the Key Point in the lesson. However, she did not articulate the aspect of ordering fractions that was critical for teachers to consider. Instead, she pointed vaguely to "focus on fractions" as the "area of concern". Even though Hannah mentioned that students were "still not good in fractions" based on "evidence" from item analysis, she did not specify what these findings were. These findings would have been useful for teachers to understand students' difficulties with the concept, which could have led to a better design of the lesson.

Moreover, Hannah went on to suggest that teachers focus on their questioning techniques, but she did not link this suggested Critical Point to students' confusion about the topic. Therefore, although Hannah referred to the Three Points, the lack of specific details prevented teachers from pinpointing students' confusion about ordering unit fractions, which could have led to a more targeted approach. Furthermore, Hannah did not offer any evidence to support her analysis. Hence, Hannah's noticing, according to adapted van Es' framework, is at the *baseline* level (van Es, 2011).

Focused noticing of the Three Points. In contrast, during the final Lesson Study discussion, both PSTs and ISTs began to notice at a *focused* level when they adopted the researcher's lens by providing specific details relating to the Three Points. However, here we only illustrate ISTs' case because of page limit. In the following vignette, Hannah was able to attend to a subtle point missed by the other ISTs. In this discussion about the use of examples and non-examples to help students recap the fractional notation *a*/*b*, the research teacher Zelina wanted to highlight the role of equal partitioning in the fractional notation. She wanted to demonstrate physically an example and a non-example of 1/4. Zelina showed two rectangles—one was divided into four equal parts and the other was not—to demonstrate what she intended to do during the lesson (Figure 2).

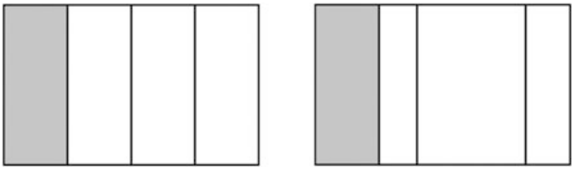

Figure 2. Zelina's representation of an example and non-example of 1/4.

To highlight the importance of equal partitioning in the fractional notation 1/4, Zelina used a detachable piece of the shaded part to show the meaning of 1/4. She removed the first shaded part and compared it to the rest of the parts of the first rectangle to show that they were equal, and hence demonstrating that the shaded part was 1/4. She then took another detachable piece (of the same area) in the second whole, and said that it was not 1/4 of the second whole *because the second whole was not divided equally*. Hannah then raised a point of clarification:

1.	Hannah	If you take the same piece, the same piece is still 1/4 of that whole.
2.	Jaslyn	This is still 1/4 of the whole… this one is not, but no… it's still 1/4 of the whole?
3.	Hannah	Yes. You must take the small one or the big one. It's still 1/4. Because it's equivalent fraction, you can subdivide that…
4.	Zelina	I don't know… make up your mind. Take or don't take?
5.	Hannah	It is still [1/4 of the whole]… you must take something that is not equal to 1/4. Because that is still 1/4 of the whole.
6.	Jacinda	… yes… yes… yes… It's still 1/4.
7.	Zelina	So, take or don't take?
8.	Hannah	You still take. But you must take a smaller or bigger piece. It's the same whole. It's still 1/4, only that we have shifted it in a way…
9.	Zelina	Where? It's not equal, right?
10.	Jaslyn	[Jaslyn shows the piece physically and compares it to the other whole which is not divided up equally] because this piece is still 1/4 of this whole…
11.	Zelina	Oh…I see.

In this episode, Hannah attended specifically to Zelina's statement that the second detachable piece is "not 1/4 because the second whole was not divided equally". This challenged the teachers' notion of equivalent fractions (Lines 2, 4, 6), and generated a useful point with regard to the choice of example (Line 5). As a result of this specific attention to mathematical details, the teachers became more aware of the subtlety of their own conceptions of fractions, and were more able to see why students might have difficulty with fractions, given that teachers themselves may also sometime struggle with the notion. For example, Jaslyn tried to make sense of what Hannah said by physically manipulating the detachable fractional piece (Lines 2, 10), and she struggled with the concept for a brief moment (Line 2) before she came to the same conclusion as Hannah that "this piece is still 1/4 of this whole" (Line 10). Consequently, Hannah's noticing highlighted Zelina's subtle error to the teachers for discussion, and they were alerted to a possible misconception that might arise as a consequence of overemphasizing the notion of equi-partitioning.

The error involved is not trivial—that the process of dividing a whole into four equal parts gives rise to an object that is 1/4 of the whole and that object can have many different pictorial representations, but it remains 1/4 regardless of any division of the same whole. The error could have occurred because of the partial conception that fractions can only involve equal parts. Unequal partitions can be challenging for students (Schoenfeld & Kilpatrick, 2008) and can be difficult even for some teachers, as suggested in this case. Therefore, as Schoenfeld and Kilpatrick (2008) have emphasized, it is important that teachers are aware of this difficulty and be fluent with the use of different representations of fractions. Hence, Hannah's noticing of Zelina's explanation can be classified as *focused* because she provided interpretative comments about the concept of equi-partitioning and highlighted how Zelina's use of the fractional diagrams might be misleading.

Teacher Noticing During Lesson Reviewing

A critical feature of Lesson Study is teachers reflecting on the lesson to generate new understanding of how students think, and connect this new understanding to broader principles of teaching and learning (Fernandez et al., 2003; Yang & Ricks, 2012). In our study, both PSTs and ISTs often engaged in less-than-effective reflection during initial Lesson Study. In the following vignette, we see that reflecting upon a lesson to gain new insights into teaching and learning was challenging, even for the ISTs.

Not focusing on student thinking. During the initial post-lesson discussion, Zelina's first and only comments were about the clarity of her instructions on the task, and not focused on student thinking. She was pleased that most students were clear about the key task of making comparison statements about fractions except for a few who picked up two equal pieces representing a tenth:

> What I saw was… my instructions were clear enough. I said, 'take out one tenth'. But when I was going around, I realised that some of them took two "tenths" instead of one. Instead of one unit fraction, they took a few more. I think they still have difficulty grasping the greater denominators and smaller fractions. They have some inkling but have not touched down yet… it's not easy… to make the whole.

Even though she gave detailed description of her observations ("… took two tenths instead of one"), she did not seem to attend to details related to the mathematical concept (comparing fractions), students' confusion about the concept (inappropriate ideas related to sizes of numbers), nor how students responded to the lesson approach (the need to reason about the size of fractions). Zelina did not seem to distinguish between what was mathematically relevant and what was not with regard to the lesson, and made general or vague statements about students' thinking. Zelina was aware that her students might not have fully understood the use of denominators to compare unit fractions ("They have some inkling…"), and might have difficulties seeing the relationship between denominators and relative sizes of unit fractions ("they still have difficulty grasping the greater denominators and smaller fractions"). However, she did not give further details on how she came to that conclusion and why that was so. Therefore, while there was evidence that she attended to some aspects of her students' thinking, the lack of detailed connections between what she observed and the 'Three Points' did not help refine ideas about the student' thinking nor the design of the tasks. Hence, Zelina's noticing is at the *baseline* level because she had begun to refer to specific events but did not provide much analysis.

Similarly, the *baseline* level of noticing is demonstrated by the other teachers when they shared their observations. Almost every one referred to an incident where Zelina tried to help her students recall the meaning of numerator and denominator through the use of a song that she composed. Zelina taught two songs in previous lessons to help students remember the definitions of key words such as fractions, numerators, and denominators. Even though the song was never discussed during the meetings, the teachers seemed to be impressed by the use of the song as a mnemonic. For example, Heidi liked how Zelina used songs to help them recall the definition without providing further evidence:

> Actually, I like how she get [sic] them to recall… the numerator and denominator… using a simple song.

Similarly, Jacinda commented that the lesson was good and liked the use of the song to "reinforce" the definitions:

> Overall, I think that her lesson was very good because I can see that her children, even though they are lower ability, they managed to get the concept very well. Like Heidi, I also like the use of the song to reinforce the fractions, the numerators and denominators…

Even though the use of the song might have counted as an instructional strategy, the teachers mostly attended to how the song was "interesting" and "catchy". All the teachers highlighted that the song helped the students remember the terms, but they did not provide any further substantiation, thus noticing at the *baseline* level.

ISTs' focused noticing of the Three Points. To illustrate how noticing directed by the Three Points can promote a *focused* noticing, we examine how Hannah generated useful pedagogical considerations from her detailed observations during the final Lesson Study session. In the following vignette, Hannah described how two students struggled with a question and highlighted that these two students were still thinking about fractions physically rather than symbolically because they used the aids to help them:

> … [the question] 1/7 is smaller than… he put 1/8. I said look again… then he look [sic] and looked. Although he put there 1/7, they still take the 1/7 fraction disc and put it on top of the representation 1/7. They want to see it … so obviously they are looking at the size, the physical size. So, they put there 1/7 and then put there 1/8, and they put it again … is it smaller, oh, it's swapped. But you can't swap it because it's already written there 1/7. Because it's not an open-ended… 1/7 is written… then they said, 'Oh no, cannot erase…' and then they panicked already… so what to do… Then later, a few minutes later… what can you do … then swapped, swapped, swapped back, but when it's swapped back, it's wrong, wrong, then stack, yeah, it's smaller… then how… then finally [Another student] said, 'take another fraction!'

Hannah's noticing contrasted with that of the other teachers in terms of the level of details given, and more importantly, how she linked her interpretations to specific instances and combined her understanding to generate a useful principle. Hannah felt that not all the students understood, and saw beyond the students' seemingly correct answers during the classroom discussion in her relatively detailed description of a particular student's thinking. She contended that students might not have seen fractions as a representation of a part–whole relationship without the physical manipulative. Moreover, Hannah also noted that the students might have problems seeing how the number of equal pieces needed to make up the whole could have been related to the size of the pieces. Therefore, even though students could have performed the task correctly, or have answered Zelina's questions correctly, they may not necessarily have understood the concept:

> They are able to do but may not be able to relate it back to the whole. Like why is the whole… I think it's logic and we assume that they know… that for the same whole, this one has many pieces and this one has lesser pieces, then this should be a smaller piece. Maybe this logic must come in at another platform… However, the children need some wait time, some thinking time, some verbalisation and articulation among themselves… You might want to hear… are they saying it?

It seemed probable that Hannah did not consider "chorus answers" to be indicative of students' ability to reason about the relative sizes of the unit fractions. Instead, her reflections highlighted the possibility that students may not understand the key idea of the lesson even though they had responded correctly to Zelina's questions. Using what she observed about the two students, Hannah analysed their thinking, and suggested that students need more opportunities to reason amongst themselves. Thus, Hannah's noticing here is at the *focused* level.

PSTs' focused noticing of the Three Points. Similarly, the PSTs were capable of focused noticing when they directed their attention to students' thinking. In a later Lesson Study session, the PSTs provided interpretive comments on students' thinking with detailed examples of students' performances and excerpts from their interactions in which students' thinking was probed. For example, in terms of the Key Point, Mary shared her observation about a student's interesting idea in composing and decomposing numbers for addition by referring to specific events and interactions. That is, during the Lesson Study, Mary demonstrated that she attended to a student's work on the question asked by the teacher, how 7 + 5 equals 3 + 3 + 3 + 3 as follows:

> Three over three and then a line in the middle and then three over three and then at the bottom he [A student, Adam] had six and six and so I ask him how many do you have all together and he said 66, I was like does 3 + 3 + 3 + 3 = 6 [and] 6 and he was like no and I was like what are you supposed to do, oh six plus six is twelve so he got that concept and then when you [The teacher, Erin] went to how does 7 + 5 equal 3 + 3 + 3 + 3 then he preceded to say ok, you have 3 + 3, which is 6 and he says you take away, you borrow one from the three. ……I don't understand why he put like a one and he was like you take one away from here and he wrote that under or next to his three and he goes ok now you have two and three and that's five and I'm like but six plus five, six plus five is not twelve and he was like no you take the one you borrowed and you add it to the six and that makes seven and you have seven plus five equals twelve.

In this data excerpt, Mary first interpreted that the student tried to solve the given problem in this way: 3 + 3 + 3 + 3 = 6 [and] 6. However, the student's solution did not make sense to Mary and she asked a question for clarification. By attending to the student's explanation, Mary was able to interpret the student's strategy to compose the two 3s and decompose one 3 into 1 and 2 to make 7 + 5. This shows that Mary had initially attended to the student's idea and understood it before she responded in a way that probed the student's idea. By investigating one student's reasoning in detail, Mary demonstrated the three processes of noticing with regard to the Key Point.

Also, in terms of Difficult Point, Mary attended to students' struggle with a question (e.g. if we have 12 kids and 24 cubes, how many cubes would each student get?), which was given after addressing some addition strategies to figure out the total number of students when there are 12 girls and 12 boys in a classroom.

> Well, like it's a lot of memorization because when you did how many, there's twelve of us and you have twenty four blocks how many do each of them get, they were both like add them it was like everybody would get two and then once you broke it down to if there's twenty four kids in the whole class how many will they get so Gabriella's well everybody would get one but she was like if you gave everybody three not everybody would have at least one so like they were going off of that and so then when I was talking to Adam about 24 all together … cause he was confused in the beginning … now there's 24 kids in the classroom and he's like 12 plus 12 is 24 …, he kept saying that, like he knew that was the it … how many will each student get and … he got it but the other two didn't get it but they all understood that 12 plus 12 equals 24.

When Mary changed the question slightly ("how many cubes would each student get if there are 12 kids and 24 cubes?"), another student, Gabriella, did not understand the reasoning behind her own solution although she gave the correct answer to the first question. Mary's response to the student's reasoning demonstrated that she tried to take on the *student lens* by relying on evidence to judge whether students clearly understood the content. Also, Mary highlighted that Adam, a very advanced student in the class, took some time to get the question while two other students did not get it although they understood 12 + 12 = 24.

Mary's remarks showed that she referred to specific evidence and instances of interaction among multiple students and elaborated them in order to provide interpretive comments on students' mathematical thinking, indicative of *focused* noticing. However, Mary did not propose any alternative pedagogical solutions to address students' challenges in learning the Key Point, which would have brought her noticing to the *extended* level.

Discussion

This study supports findings by Fernandez et al. (2003) that adopting the three critical lenses in Lesson Study is not trivial, and extends the findings by Star et al. (2011) to indicate that both experienced and beginning teachers are also not necessarily effective observers of mathematics lessons. More importantly, although Takahashi and McDougal (2016) highlight the key features of Lesson Study that may maximize the impact of Lesson Study, we have demonstrated that *what* and *how* teachers notice is critical for the benefits to be fully realized. Our findings suggest that teachers' higher levels of noticing are usually accompanied by their attention and interpretation of mathematically significant aspects of teaching and learning.

Given the wide spectrum of things to observe, it is not surprising that both groups of teachers may focus on aspects that do little to enhance their understanding of students' thinking. Without an explicit guiding focus, teachers noticed a wide variety of events and details, both relevant and irrelevant to the tasks of Lesson Study (Star et al., 2011). A vague focus, such as student mathematical thinking, also seems to be too broad for teachers to maintain their attention on noteworthy details during Lesson Study. Instead, a sharper set of focal points, such as the *Three Points*, may be more useful for teachers to guide their noticing as suggested by the findings of this research.

This study suggests that Lesson Study can be a possible means to develop noticing expertise. Even without any other professional development activities to hone teachers' noticing, both groups of teachers demonstrated some instances of higher level noticing. However, both groups did not demonstrate an *extended* level of noticing. That is, they did not focus on how their observations and interpretations were related to the instructional decisions that could have potentially enhanced students' mathematical thinking. This highlights that more attention needs to be placed on supporting fruitful teachers' noticing during Lesson Study.

A possible way to do this would be to incorporate the processes of noticing within the Lesson Study protocols. This could come in the form of questions or prompts or lesson plan templates to direct teachers' focusing. Another possible strategy is to use frameworks such as van Es's (2011) framework for noticing student thinking to guide teacher focusing. Our findings also suggest a synergistic relationship between developing teachers' noticing expertise and developing teachers' abilities to adopt the critical three lenses. If teachers noticed mathematically significant details during Lesson Study, they are more likely to make instructional decisions that promote students' thinking.

On the other hand, Lesson Study, with a special focus on studying lesson materials (Takahashi & McDougal, 2016), can offer opportunities for teachers to develop the eyes to see, the ears to hear, and the mind to think about teaching and learning. More importantly, the evidence from our study reveals the critical role of noticing in learning from Lesson Study. Although our study involved only two small groups of teachers, and the findings are limited by our methodological approach, this research warrants a need to examine, more closely, the role which teacher noticing may play in learning from Lesson Study, as well as how Lesson Study can be used to develop noticing expertise.

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Learning to Notice Student Thinking About the Equal Sign: K-8 Preservice Teachers' Experiences in a Teacher Preparation Program

Leigh A. van den Kieboom, Marta T. Magiera and John C. Moyer

Abstract In this chapter, we present our work and research related to preservice teacher (PST) noticing, describing how we provide PSTs with opportunities to notice student thinking about the equal sign and equality. We designed an instructional intervention in an integrated mathematics content and pedagogy course (with a field experience) to support PSTs in (1) learning about key mathematical ideas related to the equal sign and equality, and (2) rehearsing teacher noticing skills. Our PSTs rehearsed and reflected on their noticing skills by conducting two one-on-one clinical interviews with elementary students and participating in debriefing interviews with course instructors. Using this context, we examined (1) the extent to which PSTs attended to and further explored student understanding of the equal sign and equality, and (2) what PSTs perceived they learned about aspects of their teacher professional noticing skills and student thinking about the equal sign and equality. Our results indicate that the PSTs predominantly noticed the strategies students used to solve a task without focusing on student thinking about the equal sign and equality. In addition, our PSTs perceived that they strengthened either their own knowledge or student knowledge of the equal sign and equality while conducting their diagnostic clinical interviews.

Keywords Preservice teacher noticing · Student relational thinking about equality · Clinical interviews · Teacher preparation

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Introduction

Mathematics education reform initiatives emphasize the need for teachers to support students in developing conceptual understanding and sense making of mathematical concepts and procedures (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics, [NCTM], 2000). Teachers who use student thinking to guide instruction and whose instruction supports sense making by connecting various concepts and procedures across the discipline have a positive impact on student learning (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Kilpatrick et al., 2001). To help teachers, including preservice teachers (PSTs), learn to focus on student thinking, researchers have begun to characterize the professional noticing skills teachers draw on when they use student thinking to guide instruction. These interrelated skills include (1) attending to student strategies, (2) interpreting student understanding, and (3) deciding how to respond based on student understanding (Jacobs, Lamb, & Philipp, 2010).

Mathematics teacher educators argue that in order to prepare PSTs to use student thinking to guide their instruction, teacher education programs must support PSTs in developing their noticing skills early in their program (Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2002). In this chapter, we report on our work and research related to PST noticing, describing how we adapted Jacobs et al's. (2010) model of teacher professional noticing to provide PST's opportunities to rehearse their noticing skills. In contrast to Jacobs et al's. model, which focuses on classroom instruction, our work provides PSTs with opportunities to attend to and further explore student mathematical thinking in the context of one-on-one diagnostic clinical interviews. We use diagnostic clinical interviews as a context for our work because clinical interviews help us to support PSTs in focusing on one student's thinking about the equal sign as they rehearse their noticing skills. Interviewing one student reduces the number of distractions PSTs may encounter while rehearsing noticing skills during classroom instruction. We also provide PSTs with opportunities to reflect on their teacher professional noticing skills, thus helping to optimize their growth as effective teachers. Motivated by the need to provide PSTs the opportunity to develop their teacher professional noticing skills and to prepare PSTs to engage their future students in relational thinking about equality, then, this study examined the following questions:

(1) To what extent do PSTs attend to and further explore mathematically important aspects of student thinking about the equal sign during clinical interviews?
(2) After conducting clinical interviews, how do PSTs perceive their learning about aspects of teacher professional noticing and student thinking about the equal sign?

Preservice Teacher Noticing

Much of the research on developing PSTs' capacity to notice relies heavily on the use of video recordings of mathematics classroom instruction (McDuffie et al., 2014; Schack et al., 2013; Star & Strickland, 2008; van Es & Sherin, 2002). For example, van Es and Sherin (2002) documented how they used video recordings of mathematics classroom instruction to improve their PSTs' noticing skills. After viewing selected video clips of mathematics classroom instruction, the PSTs in their study were able to shift their noticing skills from a simple reporting of a sequence of events to identifying and analyzing the salient features of mathematics teaching and learning.

In their study with secondary PSTs, Star and Strickland (2008) specifically focused on the first component of teacher professional noticing, attending. They found that guided viewing of video-recorded mathematics instruction improved their PSTs ability to attend to the classroom environment, the mathematics content of a lesson, and the communication between teachers and students.

McDuffie et al. (2014) engaged their PSTs in analyzing video recordings of mathematics instruction using four lenses: Teaching Lens, Learning Lens, Task Lens, and Power and Participation Lens. Like van Es and Sherin (2002), they found that this structured activity assisted their PSTs in moving from a simple description of what they observed to being able to discuss significant classroom interactions.

Schack et al. (2013) specifically focused their study of PST noticing on elementary students' early arithmetic learning. In their research, Schack et al. first asked their PSTs to observe video clips of a teacher educator conducting one-on-one diagnostic interviews of students solving arithmetic problems. Then, the PSTs conducted their own diagnostic interviews to rehearse all three components of their teacher professional noticing skills. As consistent with the other studies discussed here, Schack's PSTs improved all three components of their teacher professional noticing skills.

Our work on PST noticing is similar to each of the studies discussed above in that our goal was to engage PSTs in opportunities to rehearse and improve their noticing skills. On the other hand, our work differs significantly from the first three studies described above in that we assessed our PSTs' noticing ability during face-to-face work with students rather than during virtual work with *representations* of practice (Grossman et al., 2009). In our study, the PSTs were required to attend to student strategies, interpret student thinking, and decide how to respond in the moment, i.e., instantly. The PSTs in the other studies, however, had the luxury of time as they attended to, interpreted, and decided how to respond to video recordings of mathematic classroom instruction. Our work also differs from the first three studies in that it focused specifically on one particular aspect of student mathematical thinking rather than mathematics classroom instruction. While it is true that like us, Schack et al. emphasized only one aspect of mathematical thinking, our work differs from theirs in that our work focuses narrowly on student thinking about the equal sign, while theirs focuses more generally on early arithmetic.

Student Thinking About the Equal Sign and Teacher Professional Noticing

To provide a context for the need to support PSTs in learning to notice how students might think about the equal sign and equality, we present an overview of the research on relational thinking about equality. Carpenter, Levi, Franke, and Zeringue (2005) defined relational thinking about equality as the ability to examine relationships among quantities using the fundamental properties of equality, numbers, and operations, rather than following a series of steps or procedures to solve an equation. Mathematics education researchers argue that students who have a relational understanding of equality are positioned to make a more successful transition from the study of arithmetic to the study of algebra (Knuth, Alibali, Weinberg, McNeil, & Stephens, 2005; Knuth, Stephens, McNeil, & Alibali, 2006). Despite that relational thinking about equality has been identified as an important component of student success in algebra, researchers have found that both practicing teachers and PSTs fail to *notice* the conceptions and misconceptions students have about the equal sign and equality and are often unaware that a limited understanding of relational thinking about equality has a negative impact on student learning (Asquith, Stephens, Knuth, & Alibali, 2007; Stephens, 2006). In addition, most K-8 teachers devote little instructional time to this important topic (Knuth et al., 2006).

In their work on relational thinking, Carpenter et al. (2005), Matthews, Rittle-Johnson, McEldon, and Taylor (2013), and Knuth et al. (2006) described the conceptions and misconceptions students have about the equal sign, how student thinking develops over time, and how teachers might foster relational thinking in their students. Their work can be used to support teachers in learning to *notice* student thinking about the equal sign and equality. For example, teachers would be more effective if they learned to notice the thinking of students who view the equal sign as an invitation to perform an operation. These students do not understand that the equal sign separates equivalent quantities and often interpret number sentences that are not in the form $a + b = \square$ as impossible to solve. Teachers are also well advised to notice the thinking of those students who have moved beyond thinking about the equal sign as a symbol to operate and can recognize the properties of equality, thus accepting as true number sentences in the form $\square = a + b$. Teachers should also notice the thinking of students who understand the equal sign as a symbol of equivalence that separates same quantities on either side of the equal sign. They ought to notice that to verify that the expressions on either side of the equal sign represent the same quantities, students often calculate and compare the two sides using a prescribed set of procedures. Finally, it is important for teachers to notice the thinking of students who view the equal sign relationally as a symbol that separates equivalent quantities. Specifically, these students can determine whether or not the quantities on both sides of the equal sign are the same by analyzing the relationships between the quantities, without computing. Table 1 summarizes the literature on the different ways that students think about the equal sign (Carpenter et al., 2005; Knuth et al., 2006; Matthews et al., 2013).

Table 1
Student thinking about the equal sign

Student thinking about the equal sign	Sample tasks and student thinking
Rigid Operational Thinking Thinking about the equal sign as an indicator to perform an operation, to solve the problem, find the answer	"You can't solve ☐ = 7 + 6 because the problem is backwards. The answer goes after the equal sign" "The number 9 goes in the box because 5 plus 4 is nine. Then you add 9 and 6 to make 15" (solving 5 + 4 = ☐ + 6)
Flexible Operational Thinking about the equal sign *as an operator* but accepting the symbol of equality in sentences where operations are not necessarily directly followed by the equal sign, e.g., $c = a + b$	"The number 13 goes in the box to make the sentence true. It doesn't matter, you could also do 7 + 6 = ☐" (solving ☐ = 7 + 6)
Computational Thinking Recognizing that the equal sign represents a relation between two equal numbers; carrying out calculations on either side of the equal sign to determine whether or not the quantities are the same	"Both sides need to equal twelve so 8 goes in the box, 8 and 4 is twelve, 5 and 7 is twelve" (solving ☐ + 4 = 5 + 7)
Relational Thinking Recognizing that the equal sign represents an equivalence between two quantities; comparing the mathematical expressions on both sides of the equal sign without carrying out the calculations	"The number 3 goes in the box because 10 is two less than 12 and 8 is one less than 9 so I need 3 to make both sides the same." (solving 12 + 9 = 10 + 8 + ☐)

Elementary and middle school students who resist changing their conception of the equal sign from that of "an indicator to operate" to that of a "symbol of sameness" benefit from explicit instruction that focuses on relational thinking about equality throughout the rest of their K-8 experience (Alibali, Knuth, Hattikudur, McNeil & Stephens, 2007; Carpenter, Franke, & Levi, 2003; McNeil & Alibali, 2005a, 2005b; McNeil et al., 2006). Given that K-8 students benefit from such instruction, it is important that PSTs learn to notice student thinking about the equal sign and equality and thus become equipped to identify and capitalize on, in their future practice, opportunities that might foster relational thinking about equality in their students.

Method

Context

Our work and research on PST noticing is grounded in the literature related to enacting ambitious mathematics instruction (Ball, Sleep, Boerst, & Bass, 2009; Grossman et al., 2009; Kazemi, Franke, & Lampert, 2009), which involves supporting PSTs in "actually doing the practice of teaching" (Kazemi et al., 2009, p. 12). With this idea in mind, we designed an instructional intervention that took place simultaneously in two semester-long mathematics content courses. Each mathematics course was integrated with a corresponding pedagogy course (with field experience). We situated our PSTs' opportunity to rehearse their teacher professional noticing skills and apply their knowledge of student thinking about the equal sign in the context of two diagnostic clinical interviews, which the PSTs conducted with elementary or middle school students. Designed by course instructors, the clinical interview protocol included a series of nine tasks written as a foundation from which the PSTs could elicit student thinking about the equal sign and equality (see Appendix 1).

Mathematics content courses. The two mathematics content courses (Number Systems and Operations for Elementary Teachers, and Algebra and Geometry for Teachers) provided the PSTs the opportunity to examine key mathematical concepts found in the elementary and middle school mathematics curriculum (e.g., relational thinking about equality). In both mathematics courses, the PSTs learned about the conceptions and misconceptions elementary and middle school students might have about the equal sign and equality (Table 1). They discussed specific problems that could be used to uncover these conceptions and misconceptions and they engaged in activities designed to help uncover them. They also analyzed the levels of mathematical thinking about the equal sign and equality that a given problem might evoke.

Pedagogy courses (with field experience). Students in each mathematics course were concurrently enrolled in a corresponding pedagogy (with field experience) course: Teaching Elementary School Mathematics and Teaching Middle School Mathematics. The instruction in the pedagogy courses was designed to engage the PSTs in rehearsing effective pedagogical practices and to prepare them to conduct diagnostic clinical interviews of an elementary or middle school student. In their respective pedagogy courses, the PSTs engaged in activities designed to support their understanding that the purpose of a diagnostic clinical interview is to investigate student thinking, not to teach. As a means of preparation, the PSTs also analyzed and discussed an illustrative video recording of a diagnostic clinical interview one of the authors conducted with a middle school student. The PSTs used the transcript from this illustrative interview to engage in a discussion about the potential of different types of questions to elicit student mathematical thinking during one-on-one interviews. They also used the video recording of the illustrative interview to practice their teacher professional noticing skills prior to conducting their own interviews of elementary or middle school students. To foster PSTs' reflection on their own noticing skills and thus help them to learn from their own practice (Sherin, Jacobs, & Philipp, 2011), we also engaged each PST in an individual debriefing interview conducted by course instructors following completion of each diagnostic clinical interview. The protocol for the debriefing interviews included questions that elicited the PSTs' thinking about their teacher professional noticing skills and the PSTs' interpretation of their student's thinking about the equal sign and equality.

Participants

Participants for the study were 32 PSTs, all juniors or seniors seeking Grades 1–8 teaching license, one or two semesters before their student teaching experience. Ten were concurrently enrolled in Number Systems and Operations and Teaching Elementary Mathematics. In their field placement, those 10 worked with 10 elementary students (3rd grade). Twenty-two PSTs were concurrently enrolled in Algebra and Geometry for Teachers and Teaching Middle School Mathematics. These 22, in their field placement, conducted clinical interviews with middle school students (8th grade).

Data Collection and Data Sources

Data for this study were collected via one-on-one diagnostic clinical interviews conducted by the PSTs during their field experience and via debriefing interviews conducted by the course instructors. Data sources included (1) transcripts from the 64 diagnostic clinical interviews the PSTs conducted with 3rd or 8th grade students at the middle and at the end of the semester, and (2) transcripts from the 64 debriefing interviews of PSTs, which were conducted by course instructors (one after each clinical interview).

Data Analysis and Results

After coding the data, we established validity and reliability by comparing sets of independently coded transcripts, citing specific examples, and clarifying coding themes and categories until 100% agreement was achieved. Once coded, the data were analyzed using a combination of qualitative and quantitative methods. We present the data analysis and results next, organized by research question.

Research Question #1: To What Extent Do PSTs Attend to and Further Explore Mathematically Important Aspects of Student Thinking About the Equal Sign During Clinical Interviews?

Our answer to this research question comes from analysis of the transcripts of the diagnostic clinical interview the PSTs conducted with their elementary or middle school students. The transcripts were analyzed in several steps. First, we divided each of the 64 transcripts into segments that corresponded to each of the nine interview tasks the PSTs posed for their students (i.e., 64 transcripts × 9 interview protocol tasks = 576 interview segments). We defined an interview segment as a portion of a transcript that began with a PST posing a task from the interview protocol and ending when the PST moved on to the next task.

Second, drawing on the descriptions of teacher professional noticing as a conceptual framework (Jacobs et al., 2010), we developed a rubric and analyzed each segment to examine whether the PSTs (1) did not attend to and further explore student thinking (Score 0), (2) attended to and further explored the strategy the student used to solve the task *without* an explicit focus on student thinking about the equal sign and equality (Score 1), or (3) attended to and further explored student thinking *with* an explicit focus on student thinking about the equal sign and equality (Score 2). We also further investigated each of the identified Score 2 segments to determine specifically which aspect(s) of student thinking about the equal sign and equality the PSTs attended to and further explored: (a) understanding of the equal sign as sameness of quantities, (b) use of a computational strategy to confirm sameness of quantities, or (c) use of a relational thinking strategy to confirm sameness of quantities.

In the third step of the analysis, we examined the frequency of Scores 0, 1, and 2 for each PST across both interviews, calculated the average number of Score 2's the PSTs received on interview #1 and interview #2, and conducted z-tests for proportions to explore patterns in the PSTs' noticing. To illustrate our scoring, we use segments in which PSTs #5, #9, and #2 posed the same Grade 3 interview task (Task 3 on the protocol) to their students. We further illustrate our scoring with segments in which PSTs #32, and #18 pose, the structurally equivalent Grade 8 interview Task 3.

Score 0: Did not attend to and further explore student thinking. Consistent with our rubric, we coded a PST's segment as Score 0 if, within that segment, we found no evidence that the PST attended to or further explored their student's thinking. We illustrate a Score 0 segment with the following example from PST #5:

1. PST #5: Question number 3. [13–7 = □ − 6] What number would go in the box?
2. Student: Six.
3. PST #5: Now what about this one, number four [referring to the next interview task].

As illustrated in the transcript segment, after reading the interview task, PST #5 received an incorrect answer from her 3rd grade student (line 2). In response, and despite explicit instructions to explore her student's thinking about the equal sign during the clinical interview, PST #5 simply moved on to the next interview task. Her actions provided no evidence that she attended to, or further explored, her student's thinking.

Score 1: Attended to and further explored the strategy the student used to solve the task *without* an explicit focus on student thinking about the equal sign and equality. We coded an interview segment as Score 1 if, within that segment, the PST attended to and further explored the strategy the student used to solve the task *without* an explicit focus on student thinking about the equal sign and equality. The transcript segment from PST #9's interview, shown next, illustrates this scoring category, highlighting how PST #9 focused on the counting strategy her student used to solve the task, rather than on her student's thinking about the equal sign and equality:

1. PST #9 The next question is fill in the box with the number that makes the sentence true. 13 minus seven equals box minus 6 [13–7 = □ − 6]
2. Student: 6.
3. PST #9 How did you get 6?
4. Student: I punched 7 and counted up and then I got 6.
5. PST #9: You counted up and got 6. Okay let's move on to the next question.

When PST #9's 3rd grade student provided an incorrect answer (line 2), PST #9 asked a general question "How did you get 6?" (line 3) and paraphrased the counting strategy her student used to solve the problem (line 5). Although she attended to the student's thinking about 13–7, she failed to explore the student's thinking about the equal sign. Rather than follow up with her student by asking about the box and the number on the other side of the equal sign (line 1), PST #9 posed the next task on the interview protocol. It might be that after receiving her student's response, PST #9 assumed that her student did not understand the equal sign as sameness but rather as a symbol to operate, and thus responded that 13–7 is six. However, without explicitly engaging her student in a conversation about the six on the right side of the equation, PST #9 could not be certain that this interpretation of her student's thinking is accurate. It may be, for example, that PST #9 did not even attend to the—6 on the right side.

Score 2: Attended to and further explored student thinking *with* an explicit focus on student thinking about the equal sign and equality. We coded a segment as Score 2 if, within that segment, the PST attended to and further explored an aspect of their student's thinking about the equal sign and equality. We provide examples in which PSTs attended to and further explored an aspect of their student's (a) understanding of the equal sign as sameness of quantities, (b) use of a computational strategy to confirm sameness of quantities, or (c) use of a relational thinking strategy to confirm sameness of quantities.

(a) ***Score 2: Focusing on student understanding of the equal sign as sameness of quantities***. PST #2's segment, shown next, illustrates how PST #2 attended to and further explored her student's understanding of the equal sign with an explicit focus on sameness of quantities on both sides of the equation.

1. PST #2: Question number 3 [13–7 = □ − 6] fill in the box that makes the number sentence true?
2. Student: 6.
3. PST #2: Six, so how did you get 6?
4. Student: Because I counted.
5. PST #2: You counted? How did you count? Did you count in your head or use your fingers?

6. Student: I used my fingers.
7. PST #2: Can you show me how you used your fingers?
8. Student: Thirteen, then twelve, eleven, ten, nine, eight, seven, six (says 13 then holds up one finger as she says each number).
9. PST #2: Why do you use your fingers?
10. Student: Because it's more easier.
11. PST #2: It's easier?
12. PST #2: Okay, what about this 6 over on the other side of the number sentence? Do we have to do anything with that?
13. Student: Just leave it.
14. PST #2: Just leave it? So you subtract 7 from 13 and put 6 in the box and just leave the other 6 over there?
15. Student: Yeah.
16. PST #2: Is this 6 part of the question you just solved?
17. Student: No.
18. PST #2: No? Okay, should we try the next one?

Similar to PST #9's student, PST #2's 3rd grade student provided an incorrect answer in response to Task 3 (13–7 = ☐ − 6). Like PST #9, PST #2 followed up with a general question about the strategy her student used to solve the equation (line 3), and then PST #2 proceeded to investigate her student's counting strategy (lines 4–11). After that, unlike PST #9, PST #2 further explored her student's understanding of the equal sign as sameness of quantities. In doing so PST #2 followed up with her student, asking whether she considered the number six on the other side of the equal sign when solving the equation. Once PST #2's student confirmed that she did not think that the number six on the other side of the equal sign was part of the equation (lines 17–18), PST #2 moved on to the next task.

(b) ***Score 2: Focusing on student use of a computational strategy to confirm sameness of quantities***. We also assigned an interview segment as Score 2 if, within that segment, the PST attended to and further explored whether or not their student computed on both sides of the equal sign to confirm sameness of quantities. We illustrate this situation using a transcript segment from PST #32's interview of an 8th grade student solving the equation 130–70 = $a - 60$.

1. PST #32: What value of a makes the number sentence true: 130–70 = $a - 60$
2. Student: 120?
3. PST #32: How did you get that answer?
4. Student: First I subtracted 130 from 70 and got 60. Then I added 60 to the other 60 to get 120.
5. PST #32: What other 60?
6. Student: The one here (pointing to $a - 60$).
7. PST #32: Okay….
8. Student: So I knew that if one side was 60 the other side had to be 60.
9. PST #32: How did you know the other side had to be 60?
10. Student: Because of the equal sign.

11. PST #32: What do you mean?
12. Student: The answers on both sides have to be the same, they're equal.
13. PST #32: Okay. Is there another way you could solve the problem?
14. Student: No, I don't think so.
15. PST #32: No? Okay, let's move on to number 4.

Investigating her student's computational procedure assisted PST #32 in determining that her student understood the equal sign as sameness between quantities. To find the value of *a* in the equation, PST #32's student first subtracted 70 from 130 on the left side of the equal sign and then added 60 and 60 (line 4). After operating on both sides of the equal sign, the student stated "if one side was 60 the other side had to be 60" (line 8). Attending to this line of thinking, PST #32 asked her student how he knew that both sides of the equation had to be 60 (line 9). When her student communicated that he understood the equal sign to mean that the answer on both sides has to be the same (line 12), PST #32 further investigated, asking her student if there was another way he could solve the equation (line 13). After the student stated he did not think so (line 14) PST #32 moved on to the next task.

(c) ***Score 2: Focusing on student use of a relational thinking strategy to confirm sameness of quantities.*** Finally, we also evaluated a segment as Score 2 if, within that segment, the PST attended to and further explored the relational thinking strategy their student used to confirm sameness of quantities.

1. PST #18: Okay, number 3. What value would make the number sentence true: $130–70 = a - 60$.
2. Student: Would it be 120 or 140?
3. PST #18: Which one are you going with?
4. Student: 120.
5. PST #18: How did you figure that out?
6. Student: Well, since 60 is ten smaller than 70, I would just take 10 off 130. So yea, it would be like subtracting 10.
7. PST #18: Wait, you took 10 away from 130 right? Is that what you're saying? So you took 10 away from 130 for *a*, what was that doing to your equation? What do you think that did for your equation?
8. Student: Just evened it out.
9. PST #18: Evened it out, what do you mean?
10. Student: Since 60 is ten less than 70 I took 10 off 130 to make it even.
11. PST #18: Yea, so you evened it out. Why can you do this?
12. Student: To balance each side so they would be the same.
13. PST #18: Meaning to make each side of the equation the same?
14. Student: Yea.
15. PST #18: Is there another way you could have gone about that?
16. Student: Um…..not that I would have done.
17. PST #18: Okay, well, good job. Alright, so number 4.

As this example illustrates, rather than perform calculations to determine the value of a for the equation 130–70 = $a - 60$, PST #18's 8th grade student used a relational thinking strategy, finding the difference in the magnitude of numbers (line 6). To further investigate if her student understood why he could use this strategy, PST #18 asked "What do you think that did for your equation?" (line 7). In response, PST #18's student explained "just evened it out" (line 8). PST #18 then used this response as an opportunity to further explore her student's understanding of the equal sign as sameness of quantities, asking "Why can you do this?" (line 11). Satisfied that her student's explanation of "evening out" meant that he interpreted the equal sign as a symbol that indicated the sameness of quantities, PST #18 moved on to the next task.

PST noticing skills during clinical interviews. Table 2 provides a summary distribution of the scores our PSTs received for interviews #1 and #2. The organizing feature of this summary is the percentage of interview segments for which our PSTs received each score. For example suppose a PST had the following noticing scores on interview #1, segments 1–9, respectively, 1, 1, 1, 1, 0, 0, 1, 2, 0. This PST would have 3/9 (<50%) Score 0's, 5/9 (>50%) Score 1's, and 1/9 (<50%) Score 2's. Therefore, this PST would be one of the PSTs represented in the Score 1 entry for interview #1 below: 29/32 (91%).

Table 2
Comparison of PST noticing scores on interview #1 and interview #2

% of segments	Interview #1			Interview #2		
>50% (5/9–9/9)	Score 0 1/32 (3%)	Score 1 29/32 (91%)	Score 2 2/32 (6%)	Score 0 0/32 (0%)	Score 1 29/32 (91%)	Score 2 3/32 (9%)

Table 2 shows that the vast majority of our PSTs (91%) attended to and further explored, in the context of at least half of the interview segments, the strategies their students used to solve each task. In interview #1, 91% of the PSTs received Score 1 on more than half of the interview segments, meaning that while they attended to some aspects of the strategies their student used to solve each task, they failed to explore their student's thinking about the equal sign. The same was true of the PSTs' scores on interview #2.

Overall, the analysis revealed 55 instances of attending to and further exploring student thinking about the equal sign (Score 2) in the context of interview #1 and 71 instances of attending to and further exploring student thinking about the equal sign (Score 2) in the context of interview #2. While our group of PSTs, as a whole, did not statistically significantly improve with respect to attending to and further exploring student thinking about the equal sign, more than half of the 32 PSTs (19, 59%) showed improvement in their ability to attend to and further explore student thinking about the equal sign, as demonstrated by an overall positive change in their number of segments scored as "2" from the first to the second interview. The proportion of PSTs who showed an increase in the number of score 2's from the first to the second interview was significantly greater than the proportion of PSTs who did not demonstrate an increase ($z = 2.073$, $p < 0.05$). However, on average the increase was small. The average number of segments scored as "2" in interview #1 improved from slightly less than two (out of 9) score 2's per interview $\left(\overline{M} = 1.79\right)$ to slightly more than two (out of 9) score 2's per interview $\left(\overline{M} = 2.21\right)$.

Research Question #2: After Conducting Clinical Interviews, How Do PSTs Perceive Their Learning About Aspects of Teacher Professional Noticing and Student Thinking About the Equal Sign?

To explore what our PSTs learned about teacher professional noticing and student thinking about the equal sign, we interviewed each PST after they conducted each of their diagnostic clinical interviews. With a goal of stimulating our PSTs' reflection on their learning, we asked the following questions during these debriefing interviews: (a) Thinking about your interview experience, what did you notice about the mathematical thinking of a student? (b) Thinking about your interview experience, what did you learn about yourself as a teacher? (c) Did the interview help you to develop a better understanding of relational thinking about equality? The debriefing interviews were transcribed verbatim, uploaded to NVIVO software for analysis, and then coded for themes using the open coding technique described in Straus and Corbin (1998). The themes that emerged from this analysis are presented and discussed next.

PSTs' perceived learning. Table 3 provides a summary of what our PSTs perceived they learned as they reflected on their teacher professional noticing skills and student thinking about the equal sign. The table tabulates the first response each of the PSTs provided.

As Table 3 illustrates, 80% of our PSTs who worked with 3rd grade students stated that they learned that students have a limited understanding of the equal sign. Moreover, 50% of the PSTs who worked with 8th grade students articulated they learned that students tend to use computational rather than relational strategies to

Table 3
PSTs' perceived learning

Debriefing interview question	Themes	Gr. 3 PSTs (#, %)	Gr. 8 PSTs (#, %)	All PSTs (#, %)
a. Thinking about your interview experience what did you notice about the mathematical thinking of a student?	– Students have a limited understanding of the equal sign	8/10 (80%)	5/22 (23%)	13/32 (41%)
	– Students tend to use computational rather than relational strategies	0/10 (0%)	11/22 (50%)	11/32 (34%)
	– Students think about and solve problems in a variety of ways	2/10 (20%)	6/22 (27%)	8/32 (25%)
b. Thinking about your interview experience, what did you learn about yourself as a teacher?	– Teachers must understand how students' develop relational thinking about equality	3/10 (30%)	11/22 (50%)	14/32 (43%)
	– Teachers must push students to explain their thinking	5/10 (50%)	7/22 (32%)	12/32 (38%)
	– Good questions help to uncover student thinking	1/10 (10%)	5/22 (23%)	6/32 (19%)
c. Did the interview help you to develop a better understanding of relational thinking about equality? Explain	– Yes, strengthened my personal knowledge of relational thinking about equality	3/10 (30%)	9/22 (41%)	12/32 (38%)
	– Yes, strengthened my understanding of student knowledge of relational thinking about equality	7/10 (70%)	13/22 (59%)	20/32 (62%)

solve problems that require thinking about the equal sign. The PSTs who worked with 8th grade students particularly appreciated that the interview experience increased their awareness that teachers ought to understand how students develop relational thinking about equality. Finally, all of our PSTs indicated that the interview experience assisted them in developing their own knowledge (38%) or their understanding of student knowledge (62%) of the equal sign. We interpret these results to indicate that the majority of our PSTs are positioned to actually *notice* student conceptions and misconceptions of the equal sign when they begin their teaching practice.

Implications for Teacher Preparation

Effective teachers use their professional noticing skills to guide their instruction (Jacobs et al., 2010). This is because the ability to attend to student strategies, interpret student understanding, and decide how to respond based on student understanding supports teachers in designing instruction that meets the mathematical needs of their students. Mathematics teacher educators maintain that PSTs can improve their teacher professional noticing skills and should begin to work on developing these skills early in their teacher preparation program (Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2002).

In an effort to help our PSTs develop their teacher professional noticing skills, we engaged them in conducting diagnostic clinical interviews with elementary or middle school students. We also focused our PSTs' attention on student understanding of the equal sign to increase their awareness of the role that understanding of the equal sign plays in students' transition from early arithmetic to the study of algebra. Using the context of our work we investigated (1) the extent to which PSTs attended to and further explored student understanding of the equal sign, and (2) what PSTs perceived they learned about aspects of their teacher professional noticing skills and student thinking about the equal sign. The results from our first research question provide important insights into the extent to which our PSTs attended to, interpreted, and further explored student thinking about the equal sign while they were conducting diagnostic clinical interviews. While it is valuable to learn what PSTs notice about students when viewing video recordings of classroom instruction, it is especially valuable to learn what PSTs attend to, interpret, and further explore when working face-to-face, in the moment with students.

Asquith and colleagues (2007) and Stephens (2006) have raised concerns that in general, practicing teachers and PSTs are unaware of student thinking about the equal sign and that students who have an insufficient understanding of relational thinking about equality often struggle to make a successful transition from the study of arithmetic to the study of algebra. The results of our study reinforce these concerns and draw additional attention to just how elusive student thinking about the equal sign is. In general, we found that over the course of the two diagnostic clinical interviews, our PSTs predominantly noticed (Score 1) the strategies their students used to solve a task *without* focusing on student thinking about the equal sign. Very few of our PSTs, on the other hand, did not attend to and further explore student thinking (Score 0). Furthermore, more than half of our PSTs (19/32, 59%) increased the number of Score 2's they received between interview #1 and interview #2. However, since the average number of Score 2's the PSTs received on interview #2 was still less than 3 (out of 9), their ability to attend to and further explore student thinking about the equal sign remained marginal. Although we are encouraged that our PSTs noticed and further explored the strategies their students used to solve tasks, we are disappointed that, in general, they further explored something other than their student's thinking about the equal sign. In light of these findings, we have formulated two steps that we can take to further support our PSTs in specifically attending to and further exploring student thinking about the equal sign.

First, we plan to provide an abundance of examples and counter examples of interviewers attending to and further exploring student thinking about the equal sign. We believe this will help our PSTs to better comprehend what this specific noticing skill looks like. As mentioned previously, our PSTs attended to and further explored the strategies their student used to solve a task (Score 1) but fell short in specifically investigating their student's thinking about the equal sign. This finding indicates that our PSTs may have thought they were attending to and further exploring student thinking about the equal sign, when they were in fact exploring the strategies students were using to solve a problem (e.g., counting, derived fact, or traditional algorithm). We hypothesize that our PSTs attended to and further explored (Score 1) the strategies their students used to solve a problem because these strategies were patently observable to our PSTs. For example, the PSTs could literally watch their students use their fingers to count up or down in order to solve a problem or identify errors in computation. What a student does or does not understand about the equal sign, on the other hand, was less observable, and required further exploration via careful questioning. In retrospect, we realize that while we engaged our PSTs in a variety of activities that helped them to learn how students think about the equal sign and the concept of equality (Table 1), we provided fewer opportunities to learn what it means (and does not mean) to attend to and further explore student thinking about the equal sign and equality. We believe that asking our PSTs to examine numerous examples and counter examples will heighten their awareness of what it means to attend to and further explore student thinking about the equal sign.

Second, in our current work with PSTs, we now incorporate the idea of a "missed opportunity," which provides a context for our PSTs to reflect on and analyze their own ability to attend to and further explore student thinking about the equal sign and equality. We define a missed opportunity as an instance in which a PST should have further explored their student's thinking about the equal sign but failed to do so. We now ask our PSTs to transcribe their diagnostic clinical interviews, identify a segment on the interview as a missed opportunity, and propose the actions they could have taken to enhance their teacher professional noticing skills. This provides our PSTs' additional opportunities to mentally rehearse attending to and further exploring student thinking about the equal sign and equality.

Similar to Schack et al. (2013), whose research emphasized noticing children's early arithmetic learning, our work addressed teacher professional noticing in the context of one specific issue in K-8 mathematics, namely, student thinking about the equal sign and equality. The results of our second research question indicated that all of our PSTs perceived that they strengthened either their own knowledge or student knowledge of the equal sign while conducting their diagnostic clinical interviews. These results reinforce our conviction that it is advantageous to pair issues found in the K-8 mathematics curriculum with teacher professional noticing. Several mathematics education researchers have cited student thinking about the equal sign and equality as one such issue (Alibali et al., 2007; Carpenter & Franke, 1999; McNeil & Alibali, 2005a; McNeil & Alibali, 2005b; McNeil et al., 2006). Asking PSTs to specifically investigate student conceptions and misconceptions of the equal sign via one-on-one diagnostic clinical interviews creates a viable approach to prepare PSTs to notice student thinking about the equal sign and equality upon beginning their own practice.

In conclusion, this study provides some insight into how difficult it is for PSTs to notice and further explore student thinking about the equal sign and equality. While our study yielded modest gains in our PSTs' professional noticing skills, we believe our findings can be used to guide the future work and research of the mathematics education community in supporting PSTs' learning how to notice student thinking about the equal sign and equality.

Appendix 1: Diagnostic Clinical Interview Protocol

Task #1 **Gr. 3/Gr. 8:** The arrow points to a symbol. What is the name of that symbol? What does that symbol mean? Can it mean anything else? 5 + 3 = 8 ↑	**Task #2** **Gr. 3:** What number goes in the box? 5 + 4 = □ + 6 **Gr. 8:** What value of *a* makes the number sentence true? 55 + 54 = *a* + 56	**Task #3** **Gr. 3:** Fill in the box with a number that makes the sentence true: 13–7 = □ − 6 **Gr. 8:** What value of *a* makes the number sentence true? 130–70 = *a* − 60
Task #4 **Gr. 3:** Fill in the box with a number that makes the sentence true: □ + 4 = 5 + 7 **Gr. 8:** What value of *a* makes the number sentence true: *a* + 34 = 35 + 37	**Task #5** **Gr. 3:** Fill in the box with a number that makes the sentence true: □ = 7 + 6 **Gr. 8:** What value of *a* makes the number sentence true? *a* = 700 + 600	**Task #6** **Gr. 3:** What value would make the number sentence true: □ = 25 − 12 **Gr. 8:** What value of *a* makes the number sentence true? *a* = 2500–1200
Task #7 **Gr. 3:** What value would make the number sentence true: 8 + □ = 12 **Gr. 8:** What value of *a* makes the number sentence true? 8 + *a* = 12	**Task #8** **Gr. 3:** What value would make the number sentence true: 12 + 9 = 10 + 8 + □ **Gr. 8:** What value of *a* makes the number sentence true? 120 + 90 = 100 + 80 + *a*	**Task #9** **Gr. 3/Gr. 8:** Write your own number in each box to make the number sentence true: □ + □ = □ + □

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Following a Teacher's Mathematical and Scientific Noticing Across Career Progression from Field Experiences to Classroom Teaching

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Abstract In this study, we focus on one preservice teacher's noticing of students' mathematical and scientific thinking with an emphasis on how the acts of attending and interpreting can influence decisions about pedagogical actions. The study centers on an innovative field experience approach that incorporates lesson study in order to emphasize students' thinking and its impact. Consequently, we were interested in understanding how one teacher made decisions based on her noticing at three points in her career: preservice field experiences, student teaching, and her first-year teaching. We used a case study approach to focus on one preservice teacher. Findings indicate that scaffolding PSTs to notice students' mathematical and scientific thinking influenced how she noticed and considered students' thinking while teaching. Results further indicate that supporting the development of noticing during field experiences has a positive impact on a teacher when she was in her own classroom. The study provides a unique contribution to the field as it incorporates both the mathematics and science teaching practices of the same PST from her teacher education experience into her career.

Keywords Field experience · Lesson study · Student teaching · Mathematics · Science

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What teachers perceive about students' thinking during the act of teaching and the subsequent choices about how they respond provide rich insights into the thinking of teachers. Although the benefits of focusing on students' mathematical and scientific thinking have been shown to be an important component of teacher education (Driver, Guesne, & Tiberghien, 2000; Sowder, 2007), relatively few studies have examined the long-term impact of teacher education programs that emphasize the thinking of children. In this chapter, we describe a study that examines a preservice teacher who participated in an experimental field-based course as part of the Iterative Model Building (IMB) project. Using case study methodology, we document the impact of one preservice teacher's noticing on field experiences, her initial practice as a student teacher, and her first-year teaching.

Noticing Students' Mathematical Thinking

Building on the work of van Es and Sherin (2002, 2008), Jacobs, Lamb, & Philipp (2010) introduced a special type of noticing enacted by teachers, which they termed *professional noticing of children's mathematical thinking*. Jacobs et al. posit that this type of noticing consists of three interrelated steps: (1) attending to children's strategies, (2) interpreting the mathematical understandings of children, and (3) deciding how to respond based on children's understandings.

Attending involves how teachers recall the specific details of the mathematical strategies used by children. *Interpreting* refers to the extent to which teachers' attention is consistent with the children's strategies and with research on the development of children's thinking. Finally, *deciding how to respond* describes the extent to which the teacher used her or his knowledge of mathematical thinking to determine how to react to the student. Based on their analyses of prospective and practicing teachers, Jacobs et al. (2010) found that the characteristics of advanced noticing were not as common among prospective teachers as they were among emerging teacher leaders, confirming the work of prior research (Star & Strickland, 2008) that noticing is both learned and can be developed. Our current work is based on the assumption that one's ability to notice can, and does, change over time.

Since Jacobs et al. initial description of the construct of professional noticing, a number of publications have extended their work. To classify *what* and *how* teachers notice students' mathematical thinking, van Es (2011) introduced a framework with four levels for *what* students notice, moving from focusing primarily on behavioral or teacher actions to attending to the particular strategies of students and considering the relationship between these strategies and the teaching practices. Amador, Weiland, and Hudson (2016) extended van Es' (2011) framework by further categorizing the advanced levels of noticing, including the ways teachers detail strategies, analyze evidence, and make suggestions for improvement.

Other researchers have examined the structures necessary to support teachers' development of noticing. For example, when teachers are provided with specific scaffolding questions, teachers notice at a more advanced level (McDuffie et al., 2014; Seidel, Blomberg, & Renkl, 2013). Earlier case study work has also shown that through interviewing elementary students as a formative assessment, a teacher gradually improved her ability to notice children's thinking (Weiland, Hudson, & Amador, 2014). Furthermore, new teacher education materials have been designed to help improve preservice teachers' ability to notice, such as modules designed to increase attention to children's early numeracy concepts (Schack et al., 2013). Although much of this work confirms teachers' ability to notice children's thinking change over time, there are unanswered questions concerning what impact a focus on noticing students' mathematical thinking during preservice teacher education has on the instructional decisions those educators make as practicing teachers.

The Role of Student Thinking in Teacher Education

Several research studies have shown that when teachers develop strong conceptions of students' mathematical thinking, they are better positioned to assist students by building on their thinking, adapting instructional practices, and consequently impacting student achievement (Fennema et al., 1996; Kazemi & Franke, 2004; Norton & McCloskey, 2008; Schifter, 1998). Furthermore, this type of knowledge of students' thinking is distinct from mathematical content knowledge, and content knowledge is not sufficient in order for preservice teachers to cultivate their students' conceptual understanding of mathematics (Bartell, Webel, Bowen, & Dyson, 2013). Teachers' conceptions of students' thinking may include the typical ways students think about particular problems or the common misconceptions that they employ. For example, the results of a study on Cognitively Guided Instruction showed that when teachers became aware of research-based models of students' thinking, students' achievement in regards to mathematical concepts and problem solving increased significantly (Fennema et al., 1996).

Although existing literature suggests that teacher education initiatives should focus on the thinking of students, it is less clear *what* teachers should know about students' thinking and be able to do in response to student thinking. Certainly, a commonly agreed upon action is the interpretation of student thinking (Johnson & Cotterman, 2015; Sleep & Boerst, 2012). Jansen and Spitzer (2009) suggested that preservice teachers need to describe the thinking of students with mathematical specificity and differentiate between the thinking of students in order to develop differentiated interventions. Sleep and Boerst (2012) also expected preservice teachers to elicit student thinking, whereas Harlow, Swanson, and Otero (2014) found that teachers restated the students' thinking using content-specific terminology. Furthermore, preservice teachers need opportunities to distinguish between students' conceptual and procedural understandings (Spitzer, Phelps, Beyers, Johnson, & Sieminski, 2011).

A second related question is *when* do teachers begin to truly attend to student thinking. Refuting earlier claims that teachers are unable to attend to student thinking until they begin to identify as teachers, Levin, Hammer and Coffey (2009) contend that preservice teachers can learn to attend carefully to the thinking of students. However, such experiences should be carefully framed to support the preservice teachers' analysis of student thinking. Positive outcomes from engaging teachers in student thinking activities early in the preservice teacher education program have been confirmed by others (e.g., Bartell et al., 2013; Spitzer et al., 2011), and is not dependent upon first learning mathematical content (Philipp et al., 2007).

Although there is strong evidence that teachers' knowledge of student thinking is an important component of preparation for teaching, few studies have examined teachers' longitudinal development to determine how (or whether) their collegiate coursework and field work during their preservice teacher education impact their teaching practice as a student teacher or later as a practicing teacher. This is problematic, given that research on student teaching suggests that student teachers often do not incorporate what they have learned in preservice teacher education during their student teaching semester (Moore, 2003). Santagata and Yeh (2014) found that student teaching experiences that focus on student thinking by prompting preservice teachers to reflect on the impact of their instruction on student progress were more likely to make student thinking visible and use evidence of student learning. Our study addresses the need to examine the longitudinal effects of a similar program, called IMB, by answering the following research question: As preservice teachers become student teachers and practicing teachers, how does their collegiate coursework and related field experiences, that focus on professional noticing, during a teacher education program influence their teaching practice at various points in their career progression?

Context: Iterative Model Building

The IMB approach to the early field experience includes formative assessment interviews, building models of students' thinking, and lesson study. The purpose of the formative assessment interviews is to provide preservice teachers with direct experience planning, conducting, and analyzing interviews of elementary students related to their mathematics and science thinking on specific topics (see Weiland et al., 2014). Preservice teachers then use videos and notes taken during formative assessment interviews to build models of students' thinking. Finally, preservice teachers engage in an adapted model of lesson study (Lewis, 2002), which includes planning, teaching, reflecting on, and revising lessons based on the models of students' thinking. The current study focuses on the lesson study portion of the IMB approach to gain an understanding of how one preservice teacher, Mikayla, professionally noticed while teaching and while reflecting on and revising the lesson.

The lesson study process began with preservice teachers planning a lesson in pairs. One of these preservice teachers taught the lesson, while the other co-taught or served in a support role. The remaining four preservice teachers assigned to teach in the same classroom took observation notes. Immediately after the lesson was taught, all six preservice teachers met to debrief the lesson with the classroom teacher and a university supervisor (in this case, a doctoral student in science education). This debriefing session, which we refer to as the Lesson Study Analysis Meeting, typically lasted 30 min and included reflective discussion, based on the observation notes, on what went well and what could have been improved in the lesson. The group then discussed how these reflections could inform the teaching of the next lesson (to be taught the following week). Preservice teachers usually engaged in six consecutive mathematics cycles and then five consecutive science cycles of the lesson study process throughout the field experience semester. We refer to this process as Phase One of the IMB cycle.

One year after finishing the field experience, IMB participants completed one semester of student teaching, which we refer to as Phase Two. During Phase Two, preservice teachers were observed teaching two mathematics lessons and two science lessons in their student teaching placements. We then followed the preservice teachers into their first year of independent classroom teaching, Phase Three. We then observed two mathematics and two science lessons during their first-year teaching.

Participant

For the purpose of this monograph, we focus solely on one preservice teacher, who participated in all three phases of the IMB cycle. At the onset of the study, Mikayla was in her junior year of an elementary teacher education program at a large Midwestern research university. She was concurrently enrolled in a mathematics method course, a science method course, and the associated IMB field experience (Phase One). Prior to taking these courses, Mikayla had successfully taken three mathematics content courses (Number and Operations, Finite Mathematics, and Geometry and Measurement) and four science content courses (Introduction to Scientific Inquiry, Biological Science for Elementary Teachers, Physical Science for Elementary Teachers, and Earth Sciences: Materials and

Processes). As a participant in the IMB field experience, Mikayla was assigned to teach in a first-grade classroom one day a week with her five peers, as described in the aforementioned process for Phase One. Compared to her five peers, Mikayla was above average with regard to motivation and creativity in working with the elementary students. Her assignments for the field experience course (e.g., reflections and lesson revisions) were not always submitted on time; however, she was highly engaged in the entire IMB process while in the field. During the Phase One experience, Mikayla generated a particularly strong connection to the classroom teacher with whom the six preservice teachers worked. During the Lesson Study Analysis Meetings, Mikayla spoke as often as her peers and provided good insights into the students' mathematical and scientific thinking. Therefore, we consider Mikayla to be representative of an average participant in the larger IMB project.

Data Collection and Analysis

Data collected for this chapter came from all three phases of the project, Phase One (Field Experience), Phase Two (Student Teaching), and Phase Three (Classroom Teaching). For Phase One, we analyzed Mikayla's written lesson plans for one mathematics lesson and one science lesson, her teaching (video recorded) for one mathematics and one science lesson, the accompanying Lesson Study Analysis Meetings for these lessons, and her written reflections on each lesson. For Phase Two, we analyzed Mikayla's written lesson plans for one mathematics lesson and one science lesson, her teaching for one mathematics and one science lesson based on field notes and lesson observation protocols of two research team members, and conducted post-teaching interviews after each lesson. The interviews were transcribed for analysis. For Phase Three, the data collection mirrored that from Phase Two, including analysis of lesson plans, teaching, and interviews. We intentionally focused on one participant, Mikayla, and one lesson for each subject for each year from her teaching because in Phase One she only led one of each lesson type and we sought similar data across the analyses. Further, we considered the second observed lesson in Phase Two and Phase Three to be more representative of her actual teaching because she was familiar with the observation and interview process.

Data were analyzed according to the Jacobs et al. (2010) framework for professional noticing of children's mathematical thinking. Data were initially analyzed by content area and by phase for attending, interpreting, and responding on the basis of children's thinking. For this analysis, data maps were created for both mathematics and science for each of the three phases. Figure 1 shows an example of a data map for a mathematics lesson for Phase Two.

After data maps were completed for analysis for each of the lessons for both content areas, we analyzed the data across phases for each content area, meaning we

Lesson: Making Ten (Number Sentences) Concept: Groups of Ten

ATTENDING	INTERPRETING	RESPONDING
Lesson Plan: No evidence of Attending **Field Notes and Lesson Observation:** Teacher asks students questions that are easily answered with yes and no responses. Teacher engages students in counting with their hands and then students complete problems on erase boards. She says, "Alex wrote about the ten frame, circle what Alex should have written. He said two away from ten is seven and that five and two is seven." According to the lesson observation protocol, almost the entire lesson was spent working on skill development, facts, and vocabulary without connections to related concepts. In many instances during the lesson, the teacher indicated that the content was too easy for students, but did not modify the lesson accordingly. **Interview:** Preservice teacher indicated that the lesson went very well and it helped her understand what the students already knew. Indicated that she attends to students' thinking by having them explain answers use manipulatives. "Just and interviewing students, you see that they want to have something to touch, some manipulative or something."	**Lesson Plan:** No evidence of Interpreting **Field Notes and Lesson Observation:** No evidence of Interpreting **Interview:** Preservice teacher makes interpretations about what students learned. She concluded that students learned how to look a given set of objects, line them up and formulate a number sentence based on the number of objects. "I feel like they understood how they could look at something, even if it is chips or cubes and line them up in a certain way to create a number sentence or a story problem if they wanted to. And, they really, really, took a step in that direction." When discussing how she knew student learning occurred, she noted, "When I had a student that created the ten frame and added the circles and she was able to create the number sentence and show me using the cubes and able to explain it to her fellow classmates, just really showed me that in that fifteen minutes she was able to understand what I taught her, so that was a great part." She recognized the importance of providing multiple opportunities for students, "I like to allow my students to become teachers. So, instead of just asking them to tell me, have them come up, answer, and explain why they did it and using manipulatives."	**Lesson Plan:** Four main objectives are listed, including, "Students will understand how to solve number sentences under the number 10." She provided an overview, "This lesson will be a combination of a lot of things. I will start off by using cubes and have the students use their fingers to show how to make ten and numbers under ten. Next we will talk about creating number sentences from a ten frame where students will have to tell me 5 + ? = 10. We will go through a lot of problems like that on the dry erase. Then the students will do a page in their work book." **Field Notes and Lesson Observation:** The lesson followed the description provided in the overview on the lesson plan. The teacher recognized the ease of the content for the students during the lesson, but did not make any adjustments to her original plan. **Interview:** Preservice teacher indicated the lesson was easy for students, so she concluded it went well. "I created the lesson plan based on the Envision teacher edition workbook. Changed it up a little bit based on my students and what they learn and how they learn." She notes that she will adjust future lessons for content, but did not make modifications during the lesson. "My follow-up lesson will be with bigger numbers outside of the ten frame, creating their own story problems, and writing their own number sentences, and then moving on from there."

Figure 1. Data map example.

analyzed themes from Phase One to Phase Two to Phase Three for the mathematics lessons and similarly for the science lessons. Finally, we compared analysis for the two content areas to determine similarities and differences. This process supported the intent to understand how one preservice teacher noticed students' mathematical and scientific thinking from field experience to classroom teaching.

Findings

The following presents the findings from the study, initially by content area, and then provides cross content area conclusions.

Mikayla's Noticing in Mathematics

Two main themes were evident across Mikayla's career progression when teaching mathematics. First, she placed an emphasis on students' mathematical understanding by attending to and interpreting their thinking in all three phases, but this occurred to an even greater extent in Phase Three. Second, the extent to which she adapted or modified her teaching in the moment, or her responding, differed across the three phases. The following describes these two themes, based on the three phases.

In Phase One, Mikayla was cognizant of student thinking as she planned her lessons and reflected on her lesson. She designed a lesson plan focused on *greater than* and *less than* around a game called Guess my Number and incorporated questions that would prompt student thinking. For example, in her plan, she wrote that she would ask "What did you learn from the game? Was it a hard game? Was it too easy? Did you figure out a strategy to figure out the number? Was there a better way to play the game?" After the lesson, she was able to talk generally about student understanding, "When it came to doing the game, I felt that mostly all the students understood the whole purpose of the game and that was to use the language of greater than and less than when talking about numbers." She went on to make interpretations about how well the students did with the lesson as compared to her preconceived ideas about their understandings. Although her interpretations were limited, commonly referencing whether or not students understood the concept, she made these interpretations based on what she had attended to in the lesson; however, she lacked specificity when describing students' mathematics thinking.

When responding during Phase One, Mikayla kept to her initial lesson plan and only made one minor change from her plan during the process of teaching. She asked questions that could be answered with simple responses that she deemed correct or incorrect. During the Lesson Study Analysis Meeting following her teaching, she discussed how students used the number line and had some confusion when numbers were less than or greater than other numbers. In the process of discussing her teaching with peers and knowledgeable others, she talked about what she would do to support student understanding in the next lesson. Instead of using a number line, she decided using arrows to indicate if the students' number was greater than or less than the number they were trying to guess would better support students' mathematical understanding. Despite discussing what she would do next after the lesson, it is important to remember that she only made a small change by responding in the moment.

In Phase Two, evidence of attending to and interpreting students' thinking was apparent during the interview. After teaching a lesson on making groups of ten, she indicated that she attended to students' thinking by having them explain answers and use manipulatives. She noted "And interviewing students [during the lesson], you see that they want to have something to touch, some manipulative or something." When asked specifically about student understanding in the observed lesson, she concluded that students learned how to look at a given set of objects, line them up, and formulate a number sentence based on the number of objects.

> I feel like they understood how they could look at something, even if it is chips or cubes and line them up in a certain way to create a number sentence or a story problem if they wanted to. And, they really, really, took a step in that direction.

It is interesting to note that her wording (i.e., "I feel like") suggests that at this point, Mikayla was basing her interpretation on her "sense" of the students, rather than on specific evidence of the students' words or actions. She described how she knew student learning had occurred,

> When I had a student that created the ten frame and added the circles and she was able to create the number sentence and show me using the cubes and able to explain it to her fellow classmates, just really showed me that in that fifteen minutes she was able to understand what I taught her, so that was a great part.

In these examples, Mikayla's attending and interpreting were more specific than what was seen in the evidence from Phase One.

In Phase Two, Mikayla's responding was similar to that in Phase One—she was able to discuss changes she would make after the lesson, but did not make significant changes or deviate from her plan in the moment of teaching. In her lesson plan, she wrote

> This lesson will be a combination of a lot of things. I will start off by using cubes and have the students use their fingers to show how to make ten and numbers under ten. Next we will talk about creating number sentences from a ten frame where students will have to tell me 5 + ? = 10. We will go through a lot of problems like that on the dry erase. Then the students will do a page in their workbook.

The lesson observers noted that she followed this plan with fidelity. Despite the similarity between the plan and the enactment of the lesson, Mikayla showed evidence of basing her lesson plan on past instances of attending and interpreting. For example, when asked about the lesson, she wrote "I created the lesson plan based on the enVision teacher edition workbook. Changed it up a little bit based on my students and what they learn and how they learn." Thus, there was evidence of responding based on students' thinking from lesson to lesson, but not during lesson enactment and without specificity. She went on to confirm this by noting that she would adjust future lessons for content. The present lesson had focused on numbers up to ten, but she noted "My follow-up lesson will be with bigger numbers outside of the ten frame, creating their own story problems, and writing their own number sentences, and then moving on from there." Therefore, the evidence of responding was similar to her actions in Phase One.

In Phase Three, Mikayla's ability to attend and interpret was even more specific to the students' understanding of mathematics. During the interview following the lesson, she noted that she was focused on recognizing students' errors in division with repeated subtraction. She remarked that many students had a difficult time knowing their math facts, which complicated repeated subtraction. In this way, she connected students' prior understanding (about fact families) with the content of the current lesson on dividing with repeated subtraction to come to conclusions about their understandings. This ability to attend to and interpret student thinking about specific mathematical difficulties was also noted by the observers of the lesson. One observer wrote "The teacher noticed some of the struggles the students were having and did some more modeling with the students before having them try it out on their own." The combination of the observer notes and Mikayla's comments during the interview are evidence that she was attending to students' thinking. In Phase Three, she was more specific about the mathematical understanding of the students than she was during Phase One and Phase Two.

In Phase Three, Mikayla demonstrated a notable difference in responding, as compared to Phase One and Phase Two. During classroom teaching, Mikayla modified her lesson content in the moment of teaching based on student understanding. One observer of the lesson noted

> She gave them the problem 10 divided by 5 and asked the students to show their work on their board using repeated subtraction. When the students were struggling she decided to go through another problem with the students on the board.

Following the lesson, Mikayla talked about how a few students did not meet the objective of the lesson, so she planned to repeat portions of the lesson and work with smaller groups on dividing and repeated division. She also talked about making plans to work with struggling students on their fact families because she considered this to be directly related to their difficulty with repeated subtraction. The difference distinguishing Mikayla's responding in Phase Three from Phase One and Phase Two was her ability to make changes to the lesson content during the lesson and to consider future instruction on the basis of students' thinking from the lesson.

Cross Mathematics Conclusion

Considering attending, interpreting, and responding across the three phases, Mikayla demonstrated increased ability with all three interrelated skills during Phase Three of the data collection. During Phase One and Two, she attended to students' mathematical thinking, but noted how she would make changes to future lessons on that basis. During Phase Three, she made changes from her plan in how she responded during the lesson and was able to discuss how she would respond in future lessons, based on students' mathematical understanding.

Mikayla's Noticing in Science

As in Mikayla's mathematics teaching, her ability to notice her students' scientific thinking progressed across all three phases. In Phases Two and Three, she began to attend to and interpret her students' thinking more deeply, and began to respond to students' thinking as she planned her lessons. The following describes the development of Mikayla's ability to notice in science, and how this development compares to her noticing in mathematics.

Similar to Mikayla's noticing in mathematics, in Phase One she included written question prompts to elicit students' thinking about Oobleck, for example, "How would you describe Oobleck? What are the properties that make it a liquid? What are the properties that make it a solid?" When Mikayla taught the lesson, she roved around the room, asking students to describe their observations of the Oobleck. After students explored Oobleck in small groups, she brought the students back to the carpet and asked them what they observed. One student responded that Oobleck melts, and Mikayla asked probing questions, such as "Why do you think it melts? What makes you think Oobleck is a liquid?" Through these question prompts, Mikayla attended to her students' thinking by asking them specific questions about how they were thinking about Oobleck. Some of these questions were preplanned (in her lesson plan), yet others were included in the moment of teaching. Mikayla was then able to make general interpretations of that thinking in the Lesson Study Analysis Meeting, for example, "I feel like this lesson gave them the opportunity to find both sides [solids and liquids]." As was the case in Phase One of teaching mathematics, Mikayla made general interpretations of students' thinking and began her interpretation with the phrase "I feel like," rather than citing specific evidence of the students' words or actions. However, Mikayla did connect what she observed of students' thinking in prior science lessons that semester (taught by her peers) to the lesson she taught. She began to interpret why students had been having difficulty connecting their prior knowledge of solids and liquids (i.e., how mixtures and solutions are formed) to Oobleck:

> When I asked [the student] she said, 'The sugar is a solid and when we put it in water and mix it together, it creates a liquid.' So some of them are getting that point and that's why [with the Oobleck] it was hard for them to figure out what to write about what's the same [between Oobleck and solids/liquids] without an example like water and ice, because what would you say?

In response to Mikayla's interpretation that students were having difficulty finding similarities between Oobleck and solids or liquids, she suggested that students would better comprehend the difference between states of matter if more time were spent on each lesson in the Solids and Liquids unit. The last portion of the above quote also suggests that Mikayla herself struggled to differentiate properties of solids, liquids, and Oobleck, and was confusing those properties with those of mixtures and solutions. She then responded to this interpretation when she stated

> If I were really teaching this I would have broken it down way more, the lessons on solids and liquids, and this lesson would have been way later because I feel like they got a little confused like, 'Ok, but you told me that solids are this and liquids are this, so why are these both the same?' I feel like them not really realistically knowing what are specific solids and what are specific liquids, so this changed it up a bit …but it gave them the opportunity to play a bit.

Mikayla therefore responded to her interpretation by providing a general suggestion to slow down the unit and teach each concept (properties of solids, properties of liquids, and properties of mixtures and solutions) much more thoroughly and explicitly.

Thus, in Phase One Mikayla attended to student thinking in both mathematics and science through direct questioning and probing. In the case of science, she continued to notice by interpreting students' actions and words while remaining focused on the lesson objective, which was to compare the properties of Oobleck to those of solids and liquids. While these instances of noticing were indeed related to the objective, the concept discussed within the three components of noticing was inconsistent. More specifically, the concept Mikayla attended to (i.e., students explored “both sides,” or the properties of solids and of liquids) did not provide evidence for her interpretation pertaining to mixtures and solutions. Her response did connect somewhat back to her observations of her students (attending) as she suggested that discussing “both sides” may have confused her students and therefore instruction should be “broken down.” Finally, Mikayla did not make in-the-moment changes to her lessons in order to respond to her students’ thinking.

In Phase Two of teaching science, Mikayla began to attend to student thinking during her lesson on the water cycle in multiple ways. In addition to asking students questions related to their explorations during active inquiry, she stated that she observed students interact with the content through various modalities. More specifically, she stated in her post-lesson interview that she attended to student thinking as they engaged in technology, video, a craft [making paper snowflakes], and hands-on movements. As she did in Phase Two of mathematics, in science Mikayla connected her observations to her interpretations of the students’ thinking, supporting interpretations with evidence of students’ actions and words. For example,

> I looked at how they used their hands to do [the water cycle], having them tell me what it is before I told them what it is. It helped show me that they understood … them telling me things that I even forgot we had talked about shows me that they remember. They are using the hand movements, which is fun and helps them to remember. And the journals help me to see what they learned.

In this quote, Mikayla was interpreting her students’ thinking based on her observations of their hand movements and what they told her about the water cycle. She then began to interpret this thinking by suggesting that hand movements that coincide with the content help students to remember the science concepts. However, her interpretations in science did lack specificity, as they were often limited to whether or not the students understood or remembered the content. The following example coincides with her lesson objective, “Students will learn about different types of precipitation” (from Mikayla’s science lesson plan). With regard to responding to students’ thinking, the following quote demonstrates how Mikayla described her response based on her interpretation of their thinking, yet she did not explicitly connect her interpretation to what she attended to, or observed her students saying and doing.

> I changed up the lesson from what I wrote, I added a hands-on activity, with the snowflakes, because I like to do things that are fun **(responding)**. So I added that at the last minute. *I can tell that they understand based on the one water cycle lesson last time, asking them questions and things like that, so I think it went well* **(interpreting)**. I wanted them to get a hands-on creative way to actually see what was going on—the four types of precipitation rain, snow, sleet, and hail. I can’t really make it rain in here, so I know [a snowflake] would be pretty simple to do **(responding)**.

While Mikayla stated she had observed students' hand gestures while acting out the water cycle to assess their understanding, her response during the lesson was actually based on what she had attended to in prior lessons. In this case, Mikayla had previously observed and interpreted that hands-on activities support students' science learning; she therefore responded to this interpretation by adding a snowflake activity to her water cycle lesson that focused in part on the four types of precipitation (rain, snow, sleet, and hail). Mikayla added a hands-on activity that was appropriate for first graders that allowed them to consider one type of precipitation: snow. While this response may not directly indicate a deepening of students' understanding of precipitation, Mikayla does engage in connecting the processes of interpreting and responding to students' thinking. She does this at a level that could be expected of a new teacher, as well as one who may be limited by content knowledge or knowledge of how the concept builds. This finding that Mikayla responded based on students' thinking from lesson to lesson, but not during lesson enactment, was also evident in Phase Two of her mathematics teaching. While planning this lesson, Mikayla had interpreted from a previous lesson that students effectively understood the content through this kinesthetic modality. She then responded to students' thinking by incorporating another hands-on activity (making paper snowflakes) to demonstrate one of the four types of precipitation.

In Phase Three, Mikayla again attended to her students' science understanding using various formative assessment strategies. She taught the same lesson she had taught during student teaching, relating the water cycle and precipitation. She stated

> I had students read aloud altogether so that I knew they are engaged in the reading, I used partners to discuss so that I knew what they got, I had them write down on their worksheets, and then [I had them do] the [cotton ball] activity.

Mikayla also noted that in this lesson she included yet another modality to support students' learning—a song about the water cycle. She provided a rationale for using this song, although did not directly ground this rationale in specific supporting evidence of students' thinking.

> I like to use songs because they become catchy, and the student doesn't know that they might be repeating the song over and over and basically you are learning something … songs really do help them, well my class at least.

In this quote, Mikayla suggested that, in addition to modifying the lesson by incorporating hands-on activities, she also responded by including a song, which she had previously observed "helps" her class learn. It is unfortunate that she did not state specifically how she knew songs were effective; therefore only implying (not directly linking) her interpretation to observations.

Finally, Mikayla responded to students' thinking by explicitly focusing on the four types of precipitation and incorporating instructional resources beyond the curriculum. She again noted that this response was based on her previous experience, in this case having taught the lesson as a student teacher in Phase Two.

Mikayla responded in advance of the lesson to general learning difficulties she had previously observed. She stated

> I taught this same lesson last year when I student taught, but I taught it differently this time by breaking it down a little more than I did last year. [Last year] we talked about the water cycle, precipitation, and how clouds form all in one big unit rather than breaking it down and discussing each one, we have more time this year.

Because Mikayla's changes were based on her previous experience teaching this lesson, she made changes to the planned lesson, which were not specific to her particular group of students, nor to various levels of her students. In Phase Three, Mikayla did demonstrate one example of making broad changes in the moment of teaching science, "Students are also having a hard time explaining how clouds are formed so Mikayla has the students go back to their book and reread what it says about how clouds are formed" (Researcher Field Notes). This quote demonstrates how Mikayla observed that her students were struggling to understand the content, and therefore asked students to repeat the planned activity (i.e., reading the passage). This finding correlates with Mikayla's Phase Three mathematics teaching, when she responded to students' difficulties by repeating the activity (i.e., modeling another problem on the board).

Discussion

Mikayla's noticing in mathematics and science across the phases shifted as she attended, interpreted, and responded along her career progression. The following sections are organized by the interrelated skills of noticing, and bring together the disciplines of mathematics and science (Jacobs et al., 2010). Following these sections, further discussion extrapolates the findings more broadly to relate noticing to mathematics and science content. The discussion concludes by making connections between the IMB approach for field experiences and the study findings.

Attend

While teaching both mathematics and science, Mikayla's basis for attending shifted across the varying stages of the career progression. During Phase One, for both mathematics and science, she wrote specific questions in her lesson plans and attended to the responses of those questions. As she progressed through Phase Two and Phase Three, her formats for understanding student thinking, and the related attending, shifted. In the later phases, Mikayla incorporated other forms of assessment, such as journals in science, to elicit what students were thinking. She then attended to students' understanding by focusing on what she discovered from these assessments. This shift is possibly the result of increased experience teaching and recognition that simply asking questions, as she did in Phase One, does not always provide a clear understanding of students' thinking. In the interviews for Phase Two and Phase Three, Mikayla noted that she ascertained information about students' understanding in multiple ways and she sought input about how students were thinking that extended beyond asking questions. Thus, as she progressed through the phases, her approach to gain the information she gathered, that to which she attended, developed.

Interpret

At the onset of her career progression, when Mikayla was asked about her interpretations, or what students understood, she began her responses in mathematics and science with the phrase, "I feel the students …". This terminology expressed hesitancy or uncertainty in her commitment to knowing what students understood. Furthermore, she lacked connections to evidence of student thinking. This is not surprising, given that van Es (2011) characterizes connecting interpretations with evidence as mixed or focused noticing (level 2 and level 3), which is distinguished from baseline (level 1) noticing. During Phase One, Mikayla was a preservice teacher, so it is understandable that her noticing would be at novice level and mirror that of the baseline description (van Es, 2011).

Recall that in Phase Two and Phase Three, Mikayla attended to more than just question responses to try to understand students' thinking. Despite multiple inputs for attending, Mikayla's interpretations in Phase Two and Three remained limited to what students understood or did not understand. For example, in science, she made an interpretation about what students understood, but did not ground the interpretation in evidence, and assumed an evaluative position. When she worked to interpret student thinking, the emphasis was on correct or incorrect responses, as opposed to understanding the nuances of students' thinking. One distinguishing component in Phase Two and Phase Three was rare instances when Mikayla provided information on how she knew something was correct or incorrect. Occasionally, she would evaluate what she had attended to and would then provide an explanation for how she arrived at that conclusion. This distinguished her interpreting across the career progression as she began to ground her interpretations in her observations of her students' thinking. Therefore, similar to attending, there were slight developments among interpretations across the three phases.

Respond

With attending and interpreting, the shift in Mikayla's noticing was most notable between Phase One and Phase Two, with Phase Three, in most instances, mirroring the attending and interpreting of Phase Two. In contrast, marked changes in responding were noted between Phase Two and Phase Three for both mathematics and science, distinguishing responding during classroom teaching from responding during the field experience and student teaching. In the Lesson Study Analysis Meetings following teaching in Phase One, Mikayla was able to talk about what she would do differently if she were teaching the lesson again and discussed the next lesson, or how she would respond on the basis of what happened. During her Phase Two interview, she again discussed changes she would make to the lesson post-teaching, but did not demonstrate making these changes while teaching. In contrast, during Phase Three lesson observations, Mikayla was able to deviate from her written lesson plan and make adjustments on the basis of students' thinking. Thus, Phase Three was the first instance, both in mathematics and science, where she made significant in-the-moment responses on the basis of students' understanding. These responses came after Mikayla recognized that students were misunderstanding or not comprehending the topic she was teaching, at which point, Mikayla gave students additional problems in mathematics or had students reread in science. We recognize that both of these responses do not enhance the lesson or provide students multiple entry points to the content, they simply have students repeat what was problematic (i.e., rereading or repeating problems). This provides insight into Mikayla's noticing—namely her responding. She seemed to be more cognizant of her awareness about how the lesson was progressing (Mason, 2011). Perhaps in Phase Three, Mikayla reached the point that she was able to attend, interpret, and decide how to respond in the moment of teaching and then made changes to her instruction. Jacobs et al. (2010) note that "before the teacher responds, the three component skills of professional noticing of children's mathematical thinking—attending, interpreting, and deciding how to respond—happen in the background, almost simultaneously, as if constituting a single, integrated teaching move" (p. 173). It is possible that Phase One and Phase Two provided the structured supports for Mikayla to attend, interpret, and decide how to respond and the actual first instances of responding in the moment first manifested in Phase Three. We recognize that Mikayla's responses were aligned with those of a novice and not yet an expert, but these findings suggest that she may be integrating Jacobs et al. (2010) three interrelated skills in Phase Three.

When Mikayla responded in Phase Three by adjusting her lesson in the moment of teaching, she seemed stifled or restricted to the process, format, and content she was already pursuing. For example, when students struggled with using subtraction as repeated addition, she gave them additional problems of subtraction as repeated addition instead of attending to and interpreting their understandings. There was some discrepancy between what she was attending to and how she was deciding to respond. Perhaps, the lack of interpretation (extending beyond students being correct or incorrect) constrained her ability to implement changes in instruction that would address actual content needs. Likewise in science, when students were struggling with understanding a passage they had read, Mikayla adjusted her lesson plan by asking students to reread the same passage. As teacher educators, if we expect preservice teachers and those along the career progression to be able to respond with content-specific instruction, they need to be supported to do this.

In the case of Mikayla, we recognize that an even stronger connection to the content (mathematics and science) may have supported knowledge that would result in these types of changes. It is interesting that in the case of science, she did not paraphrase or teach the content in some other way other than reading. However, we recognize that directing students back to the textbook may not be surprising if her science content knowledge is a factor and if she is unsure of other avenues for supporting students' understandings.

Mathematics and Science Content

When considering the content and Mikayla's subject matter knowledge related to noticing, it is important to note differences between mathematics and science. When Mikayla was teaching mathematics and discussed the next lesson, it was typical that she would describe larger numbers as way to further challenge students. For example, in one lesson, she was focused on numbers up to ten and said the next day she would focus on numbers up to fifteen. She demonstrated some understanding of a hypothetical learning trajectory, recognizing that students progress with numbers in a somewhat linear form (i.e., learning numbers to ten before numbers to twenty) (Clements & Sarama, 2004). In contrast, when Mikayla discussed her science plans for subsequent days, she typically focused on "breaking the concept down." To think about what students needed to know next, she thought about all of the pieces or components of a larger unit. In this way, she perceived science as more recurrent and mathematics as more linear. For example, to understand how clouds form, students would not only need to understand the tenets and phases parts of the water cycle, but would need to know about convection. From this knowledge base, students could then explore a variety of concepts to build their understanding of clouds and how clouds form (e.g., temperature). Thus, Mikayla sometimes faltered with knowing how to provide students with a variety of entry points into the science concept being taught, and simply asked students to reread the same informational passage. Perhaps Mikayla was limited by her content knowledge, as well as by her understanding of the nature of science. The nature of science is such that concepts are continually being developed and built upon [i.e., science is tentative (Lederman, 2007)], perhaps exhibiting more fluidity with the order in which topics should be taught as compared to mathematics. We recognize that our data collection did not include a specific assessment of Mikayla's knowledge, but data sources we have indicate that her knowledge level may be a factor in our findings. Thus, these discrepancies between mathematics content and science content may somewhat relate to Mikayla's choices with responding when planning lessons.

Iterative Model Building Process

Considering these findings in light of the modified field experience process used in Phase One raises questions about the incorporation of content knowledge and mathematics and science knowledge for teaching (i.e., pedagogical content knowledge) into the lesson study process (Ball, Thames, & Phelps, 2008). During Phase One there were several instances during both the mathematics and science Lesson Study Analysis Meetings in which other members of Mikayla's group raised topics related to the mathematics or science content that would support responding in future lessons. For example, in the mathematics lesson taught in Phase One, Mikayla used a number line when students were not understanding less than or greater than in that context. After discussion about multiple representations and what it meant to be greater than or less than, Mikayla considered that she might incorporate a hundreds chart into her next lesson to help students further understand the concept. We argue that these conversations and moments are important for developing the understanding and ability to attend to students' mathematical (or scientific) thinking and that experiences thinking deeply about content may be necessary for teachers to fully interpret and respond on the basis of students' thinking. This process would support the interrelated skills that Jacobs et al. (2010) deem necessary for being able to make in-the-moment teaching decisions. Moreover, the supports provided to Mikayla in Phase One through the lesson study may account for the similarities in her noticing in Phase One and Phase Two. In Phase Two she had more extensive teaching experiences, but still demonstrated noticing that mirrored that of Phase One, when it came to responding in the moment. This is reasonable given that she no longer had the collaborative support of lesson study following her teaching, as student teachers were placed in various schools that did not incorporate lesson study as a professional tool. Perhaps added collaborative supports during the student teaching portion of Phase Two, specific to mathematics and science content knowledge and how students learn these topics (Ball et al., 2008) would be helpful in supporting the development of noticing, specifically responding.

Despite the suggestions for further supporting noticing throughout Phase Two, the notion that Mikayla, as a preservice teacher, was able to attend, interpret (to some extent), and consider responding (for future lessons) during a field experience on the basis of students' mathematical and scientific thinking is notable. The structure of the IMB program provided opportunities for preservice teachers to consider that to which they attended, scaffolded interpretation through a collective group setting in lesson study, and prompted discussion about next steps and teaching that should occur on the basis of what was discovered during the lesson about students' mathematical and scientific thinking. This structure afforded Mikayla opportunities to consider these components of teaching and to practice the interrelated skills of attending, interpreting, and responding (Jacobs et al., 2010). It is possible that the scaffolded support she received in Phase One through the IMB process influenced her ability to notice in later phases.

Likewise, the post-teaching questions during the interviews in Phase Two and Phase Three likely prompted Mikayla to further consider her noticing to an even greater extent, raising awareness of awareness, which could have influenced her noticing during her teaching (Mason, 2011). More specifically, knowing that she would be asked about students' thinking following her lessons may have prompted her to consider students' thinking to a greater extent during her teaching. One could argue then that the noticing Mikayla reported is simply a feature of the structure of the data collection process; however, we propose that the opportunities for reflection built into the IMB process provide valuable time for considering noticing. We recognize that the data reported in this chapter are limited to that which we were able to ascertain from documentation (i.e., lesson plans) and data that came from observations, lesson study, and interviews, and we cannot fully describe the extent to which Mikayla was noticing. However, we do have evidence that Mikayla attended, interpreted, and responded in all three phases of the project and these skills manifested differently at different points along her career progression. She then was able to simultaneously integrate these components into a single teaching move when she decided to respond (Jacobs et al., 2010). Therefore, we argue that the IMB process supported the development of Mikayla's noticing by emphasizing students' mathematical and scientific thinking and through providing scaffolds to encourage the development of attending, interpreting, and responding.

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Noticing Students' Conversations and Gestures During Group Problem-Solving in Mathematics

Kevin J. Wells

Abstract This study investigates how attention to student conversation and gesturing can inform teacher decisions about intervention within group problem-solving in mathematics. A class of grade 5 students was videotaped during group problem-solving over the course of a school year with a purpose of examining how talk during these sessions was organized, and how changes in gesture and body language accompanied progress in the problem. It was observed that when students made progress in a problem their talk took on a cooperative, conversational structure. In addition, student gestures grew in size and became more animated as their confidence in their utterances increased. At the same time, students working cooperatively tended to echo each other's gestures and body positioning. Attending to these observed results will allow teachers to interpret how students interact in order to make more meaningful decisions about supporting group talk.

Keywords Noticing · Conversation · Gesture · Mathematics · Problem-solving

Alan Schoenfeld (2011) puts the case for noticing in the classroom succinctly when he writes "Noticing matters". A lot. (p. 223). It is less a case now, I believe, of justifying professional noticing as an area of research, and more a case of situating oneself within it. Schoenfeld (2011) goes on to ask, "Now what?" before pointing out that what a teacher sees in the classroom should shape what that teacher does. In particular, it should lead to changed practices. Also, importantly, that it is tied to the teacher's beliefs and orientations. In this research, my observations are tied to my beliefs, following Sfard (2008), that thinking is a form of communication, and that understanding, building on Wittgenstein (1957), is demonstrated as "going on conversationally" (Wells, 2014). The overt forms of this thinking, including conversational interaction, gesture, and body language, are then the noticeable clues that can help a teacher make decisions "in-the-moment". The intent of this research

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is to suggest indicators that a teacher should try to notice in a classroom setting which relate to the understanding of a group of students. While recording and using the detailed tools of conversation and gesture analysis used in this research is impractical, there is evidence to suggest that there are indicators a teacher can notice in real time in order to help recognize developing understanding amongst students. I coin the term *teaching from the sidelines* to reflect the practice of actively noticing students, unobtrusively, while monitoring their progress. This requires that the teacher is looking for, and listening to, actions that unfold through group talk. These actions, general to group talk rather than particular to a problem, can then be used to support teaching. Specifically, this research addresses the question "What features of group talk, both as conversation and as gesture, should a teacher actively be able to notice?"

Literature Review

Mason (2002) pointed out "the mark of an expert is that they notice things a novice overlooks" (p. 1). This "expertise" comes partly with experience, but professional training plays an important role alongside this experience in helping to draw the teacher's attention to what to do with what they notice. The skill to be able to make pedagogical decisions in the midst of instruction is seen as crucial in the context of educational reforms (NCTM, 2000a, b). Many researchers in the field (e.g. van Es & Sherin, 2002; Corwin, Price, & Storeygard, 1996) promote the use of video to examine classroom activities in retrospect, and use their observations to point to improved practice. Other researchers, such as Fernández, Llinares, & Valls (2012), have researched prospective teachers' analysis of student artifacts via online interactions. This reflective activity is an effective way to develop one's noticing skills over time (van Es, 2004; Jacobs, Lamb, Philipp, Schappelle, & Burke, 2007), but does place an added burden on the already busy life of the working teacher. Coles (2013), who writes "We learn about things we do not know even exist by staying alert to the detail of what we see" (p. 58), illustrates a way this can be done as part of departmental meetings in a similar vein to the broader based video clubs of van Es and Sherin (2002). Developing and employing noticing skills in the heat of classroom activity is a much more challenging, but necessary, aspect in responding to Schoenfeld's "Now what?" question. Amador (2016), for example, reported that novice teachers lack in-depth interpretive analysis about student thinking, while Choppin (2011) found that teachers who attended closely to student thinking made better decisions regarding future assignments, leading to enhanced task complexity and student engagement. Informed task selection, and attending to students' strategies (e.g. Jacobs, Lamb, & Philipp, 2010), are important aspects of teaching, but "in-the-moment" decisions are also important in order to maintain the flow of classroom talk.

A feature of the reform-based classroom since the early 1990s has been a shift in practice away from procedural understanding and towards conceptual understanding. A central focus of this shift has been promoting student talk in the classroom. Mathematical talk, which involves students' explanation of, and the defence of, ideas, is seen as a hallmark of effective teaching (e.g. Sfard, Forman, & Kieran, 2001), along with observing and listening carefully to students. Noticing what effective discourse sounds like, and how it can be used to enhance desirable outcomes, is more problematic. There are many forms of communication in the classroom, some of which can occur simultaneously. Within a whole-class discussion there may be several smaller exchanges taking place, while many students may not be engaged at all. Sfard, Nesher, Streefland, Cobb, and Mason (1998) conclude that the teacher plays a key role in the success of how classroom talk is managed, Sfard notes "There are many ways to turn classroom discussion or group work into a great supplier of learning opportunities; there are even more ways to turn them into a waste of time, or worse than that—into a barrier to learning" (p. 50). If this is the case, then it is important that a teacher is able to develop their noticing skills to be aware of what it is about classroom talk that indicates it is being productive and, as important, what indicates it is being unproductive. In addition, if the intent is to give the students space to think and generate their own solutions, the teacher needs to be away from the focus of the group but be aware of signs that indicate intervention is necessary.

It is important to notice how the stages of group conversation unfold, and to pay attention to both gesture and posture. In addition, I put forward ways the teacher can support classroom talk based on these observations. These results are part of a larger study (Wells, 2014) and more substantial arguments for these ideas can be found there.

Framework

Noticing

Goodwin (1994) investigated how members of a profession shaped events to focus their attention upon. Goodwin examined how professionals coded what they attended to into objects of knowledge, how they highlighted salient features, and how this led to what he referred to as "professional vision", an organized way of making sense of events in a particular social setting. Goodwin focussed the discursive practices of the profession and made an analogy to what Wittgenstein (1957, §7) called a language game—"a whole, consisting of language and the actions into which it is woven". Mason (2002) used "intentional noticing" in comparison to everyday noticing and "professional noticing" to refer to the action of watching someone else acting professionally. More recently, the term "professional noticing" has developed further in the work of Jacobs et al. (2010), as a progression through the interrelated phases of attending to student's strategies, interpreting their mathematical understandings, and deciding how to respond on the basis of these understandings. These phases provide a framework to help analyse students' mathematical conceptions. More recently, researchers such as Thomas et al. (2015) use a broader description of attending to involve "noting aspects of a mathematical moment as a way to gather meaningful evidence (p. 296)". Important to this research, this broader description incorporates body language, changes in inflection, and other physical manifestations of learning.

Professional noticing in the classroom should not only include looking, but also listening. Students say a great deal more, and often in a way which is more demonstrative of their thinking, than what they are usually willing to write down. A thoughtful exchange of mathematical ideas can result in an artifact which belies the effort put into it. The transient nature of sound makes noticing more challenging in a busy classroom, so establishing what to pay attention to is important. While there is a growing body of research into noticing *what* students are saying (e.g. Fernández, Llinares, & Valls, 2013), in this research I was interested more in noticing *the way* they were saying it.

Conversation

Goodwin (1994) made reference to the work of Sacks, Schegloff, and Jefferson (1974) in developing Conversation Analysis. This stemmed from the observations of Sacks, who reportedly became interested in the organization of conversation through his work at a suicide counselling hotline (Pomerantz & Fehr, 1997). He wanted to know if the seriousness of the caller could be determined from the way they engaged in conversation. In effect, he was noticing significant moments in the talk, in this case the callers' mechanism of avoiding giving their name; the at-risk caller did not respond in an expected way. Sacks et al. (1974) recognized that verbal interaction has a social structure and organization where previously it had been thought that language was simply a medium to pass on information. In a similar manner, Scheflen (1964) reported that "Configurations of posture or body positioning indicate at a glance a great deal about what is going on in an interaction", and that "such behaviours occur in characteristic, standard configurations" (p. 316).

Turn-taking, where responses between interlocutors often occur in pairs and where there is an expectation of a certain response, is a characteristic of conversation. Significantly for this research, it was seen that turns appear in sequences so that a conversation has an introduction sequence followed by a core sequence and a closing sequence (Sacks et al., 1974). It was conceived that there is an institutionalized set of conventions that provide the framework for interactions in a particular context. Being able to notice key aspects of this framework in a classroom context—a special type of social situation—can be important to a teacher. Erikson (2011) has pointed out that students are adept at noticing what the teacher notices, and that the nature of classroom talk also reflects the atmosphere nurtured by the teacher. It is important, then, that a teacher is able to notice the features of effective talk so that such talk can be supported and developed amongst the students.

Effective Group Talk

Deciding what constitutes effective group talk is a key question. What is happening during those interactions is equally important to consider. Goodwin (1994) made a link between conversation analysis and the ideas of Wittgenstein (1957), who suggested that meaning is generated in the context of conversation and not uniquely by the words uttered. Sacks et al. (1974) emphasized that there is no predetermined structure to a conversation, but nevertheless conversations exhibit an organization that can be analysed. Wittgenstein (1957) felt that understanding was present when a speaker was able to "go on" with an idea; I suggest that a conversation develops when the interlocutors are able to "go on" with their turns at talk.

Gadamer (1975) writes that "a characteristic of every true conversation is that each opens himself to the other person" (p. 347), while Davis (1996) makes a distinction between a "conversation" and a "discussion". A conversation is seen as an open-minded exchange of ideas, while a discussion consists of the articulation of pre-formed ideas. The implication is that the interlocutors in a conversation need to be willing to engage in the process; each party must be willing and able to interpret the others' utterances in a meaningful way. When examining students' talk, Sfard (2008) suggests that there should be signs of a change in their discourse about the mathematics as a basic indicator of growing understanding. When talk is reduced to discussion, as defined above, participants make statements they are unwilling to question or reluctant to change; there is little or no growth in their discourse. In this case, the interlocutors are unable to "go on" with their thinking and so to develop understanding. This is a finer grained view of conversation than is typically used but if we want classroom talk to be productive then, as Mason (Sfard, Nesher, Streefland, Cobb & Mason, 1998) points out, it needs to be within the confines of a "conjecturing atmosphere" rather than "unfocused or off-task interaction" (p. 48). Davis's (1996) distinction is really about narrowing down the term "conversation" to that part of talk which is interactive and effective. The conversation is seen as a "meeting of minds" (Davis, 1996, p. 42), and understanding as being "negotiated with others through communicative interaction" (p. 23).

Conversation implicature (Grice, 1975) is based on the belief that talk exchanges are characteristically cooperative efforts; that interlocutors generally want to make sense of what each other are saying in order to move the conversation forward. Grice (1975) outlined four maxims of cooperation, which are the hallmarks of conversation: quality, quantity, relevance, and manner. Essentially, this means only adding what you believe to be true to the group talk, and doing so in brief, unambiguous, and orderly contributions. Grice suggests that violating these maxims generally causes conversation to break down. Similarly, the Politeness Theory of

Brown and Levinson (1978) includes Goffman's (1972) notion of face, which is the social value a person effectively claims for him or herself. Face threatening acts are those that either undermine the social status of an individual (known as positive face) or a person's ability to act (negative face). Such acts inevitably cause some reaction from the person threatened and can be noticeable in terms of a lack of cooperation. I suggest that attending to these social aspects of conversation can be an important aspect of maintaining group talk.

Gesture

Further, classroom interaction requires that students listen to each other and an observable part of listening comes through bodily interactions. Gestures, and actions such as leaning-in and reaching out, are noticeable features. A shift towards an embodied view of human experience leads to a suggestion that understanding can also be exposed by subconscious gesturing. McNeil (1992, 2005) developed a continuum (later continua) ranging from the completely unintentional *gesticulation* to formalized sign languages such as American Sign Language (ASL). McNeil identified four types of gesticulation, namely *Iconic* gestures which represent an actual action or object; *metaphoric* gestures which represent an abstract idea; *diectic* gestures which point to or at something; and *beat* gestures which carry no meaning and are often timed with prosodic peaks in speech. Beat gestures can be associated with emphasis or an emotional state. McNeil later added the *performative* gesture, which enacts what it represents, such as a rolling ball indicated by rotation of the hand or arm.

In everyday talk, gestures have been considered to be an integral part of communication (e.g. Kendon, 2004; Sikveland & Ogden, 2012) and linked to speech in a semantic and temporal way, while body language plays a part in any group talk (Goffman, 1972). Goffman refers to expressive cues we use as part of the communication process, further researched by Vertegaal, van der Veer, and Vons (2000). Vertegaal et al. (2000) make a link between the amount of eye contact people give and receive to their degree of participation in group communications. In addition, Hastings (2006) describes how certain eye movements may be associated with particular kinds of thinking. Roth (2000) describes a conversation as gestures and talk, adding that gestures and words only take on specific meaning in their interaction. As such, Roth sees thinking as being shifted into the world before the listener rather than being confined "in the head" (p. 368). Radford, Edwards, and Arzarello (2009) support this position, noting that "Thinking does not occur solely in the head but also through a sophisticated semiotic coordination of speech, gestures, symbols and tools" (p. 111). Sfard (2009) observes that combining speech and gestures brings about "an obvious synergistic effect" (p. 193), adding that gestures are "crucial to the effectiveness of mathematical communication … to ensure that the interlocutors speak about the same mathematical object" (p. 197). In the realm of science education, Crowder and Newman (1993) have examined the way gestures work in sense-making talk, observing that there is a change in the manner in which students gesture between describing models and figuring things out (running a model). Describing a model uses gestures timed with speech while running a model frequently exhibits gestures which precede related verbal content. Goldin-Meadow has researched extensively into the area of gesture-speech mismatch (e.g. Goldin-Meadow, 1999, 2015) as an indicator of developing understanding. Goldin-Meadow (1999) notes that "Children who produce a relatively large proportion of gesture–speech mismatches when explaining their (incorrect) solutions to a task are particularly likely to benefit from instruction in that task" (p. 424).

Echoing

A further interesting aspect of gesture has been referred to as "mimicry" (Kimbara, 2008; Holler & Wilkin, 2011), although I prefer the term *gesture echoing*, suggested by Pimm (2014), as giving a less intentional sense of the process. If a gesture or posture is being deliberately mimicked, then there may not be a genuine connection; if the gesture is subconsciously echoed, then the connection may better reflect a sense of shared understanding. Holler and Wilkin (2011) found such gestures "appear to facilitate the mutual understanding of the particular aspect that was being referred to" (p. 143).

Coles (2013) has observed that there is sameness in how talk unfolds in the classroom, year after year, even though each year the specific patterns of talk are different; there is "stability in those patterns across the years". Such an idea suggests an organization of talk that a professional might notice. Coupling this with thinking of understanding as a dynamic process, and incorporating the ideas outlined above, I suggest that it is "understanding as a state of action" that the professional classroom teacher can notice and support. In addition to paying attention to the mathematical content of the talk, I show that being more aware of typical organization of group talk (or lack thereof), and the manner in which students interact with their bodies, can help inform us about students' mutual understanding.

Methodology

The study was focused on two grade 5 classes over the period of their school year in a Canadian school. The school is located in a city east of Vancouver, BC, and consists of a wide range of cultural backgrounds typical of the area as a whole. Immigration to this region from many parts of the world is an ongoing process and produces a broad range of English language skills in the school. All the students in this study had a working knowledge of English, but some were clearly more fluent than others. Students were observed to converse freely outside of the classroom about various social issues typical to grade 5. Such observations helped to gauge a student's general level of interaction with their peers. The school and classroom are considered to be "safe environments" in which to learn, meaning there were no obvious barriers to student participation. Classrooms are encouraged to be places where students examine their thinking and, as such, the activities captured were not presented in an atypical way to students. The classrooms were bright, with one wall being a bank of windows, and desks were arranged in groups of four. The room was colourful, with posters and student work adorning the walls. Lessons often spilled out into the corridor or common spaces around the school and students were comfortable being sent out to work in quiet places. The students in the study (n = 32) demonstrated a wide range of attitudes to their work, from showing a very motivated approach to indications of attention difficulties. No students, however, were designated as having learning disorders.

Video recordings of mathematics classes were carried out from September through to June on a weekly basis. In total, over 150 video recordings were made during this time, ranging in length from shorter clips to full-class (45-min) recordings. Recordings were also made of groups of students following the task completion, either when presenting to the class or when engaged in a full-class discussion. Each contact lesson was part of the classroom teacher's normal mathematics programme. With a few exceptions, the classroom teacher generally selected the lesson activities and taught the lesson. The intent of the research was to look for characteristics common to any mathematical classroom talk in a natural setting, rather than linked to a specific activity.

In order to capture talk and group dynamics, three cameras were used; one camera was placed close to the group to ensure clear sound recording, while the others were set back to the sides of the room to capture more of the students' gesturing. Cameras were placed on tripods, turned on at the start of the group talk, and then left to run so that adult presence was not intrusive. Additional field notes were made as the lesson progressed. The video clips were then downloaded and examined using the software *ExpressScribe* (NCH Software, Inc., n.d.), which allowed clips to be slowed down and played frame-by-frame for easier transcription and coding. While students were aware that they were being recorded, they quickly seemed to ignore the presence of the cameras.

In these classroom sessions, all material was relevant to the learning outcomes of the British Columbian grade 5 curriculum (Education, 2007), but were "problem-based" rather than instructional. The typical lesson format was of an introduction followed by group work. The groups were of two to four students, generally not selected by gender or perceived ability, but to give a variety of combinations.

Students were given time to read through the task and think about the problem without talking or writing anything down. The intent was to give the students the opportunity to think about the problem from the moment it was assigned. The students were then asked to discuss how they thought the problem should be solved and to think of more than one way to solve the problem. Occasionally, having students stand up to discuss the problem seemed to encourage more body language and gestures than when they were seated. When the classroom teacher felt the students were ready, they were allowed to retrieve pen and paper to work on the problem, or to use white boards or manipulatives as appropriate. As the lesson drew to a close, the classroom teacher would generally bring the students together to discuss their findings.

Sorting the Data

Prior to making any transcription, a recording was viewed in its entirety with the intent of noticing any general features that immediately stood out. At the same time, recordings were initially classified into one of three broad categories: groups apparently making no progress, groups that seemed to be making progress before running out of time or ideas, and groups that seemed to have moved towards a solution they were satisfied with (not necessarily the same as expected by the teacher). Attention was paid to group dynamics, the quality of the question, and the mood of the class on that particular day (for example, events such as Halloween were detrimental to activities).

While a time-consuming process, transcribing the talk gave what Psathas and Anderson (1990) have called an "intimate familiarity with its details in the (real time) temporal flow of actual sequences" (p. 77). In addition to conversational markup, recordings were viewed a further time to look for gesturing and other salient features. Gesture markup was added to the transcript to indicate the temporal location of each gesture from its starting stage, through its stroke stage, and to its completion (McNeil, 1992). In order to break down the process further, focus was placed on the opening exchanges of the groups to see what was noticeably the same or different between groups that made progress and those that did not. From a broad base of these transcriptions, selections were narrowed down further in order to look for indicators that stood out across groups as they continued to work on the problem; this selection was done to isolate a few good examples of the general process, taken from each of the earlier groupings based on observed progress. The recordings were again reviewed in order to interpret what had been attended to on earlier viewings. Throughout, the focus was on recognizing features common to group talk in a variety of situations that a teacher would be able to attend to.

Results and Analysis

Analysis of the recordings made over the course of the school year indicated several moments, which, if noticed, can give a teacher clues as to developing shared understanding. By listening to and observing students in a group setting, the classroom teacher can make a more informed choice about actions to support learning. In this section these observed key moments to attend to are first outlined, and then supported with evidence from the recordings.

Attending to the Opening of the Talk

This can be a time when there are many distractions for both the teacher and the students, but actively attending to how students open their talk can be fruitful. The results of this research indicate that taking a moment to settle the class to engage in a focused start to their group talk, and then actively listening from the sideline for certain features of the talk, can give important clues as to how the talk will progress. At this stage, it may not be a matter of what is said as the manner in which it is said, or left unsaid, that is important. The results indicated that if talk did not begin in a cooperative way, then the session did not develop mathematically; either the students were unable to develop a way to solve the problem, or there was no sense that the students had changed their discourse about the mathematics in the problem. Students who opened the session by establishing good grounds for a conversation, in the sense of creating a mutually supportive atmosphere, were more likely to engage in a mutually supportive exchange of ideas and move to the actual mathematics of the problem.

There was a clearly observable ritual nature to the opening talk that seemed to generate the conversational space needed to build understanding. When this ritual was violated, the group did not make progress beyond the introduction stage. In a typical opening start, one student would take on the role to read out the question to the others. If the question was lengthy, another student would take over when the first student paused. This seemed to be done without any predetermined agreement. Once finished reading the question, the cooperative readers were seen not to offer suggestions, but to pause to allow another to make the first contribution, even when (as was evident from ensuing turns at talk) the reader had already formed a clear idea of what to do. In each case where the student who read the question then made the first suggestion, the group failed to make progress and, significantly, no conversation was established. The following example illustrates such a violation, where italics indicate overlapping talk:

> *Simone*: Anna came across this puzzle, something times something equals six-hundred twelve.
>
> *Eric*: What might be *the missing numbers*
>
> *Simone*: *the missing numbers*. Well, so, how many solutions can you find, show all your thinking plus explanation. Well I first thought we could try doing six hundred twelve minus … well we know six times two umm equals twelve … so … twelve
>
> *Eric*: Wait, so six hundred twelve …

Simone has opened the talk by reading the question and pauses after the first line. Eric takes this as a clue to continue reading but is immediately overlapped by Simone, who finishes the question and then starts to make a suggestion. Eric's reaction to this is to aggressively interrupt and question Simone. This is rare at this stage and Eric's posture also changes; he sits back and lowers his eyebrows. From this point the exchange's turns are not cooperative and neither student supports the other's utterances. Although there are turns at talk they are more challenging in nature. The two students make no progress with the problem and eventually call the teacher over for help.

Interpreting the Opening Ritual

As an isolated case there may be a number of reasons for these two students not to work together well, but I stress this violation of the structure of the opening sequence, regardless of the individual case, could always be noticed to lead to a dysfunctional group. Violating the format of the opening sequence seems to be an affront to the face (Levinson, 1983) of the other students in the group and results in a backlash, which affects the group dynamic. The opening of the talk appears to tie in with the ideas of Grice (1975), outlined above, in terms of conversational maxims. The opening exchange sets the tone for any future talk about the problem. Without the dynamics of such a conversational space it does not seem that a shared understanding can develop. The interesting thing here is that results from this research indicated that a conversational tone is established very quickly, or not at all.

Opening politeness also extends to cases where students read the question quietly, as in the case of Alex and Nadia below:

Alex: okay, let me read the question … (15s delay)

Nadia: Well, I mean, ready?

Alex: okay.

Here, the words "okay" (in the first line) and "well" are used as markers to request a turn at talk. In this way, the talk seems to be established without a power struggle, as being a common working space. Once a communal conversational space is established, interrupts, overlaps, and completing another person's utterance are acceptable and common features of conversation.

Attending to the Manner in Which Students' Gestures Change During the Session

When students' gestures increased in size, this was observed to coincide with an improved vocalization of their thinking. Students demonstrating large gestures were, at the same time, able to "go on" with their ideas and make progress with the problem.

McNeil (1992) divides the space in front of the speaker in terms of a *centre* and *periphery*, where the centre region is the person's torso from waist to shoulder and away from the body. I refer to gestures, which move into the periphery area and beyond "big gestures". Such gestures seem to accompany confident utterances. Figure 1 (and again in Figure 6) illustrates how a student in this research uses large gestures, which fill the gesture space before her.

Student: *If you have like a pop can the bottom is like a little circle that sorta stands out that would be the perimeter*

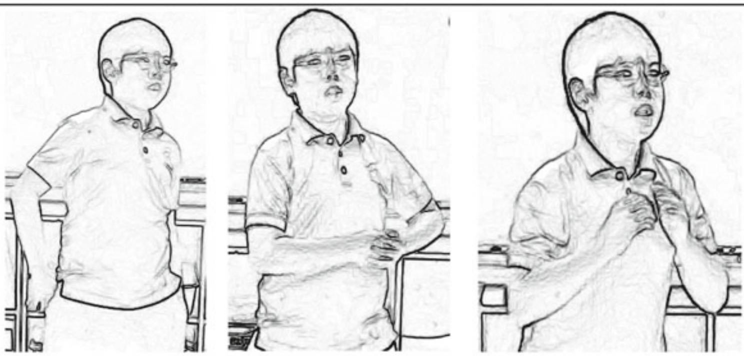
Her hands shape out the object in space and she points to its base. She then draws a large circle in the air with one hand while the other hand seems to hold the object in space.

Student: *And the inside that is like closer to the can I guess is like the area*

The gestures she is using are large and take up the centre area in front of her while also moving into the periphery area in front and to the sides.

Figure 1. Larger gestures of confident student.

Typically, the recordings demonstrated that gestures get bigger as the person is able to "go on" with the talk. This is illustrated in Figure 2, in which the original image taken from the recordings has been modified to protect the identity of the student.



These recorded images illustrate how a student who starts off an explanation lacking in confidence is often stiff in posture and holds their hands/arms fixed, in this case by hooking them in his pockets. As his confidence grows he frees his hands and they gesture in a low space around his waist. As his confidence grows further, his gesturing rises higher and his body language is more dynamic.

Figure 2. Gestures growing in size.

Interpreting Observed Gesture Size

Figure 3 illustrates gestures that were recorded when the students were working in their groups. Evidence of large gesturing seems to be an indicator that the student is confident in what is being uttered as they corresponded with clear and confident utterances. This change in the size of gesturing supports research on gesture dynamics and interaction by Gerofsky (2008), and Winter (Winter, Perlman, and Matlock, 2013), who note that gesturing size depends on the ongoing discourse; and ties into findings by Crowder (1996), who found students used larger gestures but positioned themselves further away from their gestures when presenting other peoples' ideas.

Using the ideas from watching students presenting and working in groups monitored by the camera, an inference can be made that students who use dynamic gesturing are also confident about the ideas they are expressing. Seated gestures tend to be more confined but can still stretch outside of the centre into the peripheral area.

Figure 3. Seated Group work gesturing.

Attending to Students' Echoing

One recorded aspect of group interaction was posture echoing. The more conversational students were, the more they tended to echo each other's gestures and/or posture. Echoing, particularly that of posture, was a frequently observed feature and is illustrated in Figure 4. In the first panel the two girls lean closer and adopt similar poses as they become more involved in their conversation. The girl on the left makes increasingly large hand gestures during this time. Closing the conversational space was frequently observed when students were working on a shared understanding. Figure 4, panels 2 and 3, shows that the girls in the group adopt a similar posture during their interactive talk. The boy is excluded from the talk until he adopts the same posture. Several reasons for the boy being initially excluded from the group talk can be suggested, but I again stress that this is but an example of an effect frequently enough seen across many groups to suggest that it is an important indicator of inclusion, and so important to notice.

The three girls in this group adopt a common posture while speaking as seen in panel 2. The boy in the group can only gain the others' attention when he mimics their posture.

Figure 4. Posture echoing.

Figure 5 illustrates that when individuals withdraw from the group talk they uncouple their posture. In larger groups, posture echoing between elements of the group was seen to be a dynamic process, with individuals moving in and out of the collective posture.

Another example of posture echoing. The postures change but those students who are engaged seem to adopt a common stance. The student who uncouples from the group posture self gestures before rejoining the posture of the group (panel 3).

Figure 5. Changing posture echoing.

A second, less common, indicator or shared understanding was in gesture echoing. In this case the group members adopted a common gesture, which was then used to convey a shared meaning throughout the session. On occasions where I recorded more than one group working on the same problem, a striking feature was that the group used different common gestures to represent the same thing. This is illustrated in Figure 6.

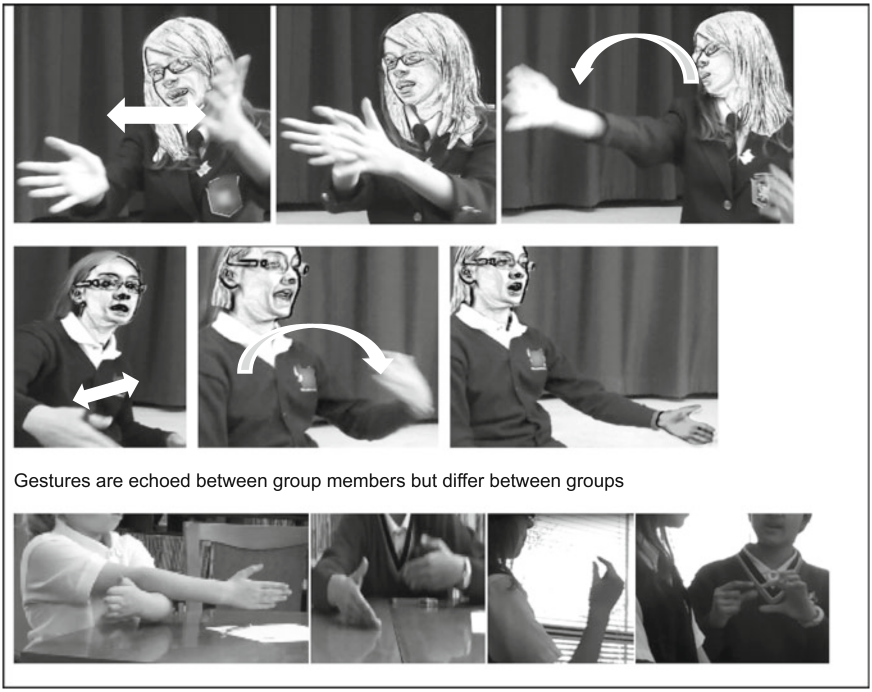


Figure 6. Gesture echoing.

Interpreting Echoing

If students are engaged in conversational talk, which will typically be accompanied by gestures, a shared understanding is being developed. This is in keeping with the idea of "exploratory talk" (Mercer, 1996), in which talk is mutually supportive when seeking to address the task in hand. If a student is constantly out of sync with his or her group, then some form of teacher intervention is necessary. When students uncouple from the group posture, this stemmed from either a loss of shared understanding and giving up, or from questioning the shared understanding and temporarily standing back in order to clarify thinking. In the former case the student's contribution to the group talk diminished; in the latter case the student typically made small self-gestures as indicators of their continued engagement.

They were then often able to re-engage with the group and make productive contributions to the group talk. Noticing this can be an important indicator to the teacher as to the level of shared understanding of the student concerned.

Attending to Developing Talk

Paying attention to the organization of the talk, as well as gesture and posture, brought out further features that were noticed. Groups that maintained a cooperative conversational space were able to progress deeper into the problem. This talk featured comments that were supported by others or were justified to the group. When fragmented talk within the group occurred it was generally an indicator of a breakdown in shared understanding. This fragmented talk is illustrated by the excerpt in Figure 7. The three boys, Aaron, Bashir, and Chan, are working on an area problem in which a field changes dimensions. Aaron is mistaking area with perimeter but continues with this despite the protest of Chan and the confusion voiced by Bashir.

The = symbol is used to indicate conjoined utterances; italic are overlaps		
24	A:	err you have to do ten times twenty first ((to Bashir))
25	B:	(Softly) then I .. then just=
26	A:	=ten times twenty is (drawing out the word ‘is’ like a prompt)
27	C:	two hundred (interrupting Aaron and Bashir’s exchange)
28	A:	two hundred and then *two hundred* plus fifty eq*uals*
29	C:	*two hundred* *NO* (overlapping Aaron’s prompt)
30	A	two hundred umm fifty minus two hundred equals fifty
31	C:	I guess that works
32	B:	You missed=
33	A:	=No this one’s right ((points to Bashir’s work)) but you just have to (*pause*) so this one’s two hundred, right ? then you subtract yeah you can do it this way too, two hundred=
34	B:	=What?
35	C:	Just write the answer to the question (sighs)
36	A:	so it depends if you reduce by fifty the area=
37	B:	=why’s it two hundred fifty? (Softly spoken)
38	A:	okay (pause) that’s *when* you *then*
39	C:	*Aaron* … *Aaron* … why are you doing it all by yourself now? It’s like its copying (soft laugh)

Figure 7. Fragmented talk typical of a discussion rather than conversation.

Interpreting the Developing Talk

In Aaron's discussion, in which ideas are being transmitted rather than worked on collaboratively, there is no connection being made for Bashir. Even when Bashir tries to add his thoughts, Aaron ignores them and continues (lines 25 and 32). Aaron also speaks over Chan's overlays without recognition (line 28). Chan's use of the word "guess" in line 31 indicates that he remains unconvinced by Aaron's help for Bashir, and perhaps even for himself. Chan interrupts Aaron more forcefully (line 34). Perhaps sensing that Aaron is still not helping, Chan interrupts Aaron's gesture space over Bashir's work by placing his hand into the gap between Aaron and Bashir, and suggests that Bashir "just write the answer" (line 35). Bashir tries again to ask for help (line 37), but Aaron continues to simply "talk". Finally, Chan stops the talk, asking more pointedly why Aaron is working by himself. Chan says what Bashir is doing is no more than copying from Aaron. Interestingly, this comment is made without a physical gesture. Chan has shown that he has been quite demonstrative throughout the session so the lack of any physical gesture here may be significant, perhaps dismissive or disengaging.

This inability to create a conversational space was typically seen when groups became unable to make further progress in the problem. By noticing when students are conversationally engaged, when their postures are echoed, and looking for signs of gesture echoing, the classroom teacher has indicators upon which they can act. In cases where a group was in a conversational mode, the arrival of the classroom teacher was seen to be detrimental to progress and it took some time for the group to re-establish their sense of shared understanding after the teacher had left. It is therefore as important that a teacher knows when *not* to intervene as much as when to do so.

Attending to the Shifts in Group Talk

The significance of the development stage of group talk is that it illustrates how the conversation can move from general talk about the problem to then incorporate the mathematics. The extension stage occurs when the students continue their conversation beyond the immediate requirements of the problem, for which they are content to have found a solution in the development stage. Figure 8 is an excerpt showing three students moving into the extension stage. They have found a solution for a problem involving ferrying cars and trucks across a local river on a boat with 42 "spaces". Each student shows clear signs of engaging in conversation as their turns at talk support and extend those of their interlocutors. Their talk is inclusive (e.g. line 14) while at the same time includes justification (line 18). Line 19 shows how this continued.

11	C:	and then that'll be thirty… and then three times the cars
12	M:	Yeah (pause)
13	C:	Okay (pause)
14	M:	Oh, no (pause), I was just thinking about like if you know that six time six equals thirty-six then if you added ten, then you would have forty-six and not forty-two (pause) so that wouldn't work (pause) Sally can you explain it?
15	S:	Yeah (drawn out). So like (pause) ah so there's forty-two vehicles and there's ten umm six trucks then you can't do it at the same time so you could put the umm trucks at two times across the river (pause) and then (pause) umm (pause)
16	M:	then the cars too like three plus *twelve is*
17	C:	*No we all*
18	S:	*no* two times six two times six equals twelve (pause) and then umm three times ten equals thirty (pause) and then add and you get forty-two, so yeah ..
19	M:	I'm trying to think of like other possible ways that you could do this

Figure 8. Moving to the extension stage of the problem.

Interpreting Shifts in Group Talk

These stage transitions can be important indicators of the students' progress and something a teacher should actively try to notice. In this transition there was a change in the talk indicated by the students posing or responding to "what if" style questions. This is an example of the conjecturing atmosphere referred to by Mason (Sfard, Nesher, Streefland, Cobb & Mason, 1998) as necessary for deeper learning to occur. Many groups stopped when they felt they had fulfilled the requirements of the question and needed further prompting to think more deeply about what they had found. Noticing when students shifted between stages, and intervening to prompt further enquiry, was seen to prevent group talk degenerating and becoming unproductive.

Conclusions and Reflections: Teaching from the Sidelines

Noticing is about being aware of details the casual observer looks past, as Mason (2002) pointed out. The results from this research suggest that there are such details a teacher can attend to in real time in order to help promote productive talk in the classroom. Such productive talk may be seen as an indication of developing mutual understanding amongst students. The third aspect of professional noticing involves deciding on effective tactic drawn from the interpretation of the classroom events (Jacobs et al., 2010). The concept of *teaching from the sidelines* is that the teacher stands far enough away from a group so as to minimize his/her influence on that group. At the same time, the teacher can be aware of the progress of the group by watching and listening to the group members.

Results from this research highlighted how it was possible to stand back from any group in the room and yet tune into the group talk unobtrusively. While a teacher may develop and enhance their noticing skills through experience, being more aware of key points to notice in group talk will develop these skills further. This research draws attention to the gestures and postures of the students as they interact with one another, something that previously may have been done subconsciously and unresponsively. It is possible to recognize, for example, when students are engaged in mutual activity from their posture and/or gesture echoing. An unproductive group—one that may be helped by intervention—is apparent by a lack of such echoing. An individual who is not engaged in a larger group is similarly noticeable. The confidence a student has in their utterances is often indicated by the nature and size of their gesturing. The absence of gesturing, or when gesturing is small or a mismatch to talk, can be a noticeable sign of a lack of confidence in an utterance and as such be a good time to offer support.

By listening carefully to how groups begin to solve a problem, a teacher may choose to intervene quickly if the student who reads the problem violates the polite turn-taking aspect that characterizes the introductory stage of the group talk. Similarly, flouting any of Grice's (1975) maxims of quality, quantity, relevance, or manner can stall the functioning of the group. Working with students who routinely flout these maxims might be considered as a way to better integrate them into the class. It may be the case that students are expected to know how to function in a group setting, and while this is a skill many have developed, there is work that can be done to improve this. Conversation may be a skill teachers need to teach, for example, and foster in their classrooms if they are to have success in non-traditional ways of teaching. This, I suggest, is where professional noticing can be of great importance, for students cannot learn these skills unless teachers can intervene at the appropriate time to support them.

Students who are functioning cooperatively in a group are best left alone as the casual "dropping in" of a teacher may break the pattern of conversation and disrupt the understanding process. Conversely, it is important that a group be monitored so that they remain on-task and develop a mathematical understanding that meets the intended outcome of the problem (or extends beyond it or beside it in a productive manner). Learning how to listen to and notice productive indicators of success/failure can be an important habit of mind for teachers to develop. If talk is seen to be "central to the meaning making process and thus central to learning" (Mortimer & Scott, 2003, p. 72) then it is important that classroom teachers are able to manage this talk. This should also mean that the teacher notices important features of successful talk.

The results of this research suggest features of group talk which can be attended to and subsequently allow for in-the-moment interpretations and decisions about how best to support the group talk. Such features may not be noticed by casual observation, or if attention is paid to the content of the lesson alone. By carefully noticing the organization of group talk, and paying attention to student posture and gesture, there is much more that the teacher can learn about students' shared understanding.

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Part III
Extending Equitable Practices in Teacher Noticing

Extending Equitable Practices in Teacher Noticing: Commentary

Cindy Jong

Abstract In recent years, equitable pedagogy and professional noticing have intersected in mathematics education research (Erickson, 2011; Hand, 2012; Wager, 2014). Teachers can make assumptions about students from non-dominant races, cultures, languages, and low socioeconomic status that are deficit-oriented (DiME, 2007). Thus, it is critical for equity to be central to professional noticing to provide all students with high quality learning opportunities. Hand (2012) emphasized the significance of teacher disposition in equitable instruction and developed a model consisting of three practice features to include: promoting dialogic space in classroom interactions, blurring distinctions between mathematics and cultural activity, and reframing the system of mathematics education. However, questions continue to be raised about what noticing for equity looks like in diverse classroom contexts. While there was agreement in this section on professional noticing consisting of the three interrelated components of attending, interpreting, and deciding (Jacobs, Lamb, and Philipp, 2010), the authors provided varying perspectives on how to embed equity. In this commentary, the following are highlighted: (a) equity frameworks, (b) teacher disposition and identity, and (c) classroom-based practices. Then, final thoughts are presented to connect topics in these chapters with further questions and considerations for the field.

Keywords Equity · Teacher disposition · Status · Positioning · Mathematics identity

In mathematics education research, equitable pedagogy and teacher noticing have flourished over the last two decades (D'Ambrosio et al., 2013; DiME, 2007; Gates & Jorgensen, 2009; Gutierrez, 2002; Sherin, Jacobs, & Philipp, 2011; Stahnke, Schueler, & Roesken-Winter, 2016; Strutchens, et al., 2012). Yet it is only in recent years that equitable pedagogy and teacher noticing have intersected in mathematics education research (Erickson, 2011; Hand, 2012; Wager, 2014).

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Erickson (2011, p. 28) referred to teachers' "pedagogical commitments" as "basic ontological assumptions, both tacit and explicit, concerning manifold aspects of teaching and learning activities," to illustrate how such assumptions inform equitable (and non-equitable) classroom practices. For example, teachers' views on learners' abilities ("low" or "high") and/or effort ("works hard") can influence their own expectations, the support they are willing to provide, and the tasks they select. Teachers often have assumptions about students from non-dominant races, cultures, languages, and low socioeconomic status that are deficit-oriented (DiME, 2007). Thus, it is critical for equity to be central to teacher noticing if the goal is to provide all students with meaningful learning opportunities and experiences. Hand (2012) emphasized the significance of teacher disposition in equitable instruction and developed a model consisting of three practice features to include promoting dialogic space in classroom interactions, blurring distinctions between mathematics and cultural activity, and reframing the system of mathematics education. This model offered tangible ideas of equitable teaching, but questions continue to be raised about what noticing for equity looks like in diverse classroom contexts. While there is agreement in this section on professional noticing consisting of the three interrelated components of attending, interpreting, and deciding (Jacobs, Lamb, and Philipp, 2010), the authors provided varying perspectives on how to embed equity. In this commentary, I highlight (a) equity frameworks, (b) teacher disposition and identity, and (c) classroom-based practices. Then, I close with final thoughts to connect topics in these chapters with further questions and considerations for the field.

Equity Frames

It is well established that equity in mathematics education is complex and multi-layered. Several scholars have recognized that the term equity consists of a range of concepts to include access, teaching for social justice, culturally relevant pedagogy, funds of knowledge, and status and participation (DiME, 2007; Jong & Jackson, 2016; Wager & Stinson, 2012). Some of these concepts have also been discussed in terms of distinct levels where *access* is moderate and *challenging structural inequities* is radical (Gates & Jorgensen, 2009). Thus, it is important to note the equity frames that are used within these chapters. Kalinec-Craig and Baldinger centered their work on status as it connects to student participation and positioning. Specifically, Kalinec-Craig drew on the sociological theory of expectation states and complex instruction to explain status. Similarly, Baldinger discussed status using complex instruction, but placed an emphasis on the social organization of the classroom and power dynamics. While van Es, Hand, and Mercado did not explicitly use a status framework, they noted the role expectations play in what teachers attend to during instruction and student participation; furthermore, a theme from their findings showed that the participating "teachers all attend to issues of *status and positioning*." (p. 266) Informed by Erickson's (2011) *pedagogical commitments* and building on Hand's (2012) model of equitable instruction, van Es et al. focused on teacher dispositions as they relate to equitable mathematics teaching practices.

The aforementioned equity frames, undoubtedly, informed the findings and implications. For example, Kalinec-Craig found that preservice teachers noticed characteristics of status and issues of participation within their field placements and classroom videos viewed in their mathematics methods course. A key finding emphasized was that "the process of equalizing students' status is not a process by which raising the status of one child means the teacher must lower the status of another" (p. 226). In Baldinger's chapter, she showed how a coach can support a teacher's noticing for equity and suggested the use of code profiles as a method to analyze discussions. The code profile of the teacher in this study implied that conversations with the coach promoted a shift from noticing *compliance* toward the *social organization of the classroom* and *mathematics learning*. The findings in van Es, Hand, and Mercado's chapter revealed clear relationships between teachers who noticed for equity and how it informed their instructional decisions. Along with status and positioning, teachers "attended to individual student histories" and "noticed the energy and flow of the students and the class," which indicated a "multi-layered nature of noticing for equity" (p. 266).

Teacher Identity and Disposition

Whether explicit or implicit, *identity* was a common factor in the studies in this section. Within these chapters, the spectrum of teacher development is represented, including preservice elementary teachers, a secondary mathematics teacher and coach, and expert secondary mathematics teachers. The authors all note that an equitable teacher disposition is central to promoting equity by having high expectations, valuing students' cultural knowledge, or connecting mathematics content with students' interests. Similarly, teachers' identity and their awareness of student identity shape their instructional decisions (Hand, 2012; Jong, 2016). In her literature review, Baldinger discussed how the learning opportunities teachers provide shape the development of students' positive identities as creators of mathematical ideas and capable learners. Kalinec-Craig's study focused on how Mexican-American immigrant preservice elementary teachers noticed and addressed issues of status and participation in their own prior experiences, courses, and field placements. They were able to identify with students in their field placements who were primarily Spanish speakers and emerging bilinguals, yet still able to attend to students who were different than themselves. This perspective is one that is rarely captured, because the majority of teachers in the U.S. are white while the student population continues to increase in racial and ethnic minorities (Museus, Palmer, Davis, & Maramba, 2011). Similarly, students of color face stereotype threat and lowered teacher expectations, and often attend schools that have more unqualified teachers and fewer resources (Museus, et al., 2011; Stinson, 2009).

In response to such inequities, van Es, Hand, and Mercado aimed to understand how secondary mathematics teachers, “come to notice the activity of their mathematics classrooms in ways that enable them to interrupt these deficit perspectives and processes in support of their learners” (p. 252). While three of the four teachers in this study were white, they were selected based on demanding criteria that clearly demonstrated their commitment to and success with promoting equity. Their results confirmed that the teachers had an “equity lens” that informed how they attended to students, interpreted experiences in the mathematics classroom, and made instructional decisions. As Hand (2012) explains, “dispositions of mathematics teachers are critically important because they underlie distinctions teachers are likely to make in moment-to-moment classroom activity” (p. 234). For example, a teacher may interpret a student’s seeming disinterest as one who lacks motivation or aptitude rather than one who needs to connect the content with his/her cultural background or interest.

Classroom-Based Practices

At the heart of noticing for equity is making instructional decisions that will positively influence students’ achievement, experience, and identity. It was promising that several findings and implications in these chapters included pedagogical moves that promote equity, which provide more clarity on what noticing for equity looks like in the classroom. van Es, Hand, and Mercado found five teacher practices that promoted equity: leaving students to grapple with mathematical ideas, making norms explicit for doing mathematics, supporting students in developing mathematical identities, connecting with students to honor individual strengths, and making systems of schooling explicit. They elaborate on these practices with rich descriptions and supportive examples. Baldinger argued that teachers can be more attuned to status issues if they notice the social organization aspects of the classroom (e.g., group dynamics, who is participating in the discussions) as opposed to compliance (e.g., who is following instructions). By doing so, the goal is to have a greater focus on engaging students with the mathematics of the lesson. This aligns with an example where Schoenfeld (2011, p. 229) noted, “The teachers were so focused on issues of order and discipline that they failed to notice that the students were amazingly competent!” In Kalinec-Craig’s context, preservice teachers made the following instructional decisions to promote equity in the classroom: using students’ native language, providing opportunities for all students to communicate their thinking, and encouraging the participation of individuals who were perceived to have a lower status.

While all the equitable practices presented in this section were deemed valuable, there is variation in the skills and knowledge required for implementation of these practices that may be aligned to a developmental progression, to a certain extent. For example, preservice and novice teachers are more likely to take up practices such as encouraging participation of individuals as opposed to making systems of schooling explicit, which might be achieved with more experience and a more complex level of noticing for equity. My point here is to say that context and teacher development are two critical factors for mathematics educators and researchers to take into consideration. While both factors, context and teacher development, have been discussed in teacher noticing research, there has been more attention on teacher development in terms of what is required to notice at various levels (Schoenfeld, 2011; van Es, 2011). So it may very well be the case that noticing for equity contributes to the field by paying particular consideration to the contexts of classrooms and schools, as these authors have shown. In addition, deliberate attention extends beyond students' mathematical thinking to include their positioning, whether they are making personal connections to the content, and how they are interacting with their peers and the tasks. As van Es et al. note, there is a distinction regarding equitable teaching practices that include "issues of status, culture and power in the mathematics classroom" that surpasses "just good teaching" (p. 268). Correspondingly, Cochran-Smith et al. (2009) make a case for teaching for social justice by directly addressing critiques of it as *just* or *simply* "good teaching," because it is viewed as an "ambiguous concept that is widespread but undertheorized" (p. 347). To make such a case, evidence was provided of preservice teachers who had both a thoughtful understanding of teaching for social justice and classroom practices that reflected the following four characteristics: focusing on all students' learning, building relationship with students and respecting their families and cultures, being an activist by advocating for students and engaging in community work, and recognizing inequities related to race, class, or resources. In this section, additional characteristics of equitable pedagogy were presented to strengthen the case for noticing for equity in mathematics classrooms.

Final Thoughts

There are two common features I found in reviewing these chapters that warrant further discussion as equity and teacher noticing intersect. The first is that the *contexts* were all in low-income schools and mathematics classrooms with students who are racially, culturally, and/or linguistically diverse. Second, the *research methods* were qualitative in nature and drew primarily on interview and observational data.

The authors in this section all recognize that equitable teaching is essential for students who are racially and culturally diverse, because they can experience school in ways that are quite distinct from students and teachers in dominant groups. There was also consensus for the need to notice equitably in low-income schools, because there might be fewer resources and structures that limit learning opportunities (e.g., tracking, larger class size). While I completely agree with the authors on these accounts, I could not help but wonder: *What does noticing for equity look like in a suburban school with mostly white students?* There are some equitable teaching practices, such as supporting students in developing positive mathematics identities, which can certainly be beneficial to all students; however, I suspect that there are teaching practices that specifically apply to students in predominantly white schools where an awareness of privilege and inequities are integrated into the mathematics curriculum. This might also look different depending on students' socioeconomic status or gender. I do not have any clear answers to this question, but think it is worth more consideration. Another pedagogical question related to the mathematics classroom context I pose is *Can equitable teaching exist in classrooms where reformed-based curricula are not used?* I raise this question because van Es, Hand, and Mercado required "skilled use of reform-based mathematics curriculum" as a criterion to select "exceptional equitable teachers" (p. 256). It is certainly challenging for me to envision equitable mathematics practices in a teacher-centered classroom where the focus is on rote learning and students are not given an opportunity to discuss mathematical ideas. The case has been made that reform-based approaches promote equity (Boaler, 2002; Secada & Berman, 1999), but whether they are inherent to equitable teaching is a topic that can, and should, be further explored.

In this section, the qualitative research methods were appropriate to the questions and aims in the chapters. There was variation in the extent to which the coding and analytical methods were detailed, but some common themes were apparent. Baldinger suggested the use of code profiles to examine potential changes in noticing; however, the three broad codes might not be specific enough to disentangle the nuances that exist in equitable teaching. A methodological question I have is *Can an instrument be developed to measure noticing for equity in mathematics?* While it is possible (and there might be one that exists), agreeing upon a clear purpose and common characteristics to measure would not be a simple process. For example, measuring whether teachers demonstrate equitable dispositions versus practices would look quite different. Dispositions can more appropriately be measured by a survey or open-ended interview questions in comparison to practices where an observation protocol would be valuable. An instrument that applies to a variety of contexts (e.g., elementary vs. secondary, urban vs. rural) and stages in teacher development (novice to expert) might be so generic that it is of limited value *or* might be so extensive that it becomes taxing to use. I raise these methodological and measurement questions for scholars to consider ways to further validate the construct of noticing for equity. Research questions about equity and teacher noticing that are of interest to scholars in the field may not lend themselves to the positivist paradigm. However, it might be fruitful for noticing for equity research to take a more critical theory approach by exploring participatory design research with the aim of developing more socially just systems (cf. Gutiérrez, Engeström, & Sannino, 2016).

As research on noticing for equity continues to grow, scholars need to take careful consideration of contextual factors and develop purposeful research designs. In addition, selecting equity frames that apply to classroom practices, and attending to teachers' disposition and identity are critically important, as the authors in this section have demonstrated.

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“Everything Matters”: Mexican-American Prospective Elementary Teachers Noticing Issues of Status and Participation While Learning to Teach Mathematics

Crystal Kalinec-Craig

Abstract When prospective teachers learn to teach mathematics, they develop an understanding of content and pedagogy, which also includes strategies that encourage all children to participate in their learning. There is research that shows that issues of status and inequitable participation can hinder children’s access to learning mathematics and can give children the impression that only some students can do mathematics. The following book chapter presents the experiences of three Mexican-American immigrant prospective teachers as they learn how to teach elementary mathematics and to notice issues of status and participation in their fieldwork. Data sources include coursework artifacts from the methods classroom, observations in the field and semi-structured individual interviews with the participants. Using the professional noticing framework, the findings suggest that the prospective teachers attended to, interpreted, and acted upon moments of unequal status and participation with children in the field over the course of the semester. Implications for teacher education and future research will be discussed.

Keywords Elementary mathematics · Teacher education · Status · Equitable participation · Professional noticing framework

> After watching this video I realized that everything matters: how we [as teachers] stand, where we stand, how we talk, and who we talk to… We decided to pick Jordan to come up front because we realized that he had good strategy, and *I am glad we did because it gave him the opportunity to participate in the class despite [the fact that] he is not proficient in Spanish yet, since this is his first year at Douglas [Elementary]*” (Maricela,[1] italics added for emphasis).

[1]All names have been changed to maintain the confidentiality of the participants.

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Introduction

Learning to teach mathematics is a process that is challenging, dynamic, and iterative. Prospective teachers learning to teach mathematics must attend to multiple aspects of their practice, including the mathematical thinking of their students and pedagogical strategies that elicit and extend this thinking (Berk & Hiebert, 2009; Stein, Engle, Smith, & Hughes, 2008). Yet, there are other aspects of teaching, such as affording children the opportunity to participate in the process of learning mathematics, which may hinder (or support) all children to learn mathematics. For example, traditional classrooms may not incorporate children's diverse knowledge, experiences, and resources that they bring to the classroom and as a result, not all children may have an opportunity to learn mathematics (Featherstone et al., 2011; Moschkovich, 2013).

In an effort to open more opportunities for children to learn mathematics, there is growing research about how a child's status (a perceived social ranking) (Cohen, Lotan, & Catanzarite, 1988) in the classroom can influence how (and/or if) they take an active role in the classroom (Featherstone et al., 2011). When children have equal status (e.g., academic, social, linguistic statuses) in the classroom, teachers can work toward a goal of promoting equity for all students in their classrooms (Boaler, 2006; Cohen & Lotan, 1995). The purpose of this chapter is to use the professional noticing analytic framework (Jacobs, Lamb, & Philipp, 2010) to explore the experiences of three Mexican-American prospective teachers (PTs) as they noticed issues of status and participation in their field experience classrooms. The chapter will begin by discussing what is already known about mathematics teacher preparation regarding status and participation. Ultimately, this chapter will return to Maricela, one of the participants in this study, to consider her conclusion that "everything matters" when it comes to recognizing issues of status and participation while learning to teach mathematics.

Background to the Problem

PTs come to their teacher preparation programs with a variety of skills, knowledge, experiences, and beliefs about what it means to teach mathematics (Hammerness et al., 2005). For many PTs, their prior educational experiences directly inform their vision for teaching mathematics (Lortie, 1975) and it can be challenging for PTs to negotiate their prior experiences with what they learn in their teacher preparation program (Hammerness et al., 2005). As reported by Ball (1988), teacher preparation programs should be places where all PTs are "unlearning to teach mathematics" so that they can adopt a more inclusive vision for what it means to teach mathematics.

The need for teachers to adopt a more inclusive vision for teaching mathematics is a pressing issue given the changing demographics in our classrooms. Our schools are receiving more immigrants and children who speak a native language other than English (Kena et al., 2015; Passel & Cohn, 2008), all of whom bring a wealth of mathematical knowledge and experiences. Unfortunately, traditional teaching strategies are typically effective for only a small subset of our student population (Grossman, Schoenfeld, & Lee, 2005). Traditional teaching strategies might ignore the mathematical resources of many other students and ultimately limit the opportunities for all children to participate and succeed when learning mathematics (Featherstone et al., 2011). The work of Funds of Knowledge (Moll, Amanti, Neff, & Gonzalez, 1992) and Cognitively Guided Instruction (Carpenter, Fennema, Loef Franke, Levi, & Empson, 1999), for example, describe frameworks that help teachers to adopt a vision for teaching mathematics so that all students participate in learning mathematics while helping students to utilize their mathematical resources (Gay, 2002; He & Cooper, 2009; Turner et al., 2012).

Some PTs already hold a vision for teaching mathematics that elicits and incorporates students' diverse needs, knowledge, and experiences. PTs who learned mathematics in a second language (Gomez, Rodriguez, & Agosto, 2008) and/or share the cultural background of their students (Vomvoridi-Ivanovic, 2012) may be familiar with strategies that encourage all children to be active participants in the learning process. For example, PTs like Cavazos (Cavazos, 2009), an immigrant who learned mathematics in a second language, openly resisted and dismantled the low expectations that her colleagues assigned to immigrant students. A note of caution though: it is naïve and shortsighted to expect that students will learn more mathematics only when their teacher shares in their cultural and/or linguistic background (Achinstein & Aguirre, 2008). Nonetheless, there is a need to understand how PTs learn to respond to the needs of their students so that more students can participate and be successful in mathematics.

Therefore, this particular book chapter poses the following research questions: (1) Using the professional noticing framework, in what ways do Mexican-American prospective elementary teachers recognize issues of status and participation in their prior experiences and address similar issues that might arise with their students in the field placements? (2) How might PTs' attention to status and participation inform their general vision for teaching mathematics for all students?

Conceptual Frameworks

This study used the following conceptual frameworks: (1) sociocultural learning theory, (2) status (based on the sociological theory of expectation states), and (3) professional noticing. First, sociocultural learning theory in mathematics education posits that people learn and do mathematics by interacting with each other, the curriculum, and the norms established in the classroom or context (Atweh, Forgasz, & Nebres, 2001; Lave & Wenger, 1991). The language, symbols, and tools, which could denote the “culture” of the classroom, are established among teachers and students throughout the learning process (Vygotsky & Cole, 1978). Furthermore, a sociocultural learning perspective argues that students leverage their experiences, backgrounds, and interactions with others to construct meaning in their learning. This theory was selected because it specifically aligns with the second framework regarding expectation states theory and more specifically, issues of status.

To address issues of status and participation in classrooms, the study used the research of expectation states theory (Berger, Cohen, & Zelditch, 1972) and the research of Complex Instruction (Boaler, 2006; Cohen, Lotan, Scarloss, & Arellano, 1999). As described by Foddy (1988), the expectation states of a person are based on “beliefs that group members hold about each other’s abilities to produce good or poor task performances” (p. 232). Status characteristics, which are fluid and differ based on the particular task at hand, include three types: (1) diffuse status characteristic (e.g., racial identity, native language); (2) specific status characteristics (e.g., based on one’s specific profession or skill set); and (3) local status characteristics (e.g., status assigned to a peer or based on a school culture) (Featherstone et al., 2011; Horn, 2014). Simply put, expectation states theory claims that a child who has a low status characteristic is likely to be assigned a low expectation for their performance and therefore may adopt a similar perspective about their own potential to contribute to the task (Webster & Foschi, 1988). The research of Complex Instruction leverages the research of status by describing the ways in which teachers can carefully design tasks that foster equitable group work, build on students’ particular mathematical strengths, and equalize student status (Cohen et al., 1999).

The professional noticing framework (Fisher et al., 2014; Jacobs, Lamb, & Philipp, 2010), based on the teacher noticing framework (van Es & Sherin, 2002), can help (re)direct teachers’ attention toward particular aspects of classroom instruction. In general, the noticing framework is more than simply a tool to examine what teachers tend to notice about teaching, but to help teachers *analyze* what they saw, to make *connections* to some larger pedagogical implications, and to provide *suggestions* for future practice (van Es & Sherin, 2002). And as Sherin, Jacobs, and Philipp (2011) argue, “the word ‘noticing’ names a process rather than a static category of knowledge” and implies that teachers should constantly work to notice new aspects of classroom instruction as a means of developing their practice and expertise (p. 5). The professional noticing framework explicitly foregrounds the iterative ways in which teachers can use what they see and know about teaching in order to drive future instructional decisions (Jacobs, Lamb, & Philipp, 2010). Although this particular framework has been used to help teachers notice aspects of children’s mathematical solution strategies (Fisher et al., 2014; Jacobs et al., 2010), this chapter used the professional noticing framework while elementary prospective teachers learned to teach mathematics in their methods coursework and fieldwork.

Methodology

Overview

The study is a subset of a larger qualitative phenomenological case study (Creswell, 2007) about the ways in which three Mexican-American PTs learned to teach mathematics for understanding (Hiebert et al., 1997) and incorporate children's out-of-school mathematical knowledge and experiences (Moll et al., 1992). The participants in the original study (the three female immigrants from Mexico and one American student who identified as Latino) were enrolled in a mathematics methods course[2] within a teacher preparation program in a large urban university in the Southwest part of the United States. For the purpose of the research questions, this study is limited to only the experiences of the three Mexican-American PTs with respect to how they noticed and addressed (if at all) issues of status and participation with their students, many of whom were also immigrants and native Spanish speakers.

During this semester, the three PTs (Sara, Miria, and Maricela) were concurrently enrolled in other methods courses (e.g., science, social studies, reading, writing) and completed fieldwork hours at an elementary or middle school where the majority of the students qualified for free and reduced lunch. Sara, Miria, and Maricela were placed with cooperating teachers (Ms. Arevalo, Mr. Cruz, and Ms. Cabrera, respectively) who were Mexican-American and native Spanish speakers and whose students were mostly Mexican-American and emerging bilinguals. More specifically, Sara and Maricela's field experience classrooms were designated as dual language, which meant that the teacher instructed in Spanish, for at most, 70% of the class time.

Data Sources and Analysis

Over the course of the 16-week methods semester, the following data was collected from the three PTs: mathematics autobiography, a series of interviews about problem-solving and children's funds of knowledge, reflections from lesson plan implementations, and video analyses of classroom instruction. Four semi-structured interviews and observations from the field were also conducted with respect to what the PTs were learning in their methods coursework and field experiences.

[2]This particular mathematics method course was a part of a larger research project, TEACH Math (Teachers Empowered for Advancing Change in Mathematics) that was supported by the National Science Foundation under Grants No. 0736964 and 1228034. For more information about how TEACH Math research team conceptualized the assignments described in this chapter, please see Aguirre et al. (2013), Roth McDuffie et al. (2014), and Turner et al., (2012).

Using a content analytic framework (Krippendorff, 2012), the data sources were first divided and renamed as sampling units. Within these sampling units, recording subunits were created when the PTs attended to or highlighted (Jacobs et al., 2010; Goodwin, 1994) an issue of status or participation (Cohen & Lotan, 1995). For each recording subunit, a memo was generated that described the context in which the PT described the issue of status or participation. Table 1 shows examples from three of Maricela's recording subunits.

Table 1
Example from Maricela's recording subunit coding scheme

Mathematics autobiography	Maricela described a moment during her college class when she felt as though she did not have an opportunity to participate in the mathematical discussion because she was unfamiliar with the mathematical terminology in English
Problem-solving interview case study	During the Getting to Know You Interview, Maricela noted she learned about Jordan's struggles to learn mathematics in Spanish because he was not yet fluent in that language. Maricela documents specific strategies for helping Jordan to use his native language so that he communicated his thinking and to bridge this knowledge in Spanish. She specifically talked about how these strategies can help him participate and learn more mathematics
Whole group mathematics lesson	During Maricela's whole group math lesson, she described how she opened opportunities for Jordan to participate in the whole group sharing. She recognized that Jordan could contribute to the mathematical discussion and helped Jordan to communicate this thinking in his native language

The process of creating recording subunits and memos continued until the data was coded for each PT. Once the recording subunits were analyzed for each PT, the coding and memos were compared *across* the PTs and a single data source (e.g., the coding for Maricela's mathematics autobiography was compared against Miria's mathematics autobiography) in order to further refine the analysis across the PTs. In all, the data was analyzed in three ways: within the data sources from a single PT, across a single piece of data from all three PTs, and finally, across all PTs and all data sources. This process was replicated in order to achieve an accurate interpretation of the PTs' experiences during the semester.

As with all research, threats to validity (Maxwell, 2013) should be addressed when discussing any study's limitations. The larger portion of this particular study examined the PTs' perceptions of what they noticed in the classroom and their prior experiences and a threat to these reported perceptions and interpretations lies in the fact that some of the field experience teachers did not consent to be interviewed for the study nor permitted the researcher to conduct multiple observations of the PT in the field. Therefore, not all of the PTs' perceptions could be completely verified with what happened in the field. Finally, the notion of status and participation emerged from the data after the larger study concluded. Therefore, because the PTs were not consistently asked to attend to issues of status and participation throughout the semester, the findings do not assume that a PT who talked about status and participation during their interviews and reflections were the only ones who happened to notice these issues during the semester. Nonetheless, this study acknowledges the limitations of the PTs' report from the field and aims to not overgeneralize the findings and implications.

Researcher Positionality

As a means of acknowledging the lens that the researcher used to conduct this study, the author identifies as a White scholar in the field of mathematics teacher education. Her goals as a mathematics teacher educator at a Hispanic Serving Institution in a large urban university is to help her PTs develop not only pedagogical content and content knowledge, but also to develop a lens for noticing issues of status and equitable participation in their classrooms. The experiences of Sara, Miria, and Maricela motivate her to constantly search for new ways to elicit and honor the resources that her PTs bring to their teacher preparation program.

Findings

The following section first describes what the PTs noticed about status and participation as mathematics students—narratives that draw from the PTs' mathematics autobiographies and initial interview. Next, the section describes what each PT noticed about status and participation as they completed their mathematics methods coursework and field experience. Finally, the section briefly describes a cross-case comparison of Sara, Miria, and Maricela.

Noticing Issues of Status and Participation in Prior Experiences

All three PTs were young adults and teenagers when they first immigrated to the United States: Sara and Maricela were both 18 and Miria was 14. The PTs similarly described how they struggled to learn mathematics because their teachers in the United States did not use effective linguistic strategies. As an example, Maricela recalled how she struggled to learn mathematics in college because many of her professors communicated mostly in English and rarely elicited her existing mathematical knowledge in Spanish. Maricela stated:

> It is not easy to take a college math class for the first time if you are not a native English speaker, so I had a hard time figuring out the math terminology in English. Some terms are very similar to Spanish, but some of them are completely different.

Maricela attended to how her teacher did not necessarily use strategies to help her bridge her existing mathematical knowledge in Spanish to learn mathematics in English and felt limited in her opportunities to participate. Sara reported a nearly identical experience in college as well. The diffuse high status characteristic of English fluency (mostly for students in the United States) was not explicitly assigned to Sara and Maricela, both native Spanish speakers, in their college classes.

On the other hand, Miria viewed her English fluency as an indication of her overall potential to succeed when learning mathematics. In one poignant instance, Miria described how she struggled to communicate her confusion about a mathematics problem to her high school mathematics teacher. She recalled

> I couldn't explain to [my teacher] what was my problem and he didn't do anything to help me. I got out of his classroom feeling really miserable. That day I understood that if I couldn't communicate in my new language I was never going to be good at math and it also made me start hating math.

Miria recognized the challenge she faced when trying to communicate her confusion to her teacher and she interpreted this challenge as a defining moment for her own self-efficacy in mathematics—English fluency was a high status characteristic in mathematics classrooms in the United States. Later in her reflection, Miria described a goal for herself as a teacher in response to what she experienced when she wrote "I also want to have strategies to help the students who don't speak English, because that way they will be more engage[d] during the activities as well as more comfortable." Each of the PTs came to the United States with varying levels of English fluency and experienced challenges when learning mathematics in a second language. The PTs began their methods coursework and field experiences with the overall goal that they would respond to the needs of *all* of their future students, not just those who happened to be fluent in English, the dominant language of instruction.

Noticing Issues of Status and Participation During the Mathematics Methods Semester

Over the course of the mathematics methods semester, Sara, Miria, and Maricela each recognized various issues of status and participation by reflecting on their own experiences and by observing the practice of other teachers. In some cases, PTs like Sara typically noticed issues of status and participation when she was prompted by a course assignment or interview question. Yet on the other hand, PTs like Miria and Maricela noticed issues of status and participation that arose in their field experience without being prompted. The findings will describe the individual PTs' experiences as well as the similarities and differences across their experiences.

Sara. Sara first attended to and interpreted a moment of inequitable participation when she and her classmates were asked to analyze a video clip from the

Annenberg Learner Video online series. In the “Marshmallow” video, a second-grade teacher posed a problem-solving task to her students to decide how many children can eat from a bag of marshmallows if each child will eat six marshmallows. As the PTs watched the video, they were explicitly asked to analyze the video by considering, “Who participates? Does the classroom culture value and encourage most students to speak, only a few, or only the teacher?”[3] In the quote below, Sara specifically noticed one child in particular, Marisa, who appeared to have a high academic status:

> Marisa is the one that was assumed to know the answer. She was always participating and jumping into the discussion. And I put that the students were assigned low and high status because, you know, when she [the teacher] ask[ed], “did you choose a spokesperson?” and they all point to this girl [Marisa.]

Although Sara attended to Marisa’s high status and role as a spokesperson, Sara did not explain the status characteristic that Marisa held in order to be selected for this role nor did Sara provide a suggestion as to how she might balance the status among the other children in the video. Nonetheless, Sara continued to attend to and interpret the teacher’s use of English and Spanish as one way to help more children participate in the task when she stated:

> …all students are encouraged to participate and their feeling of community and students are encouraged to participate and share their ideas. Even one girl [speaks] in Spanish and the teacher translated for the rest of the group so this allowed for the students’ opinions to feel valued and appreciated.

Even though Marisa appeared to have a high academic status in the video, Sara still interpreted the teacher’s use of the children’s native language as a helpful move that afforded more children the opportunity to contribute to the mathematical discussion and learning.

Near the end of the semester, when Sara presented to her classmates about her whole group mathematics lesson plan, she showed a video clip of her lesson about creating equivalent fractions. After Sara showed a video clip of her lesson, she discussed about the ways that she attended to the needs of her emerging bilingual students in the classroom when she stated “I used English and Spanish because he [Sara’s student] is an ELL [English Language Learner] and sometimes has a hard time understanding English. That’s why I switched to Spanish.” Sara attended to a small group of students who were struggling to understand the objectives of the lesson as it was presented in English. She interpreted this issue as an opportunity to use her Spanish fluency to help her emerging bilingual students participate in the mathematics task. Sara then responded to her student’s needs by communicating the details of the task in Spanish, the child’s native language. Sara practiced using the responsive strategies that she was familiar with as an emerging bilingual student herself. The findings suggest that Sara did not report about a child’s perceived

[3]For more details on the lenses that focus PTs’ attention to different aspects of classroom instruction while watching video clips, please see Roth McDuffie et al., (2014).

status unless she was specifically prompted by an assignment or interview question (i.e., Marisa's high status in the video clip and her reflection of her whole group mathematics lesson). Sara typically reported about how the ways in which she opened opportunities for all of her students to participate and communicate their thinking.

Miria. Similar to Sara, Miria noticed issues of status and participation when specifically prompted to do so in her practice and the practice of others. When Miria watched the same marshmallow video, she noticed Marisa's high status, as did Sara. More specifically, Miria noticed how the teacher used the rug as a means of facilitating a whole group mathematics discussion:

> So I think most of the instruction took place at the rug. And this is because, I think, it's the, like when she [the teacher] did the graph, it was easier for the students to see the graph. It was easier for them to know [sic] what she was asking for. And it was a way to control more participation…[for the teacher] to hear the students and [for the students] to hear her.

Miria attended to and interpreted the opportunities for children to participate in the video given that the physical object of a rug both helped the children to see the graph and participate in the whole group conversation. Miria perceived the rug as a tool for facilitating equitable participation among the students in the video.

During an interview later in the semester, Miria noticed how sometimes whole group discussions could limit some children's opportunities to participate in that discussion. I asked Miria to elaborate about the perceived status of her students and how Mr. Cruz, her field experience teacher, facilitated whole group mathematical discussions. In the following quote, Miria attended to an issue that arose when the children were positioned on a rug and engaged in a whole group discussion except for children like Letty, a child who received special services, rarely had an opportunity to contribute:

> …the same ones that answer all the time are the ones [who] raise their hand and speak out. And I think I would [call on all of them]. I know the little girl [Letty] has an aide too. She understands [the question asked] if you explain to [Letty]. I think she's really good if you help her to look at you and explain to her what you're saying, but she sits in the back… I don't know why [Mr. Cruz] does that….Maybe [I will] make sure that the special needs [children] understand what they are doing by asking them questions, make sure the ones that are learning English are understanding too. Not just [assume that if] one kid [speaks up] and because that kid understands, then everyone understands.[4]

Unlike Sara, Miria attended to Letty's physical positioning and interpreted Letty's isolation as a specific student status issue in Mr. Cruz's classroom. As a means of equalizing the students' status in the class, Miria suggested that she would call upon all students to contribute. Miria's vision for teaching mathematics, extended beyond noticing issues of emerging bilinguals, Letty, a native English speaker who received special services, was also in Miria's purview.

[4]Some of the PTs' quotes were edited in order to provide more clarity and context based on extended interviews.

Maricela. Maricela placed the needs of emerging bilinguals at the forefront of her vision for teaching mathematics, as did Sara and Miria. Yet unlike the previous PTs, Maricela spent much of her methods coursework and field work focusing on the needs of one particular child, Jordan, a student in her field experience class. Jordan was a White, native English-speaking child in Ms. Cabrera's third-grade classroom, Maricela's field experience classroom. Ms. Cabrera's classroom labeled as dual language where 70% of the instruction was in Spanish and 30% was in English. Maricela noticed that Jordan, a child who she perceived to have low academic status, experienced limited opportunities to participate in the mathematical discussions. Maricela reflected on what she noticed about Jordan and Ms. Cabrera.

> …And my teacher is like assuming that he doesn't deserve even a 1 [a basic proficiency score], he's not trying to do it because he doesn't know how to do it. Cause when I, when we solve problems together, he really knows, he really knew what to do and everything and he got them right. This kid knows, he can learn and everything and my teacher [says] that he doesn't even deserve a 1.

In the excerpt above, Maricela noticed that Jordan was assigned both a low diffuse status characteristic because he was an emerging bilingual and a low academic status characteristic because he struggled to communicate the mathematics that he did know. In Maricela's final report about Jordan, she noticed that Jordan, a second language learner, needed help communicating his thinking in class.

> What is happening to this child [Jordan] is similar what is happening right now with English Language Learners; the only difference is that he is a Spanish Language Learner. The teacher needs to use sheltered instruction techniques to help him understand, such as posters, word walls, videos, interactive games, etc. Things that can help Jordan make connections and have a better understanding of what is going on. The teacher needs to get to know him better to understand his needs, and make math and all the other subjects meaningful to him.

Maricela interpreted Jordan's fluency in Spanish as a contributing factor to his limited opportunities to participate in the lessons. Maricela responded to Jordan's situation by suggesting Ms. Cabrera utilize more explicit strategies with Jordan to support his learning. And for much of the semester, Maricela continued responding to Jordan's low status by helping him to learn mathematics in Spanish using his existing knowledge in English. When Maricela reflected on her whole group mathematics lesson, she talked about how she publicly reassigned Jordan to have high academic status by encouraging him to participate in the mathematics discussion:

> After watching this video [of me teaching my whole group lesson] I realized that everything matters: how we stand, where we stand, how we talk, and who we talk to...We decided to pick Jordan to come up front because we realized that he had good strategy, and I am glad we did because it gave him the opportunity to participate in the class despite he is not proficient in Spanish yet since this is his first year at Douglas elementary…Later you don't like think about all those things that can really affect the way you teach and the way your students perceive math.

As a college student, Maricela knew what it felt like to struggle to learn mathematics in a second language, much like Jordan, and as such, she responded to what she noticed by utilizing strategies that could support Jordan and reassign his status among his peers. What is important to note about Maricela is that she responded to the linguistic needs of Jordan, a child who would not typically fit the demographics of an emerging bilingual in the United States.

Discussion and Conclusion

Sara's, Miria's, and Maricela's experiences of learning to be respond to the needs of their students by leveraging their prior experiences is similar to what others have found in their research (Drake, Spillane, & Hufferd-Ackles, 2001; Gomez et al., 2008). The experiences of Sara, Miria, and Maricela complicate Lortie's (1975) assertion that PTs typically adopt similar strategies that their teachers employed with them. The three PTs in this study noticed when their teachers did not use effective strategies that bridged their mathematical knowledge between their native and second languages. While the PTs learned to teach mathematics, they considered and utilized strategies they thought would respond to the needs of emerging bilinguals like themselves. The strategies used by the three PTs encouraged more children to participate in the mathematical learning, not just those who are fluent in the dominant language of instruction.

The stories of Sara, Miria, and Maricela add to our understanding of how PTs learn to attend to the resources that students bring to the classroom (Aguirre et al., 2013; Fisher et al., 2014; Rodríguez & Kitchen, 2005). The findings from this study suggest that TEACH Math assignments such as a case study of a child's mathematical thinking and experiences (Turner et al., 2012) might also help PTs to notice issues of status and participation with their field work students.

Furthermore, the findings from this study also suggest that Miria and Maricela noticed issues of status and participation when it involved a child who was from a background *different than their own*. Miria described Letty, a child who received Special Education services, even though Miria never claimed to identify herself as someone who also received Special Education services. Maricela focused on the needs of Jordan, a White native English-speaking child in her field experience classroom. Without prompting, Maricela continued to perceive Jordan's low academic status and experienced limited opportunities to participate. Maricela designed and implemented a task so that she could publicly assign competence to Jordan, even though she did not perceive him as one who was expected to contribute to the classroom discussions.

The findings from this study also suggest that equalizing students' status is not a process by which raising the status of one child means the teacher must lower the status of another (Cohen & Lotan, 1995). As Cohen and Lotan (1995) suggest, "Participation however is not a zero sum game." (p. 26). In order to raise Jordan's status, Maricela did not need to lower the status of another child, but instead Maricela opened *more* opportunities for Jordan to be assigned a high status.

The implications from this study suggest that there is more to be learned about the ways in which PTs' prior experiences inform their vision for teaching mathematics, particularly with PTs emerging bilinguals and/or are immigrants. Because mathematics is culturally situated, "traditional" algorithms used in the United States may be unconventional elsewhere in the world (Gonzales et al., 2008). Therefore, it is imperative to explore the ways new teachers leverage their particular set of knowledge and experiences as a means to notice how all children can communicate their mathematical thinking, especially due to the increase of immigrants who need mathematics teachers that are prepared to respond to their needs (Passel & Cohn, 2008).

Furthermore, there is still more to learn more about how new teachers learn to consistently notice issues of status and participation in their practice as mathematics teachers (Boaler, 2006; Featherstone et al., 2011). How might teacher educators support PTs to sustainably notice issues of status and participation within multiple contexts? How can PTs learn to use their lens for noticing status and participation to drive future instructional decisions? How can methods instructors create spaces so that PTs and field experience teachers can learn more about how their instruction limits or encourages all children to be seen as smart in mathematics? As Maricela so eloquently stated, "everything matters" when learning to teach mathematics including learning to notice how nuanced issues like status and participation can play a role in our mathematics classrooms.

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"Maybe It's a Status Problem." Development of Mathematics Teacher Noticing for Equity

Evra M. Baldinger

Abstract This chapter proposes an aspect of *teacher noticing for equity,* bringing together ideas from literature related to educational equity and to the social nature of teacher learning. It argues two points and offers methods for empirical study to investigate them. First, it argues for an important direction for the study of teacher noticing that supports equitable instruction: noticing of the social system of the classroom within which power dynamics operate. Second, it argues that the development of this type of noticing for equity can be supported through purposeful, work-embedded interactions. It offers methods for the study of this development, and exemplifies those methods using data from a case study of teacher learning through conversations with an instructional coach, which take place in the context of an equity-focused professional development project.

Keywords Equity · Discourse · Teacher learning · Sociocultural theories of learning · Discourse analysis

This chapter considers teacher noticing in light of lessons learned from scholars concerned with, first, educational equity, and, second, with the social nature of teacher learning. It builds on the work of scholars concerned with educational equity, who have focused our attention on inequitable distribution of power, which takes place within classrooms and creates barriers to meaningful learning for some students (Boaler, 2008; Boaler & Greeno, 2000; Cohen & Lotan, 1997; Nasir & Hand, 2008). The chapter also builds on the work of scholars who have focused our attention on teachers learning in and from interactions that are intimately tied to their own teaching practice (Grossman, Wineburg, & Woolworth, 2001; Horn, 2005; Little, 2002; McLaughlin & Talbert, 2001; Wenger, 1998). Bringing these ideas together, this chapter argues two points and offers methods for empirical study

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to investigate them. First, I argue for an important direction for the study of teacher noticing that supports equitable instruction: noticing of the social system of the classroom within which power dynamics operate. Just as noticing of student thinking allows teachers to build appropriate responses to that thinking (Sherin et al., 2011), noticing of the social organization of the classroom supports teachers to respond appropriately. When teachers recognize classrooms as social systems within which power dynamics operate (rather than just collections of individual students and a teacher), they can attend to reconfiguring these social systems in ways that create more equitable access to opportunities for students to learn and to construct identities as competent doers of mathematics. They can intervene in status and power issues only when those issues are recognized for what they are (and not, for example, interpreted as individual students lacking motivation or desire to learn).

Second, I argue that the development of this type of teacher noticing for equity can be supported through purposeful, work-embedded interactions. I offer methods for the study of this development, and exemplify those methods using data from a case study of teacher learning in the context of an ongoing, equity-focused professional development project.

Background and Theoretical Perspectives

Teacher Noticing

Sherin et al., (2011), in their summary of the field of mathematics teacher noticing to date, describe teacher noticing as consisting of two interrelated and cyclic processes in which teachers engage: (1) selecting particular phenomena for attention "from the blooming, buzzing confusion" of classroom life and (2) making sense of those phenomena. Some scholars (Jacobs, Lamb, Philipp, & Schappelle, 2011; Kazemi et al., 2011) further articulate the sense-making process as encompassing subprocesses of interpretation and response. These conceptualizations offer a three-part understanding of teacher noticing: (1) selecting particular phenomena for attention, (2) interpreting those phenomena, and (3) responding accordingly. It is important to note that teacher noticing is not understood as a passive process;[1] rather it takes place as teachers act and interact in and out of classrooms. And, while it may be analytically useful to consider the three

[1]These processes are also deeply situated; they are done by individuals (alone or together), each of whom carry particular constellations of resources, orientations, and goals (Schoenfeld, 2010), and whom are embedded in classrooms, schools, local and extra-local cultural and historical contexts, each of which must bear heavily on noticing that takes place in classrooms. This chapter foregrounds processes of noticing as they take place in the context of teachers' work-embedded interactions, and thus backgrounds psychological or cognitive conditions that undergird the noticing that takes place.

subprocesses of teacher noticing separately, the relationship among them has yet to be established, as does their degree of mutual distinction. They are seen as interrelated and cyclic (Sherin et al., 2011) and it stands to reason that shifts in any of these three component processes of noticing may have implications for the others. For example, if a teacher comes to interpret a particular phenomenon in new ways, she may also choose new responses to that phenomenon. These changes may lead to changes in which phenomena she selects for attention in the future. In fact, research does show that when teachers become skilled at attending to and interpreting student thinking, they become more adept at designing instructional responses that build upon and extend that thinking (van Es & Sherin, 2008).

Furthermore, research suggests that professional vision (Goodwin, 1994) and in particular teacher noticing (van Es & Sherin, 2008) is, at least to some degree, trainable. That is, practitioners can be supported purposefully to develop new and more productive ways of noticing. As Sherin and colleagues point out (Sherin et al., 2011), there is much more work to be done to uncover potentially productive ways to support the development of various aspects of teacher noticing.

In the following sections, I describe findings from literature related to *equity* in education and to *teacher learning through interaction* that illuminate the utility of *teacher noticing for equity* as a construct of focus.

Equity

For many years, researchers have exposed gaps between demographic groups in various measures of achievement. Federal education policy (No Child Left Behind [NCLB], 2003) brought these achievement gaps to the center of mathematics education conversations on every level, from faculty meetings in school libraries to conversations that shape local, state, and federal education policy. However, focus on these sorts of gaps (or “gap-gazing”) has been critiqued as reifying discourses that position students from nondominant groups as deficient and offering little guidance for policy makers or practitioners concerned with improving teaching and learning and working toward equity (Gutiérrez, 2008; Martin, 2003).

In the late 1980s and early 1990s, scholars offered a shift in focus from *achievement gaps* to *opportunity gaps* (Gamoran, 1987; Oakes, 1990), exposing patterns of students’ unequal access to resources such as advanced courses, qualified teachers, adequate facilities, and textbooks. These scholars expanded the field’s focus from distribution of desirable outcomes to include distribution of supportive inputs (i.e., various kinds of opportunities for learning). This broadened view supports the design of policy-level responses that involve the redistribution of access to the opportunities that are identified as important and inequitably distributed.

As the field became interested in understanding the opportunities for learning available (or not available) to students, scholars began to investigate how these opportunities are afforded and distributed within classrooms. Key findings suggest that widely distributed access to meaningful learning can take place when students work together on challenging tasks, when they are held accountable to their own and each others' learning, and sense-making is valued over answer getting and prior achievement (Boaler & Staples, 2008).

Other scholars began to attend to opportunities students were afforded to construct particular kinds of identities in mathematics classrooms and the relationships between the social structure of classrooms and the distribution of these opportunities (Boaler & Greeno, 2000; Horn, 2008; Nasir & Hand, 2008). For example, Boaler and Greeno (2000) found that discussion-based mathematics classrooms in high schools supported students to author identities as creators of mathematical ideas and to choose to continue their studies of mathematics. In contrast, students in didactic classrooms tended to author identities as received knowers of mathematics (which was generated outside of themselves) and fewer of these students chose to continue their mathematical studies. Nasir and Hand (2008) found that classroom-level supports such as clear expectations and feedback, opportunities to take on integral roles, and opportunities for self-expression supported students to view themselves as competent members of the domain (the domains in their comparative study were classroom mathematics and participation in the activities of a high school basketball team) and that these sorts of opportunities were unevenly distributed among students. From these scholars, we learn that the ways in which classroom environments are structured, and the supports that these structures offer for students, matter for the distribution of opportunities for students to learn and construct positive disciplinary identities.

Unequal distribution of status and power in classrooms is a significant barrier to equitable access to opportunities both for learning and for developing positive identities. Cohen (1997) argues that societal structures, such as unequal power relations and hierarchical narratives of competence, travel with students and teachers into classrooms. Students and teachers enter classrooms with differential expectations for their own and each other's competence. These expectations are deeply cultural in that they are rooted in the cultural discourse relevant to particular communities (rather than being the creative products of isolated individuals). They influence patterns of participation and have important consequences for students' opportunities for learning and for developing identities of competence.

This perspective suggests that classroom-level interventions aimed at increasing equity must attend to the social and cultural nature of classroom environments and the ways in which status and power operate in these environments to afford and constrain important opportunities for students. Supporting teachers to attend to and make sense of the social and cultural nature of classroom life is an important aspect of an agenda for improving teaching and learning. This chapter considers ways in which work-embedded interaction can support teachers to learn to notice the social organization of classrooms. The following section considers learning and, in particular, teachers’ learning in the context of work-embedded interaction in order to ground the empirical investigation of the development of teacher noticing that follows.

Teacher Learning in Work-Embedded Interactions

Lave and Wenger (1991) focus the attention of learning scientists on the deeply situated, cultural nature of learning. Wenger (1998) further articulates a theory of learning that includes the ongoing negotiation of meaning, in which people, in the context of communities of practice, continually negotiate and reify meaning. In Wenger’s theory, the negotiation of meaning in and about practice among participants engaged in that practice takes place continually and is an essential component of learning. It is important to note that this omnipresent negotiation of meaning is unpredictable in nature; there is no guarantee that the meanings that are negotiated will be of any particular sort. It follows, then, that people embedded in practice (which all people are) continually engage in learning and that only some subset of that learning will satisfy observers as “good” learning, or the learning that we might hope takes place to support any particular outcome.

Scholars concerned with teachers’ learning, and with instructional improvement more generally, have distinguished teachers’ work-embedded interactions as important sites for their learning (Grossman et al., 2001; Horn, 2005; Little, 2002; McLaughlin & Talbert, 2001). Consistent with Wenger’s ideas about learning, scholars have found that attending to teachers’ ongoing negotiation of meaning in the context of work-embedded interactions is fruitful for understanding the opportunities that teachers have to learn in productive ways. For example, we know that the nature of work-embedded interactions has consequences for the opening or closing of important opportunities to learn (Little, 2002) and that particular norms for interaction are consequential for the kinds of learning available to participants (Grossman et al., 2001; Louie, 2016).

Also consistent with Wenger’s articulation, scholars have found that teachers’ learning in the context of naturally occurring, work-embedded interaction does not always support excellence or increased equity in classroom instruction. McLaughlin & Talbert (2001) and Horn (2005) found that teachers’ work-embedded interactions in some cases support resistance to reforms or more exclusive and less equitable instruction.

These findings teach us that it is important to be clear about what kinds of learning we mean when we talk about teacher learning. Certainly, while teachers' solidifying their tendencies to label students as fast, slow, or lazy (Horn, 2007) is a kind of learning, it is not the sort of learning that supports instructional improvement. In this chapter, I investigate processes by which teachers learn to notice classroom phenomena in ways that position them to offer more equitable instruction. The learning attended to here, then, is the ongoing negotiation of meaning likely to support this sort of instructional improvement. As we glean from the equity literature, an important goal for this kind of teacher learning relates to developing teachers' facility with noticing the social and cultural dimensions of the classroom. This chapter investigates learning consistent with this goal.

Development of Mathematics Teacher Noticing for Equity

In this chapter, I argue that when teachers learn to notice status and power at work in their classrooms, they are positioned to intervene constructively and to reshape patterns of inequitable access among students to meaningful learning. Further, I argue that the development of this type of teacher noticing can be purposefully supported, just as an experienced archeologist can support a novice to develop constructive ways to "see" dirt and skillful attorneys can support jurors to "see" police officers' use of force as professionally appropriate (Goodwin, 1994).

In the following sections, I look closely at work-embedded interactions to investigate the following questions. (1) How can we identify patterns in aspects of teacher noticing for equity, and in particular teacher noticing of the social organization of the classroom, in the context of teachers' professional conversations? (2) How can professional conversations purposefully support development of teachers' noticing of the social organization of the classroom? The close examination of work-embedded interactions gives us some access to all three subprocesses of noticing (selecting phenomena for attention, interpreting those phenomena, and responding accordingly), but primarily to the second, interpretation. The conversations examined in the research reported here take place after phenomena have been selected for attention and reveal interpretive work taking place (i.e., Wenger's negotiation of meaning) and, in some cases, also reveal potential responses as they are conceived.

Methods

This chapter's purposes are (1) to argue for the importance of teacher noticing of the social organization of the classroom and (2) to offer emergent methods for the study of the development over time of aspects of this type of teacher noticing, as evident in work-embedded interactions. This section introduces methods to support the study of the development of teacher noticing of the social organization of the classroom. It begins by outlining the study within which the methods were developed and then it shares and exemplifies the methods.

The Study Context

This study examined the interactional work done by teachers with their instructional coaches in the context of an extensive, ongoing professional development project in Complex Instruction (CI) (Cohen, Lotan, Scarloss, & Arellano, 1999; Pescarmona, 2010) for secondary mathematics. CI is a pedagogical approach that focuses on providing equitable access to rigorous, student-centered learning experiences by preventing, identifying, and addressing status problems that stem from hierarchical and elitist notions of who can be “smart” in academic environments. CI takes as a foundational assumption the idea that all students are capable of participating in rigorous learning, and that teachers can support participation and learning for all students by intervening when status problems arise and by working to create classroom cultures in which “smartness” is understood in inclusive and expansive ways.

Two teachers who worked with one coach were selected for close analysis. These teachers worked at the same urban, continuation high school serving low-income students, and shared many of the same contextual supports and challenges. Video and audio records were collected of the interactions that took place between each teacher and the coach. Data shared here come from records of three debriefing conversations (after lesson observations) that took place between Mr. Shaw (a pseudonym) and his coach. (I am the coach involved in the conversations in this study. While a thorough discussion of affordances and limitations of participant-observation is outside the scope of this chapter, I mention a few issues briefly as they are relevant to the methods discussed in the following section.)

Analytic Methods

Conversations were recorded and then transcribed and organized into a two-column format in order to foreground the flow of conversation between the two participants (Ochs, 1979). Talk was segmented by breath, or meaning group (Chafe, 1994), with a new unit (referred to as "line") of talk beginning when (1) a new speaker began to speak or (2) a speaker paused and took a breath.[2]

Coding

Codes were developed to answer the following questions about noticing: (1) What do participants in the conversations attend to and interpret as successful and/or challenging in the lessons? (2) What do they name as goals or targets for the development of future instruction? In other words, what responses do they develop or envision?

Using open coding and grounded theory (Strauss & Corbin, 1994), the following categories of noticing emerged and were coded for across the data corpus: (1) Talk that related to noticing of *students' mathematical thinking and learning* (such as, "I was happy that they all seemed to get the main idea.") or to mathematical goals for instruction (such as, "What math do we want them to be learning?") was highlighted using purple (medium gray for this printing); (2) Talk that related to noticing of *the social organization of the classroom or of learning* (such as, "I saw really strong group work today.") was highlighted with pink (light gray for this printing); and (3) Talk that related to noticing of *student compliance*—whether and how students were "doing what they were supposed to do"—was highlighted with blue (black for this printing). Color-coding was critical in the creation of *code profiles*, which are discussed in the following section, and colors were chosen to provide visual contrast. (For this printing, coding is done in gray scale, with shades of gray chosen to support readers to see patterns discussed here. I ask the reader to imagine ways in which color makes patterns available to visual perception differently than can shades of gray.)

My participation as a coach in these conversations allowed me interpretive power in that I was able to check the results of my analyses against my assessments as a practitioner. It also forced me to seek out opportunities to ensure that the inferences I was making were warranted *in the data* and not unduly influenced by my impressions and biases. I did this by involving a research assistant who had not been present during the coaching conversations and had never met the teachers in the study. Together we combed carefully through the data to ensure that we were consistent in our interpretation of transcript and application of codes. Throughout my analysis and in this paper I have referred to myself in the coaching role as "the coach" as I have found that this choice helps to maintain an analyst's, rather than a practitioner's, perspective and voice.

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[2]Traditionally, researchers who have looked for a low-inference method for segmenting talk have used turns or grammatical structures such as sentences or phrases. Chafe (1994) introduced the idea that *breath* or *meaning groups*, segments of talk that take place between breaths taken by a speaker are units of talk that carry meaning for participants in conversation.

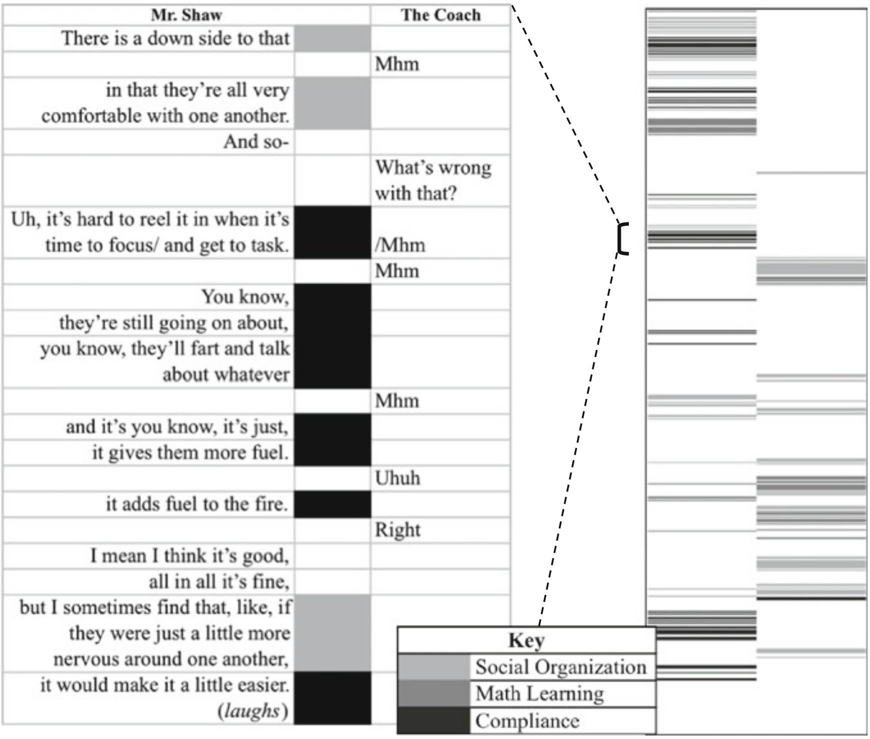

Figure 1. Transcript, Mr. Shaw 10/13/11, Lines 183–202.

Application of Codes to Transcript and Formation of Code Profiles of Conversations

Color codes were applied to breath-group segments of transcript using Microsoft Excel. Topic shifts were noted with horizontal lines. Text was then removed and the row height for each breath group was standardized. This process, adapted for gray scale and exemplified in Figure 1, yields representations called *code profiles*. Note that, because of the standardization of heights of each breath group in the code profile, the height of each strip of color is proportional to the number of breath groups receiving that code. (This is therefore independent of the width of the columns and the number of words within a breath group. For readability, this standardization is not possible in the transcript itself.)

Discussion of the Data Analysis

Code profiles are examined for patterns. The patterns that emerge are investigated using appropriate methods. For example, observations related to relative frequency of various kinds of noticing revealed in talk within or across participants can be investigated with counts and relevant calculations. Other observations suggest patterns of interaction that may prove instructive, and these observations can be investigated qualitatively, by looking closely at particular parts of the data. To understand the utility of code profiles, it is important to consider what they reveal that might otherwise remain hidden. While simple frequency counts can certainly be conducted without the support of these visual representations, such counts do not reveal ways in which coded talk unfolds between participants across time. Code profiles allow for the examination of such unfolding of coded talk and suggests to the analyst interactional phenomena that may be of particular import and worthy of further investigation. Both types of observations (those that rely on the code profiles and those that do not), along with investigations resulting from each, are exemplified in the following sections.

Illustrative Findings: The Case of Mr. Shaw

This section demonstrates the utility of the methods described above for the identification of the development of teacher noticing. The following two findings are discussed: First, examination of code profiles and subsequent numerical analysis revealed that Mr. Shaw's noticing of the social organization of the classroom developed over time, as evidence by his relevant talk in conversations with his coach. Second, code profiles revealed patterns of response by the coach to Mr. Shaw's talk about compliance that suggest successful efforts to support (or *apprentice*, as discussed in Goodwin (1994)) development of his noticing for equity. Qualitative analysis of relevant sections of these coaching conversations support this interpretation.

Finding 1: Code Profiles Reveal Development of Mr. Shaw's Noticing for Equity

Figure 2 shows code profiles for the three conversations between Mr. Shaw and the coach. In each code profile, Mr. Shaw's talk is represented on the left and the coach's talk on the right.

Examination reveals decreased presence of black (talk about compliance) in the left-hand columns across the three code profiles. This suggests that over the course of the three conversations, Mr. Shaw's talk revealed less noticing of compliance

Figure 2. Code profiles of conversations between Mr. Shaw and the Coach.

Table 1
Relative frequency of codes for Mr. Shaw's and Coach's noticing over time

	Mr. Shaw in conversation			Coach in conversation		
Noticing	1 (n = 106)	2 (n = 42)	3 (n = 35)	1 (n = 83)	2 (n = 129)	3 (n = 89)
Compliance	48% (51)	55% (23)	31% (11)	6% (5)	0	7% (6)
Social aspects	37% (39)	45% (19)	43% (15)	70% (58)	98% (127)	40% (36)
Math thinking	16% (16)	0	26% (9)	24% (20)	2% (2)	53% (47)

and more noticing of the social organization of the classroom and of mathematics learning.[3] This pattern is confirmed by simple comparison of relative frequencies of occurrences of each code. The left-hand section of Table 1 shows that the relative frequency of each type of Mr. Shaw's noticing changed across the three conversations in ways that confirm that pattern in the code profiles that was identified visually. (The coach's noticing, shown in the right-hand section of Table 1, is discussed in relation to Finding 2 in the following section.) In Table 1, n represents the total number of lines of talk that were coded for any type of noticing for each participant in the conversation.

While Mr. Shaw's noticing of compliance decreased over time, his general topics of concern stayed relatively stable. In particular, across the three conversations, he maintained concern for the extent to which his students were engaged with the mathematics of the lesson. However, the ways in which he talked about this concern shifted to include more concern for the ways in which the social organization of the classroom supports this engagement.

Finding 2: Code Profiles Reveal Ways in Which the Coach Supports Mr. Shaw's Development of Teacher Noticing

Examination of the code profiles (Figure 2) for all three conversations yields an interesting pattern of interaction between Mr. Shaw and the coach. Almost every time that Mr. Shaw's noticing talk is coded with blue (black here, signifying noticing of compliance), the next coded talk of the coach is pink (light gray here,

[3]One might also note that the *amount* of coded talk in each conversation is not consistent. In particular, there is much less coded talk in the third conversation. This happened because a larger part of the third conversation consisted of talk that did not give clear information related to these codes. Some examples of the types of talk that were not coded are when Mr. Shaw (1) reflected on the structure of the math task he used; (2) discussed individual students and his interpretations of their motivations as they related to his observations of their behaviors in the lesson; (3) talked about his need to plan future instruction and his struggles to meaningfully connect the mathematical content of his lessons to the other work that his students do; (4) brainstormed ideas for math activities for future lessons; and (5) reflected on those aspects of Complex Instruction that he found relatively easy compared with those that were more challenging for him.

signifying noticing of the social organization of the classroom). Closer examination reveals that of the 24 topic segments across all three conversations that contain any code on Mr. Shaw’s side, his talk is coded with blue (black here signifying compliance) in 12 topic segments. In eight of these, the coach has some coded talk that follows and in every case but one that talk is coded pink (light gray here signifying social organization of the classroom). This suggests that the coach may be consistently re-interpreting issues that Mr. Shaw interprets in terms of compliance in terms of the social organization of the classroom.

This pattern suggests that the coach’s talk was responsive to Mr. Shaw’s. That is, her patterns of response appear to be purposeful and they shift as Mr. Shaw’s talk

Table 2
Instances in which Mr. Shaw’s compliance talk was followed by coded Coach talk

Conversation	Summary of Mr. Shaw’s noticing of compliance	Summary of Coach’s next talk
1	Students’ high degree of comfort with one another is problematic as it pulls them off task.	This comfort level is actually a positive and can be built on in his search for better focus.
1	I might need “more structure” in my tasks to address my concern for the lack of focus and production from students.	“More structure” should be about structuring clear expectations for group work, and not structuring the mathematical thinking in which we hope students engage.
1	He talks about a particular student who is “not engaged”.	She suggests solutions that relate to using an instructional strategy to give the student a clear role to play in his group’s success.
2	He talks about a student who is “willing to work” in other arenas outside of math class, but not in math class.	She reframes this as the student having high status in other arenas and lower status in relationship to math.
2	He asks a question relating to allocating responsibility for getting work done among members of student groups.	She reframes the conversation to being about how to make it clear to students that they are responsible for making sense together of the content of the lesson.
2	He talks about a particular student who is “not engaged”.	She suggests solutions that relate to making the student feel needed and promoting group interdependence.
3	He talks about a group that never really “connected” or “engaged” and wondered about whether the use of team roles might have helped with that.	She agrees that it is useful to think about team roles in relation to this group.
3	He talked about students being “willing to engage” but then reframed the same issue in terms of risk-taking and safety.	She agreed and did not reframe.

also shifts. In particular, the analysis suggests that the coach may have worked to support Mr. Shaw to shift away from noticing compliance and toward noticing both the social organization of the classroom and mathematics learning.

As is evident in the right-hand side of Table 1, the coach gave little attention across the three conversations to issues of compliance. Her focus on the social organization of the classroom and on mathematics learning varied somewhat, with the focus on the social organization of the classroom primary for the first two conversations and the focus on mathematics learning catching up in the last one.

To investigate patterns in the coach's responses to Mr. Shaw's noticing of compliance, all eight topics were examined in which (1) Mr. Shaw had some compliance talk and (2) there was some subsequent coded talk for the coach. These eight instances are summarized in Table 2.

To give the reader a sense for what this reframing sounded like, the transcript in Figure 3 below is taken from the conversation on October 13, 2011 (conversation 1), which is described in the third row of Table 2. Here Mr. Shaw talks about a particular student who he describes as "not engaging." The coach suggests solutions that might give the student a clear sense of his own role in the group's potential success.

Here we see that the coach's response to Mr. Shaw's compliance-focused concerns was to suggest what he might do and say that would encourage students to

Mr. Shaw:	um... And Malik [pseudonym] was just not... you know, I didn't want to make a big deal out of it. Cause I felt like... I would go by at some point and he was just kind of like singing to himself, or like not engaging, and um… you know maybe a little bit like you know the big idea, maybe weight loss and how they can present it, and then he'd just back off again. Cause he was supposed to be the recorder and Jacob was recording. And I said something at first. I was like, make sure we're doing our roles, without pointing anyone out. I was hoping they would self moderate. And… in the end they didn't. They just kinda like-they figured, whether they were conscious of it or not, that this person's not going to do their job, so I'm gonna step up
Coach:	We'd better do it if we want it to get done
Mr. Shaw:	Right. And luckily with that group the three other kids are all very selfless, in that they'll do whatever needs to get done, which sometimes is great and other times is not.
Coach:	I like that you didn't want to call him out. One thing you can do too is to call out recorder-reporters, like you can, like, "hey, everybody, I need you for a second" if you feel like it's important. Like, "recorder-reporters raise your hand" and then you can get all three of them, including him. "It's really important that you be-… I'm hearing lots of good ideas. I'm not sure they're getting written down really well. It's really important that you be writing things down really clearly and I'm going to check in with you in a few minutes," or something like that.

Figure 3. Condensed transcript 10/13/11, Lines 506–546.

take responsibility for the engagement of their team members, promoting interdependence.

This examination of this pattern of reframing across the conversations yields two observations that are useful in understanding Mr. Shaw's learning to notice. First, we see that the coach consistently responds to Mr. Shaw's compliance-related talk by focusing attention on issues related to the social organization of the classroom. Whether or not this was connected with an intentional effort by the coach to support the development of Mr. Shaw's noticing for equity, evidence suggests that this is what happened. Table 1 shows the proportion of talk that is coded for (1) compliance and (2) the social organization of the classroom AND math learning for both the coach and Mr. Shaw. It reveals that his focus on compliance versus other areas did approach hers over time.

Second, and similarly, we see that the last two topics summarized in Table 2, both of which took place during the third and final conversation in the study, show a markedly different pattern of interaction from the ones in previous conversations. In the second to last topic segment, Mr. Shaw talked about the extent to which the group "connected" or "engaged," which frames the issue at least partially in terms of group dynamics. He then considered whether the use of team roles, a strategy for managing group dynamics, may have made a difference for the group. The coach did not reframe his talk here, but affirmed his focus on considering group dynamics. Here, we see an example in which the teacher's interpretation of the issue in terms of group dynamics made space for him to consider constructive instructional responses.

In the eighth and final topic segment considered in Table 2, Mr. Shaw reframed *his own* talk about students being "willing to engage" in terms of issues of risk-taking and safety. Here the coach did not need to reinterpret the phenomena in terms of the social organization of the classroom, as he did so himself. The temporal order of these final two examples lends credence to the interpretation that Mr. Shaw has been learning to notice in productive ways across these conversations. In particular, he seems to be shifting away from noticing compliance and toward noticing the social organization of the classroom.

The code profiles also reveal clear evidence that the coach is crafting her responses to Mr. Shaw in relationship to his patterns of talk. In the third and final conversation in this study, the coach's emphasis on the social organization of the classroom was considerably reduced, and her emphasis on content learning increased significantly. Taken in light of the findings above, we might understand this in this way: as Mr. Shaw began to shift away from noticing compliance toward noticing the social organization of the classroom, the coach no longer needed to work so hard to support that noticing. She was therefore able to begin to suggest noticing of content learning. Limitations of data collection for this study prevent us from being able to follow the development of their conversations further to investigate whether Mr. Shaw considered the social organization of the classroom more consistently in subsequent conversations, or whether his noticing of content learning continued to develop.

It is important to note that there are limitations in our ability to generalize from Mr. Shaw's case to draw conclusions about what is likely to happen for other teachers working with other coaches. The point of this chapter is not what happened for Mr. Shaw and the extent to which the same thing might happen for other teachers, but that findings in this case help to illustrate both the utility of the methods and potential for coaches to support the development of teacher noticing for equity.

Discussion

In his seminal 1994 paper, Goodwin names three practices by which participants in communities of practice build and contest *professional vision,* "which consists of socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606): (1) coding, (2) highlighting, and (3) producing and articulating material representations. He goes on to explain that "…the ability to see a meaningful event is not a transparent, psychological process but instead a socially situated activity accomplished through the deployment of a range of historically constituted discursive practices" (p. 606). He shows ways in which experienced professionals (an archeologist and a legal defense team) "train" the vision of novices to their fields. A new archeologist comes to see relevant color and texture distinctions that transform what had been a pile of dirt into a rich source of archeological evidence and a jury comes to see police actions that had been unprovoked violence against a defenseless man as sensible responses to the drug-fueled actions of a dangerous man who was, in fact, in control of the situation. Through these examples, Goodwin demonstrates that professional vision is both (1) deeply consequential for the actions that are available and sensible to people engaged in practice and (2) an active, socially negotiated, and situated process (or set of processes) into which people can be apprenticed.

This chapter suggests that teacher noticing for equity, and in particular teacher noticing of the social organization of classrooms is a particular type of professional vision that is consequential for teaching practice and consists of a set of active processes into which teachers can be apprenticed. I stipulate that patterns of inequity in classrooms have persisted in part because many people, including teachers, do not yet "see" them. The data presented here supports the extension of Goodwin's ideas to suggest that this type of teacher noticing can be purposefully supported by expert practitioners and that in-service teachers can be apprenticed into noticing for equity. The data presented here suggest that the coach in this study had a professional vision of equity such that she noticed the social organization of the classroom. The teacher did not yet have this vision. However, through interactions in which the coach prompted Mr. Shaw to attend to his classroom in a particular way, he began to engage in this type of noticing without prompting.

As teacher educators work to support teachers to create equitable classrooms, it will be important to consider and design opportunities to support teachers' noticing of equity and inequity as they play out in the social environment of the classroom. Data here suggest that one way to do this is through the support of coaches or other practitioners who are well versed in noticing for equity and who work purposefully to support teachers in this development.

These ideas are offered with the hope that other scholars will weigh in about other aspects of teacher noticing that may be important for equity and about how those aspects of noticing might be productively studied. For example, some of the work of Gutiérrez (2002, 2007, 2013) may suggest that there are important aspects of teacher noticing for equity related to the cultural and political contexts of the schools and communities within which teaching and learning take place. Teachers may need to develop particular kinds of noticing to be prepared to act as effective change-makers on behalf of their students.

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Making Visible the Relationship Between Teachers' Noticing for Equity and Equitable Teaching Practice

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Abstract This study examines mathematics teachers' *noticing for equity*. Noticing for equity is a critically important practice given research that documents how particular groups of students feel more or less empowered to take up ambitious mathematics practices. We conducted classroom observations and a series of noticing interviews with four secondary mathematics teachers nominated as exceptional equitable mathematics teachers. Using qualitative methods, we conducted a cross-case analysis to identify common instructional practices these teachers enacted to close participation gaps in their classrooms, as well as the associated ways of noticing during instruction. These findings document the intricate relationship between what teachers committed to equitable mathematics instruction attend to, how they reason about observed phenomena, and how they use this information to make instructional decisions.

Keywords Equity · Secondary mathematics teaching · Noticing · Mathematics knowledge for teaching · Teacher education

The complexities of mathematics teaching have motivated a large body of research detailing core instructional practices and the professional vision that supports the enactment of these practices in daily classroom life (Goodwin, 1994; Lampert et al., 2013; McDonald, Kazemi, & Kavanagh, 2013; Sherin, 2007; Sherin, Jacobs, & Philipp, 2011). Studies show that professional vision that involves attending closely to qualities of students' mathematical talk and inscriptions supports student engagement in powerful mathematical argumentation and reasoning

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(Franke, Carpenter, Levi, & Fennema, 2001). Little is known, however, about how mathematics teachers notice aspects of classroom mathematical activity that have consequences for whether or not particular groups of students feel more or less empowered to take up these practices, or what Erickson (2011) calls, *teacher noticing for equity*. Noticing for equity is critically important, given growing empirical work that documents how culture and power permeates every aspect of classroom mathematics learning (DiME, 2007), and that shows that students must negotiate racialized narratives based on stereotypes about which groups of students can and cannot do mathematics (Nasir & Shah, 2011). It is yet unclear how mathematics teachers, who in the secondary context are primarily white, come to notice the activity of their mathematics classrooms in ways that enable them to interrupt these deficit perspectives and processes in support of their learners. The purpose of this study is to begin to examine the relationship between the nature of teacher noticing for equity and equitable instructional practice. In particular, we investigate what teachers who are committed to achieving equity in mathematics teaching attend to during instructional interactions and how these ways of noticing influence their decision-making. The results of this study contribute to theories of equitable mathematics teaching and have implications for understanding how to create more equitable learning environments for learners.

We report on a two-year study of teacher noticing for equity in secondary mathematics classrooms. The study identified common noticing practices of teachers who were committed to and successful (to different degrees) at engaging learners from less-dominant ethnic, racial, linguistic, and socioeconomic backgrounds in mathematics learning, and thus at narrowing gaps in participation that often occur among groups of students in mathematics classrooms. Our analysis in this paper focuses on teachers' reflective noticing of video clips of their classroom instruction. This focus enables elucidation of the *particular* aspects of students' individual and collective participation that teachers attended to, how they reasoned about these observations to draw inferences about students' access and opportunity in their classrooms, and how they used this information to inform their instructional practice. The central claim we make is that equitable instructional practice relates to particular forms of noticing for equity. To support this claim, we identify and characterize shared instructional practices of the teachers that supported students in ambitious forms of mathematical participation and the associated ways that teachers noticed classroom activity. Our goal is to elucidate the layered, multifaceted, robust nature of noticing for equity.

Theoretical Framework

We situate our study within the professional noticing literature. It is widely agreed that noticing for teaching consists of three parts: *attending* to features of classroom interactions as they unfold during instruction, *reasoning* about what is observed, and using these inferences to *decide* what to do next (Jacobs, Lamb, &

Philipp, 2010; Mason, 2002; Seidel & Stürmer, 2014; Sherin, Jacobs, & Philipp, 2011). Research on teacher noticing in the last decade has been largely focused on teachers' attention to and sense-making of student thinking (Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin & Han, 2004; Sherin & Russ, 2014; van Es & Sherin, 2008). This work is informed by studies that document that attending closely to student ideas can result in increased teacher and student learning (Franke et al., 2001) and that continual reflection on the relation between teaching and student learning can promote learning in and from one's practice (Hiebert, Morris, Berk, & Jansen, 2007; Lampert, 2010). Research on equitable teaching suggests that inquiry into teachers' noticing of issues related to participation and access is equally important for understanding how to motivate and sustain broad-based classroom mathematical activity (Hand, 2012; McDuffie et al., 2014; Wager, 2014).

Teachers' Pedagogical Commitments, Noticing, and Instruction

Synthesizing 30 years of research on teacher noticing, Erickson (2011) concludes that the expectations and perspectives that teachers have formed about the relation between teaching and students enable teachers to, "put [what is going on in the classroom] all together" (p. 26). These expectations and perspectives guide what teachers attend to in the midst of instruction, and the kinds of interpretations they will make about the relation between their own instructional practices and students' behavior, participation, mathematical understanding. For example, a teacher might notice that a student has their head down in class and interpret this as an instance of "not paying attention." In contrast, the teacher could view this as a signal that the student may have an unmet emotional or physical need. These interpretations stem from different kinds of dispositions towards teaching and students that teachers are developing over time by virtue of their experiences both inside and outside of the classroom.

These dispositions, or what Erickson describes as pedagogical commitments, can be characterized in terms of

> … the teacher's "philosophy of practice"–basic ontological assumptions, both tacit and explicit, concerning manifold aspects of teaching and learning activity, e.g. the nature of learners (high, medium, or low in ability; tries hard or doesn't try hard), of subject matter (easy, difficult; inherently interesting or boring but necessary), of social relations (threshold levels of disruption, concern for face threat), of how semiotic systems communicate meaning ("If I said it clearly [or wrote it on the board] they should understand it.") (Erickson, 2011, page, 28).

As Erickson describes, these dispositions are tied to fundamental assumptions about phenomena like the nature of intelligence, rules for social behavior, and systems of meanings. As such, teachers' dispositions are necessarily shaped by their membership and histories of participation in sociocultural communities (Hand, 2012). As we described earlier, many of the teachers of secondary mathematics do not participate in the same sociocultural communities as their students, and thus, may hold assumptions that are different from them. Importantly, as a teacher responds to classroom activity based on their developing disposition, the classroom acts as a feedback mechanism to either reinforce or challenge their interpretations. Given the hierarchical nature of the classroom, and the limited opportunity for students to provide their interpretations of the ongoing activity, teachers' dispositions often remain fairly static (Erickson, 2011).

In this paper, we are centrally concerned with teachers' dispositions for noticing as they relate to issues of equity. Noticing for equity necessarily entails pedagogical commitments that do not perpetuate deficit (or privileged) perspectives about groups of students. It is unclear, however, what mathematics teachers who are disposed to enact these commitments attend to in moment-to-moment classroom interaction. What do they notice about students as they participate in mathematical activity? To what other features of the classroom do they attend? In what ways do forms of noticing for equity take into consideration how and which students are participating in the classroom and the relation of their participation to broader sociocultural processes and structures? How does what teachers notice inform their instructional decision-making, both in classroom moments and over time? Capturing the noticing practices of mathematics teachers who *notice for equity*, from the perspective of classroom participation, may illuminate how teachers' pedagogical commitments unfold in daily classroom life.

What Do We Know About Equitable Noticing?

Within research on teacher noticing, studies of noticing for equity are still in their infancy, yet researchers concerned with issues of equity in mathematics education argue that it is a central component of ambitious and equitable mathematics teaching (e.g., Hand, 2012; McDuffie et al., 2014; Turner et al., 2012; Wager, 2014). One finding from this early research is that teachers who are not yet noticing for equity tend to view classroom mathematical activity as separate from other forms of classroom participation. For example, Turner et al. (2012) found that beginning teachers attended to children's mathematical thinking and forms of knowing related to their culture/community, but had difficulty seeing the two as intertwined with each other. The authors describe the teachers as having "fragmented awareness" (Mason, 2008). This type of noticing is consistent with a disposition towards culture in the classroom as located within particular groups of children instead of in the interactions of all classroom participants (including the teacher).

A second finding is that teachers tend to notice classroom participation in general terms, with less attention to differences among students and how these relate to their instructional choices. In a study of mathematics teacher noticing that took place within the context of professional development, Wager (2014) found that, on the whole, teachers were more likely to describe what occurred in their classroom with holistic snapshots about student engagement in relation to the effectiveness of instructional moves. For example, teachers described how a particular activity allowed all of the students to share their mathematical ideas, or how another one elicited ideas from a range of students in the class. Teachers were less likely to notice details of individual student participation and the relation of this participation to both immediate and longer range instructional practices. This is consistent with other research that finds that teachers look at classrooms more holistically and often propose overly simplistic, general inferences about what they observe, without attending to the detailed ways that individuals and groups interact around mathematical ideas (Jacobs, Lamb, & Philipp, 2010; Miller & Zhou, 2007; van Es, 2011).

In contrast, teachers who notice equitably perceive mathematical and interpersonal activity as inextricably linked and attend to how these play out in the participation of individual (and groups of) students. For example, Wager (2014) found that teachers who engaged in what she calls more frequent noticing of classroom participation not only tended to provide more nuanced interpretations for what they observed, but they also thought more deeply about issues of equity. In particular, these teachers described equity in terms of the privilege and marginalization of particular ethnic and racial groups through schooling experiences. These findings are consistent with Hand's (2012) study of teachers who were successful at narrowing participation gaps among groups of students in diverse mathematics classrooms through noticing opportunities to create dialogic space in the classroom, to blur the lines between mathematics and cultural activity, and to frame mathematics education as a part of a broader sociopolitical system of schooling. McDuffie and colleagues (2014) also find that providing structured protocols supported preservice teachers in developing attention to multiple aspects of equitable instruction and deeper analyses of what they observed, suggesting that learning to notice equity can be cultivated through participation in a carefully designed learning experience.

The purpose of this study is to extend the research on noticing for equity in two important ways. First, we seek to build on prior accounts of noticing for equity by providing more detailed, nuanced accounts of how teachers see issues of participation and access in their classrooms. In addition, we examine how teachers notice learner's participation in classroom activity and how their noticing is tied to particular instructional practices. Our goal is to uncover dimensions of teachers' noticing as they are tied to classroom mathematical activity and the ways that teachers' noticing is inextricably tied to their commitments to promoting equity in their classrooms.

Study Context and Data Collection

Subjects in this study were six secondary mathematics teachers nominated as being exceptional equitable teachers from three individual school sites in southern California and one large urban school district in Colorado. Teacher educators and district leaders nominated teachers based on the following criteria: (1) effective at narrowing participation gaps in classrooms, (2) skilled use of reform-based mathematics curriculum, and (3) district-level recognition. Data consists of classroom observations and interviews that took place over the course of a six-month period in one school year, from December to June. During this time, the research team conducted week-long observations and videotaped three additional lessons in the six teachers' classrooms. We also conducted three *noticing interviews* with each teacher. The research team identified selected segments from videotaped observations that had implications for equity, what we refer to as *noticing clips*. Within one week of observing and videotaping in the teachers' classrooms, we conducted the noticing interview. The teachers viewed the preselected noticing clips and commented on what they were noticing during the instructional episodes. These interviews lasted 45–60 min. Each interview was transcribed.

We focused this analysis on four teachers' instruction and noticing interviews (see Table 1)—two from each research site. These teachers all demonstrated efforts to enact reform-based mathematics teaching and expressed an explicit commitment to narrowing participation gaps in their classrooms. The two other teachers, whom we did not include in the analysis, also shared a commitment to equitable practice but classroom observations revealed fewer opportunities for broad-based participation among students. As a result, they were excluded from this analysis.

Table 1
Profiles of teacher participants

Teacher	Ethnicity; gender	Class(es) observed	Grade level	Student demographics
Parker	Caucasian; female	IMP 2	11th	100% 1st and 2nd generation Latin@; low-income
Julie	Caucasian; female	IMP 2	10th–12th	Highly diverse (ethnicity/race/language); middle to low-income
Carter	Caucasian; male	Geometry 1	9th	100% 1st and 2nd generation Latin@; low-income
Raymond	Japanese/Mexican; male	Core concepts	9th–12th	Highly diverse (ethnicity/race/language); low-income

By highly diverse, we mean that at least three ethnic, racial or linguistic groups were represented in significant proportions

Analytic Methods

Data analysis was qualitative in nature. Through an iterative process, we developed characterizations of each teacher's instructional practice (Saldaña, 2009). Using the constant comparative method (Strauss & Corbin, 1998), we reviewed the field note data and generated a coding scheme that captured dimensions of teachers' practice related to advancing students' mathematical understanding, as well as attending to individual student's and the class's patterns of participation, culture, identities, and positioning. We began with one set of field notes from one teacher and collaboratively coded line by line, identifying practices that reflected those documented in the literature as supporting substantive engagement with mathematical ideas, opened up opportunities for students to engage with the mathematics, and responded to students in positive and inclusive ways. We then separately reviewed field notes from two observations for this same teacher and refined the code list. After reviewing three sets of field notes from one teacher, we then turned our analysis to the field notes for a second teacher. We drew on the codes that emerged from the first round of analysis and continued to add additional codes. After open coding field notes for two teachers, we worked together to generate categories of codes that identified dimensions of practice across the two teachers. With this new framework, we returned to the data and individually reviewed one set of field notes from two additional teachers and further refined the coding scheme. We felt confident we had an exhaustive list of codes that reflected the instructional practice across the four teachers. Throughout the coding, we continued to revise the categories of shared practices across the four teachers. The categories that emerged from this coding included: *teacher moves* (e.g., elicit, press, invite contribution, open third space); *student responses*; *student and teacher moves* (e.g., reasoning and explaining); *positioning* (e.g., authority over the math, accountable to peers, self, task, community); *positioning of the mathematics* (e.g., relevance, set of procedures, multiple perspectives); *positioning requirements for doing math* (e.g., getting/being organized, being willing to grapple with challenges); *activity structures* (e.g., warm up, group work, sharing work); *resources* (e.g., manipulatives, tools to represent ideas); *physical moves* (e.g., pulling back, being close to a student or group); *class structure (*e.g., desks in rows or groups, random or purposeful selection of students); and *interactional tone* (e.g., warm, sarcastic, encouraging). With this framework in mind, we returned to the field notes for all four teachers and generated analytic memos (Miles, Huberman, & Saldaña, 2013) that captured the nature of instruction we observed across the field note data.

We used a similar approach to analyze the noticing interview data. We selected an interview transcript for one teacher, Parker. Together, we examined what she claimed she was attending to in the interaction that unfolded during the clip we presented and how she interpreted the meaning behind what she observed. As she discussed what she noticed, she also discussed the subsequent teaching moves she made, and in some instances, highlighted features of the interactions that stood out to her upon viewing the video clip but that she may not have noticed during instruction. We identified four categories for our analysis: event noticing, interpretation, teaching move, and general noticing. Event noticing referred to the particulars of the events that the teacher attended to, while general noticing referred to broader categories of classroom activity that framed teachers' noticing. This analysis was informed by prior research on teacher noticing (van Es & Sherin, 2008) and noticing for equity (Erickson, 2011; Hand, 2012; Wager, 2014).

Similar to the field note analysis, we conducted a cross-case analysis of the noticing interview data. We began by reviewing one noticing interview transcript for Parker, highlighting what she identified as noteworthy in the classroom interactions (event and general noticing), how she interpreted what she saw, and the associated teaching moves. We constructed a list for each dimension. We then selected one noticing interview transcript for each of the three additional teachers—Julie, Raymond, and Carter—and individually examined their noticing and instructional practices to further develop the list of emergent codes. We discussed this list of codes, and subsequently returned to additional interview transcripts for each of the teachers and identified any additional codes and categories of noticing across the four teachers. Using these categories and related codes, we returned to the coded transcripts and individually constructed analytic memos for each teacher to characterize the nature of their noticing of classroom activity. In the final phase of analysis, we examined the analytic memos for all four teachers and jointly identified a set of shared instructional practices, and associated ways of noticing with respect to those practices.

Results

Our analysis revealed that these teachers had several shared practices and that they noticed classroom interactions in similar ways that reflected a commitment to promoting equity in their classrooms. We begin by presenting a framework of teachers' practice and associated noticing and use two cases to examine how these teachers' noticing informed their instruction to create equitable opportunities to learn. We then conclude by discussing the implications of this framework for both research and teacher education.

Shared Teaching Practices and Associated Noticing for Equity

First, we identify five common teaching practices and associated noticing among the four teachers (see Table 2). We identify which teachers we observed engaging in each practice. To illustrate how teachers' noticing informed their instructional decisions, we select two practices and used two cases to describe them in greater detail.

Table 2
Teacher practice and noticing for equity

Teaching practice	Associated noticing	Teachers observed
Leave students to grapple with mathematical ideas	• How students can be resources for each other in the moment • The demands and contours of the task and how these relate to what a student is doing in the moment • The effect of teacher presence on what students are doing • If students have sufficient resources to move forward • Who is taking up space in the group/how well they are working together • Students emotional state as a sign to give space to participate in less productive ways (for example, because of things going on in their lives at that moment) • When students need some kind of reassurance that they can succeed	Julie, Parker, Carter
Make norms explicit for doing mathematics	• How well students are attending to classroom norms and practices • How well particular students have gotten used to the norms • How well group members support each other in learning/reinforcing the classroom norms • Students' use of tools and resources and how suggested adjustments help them take advantage of various affordances for learning	Julie, Parker, Carter
Support students in developing mathematical identities	• Student connection and/or disconnect with the task and with each other • What is going on with individual students and what is going on with a group • Different aspects of the students' lives and how to relate to them mathematically	Julie, Parker, Raymond
Connect with students to honor individual strengths	• Cultural ways of being • How particular students like to interact with others • What makes each student unique • Patterns within families, within social groups, and within youth culture • When students' backchannel conversation begins to pull away from main line of classroom inquiry and whether to pursue it and create a third space • When a student needs to take a pause	Parker, Raymond
Make system of schooling explicit	• Relation between student work and content of testing system • Experience in school as a point in time in students' lives • Relation between the mathematics of focus and who students are as people	Parker, Raymond

Leaving the students to grapple with mathematical ideas. The first shared instructional practice, creating opportunities for students to grapple with the mathematics on their own, was a pedagogical tool for three of the four teachers. This practice involves giving students a sufficiently scaffolded high-level mathematical task and establishing classroom norms around making sense of mathematical ideas either individually or in groups. We often observed the teachers providing tasks that required students to reason mathematically, and responding to the students' questions about the task by pressing them to figure out what the task was asking, what they know about it, and what tools they bring to the process of solving it. Once the teacher noticed that the students were on track, they would quickly walk away.

Our analysis indicates that the teachers' noticing practices around this instructional move reflected attention to both the informational and interpersonal aspects of the students' participation. First, the teachers attended closely to the demands and contours of the task and how these relate to what a student was doing in that moment. For example, teachers would notice whether a student understood the task demands, were grappling with the heart of the mathematics, or were simply missing procedural cues. The teachers also noticed the kinds of resources that were available to the student(s) in the form of other people and their previous mathematics learning. Similarly, they noted if students were getting too dependent on them for help or, alternatively, if they were too afraid to ask for it. They also attended to socioemotional aspects of the interaction as well, such as how the group was functioning together (e.g., if one person was dominating or distracting the others), or a student's level of confidence and the kind of reassurance s/he might need to keep pressing forward. Finally, they noticed a student's emotional state, for example, if a student was having a bad day/week, and when it was appropriate to give the student a little space or reassurance.

Making norms explicit for doing mathematics. The second shared instructional practice revolved around establishing and making norms explicit for how to do mathematics in their classrooms. As with the first instructional practice, we observed all but one of the teachers (Raymond) engaged in this type of pedagogy. In particular, we observed the teachers structuring norms for approaching mathematical tasks, using resources, and having group and class discussions. They relied on these norms to maintain a high level of rigor in classroom inquiry and to keep the class flowing smoothly. The teachers did little instruction at the front of the class; instead, they relied heavily on presentations of student work and group work.

In their noticing interviews, these teachers described how they were constantly attending to how well the norms were functioning to support both group and individual learning. They analyzed student participation with respect to these norms, for example, how well students were adhering to the norms in their work together and the resulting level of mathematical work. They paid attention to which students were comfortable and familiar with the norms and which ones needed more information about them. When students began to stray from the task, the teacher often interpreted the situation in terms of the students needing to be reminded of the norms, instead of reprimanding student behavior. They also paid attention to whether the class was functioning as expected, for example, if the students were willing to volunteer their ideas or if there was something getting in the way of the students meeting this expectation. Finally, they attended to whom to place in particular groups and how coordinating students in particular ways would influence the level of productivity of the group. They noticed which students were "getting along" or who recently had conflicts, as well as who needed more support in terms of language or level of comfort, using this information to help them decide how to group students—either by purposely avoiding or carefully selecting particular students to work together. They also carefully attended to how these groups functioned, paying attention to whether the students offered support to each other in ways they anticipated or if they needed to make additional adjustments to promote student success.

Supporting students in developing mathematical identities. The third instructional practice, supporting students in developing their mathematical identities, was observed in three of the four teachers' classrooms, with Carter being the exception. These teachers made explicit efforts to create opportunities for students to express what they know and understand both about the mathematics and their worlds and used what they learned about students to create meaningful learning spaces. For these three teachers, the creation of meaningful learning spaces revolved around enabling students to "take up space" in the classroom. Teachers supported students in taking up space in several ways having them direct the flow of the lesson, either by calling on a wide range of students to share ideas; showing unsolicited student work with the class; responding to an idea that may have at first appeared to be irrelevant to the lesson; and allowing individual or groups of students to participate in ways that did not look anything like mathematical activity, but appeared to support them as developing adolescents. Such teaching practices served to develop confident mathematics learners—students who felt safe in the mathematics classroom to express themselves as individuals who are members of various communities—both in school and in society—and to express themselves as learners who have questions, confusions, and important and worthwhile ideas to contribute.

Teachers' noticing associated with this practice tended to *connect* interpersonal aspects of students' participation with their mathematical activity. For example, all of the teachers attended to the relation between students' individual histories both inside and beyond the classroom and how these factored into their classroom interaction. They noticed if a student was ready to take a lead mathematically among their classmates, but was hesitant to take the risk socially, and sought out opportunities in which the student might feel safe in this position. They also paid attention to situations where students seemed to connect with the task (and sought out tasks that they thought would support this) and situations where the task might not make sense or be relevant from a cultural perspective.

They also attended to the relation between a student's mathematical participation in the moment, and its relation to what they knew about the student's personal experiences—at home, school, or more broadly, as a member of a particular racial, linguistic, or ethnic community. They might see an individual student's lack of engagement as a sign that the student was tired from a new job or was having to move to a new home. They also noticed students in terms of their broader social and cultural communities (e.g., basketball team, Ethiopian students, Latinas) and made sense of a student's participation in terms of the histories of these communities in US mathematics schooling. Generally speaking, when they did this, however, the interpretation would focus less on linking specific behaviors to the various groups, and more on how the individual student's membership could be playing a role in their schooling experience. Whatever the case, the interpretation of students' participation in this way would be in service of enabling them to see themselves as mathematics learners. The teachers were on the lookout for the possibility to support their students—either through individual attention or classroom participation structures—in ways that enabled them to negotiate the classroom practices with respect to these (personal and broader) histories.

Connecting with students to honor individual strengths. The fourth instructional practice involves connecting with each student in sincere ways that recognize and honor who they are as people. This is slightly different from the practice described above, in its focus on *relating* with students, rather than simply supporting their negotiation of classroom mathematical participation. We regularly observed two of the four teachers noticing opportunities to make connections with students either on an individual level or as a whole class. These teachers came to know their students and either designed lessons based on what they knew about their students in order to engage them in the mathematics or they adjusted a lesson during instruction to allow for students' strengths to be made visible to the class. In addition, they provided space for "playing around" with the students—engaging in humor and play—as a way to connect with students on their terms and honor them as individuals.

To connect with students in these ways involved attending to and understanding how to draw on their cultural ways of being to be able to use them as resources for mathematics learning. This involved, for example, finding ways to see activities or practice from the youths' lives and interpreting them as resources for learning the mathematics at hand. These teachers also attended closely to how students interacted with them as well as with other students and what unique attributes they brought to these interactions—were they shy, serious, funny, etc. By attending to how individuals interacted with each other, they were able to notice patterns of participation in class and interpret these patterns as being resources for learning or they gave students space to be who they were as people in class. Students who might have a tendency to shout out answers were not viewed as students who caused trouble or as being disruptive; rather, the teachers viewed this tendency in terms of the broader picture of the students' lives and listened carefully to what they said to take them seriously as people. They also attended to how the students talked about themselves in relation to their families and friends, as well as within broader youth culture, and drew on that knowledge to help students become mathematics learners. Finally, they attended to when students' backchannel conversations started to pull them away from the main line of classroom inquiry and considered whether to encourage dialogic (or third) space (Gutiérrez, Baquedano- López, Alvarez & Chiu, 1999) to emerge around the students' interests or whether to redirect to the mathematical task. Similarly, they noticed students' body positioning and verbal responses when a teacher may be pressing them to do more and had an awareness of when to stop pressing and give the students space to think and work on the task or in some instances, space to take a break and then reengage.

Making the system of schooling explicit. The last practice we observed in both Parker's and Raymond's instruction involved making the system of schooling explicit. Though this was less prevalent in our observations, we found evidence of these two teachers talking about the broader testing system in which students' mathematical work was embedded, relating the mathematical work students did in school as a point in time in their lives, and relating the mathematics to who they are as people. In their interactions with students, Parker and Raymond highlighted, for instance, how a student's answer to a mathematical problem would be scored on a state test and what in particular about the answer would yield that score. To be clear, the focus was less on meeting the benchmark of the state test; however, the practice made visible to the student how her work would be interpreted in the broader testing environment. For both Parker and Raymond, it appeared that they wanted to help students see how their mathematical work fits into a broader system of schooling, as well as how their mathematical engagement linked to whom they were becoming as adolescents.

Teachers' noticing related to this practice included attending to student participation within a lesson and over the course of a year and inferring how their participation would have potential impact on their future selves. It also involved noticing students' level of awareness of what school could do for them. In other words, these teachers seemed to be cognizant of how students treated opportunities available to them and also attended to how students' experiences at this point in time—the time in which they were in these teachers' classroom—was only one experience in a student's broader schooling and life experience. They used this information as a way to consider how much and what kind of support to offer students, to ensure that students had access to tools to navigate the school system, and to help students envision who they could become as people in the world.

We present the following two cases, taken from the *noticing interviews*, to illustrate more deeply the relation between teachers' instructional and noticing practices.

The Case of Parker: "Walk Away"

Instructional Practice. Leave students to work independently or in groups and grapple with mathematical ideas.

Noticing Clip. The students were working in groups on a task that involves solving and graphing inequalities. Parker is circulating the classroom when she approaches a student, Jesus, who tells her that he needs some help. She has a brief conversation with him and then starts to move to another group. As she leaves, she asks the student next to him, Rosa, to help him on the graph if he needs it.

Noticing Interview. When asked what she was attending to when enacting her instructional moves, Parker described several features of the interaction. Parker mentioned that Rosa had already solved the inequalities for the graph that Jesus was working on, and that she was chosen to help Jesus because, “I really try to look for kids that are newly successful at something.” She also noted that Jesus was fairly new to the classroom, and “he’s still in this process of being able to ask for help … and isn’t as good at it as some of the other kids.” She emphasized that,

> Instead of just like [simply saying] “Hey you guys work together”, asking Rosa very specifically to help him with a very specific…and finite task. Like it only takes a minute or two. So instead of just shoving them together, she has one responsibility, which was helping him solve for p.

When asked about this move in relation to others like it, Parker reflected on her intention to position students as accountable for grappling with the mathematics.

> I do walk away a lot and don’t hover. I give them a direction and I move on. Hopefully I convey two things. One is, “I have the confidence that you can do this on your own and you don’t need me to do it for you.” And number two is that they’ll just keep saying things to me, and not really thinking about the problem if I stand there. There are often times, and I feel like in class, quite honestly, [that] I’m just like cruising around town like I’m a cat or something, trying to not get stuck any one place for too long, because, they almost stop thinking when I’m there.

There are several aspects of her noticing that we find interesting. The first is that she is aware of the extent to which students in class have taken up the norms that she is co-constructing with them and that she needs to structure interactions to support these norms. She is also noticing opportunities to position students who are newly successful as a mathematical authority. Similarly, she is cognizant of the need to communicate to students what they are authorized to do when helping each other. In terms of her general noticing, Parker continually attended to the effect of her presence on students’ persistence with the mathematics and sense of confidence around it. She looked for opportunities to turn the mathematics over to students, and was aware of the needs of individual students for taking up these opportunities.

The Case of Raymond: “You Started Off the Period a Knucklehead”

Instructional Practice. Connect with each student in sincere ways that recognize and honor who they are as people.

Noticing Clip. At the end of the class period, Raymond asked to speak briefly to Javier. He told Javier, “You started off the period a knucklehead, right? Okay, but then you ended the period, what, strong (i.e. helping other students, answering questions).” Raymond continued and emphasized to Javier that he should care about his own education for the sake of his future and represented this by writing an inequality on the board, "You need to be the one that cares more about your education than the teacher … or at least greater than or equal to."

Noticing Interview. When asked about what prompted the need to speak with Javier after class, Raymond described how he generally approaches engaging Javier in mathematics. In doing so, he depicted Javier's general qualities and potential to be a great student when given attention. He explained

> You know someone like him can become a real leader in the classroom. He can really help out 6 or 7 kids in the classroom… You know, he is a listener. So he is respectful. He is polite. So he is someone I could talk to… I could use math along the way with my lectures in the kindest way possible. You know, I care about you. And if I care about your grade more than you, there is an issue here.

There are several aspects of Raymond's noticing that we find interesting. The first is that he was aware of how his students were engaging in the lesson. If Raymond perceived that a student was not participating in learning, he made an effort to speak with the student about his or her effort (or lack of effort). For instance, in the case of Javier, Raymond had to "call him out." By doing this, Raymond demonstrated that he cared about his students' participation and learning. Similarly, Raymond was cognizant of the need to have one-on-one conversations with students. He was aware of the necessity to give students individual attention and encouragement, and he was continually attending to the effect of his awareness of students' participation on students' potential success and confidence in mathematics. Thus, he looked for opportunities to keep all students engaged in learning within and outside of his mathematics classroom, by attending to how individuals participated in class and helping them see how their participation that day, week, and year is linked to their future selves.

In both of these cases, the teachers were attending not only to how the class and/or particular students were engaging in mathematics, but perhaps more importantly, the relation of this engagement to other factors they could discern. In other words, noticing for equity appears to involve actively seeking clues from a variety of sources (e.g., the mathematics, the classroom social structure, students' home lives, students' youth and broader cultures, etc.) to make sense of, promote, and sustain student participation.

Discussion and Conclusion

The goal of this study was to make visible particular forms of noticing related to equitable instructional practice. Attention to practices that promote equitable opportunities to learn is a priority in mathematics education (Gutiérrez, 2007; Martin, 2003; Nasir & Cobb, 2007; NCTM, 2000). We contribute to this body of work by examining how teachers who promote equity *notice* classroom activity and by documenting the relation between these teachers' noticing and instructional practice. Our findings reveal how teachers who notice for equity see phenomena through an equity lens and use that lens to interpret these phenomena and inform their instructional decisions. Thus, we extend research on teacher noticing, which for many good reasons has focused on noticing student thinking (e.g., Goldsmith & Seago, 2011; Levin, Hammer, & Coffey, 2009; Sherin & van Es, 2009; Walkoe, 2014), to characterize the layered, multifaceted nature of noticing for equity.

One of the central findings of this study is that teachers who promote equity not only engage in shared instructional practices but also demonstrate commonalities in terms of their noticing. More specifically, we see a number of general themes emerge with respect to what the teachers attend to, how they reason about these phenomena, and how they use what they learn to inform their instructional choices. First, the teachers all attend to issues of *status and positioning*—how groups function and how students support one another in group work; who is participating and taking the floor during whole class discussion and how different forms of student participation afford opportunities for others' learning; and how the teacher constructs opportunities for students to take up space. In addition, the teachers attend to *individual student histories* both within and beyond the class to inform their interactions during instruction. These teachers were acutely aware of who their students were as people—as individuals and as members of other communities (e.g., youth and cultural communities)—and they attended to students' culture and community as it played out during instruction. The teachers also noticed the *energy and flow of the students and the class*. That is, they noticed how and when students took up ideas, when students sought to move conversations in a new direction, and when students could sustain mathematical inquiry or needed to take a pause and reengage. As such, they attended to how choices they made in the moment, and in response to the energy of the students and class, influenced the direction of a lesson and the potential consequences for moving a conversation in a direction of students' interest versus maintaining the central line of mathematical inquiry. To be clear, these teachers engaged in forms of noticing previously identified in the literature—careful attention to the mathematics as it unfolded in a lesson, as well as student thinking and understanding of the mathematical content (see for example, Jacobs et al., 2012; Sherin & van Es, 2009). However, these teachers also attended to dimensions of instruction related to participation, access, and opportunity. We propose that this attention to these different dimensions of classroom activity is what constitutes the multilayered nature of noticing for equity.

We also found that the teachers in our study did not all notice the same dimensions of classroom activity, nor did they interpret all interactions in the same way. For instance, Julie and Carter attended to the extent to which they were needed or not to help students make progress on the mathematics and how their input could support or limit students' mathematical work. At the same time, these two teachers were aware of how classroom norms influence students' access, and they attended to how well students took up the norms and if students appropriated norms for group work to support one another's learning. These teachers had a keen sense of how students and groups worked with tasks and were constantly monitoring how individuals and groups coordinated their activity. On the other hand, Raymond was focused on individuals' cultures and communities and the broader system of schooling. Thus, he attended to the ways that the contexts of their students' lives, both related to schooling and outside of school, interfaced with their opportunities to learn and engage in mathematical activity. Parker was the only teacher who attended closely to both opportunities for students to reason mathematically and to take up space in her classroom, and did so in a way that made participation gaps very uncommon in her classrooms. We see these variations in noticing as providing affordances and constraints for engaging in equitable practice. That is, attention to some facets of classroom activity will inform particular instructional choices; whereas inattention to other dimensions may limit teachers' ability to construct equitable learning environments. A subject for future inquiry concerns understanding the influence that noticing different features of classroom activity has on cultivating equitable mathematical learning environments for students.

At the same time, we propose that each of these teachers had particular strengths for promoting equity that can be leveraged and used as opportunities for teacher learning. Participation in professional development like video clubs or lesson study, where teachers make their practice public and talk about their instructional choices, can become spaces for teachers to learn from each other what is worth noticing and how to use what they see to promote equity in their classrooms. Analysis of Parker's and Raymond's instruction, for example, can highlight how teachers attend to students as individuals and the broader communities in which they participate; whereas, analysis of Parker's, Julie's, and Carter's instruction, and associated noticing, can reveal how teachers carefully structure and monitor group work, such as, how particular students work together and support each other or when group members do not support each other and inhibit each other's learning.

Finally, the findings of our study have important implications for teacher education and teacher development. By making visible what and how teachers notice for equity, we can contribute to recent efforts to develop a pedagogy for teacher education (Grossman, Hammerness, & McDonald, 2009). Much of the work on improving teacher education has focused on identifying high-leverage practices and preparing teachers to engage in these practices early in their careers (McDonald et al., 2013). We propose that developing a stance toward equity is also a central goal for teacher preparation and teacher education (Gutiérrez, 2002). Thus, research is needed to explore the kinds of tasks and activities that will enable preservice and practicing teachers to learn to notice classroom activity from an equity frame.

We recognize that there are several limitations to this study. One issue concerns the short duration of the study. We observed these classrooms over a few months in the middle of the school year, and thus, may have missed important aspects of students' participation and the teachers' instructional practices. Similarly, because we captured teachers' noticing in the moment, we did not necessarily have access to patterns in their noticing over time, and how these related to their instructional choices. A longer study would have also enabled us to relate observations of their teaching and noticing to their developing dispositions, providing insight into the influence of teachers' pedagogical commitments on their ways of noticing and practice.

Another issue centers on the distinction between these teachers' instructional practice as equitable as opposed to "just good teaching." Since our conceptualization of equity focuses on narrowing classroom participation gaps in classroom mathematical activity, there may be significant overlap with the practices of reform mathematics teaching. However, unlike that body of research, this study attends to issues of status, culture, and power in the mathematics classroom, and was conducted in classrooms in which issues of equity often emerge. That said, we find it problematic that all but one of the teachers that were identified by district personnel as being exceptional at equitable mathematics teaching were white. This is despite the fact that the student demographics of these districts were highly diverse or even hypersegregated. We are concerned that this process of teacher recruitment may have severely limited our access to teachers from nondominant backgrounds or white teachers who are highly aware of the role of race and racism in education, who might have approached equitable mathematics instruction in markedly different ways. (We see some evidence of this in Raymond's instructional practice.) Despite these limitations, we view the findings of this study as taking an important first step towards identifying practices of *noticing for equity* tied to mathematics instruction that affords greater access and agency to a wider range of our learners.

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Part IV
Complexities in Measuring Teacher Noticing

Complexities in Measuring Teacher Noticing: Commentary

Victoria R. Jacobs

Abstract With the growing research base on teacher noticing has come a similar expansion of methodologies used to measure teacher noticing. The six chapters in this section reflect a range of methodologies, and this commentary is organized around three methodological considerations showcased in the chapters: (a) adoption of a conception of teacher noticing, (b) design of data-collection tools, and (c) choice of data analysis lenses.

Keywords Noticing conceptions · Noticing measures · Noticing analyses · Group noticing · Failure to notice

Research on teacher noticing has been rapidly increasing in mathematics education as well as in other content areas like science education (e.g., Russ & Luna, 2013; Talanquer, Tomanek, & Novodvorsky, 2013) and literacy education (e.g., Rosaen et al., 2010; Ross & Gibson, 2010). With this growing research base on teacher noticing has come a similar expansion of methodologies used to measure teacher noticing. The diverse, currently somewhat disjoint, nature of these methodologies reflects the multiple conceptions and dimensions of teacher noticing and the fact that measuring this construct is not straightforward. Because teacher noticing refers to an in-the-moment practice that takes place when teachers are attending to and making sense of particular events in an instructional setting, but before actually responding to those events, this practice is—by definition—invisible (Sherin, Jacobs, & Philipp, 2011). The complexity and invisibility of teacher noticing provide numerous methodological challenges, and this commentary is organized around three methodological considerations showcased in the chapters in this section: (a) adoption of a conception of teacher noticing, (b) design of data-collection tools, and (c) choice of data-analysis lenses. Across the six chapters, these methodological considerations are discussed in relation to the noticing of a range of individuals: prospective and practicing mathematics teachers at both the

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elementary and secondary levels. For convenience, the term *teacher* will be used to indicate all these individuals throughout the commentary.

Adoption of a Conception of Teacher Noticing

At first glance, conceptualizing teacher noticing may not seem like a methodological consideration, but teacher noticing is an emerging construct without an established definition (see Jacobs & Spangler, in press, for a summary of conceptualizations). Stockero and Rupnow acknowledged these varied conceptualizations and described their common feature as “honing in on a key aspect of or instance that occurs during a lesson and engaging in reasoning to make sense of it” (p. 282). However, what constitutes this “reasoning” is variable across conceptualizations.

For example, compare two oft-cited conceptualizations: van Es and Sherin (2002) focused on teachers’ making connections to principles of teaching and learning, whereas Jacobs, Lamb, and Philipp (2010) focused on teachers’ making decisions about how to respond (instructional next steps). Different data-collection tools and analysis lenses are needed for each focus. Similarly, Spitzer and Phelps adopted the Jacobs and colleagues’ (2010) focus on teachers’ decisions about how to respond, but their conceptualization (and thus analysis lens) allowed for an expanded view, which included both instructional next steps and reflections on the cause–effect connections between teaching and students’ mathematical thinking. Researchers who choose to focus on only one of these pieces might use different prompts to elicit teacher noticing and different analysis lenses. These examples are meant to illustrate important connections between theoretical conceptualizations and methodological decisions, and even this small sample of conceptualizations should provide readers with a sense of the many methodological challenges involved with measuring teacher noticing.

In addition to illustrating a range of conceptions of teacher noticing, these chapters sometimes linked teacher noticing to other constructs of interest. In particular, the idea of teacher knowledge was central to three of the chapters, but in different ways. Beattie, Ren, Smith, and Heaton measured separately teachers’ noticing expertise and teachers’ Mathematical Knowledge for Teaching (MKT) and then looked for connections. Dick worked to integrate the measurement of teacher noticing with a subset of MKT (specialized content knowledge). Specifically, she looked for evidence of teachers’ MKT related to multidigit addition and subtraction within each component of teacher noticing that she studied. Stürmer and Seidel also chose to integrate the measurement of teacher knowledge in the design of their Observer Research Tool, which tracked teachers’ representations of their “theory–practice integrated knowledge” (p. 363) through ratings of classroom video. In summary, the chapters in this section provide examples of how methodological decisions can be best understood when researchers define their conceptions of noticing and are explicit about connections to other constructs.

Design of Data-Collection Tools

Teacher noticing is situated in and integrally tied to instructional settings, and, thus, data must be collected in a contextualized way. Ideally, teacher noticing should be measured in the midst of instruction, but this type of assessment is hard to accomplish without disrupting both instruction and the practice of noticing itself. Recent work has come closer to measuring real-time, authentic noticing by having teachers use wearable cameras, while teaching, to capture instances in which they felt they had noticed something significant (Sherin, Russ, & Colestock, 2011). However, this research is still in the early stages, and teacher noticing has more typically been studied in one of three ways: (a) teachers engage with researcher-selected artifacts of practice from other teachers' classrooms, (b) teachers engage with artifacts from their own classrooms after having taught a lesson, and (c) researchers infer teacher noticing from instructional episodes (Jacobs & Spangler, in press). The first two approaches are most common in the field, and the following sections showcase two decisions—selection of artifacts and elicitation of teacher noticing in relation to those artifacts—that are integral to both approaches.

Selection of artifacts. Many researchers use artifacts to represent practice—generally video or student written work—and then ask teachers to notice in relation to those artifacts. Artifacts from the teachers' own classrooms enable teachers to use their insider knowledge about students, but comparison across teachers is challenging because, with different artifacts, teachers have different opportunities to notice. In contrast, artifacts from researcher-selected classrooms facilitate comparison across teachers because everyone engages with the same artifact, but some of the authenticity may be lost because teachers are missing the contextualized knowledge they have about their own classrooms. In both cases, selection of artifacts is complex and is often a focus of study itself (Goldsmith & Seago, 2011; Sherin, Linsenmeier, & van Es, 2009).

Throughout these chapters, readers will find explicit choices related to artifact selection. For example, Stockero and Rupnow purposefully left their videos unedited, and Stürmer and Seidel outlined three criteria for their video selection: (a) teachers should perceive the videos as authentic examples of classroom practice, (b) the videos should serve to activate teachers' knowledge by being stimulating but not overwhelming, and (c) experts in the field should view the videos as examples of the target practice. Nickerson, Lamb, and LaRochelle outlined different criteria, selecting video that depicted content that all their teachers had experience teaching and that provided multiple opportunities to notice features of students' mathematical thinking. They also cautioned that having criteria may not be sufficient when resources are sparse, and they shared their challenges with finding secondary-level video in which a variety of students' mathematical ideas were visible (e.g., multiple strategies and representations).

Elicitation of teacher noticing. Giving teachers an opportunity to notice in relation to artifacts of practice is only one step in measuring teacher noticing.

Researchers must also elicit and record teachers' noticing to preserve it for later analysis. The format of these data has often included teachers' verbal or written responses to open-ended questions or video-recorded conversations among groups of teachers about the artifacts. However, technology is playing an increasingly important role in the collection of teacher noticing data, and these chapters provide several examples. For instance, Spitzer and Phelps studied teacher noticing that was captured in debates in online discussion boards, and Stockero and Rupnow examined teacher noticing reflected in teachers' use of Studiocode video analysis software to identify and justify their selection of mathematically important teaching moments in video-recorded instructional episodes.

Selection of data sources and formats can have implications for the teacher noticing that can be measured. For example, Stockero and Rupnow highlighted the difference in precision of language used in written versus oral communication as well as the benefits afforded by follow-up probing during interview situations. Stürmer and Seidel faced a different challenge in their quest to create a "standardized, yet contextualized" (p. 365) instrument that could be used on a large scale. They preserved the situated nature of noticing in their instrument by including video of classroom episodes, but they chose a closed-response format (ratings) rather than the traditional open-ended format because of the time-consuming nature of scoring open-ended responses for large samples.

Choice of Data-Analysis Lenses

After the teacher-noticing data have been collected, researchers must choose how to make sense of the data. In these chapters, the scope of the mathematics targeted seemed to be linked to the analysis lenses chosen. For example, some chapters were focused on narrow mathematical topics (e.g., Dick focused on multidigit addition and subtraction, and Nickerson and colleagues focused on algebraic generalization), and these researchers used analysis lenses linked to research on students' thinking about those mathematical topics (when available). In contrast, in other chapters, researchers purposefully chose to look across content and contexts, focusing on a generalizable approach for analyzing teacher noticing (e.g., Spitzer and Phelps focused on the process of decomposing mathematical-learning goals, and Stockero and Rupnow focused on the characteristics of classroom instances that are most likely to promote student learning). These different approaches to measuring teacher noticing in relation to narrow mathematical topics or across mathematical topics are not mutually exclusive but rather differ in what is foregrounded. The link between the scope of the mathematics targeted and the researchers' analysis lens is further illustrated in the following sections which identify four analysis lenses represented in the chapters: (a) frameworks of students' mathematical thinking, (b) frameworks of teacher noticing, (c) mathematical-learning goals, and (d) comparisons to expert noticing.

Frameworks of students' mathematical thinking. Nickerson and colleagues underscored the importance of using frameworks of students' mathematical thinking or learning trajectories (sometimes called learning progressions) as a basis for measuring how teachers make sense of the relative sophistication of the students' understandings. These frameworks are linked to specific mathematical content and students' mathematical thinking about that content. For example, they cited the Jacobs and colleagues' (2010) study based on frameworks from Cognitively Guided Instruction and the Schack and colleagues' (2013) study based on Steffe's stages of early arithmetic learning. They also noted that frameworks and learning trajectories are not equally available for all content and, in particular, are sparse for the content covered at the secondary level. They made a plea to the field to develop a stronger research base about students' conceptions (and their development) at the secondary level while also acknowledging the challenges inherent in this task given that secondary students' understandings may be extremely broad because of their divergent earlier school experiences.

Frameworks of teacher noticing. Beattie and colleagues adapted the van Es (2011) framework for learning to notice student thinking to characterize how teachers' written responses reflected their expertise in noticing students' thinking. This framework is different from frameworks of students' mathematical thinking in that it describes the development of teachers—not students—while they gain expertise. The researchers also used levels of teacher-noticing expertise to create profiles of teachers, which they then discussed in relation to potential supports that could be used to help teachers in particular profiles progress.

Mathematical-learning goals. Spitzer and Phelps used a lens of mathematical-learning goals to analyze teachers' noticing because they argued that teacher noticing is most effective when linked to lesson goals. Specifically, they separated each learning goal into key mathematical subgoals that were then used to code the teachers' noticing data and, in particular, distinguish responses that were at different levels of depth. They preferred this focus on the mathematics because of its generalizability to other mathematical topics, tasks, and student-work samples in contrast to other lenses that may be more likely to be linked to idiosyncratic features of specific lessons or lesson artifacts.

Comparisons to expert noticing. Stockero and Rupnow compared the instances teachers noticed as mathematically important teaching moments to the instances that experts—in this case, their research team—noticed. Thus, they identified a target ("correct") noticing of what they called *Mathematically Significant Pedagogical Opportunities to Build on Student Thinking* (MOSTs)—classroom instances, in any domain, that simultaneously involve students' mathematical thinking, significant mathematics, and pedagogical opportunity (Leatham et al., 2015). They view MOSTs as *high-leverage* student thinking in that these instances have strong potential for supporting student learning. In addition to recording whether the teachers noticed the researcher-identified MOSTs, Stockero and Rupnow also underscored the importance of considering the teachers' rationales for identifying particular instances as mathematically important teaching moments because their reasoning was not always consistent with the reasoning of the researchers.

Final Thoughts

In reviewing these chapters, I found the range of methodologies most notable. This finding is perhaps unsurprising given that mathematics teacher noticing is a relatively new field, and, thus, norms for measuring teacher noticing are still emerging. In this time of expansion, researchers need to be creative with new methodologies and, in particular, capitalize on new technologies. In closing, I highlight two ideas raised in the chapters that I believe are under-researched and warrant further consideration while the methodologies for measuring teacher noticing continue to evolve.

First, I was intrigued with the idea raised in several chapters (e.g., Spitzer & Phelps; Stockero & Rupnow) that researchers should be concerned not only with what teachers notice but also with what they fail to notice. This idea has strong face validity because everyone has had faulty noticing experiences in which particular events or opportunities were missed, generally with negative consequences. However, identifying what teachers miss is challenging methodologically. First, identifying something as *missed* indicates that a *correct* noticing exists and that it was not achieved. This perspective is in contrast to the views of some researchers who consider multiple paths and responses as correct, thus rendering what counts as *missed* almost meaningless. Second, when teacher-noticing data are collected in written form, without opportunities for follow-up questions, the challenge is even more daunting. When teachers do not report noticing something of interest, did they fail to notice it or simply fail to report noticing it? Researchers can be confident when teachers provide evidence that they noticed something. However, without such evidence, researchers must find ways to tease apart when teachers failed to notice something versus when they simply did not provide evidence that they had noticed it.

Second, most of the chapters, like most of the work in teacher noticing, were focused on measuring the noticing of individual teachers, but Stockero and Rupnow also explored the noticing of a group (cohort) of teachers. They found differences in what teachers could notice alone versus in groups. This information could be particularly useful to professional developers and university faculty who often work with groups of teachers. However, focusing on the noticing of groups also raises methodological challenges in terms of what researchers can (and cannot) learn about an *individual's* noticing expertise inside and outside of that group setting.

While researchers pursue these and other ideas in the development of new or refined methodologies for measuring teacher noticing, they need to be purposeful in their methodological decisions and clearly communicate these decisions and their rationales to readers. Only by continued conversations can the field consolidate the advantages and disadvantages of various approaches. These chapters provide a basis for this work through examples of a range of methodologies and decisions researchers make when measuring the complex and invisible practice of teacher noticing.

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Measuring Noticing Within Complex Mathematics Classroom Interactions

Shari L. Stockero and Rachel L. Rupnow

Abstract Drawing on our work focused on developing prospective teachers' ability to notice using unedited classroom video, we explore various ways that we might measure teacher noticing within a complex classroom context. Our work has a goal of helping prospective teachers notice high-leverage instances of student mathematical thinking that could be built upon to support student learning during a lesson. To measure changes in noticing, one approach we have used is categorizing what is noticed using the components of our noticing goal: individual students, mathematical thinking, and student-centered teacher responses. We have also measured the extent to which teachers' noticing combines these components. Another approach we have used is measuring noticing against a set of instances that meet defined criteria—instances that the research team identified as high-leverage instances of student mathematical thinking. We discuss what each of these measurements tell us about teacher noticing and what they do not. Our goal is to raise issues for the field to consider in order to advance the work of teacher noticing.

Keywords Teacher noticing · Student mathematical thinking · Teachable moments · Preservice teachers · Learning outcomes

Studies of teacher noticing have varied along a number of dimensions. Some are a one-time documentation of noticing (e.g., Jacobs, Lamb, & Phillip, 2010; Roller, 2016), while others focus on changes in teachers' noticing as the result of an intervention (e.g., Sherin & van Es, 2009; Stockero, 2014, 2017). The medium used to ground noticing ranges from student written work (e.g., Fernández, Llinares, & Valls, 2013) to full-length classroom video (e.g., Stockero, 2014, 2017). The goal for noticing is often related to students' mathematical thinking (e.g., Jacobs et al.,

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2010; van Es, 2011), but studies have also focused on noticing such things as students' participation (Wager, 2014) and teacher–student interactions (Scherrer & Stein, 2013). The content focus of noticing interventions has varied from a relatively narrow topic, such as early arithmetic reasoning (Schack et al., 2013), to a broad range of topics in interventions that use video from participants' own classrooms (Barnhart & van Es, 2015; Sherin & van Es, 2009).

These variations in how teacher noticing is developed and studied raise interesting questions related to measuring teachers' ability to notice. What constitutes more or less sophisticated noticing? What measurements most accurately represent the quality or development of teachers' noticing within specific contexts? How might we also consider what teachers do not notice? In this chapter, we use data from an ongoing study of prospective teacher noticing to explore multiple ways that noticing might be measured, considering advantages and disadvantages of, as well as interactions among, these various approaches. First, however, we examine how others have measured noticing and briefly describe the context of our work.

Methods of Measuring Noticing

Just as studies of noticing have varied, so too have the ways in which teacher noticing has been measured or analyzed. Many of these variations are due to differences in the definition of noticing adopted, the nature of noticing interventions, and what is valued as important to notice. Although the methods of measuring noticing are not mutually exclusive—many are used in tandem—we discuss them separately to make distinctions among them. First, however, we discuss the definitions of noticing that often frame how noticing is measured.

Two definitions of noticing are dominant in the literature. The first is van Es and Sherin's (e.g., 2002) *Learning to Notice Framework*, which defines noticing to include "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions" (p. 573). The second is Jacobs et al.'s (2010) definition of *professional noticing of children's mathematical thinking*: "a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 171). Common between these definitions is that noticing involves honing in on a key aspect of or instance that occurs during a lesson and engaging in reasoning to make sense of it. The primary difference is that van Es and Sherin focus on making connections to principles of teaching and learning, while Jacobs et al. focus on making a decision, a next move, based on what has been observed and analyzed. Some noticing analyses have included all three components of one of these definitions (e.g., Barnhart & van Es, 2015; Fernández et al., 2013), while others have focused on only some aspects of noticing (e.g., Star & Strickland, 2008; Stockero, 2014, 2017), typically identifying and making sense of important classroom instances.

Measurement Using Categorization of Instances

Studies that draw on the *Learning to Notice Framework* (e.g., van Es & Sherin, 2002, 2008) typically categorize instances of teacher noticing and look for significant changes in the frequency of noticing within categories at different points in time. van Es and Sherin (2008, 2010), for example, segmented video club discussions by topic and coded what each participant noticed using five dimensions: actor, topic, stance, specificity, and video focus. Changes in noticing from an early to a late video club meeting were examined using the percentage of teacher comments that fell into subcategories of each dimension. They documented significant changes in several key areas of noticing, including focusing on students and their mathematical thinking. Mitchell and Marin (2015) examined participants' noticing using an adaptation of van Es and Sherin's coding scheme that included only the actor, topic, and stance. They also found that teacher noticing changed to become more focused on students, but documented a split in the topic of noticing between mathematical thinking and pedagogy, reflecting the goal of their intervention: to notice both students' mathematics and ways that teacher moves can foster student mathematical engagement. The difference in these findings highlights the importance of measuring and interpreting teacher noticing with a clear sense of the noticing goal.

Measurement Using Point or Ranking Systems

Another means of measurement is using point or ranking systems to score noticing. A number of studies have numerically scored teachers' responses to prompts focused on the components of noticing—attending, interpreting, and responding to student work (or, alternatively, making connections). In such studies, scores are assigned according to the extent to which responses show evidence of each noticing component (e.g., Jacobs et al., 2010; Schack et al., 2013). In science education, Barnhart and van Es (2015) did not score numerically, but used a parallel method by defining levels of sophistication to classify teacher responses as low, medium, or high with regard to the noticing components. Using categories more closely aligned with their noticing goal, Scherrer and Stein (2013) scored teacher responses for three noticing targets: noticing student–teacher interactions, using one of the codes from their intervention, and discussing the relationship between students' opportunities to learn and teachers' use of student responses. Their study was different than the others in that it scored based on inclusion of specific elements, not on the quality of responses. Studies using points or ranks typically look for significant differences in noticing, either pre- to post-intervention (Schack et al., 2013; Scherrer & Stein, 2013) or among different teacher groups (Barnhart & van Es, 2015; Jacobs et al., 2010).

Scoring has also been used to measure noticing by rating the level of teachers' discussions of noticing. Using two main categories of what teachers notice and how teachers notice, van Es (2011) defined four levels of noticing—baseline, mixed, focused, and extended—that allowed her to examine the trajectory of how teachers' noticing developed. Roth McDuffie et al. (2014) adapted this trajectory to their work on noticing of equitable mathematics teaching practices, using the categories baseline, attention, awareness, and making connections. Although these researchers discussed their findings in terms of teachers' initial and final noticing, van Es' analysis of how noticing developed during her intervention could easily have been paralleled.

Measurement in Relation to a Standard

Noticing has also been measured in relation to defined standards. One standard that has been used is what is known about students' mathematical thinking in particular areas of mathematics. For example, Fernández et al. (2013) rated participants' noticing of student written work at one of four levels that indicated the extent to which the participant was able to discriminate between proportional and additive reasoning and develop a student profile based on the written work. Similarly, Schack et al. (2013) rated noticing based on inclusion of references to specific Stages of Early Arithmetic Learning. Each of these studies focused teacher noticing on artifacts of practice related to a relatively narrow mathematical topic, allowing the researchers to measure noticing in relation to what is known from research about common student misconceptions about that topic or how student thinking about that topic generally develops.

Other studies have measured noticing in relation to a defined framework. One example is Walkoe (2015), who analyzed whether prospective teachers' discussions of their noticing were implicitly or explicitly connected to the Algebraic Thinking Framework that was used to frame their noticing. Another example is Mitchell and Marin (2015), who measured noticing in relation to how well participants' noticing using the Mathematical Quality of Instruction (MQI) framework aligned with that of the researchers. In this case, it was not references to a framework that were measured, but how the participants' noticing met a specific standard—the noticing of an expert, as framed by a particular viewing instrument. In general, having some type of framework that defines clear parameters for noticing allows for different types of measurements than more general interventions that focus broadly on noticing student mathematical thinking.

Framing Our Work

In our work, we follow Jacobs et al.'s (2010) definition of *professional noticing of [student]'s mathematical thinking* to include the three interrelated skills of attending to student thinking, interpreting the student thinking, and deciding how to respond. Our choice of this definition stems from its close connection to our goal of helping prospective teachers learn to enact instruction that is responsive to students' current understanding of the mathematics, what Lampert, Beasley, Ghousseini, Kazemi, and Franke (2010) have called *ambitious teaching*.

It is our perspective that not all instances of student mathematical thinking have the same potential to enhance student learning. We place value on noticing those instances of student thinking that have significant potential to be used during the lesson to support students' mathematical learning. We draw on Leatham, Peterson, Stockero, and Van Zoest's (2015) construct of **M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking [MOSTs], which they define as occurring at the intersection of three characteristics: (a) student mathematical thinking, (b) significant mathematics, and (c) pedagogical opportunity. In the MOST Analytic Framework, two criteria are used to determine whether an instance of student thinking embodies each characteristic (see Figure 1 for criteria and key questions associated with each). If all six criteria are satisfied, an instance is determined to be a MOST (for more details, see Leatham et al., 2015 or Stockero, Leatham, Van Zoest, & Peterson, this volume). If an instance is a MOST, they define the most productive teacher move as *building*, making the student thinking an object of discussion for the class in order to support them in making sense of the mathematics. We used the MOST Analytic Framework as a tool to focus participant noticing.

MOST Characteristics	Criteria	Key question for criteria
Student mathematical thinking	Student mathematics	Can the student mathematics be inferred?
	Mathematical Point	Is there a mathematical point closely related to the student mathematics?
Significant mathematics	Appropriate mathematics	Is the mathematical point accessible to students with this level of mathematical experience, but not likely to be already understood?
	Central mathematics	Is understanding the mathematical point a central goal for student learning in this classroom?
Pedagogical opportunity	Opening	Does the expression of the student mathematics create an intellectual need that, if met, will contribute to understanding the mathematical point of the instance?
	Timing	Is now the right time to take advantage of the opening?

Figure 1. MOST characteristics and criteria (Leatham et al., 2015).

Context of Our Work

The participants were prospective mathematics teachers (PTs) who voluntarily enrolled in a special section of a field experience course early in their teacher education program. Here we examine data from seven PTs who participated in a 10- or 11-week intervention during the fall 2013 (n = 4) or fall 2014 (n = 3) semester, referred to as Cohorts A and B, respectively. Mathematics lesson videos recorded by the PTs in local secondary school classrooms were used as the basis of the learning-to-notice activities. Efforts were made to collect video from a range of grade levels (6–12) with varied mathematical topics. The instructional portions of the video were left unedited for analysis, with portions in which students could not easily be heard removed.

The PTs and researchers used the Studiocode (SportsTec, 1997–2015) video analysis software to individually analyze one video each week, marking mathematically important moments a teacher should notice in the classroom. The PTs included in their analyses a description of why they chose each moment. Prior to a weekly group meeting with the PTs, the researchers met to agree on instances that were MOSTs, discuss the instances PTs had identified, and select instances that would be discussed with the PTs. The weekly group meetings were facilitated by the first author, with a goal of helping the PTs learn to notice instances of student mathematical thinking that might be built upon during a lesson to support student understanding.

The specific learning to notice activities varied by semester, consistent with a design experiment approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Both cohorts were initially prompted to identify *mathematically important moments that a teacher should notice during a lesson*; this construct was left ill-defined to provide baseline data for PT noticing. In weeks 3–4 of their 10-week intervention, PTs in Cohort A codeveloped labels to describe and categorize types of mathematically important moments (e.g., student realization, student question, higher order wrong answer). After the labels were created, the PTs assigned them to moments they noticed in subsequent videos. Both cohorts were introduced to the MOST framework (Leatham et al., 2015), Cohort A after week 5 and Cohort B after week 2, as a way to focus their noticing and aid in their analysis of video instances. The PTs were then expected to identify instances that were MOSTs and describe why they met the MOST criteria. Late in the intervention, the PTs were also asked to propose a teacher response to each identified MOST. Since the MOST framework focuses on instances that could be built upon by a teacher, responses that would engage students in mathematical thinking were what was valued in the study.

At the end of the intervention, PTs engaged in an individual interview in which they identified MOSTs in a 12-min clip from a publicly available video (UCLA and the Carnegie Foundation for the Advancement of Teaching, 1999). They identified moments in real time, as the video was playing, to document their ability to recognize and make sense of instances as they occurred in a lesson, rather than after replaying a video. When a PT identified an instance as a MOST, the researcher stopped the video to allow the PT to discuss their reasoning about the instance. When necessary, the researcher prompted the PT to elaborate on their thinking.

The data for the study were the PTs' individual video analyses—both the instances they noticed and their written explanations—video recordings of the group meetings, and video-recorded post-interviews. We discuss the various ways the data were analyzed to make sense of how noticing changed as a result of the intervention in subsequent sections of the chapter.

Measuring Changes in Teachers' Noticing in Our Work

We now consider various methods we have used to measure changes in the noticing of both cohorts and individual PTs, highlighting what each analysis tells us about PTs' noticing and what it does not. In this discussion, *initial* refers to the PTs' noticing in the first two videos each semester, before we attempted to influence their noticing. *Final* refers to the PTs' noticing in the last four videos each semester—an indication of their most refined noticing. We use four videos to report the final noticing because the PTs noticed significantly fewer instances in these later videos. In fact, PTs' noticing decreased from noticing 13.1 instances per video initially to 4.1 instances per video in the final videos, perhaps reflecting a more refined perception of what is important in a classroom lesson. *Post* refers to the PTs' noticing in the post-interview.

Categorization of Noticing

One way we measure noticing is by categorizing instances that teachers notice based on the aspects of noticing that we value in our research, consistent with studies that have used variations of van Es and Sherin's *Learning to Notice Framework* (e.g., 2002, 2008). We value the noticing of individual students and their mathematical thinking, and want the PTs to attend to the specific mathematics that is embedded in the student thinking, so these are what we choose to analyze. To analyze the PTs' individual noticing, each instance a PT identified is our unit of analysis and we use their reasoning about the instance to assign codes that characterize their noticing. Consistent with prior research (e.g., Stockero, 2008; van Es & Sherin, 2008), we code for whom the PT attended to (*agent*) and the *specificity* with which they discussed the mathematics (see Stockero, Rupnow, & Pascoe, 2017, for more detail about these categories). Because we are primarily interested in how PTs attend to students, we combined van Es and Sherin's (2008) topic and stance categories to code the *focus* of the PTs' noticing in instances where students are the primary agent. When the primary agent is not student, we do not assign a focus code.

Agent The agent of the PTs' noticing is related to the attending component of noticing. Table 1 summarizes the PTs' noticing according to whether the teacher, the students, or the mathematics itself was the primary agent. Note that in some instances, both the teacher and the student were agents; these were coded as such, with one being primary. These are included in the values in Table 1. Initially, the PTs largely attended to the teacher in the video. We see that one participant in each group (A1 and B3), however, displayed a fairly strong emphasis on students from the start; thus, although noticing the teacher may be a natural tendency for PTs, this is not always the case. In the final data, the PTs' overall noticing had shifted primarily to the students in the video, although some individual participants (B1 and B3) still displayed a fairly significant amount of teacher-centered noticing, which accounts for the lower percentage of student noticing for Cohort B. This finding is important because it allows us to see that even though the overall student-centered noticing was lower for Cohort B, B2's noticing was more consistent with the members of Cohort A, indicating that there was potential for the intervention to support high levels of student-centered noticing for this cohort. However, the small size of the cohorts makes it easy for the data to be skewed by anomalous individuals, which may be the case here. In the post-interview, all of the PTs' noticing in both cohorts had a primary student agent.

To make more sense of the agent data, we broke down the PTs' primary student noticing to determine to what extent the PTs were attending to individual students—a goal of our intervention since MOSTs necessarily come from an individual student—versus groups of students or student–teacher interactions (Table 2). Of the primarily student-directed noticing in the initial data, all of the PTs concentrated 50% or less of their noticing on individual students. Cohort A did have a fair amount of emphasis on individual students at the start (39%), whereas Cohort B hardly attended to individual students. By the end, Cohort B attended to individual students more often than Cohort A, indicating a significant shift for Cohort B, but a Cohort A member, A3, was the only PT who attended solely to individual students in the final

Table 1
Participant noticing by primary agent

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined[a] (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Student primary	Initial	47	8	16	13	19	6	11	60	28
	Final	100	95	100	100	98	75	100	82	86
Teacher primary	Initial	53	80	39	81	59	91	50	40	63
	Final	0	5	0	0	2	25	0	18	15
Math primary	Initial	0	12	45	6	22	3	39	0	9
	Final	0	0	0	0	0	0	0	0	0

[a]Computed by taking the sum of all PT instances coded in category divided by total instances identified by PTs

Table 2
Percentage of participants' primary student noticing directed at individual students

	Cohort A					Cohort B			
	A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Initial	38	50	33	50	39	0	0	5	4
Final	45	83	100	77	80	80	89	86	85
Post	67	38	43	50	46	33	67	67	58

Table 3
Specificity of participant noticing

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Specific math	Initial	82	48	76	81	71	26	83	6	30
	Final	100	67	94	92	86	100	100	82	95
General math	Initial	18	44	24	13	26	23	6	51	31
	Final	0	33	6	8	14	0	0	18	5

data. In the post-interview, Cohort B attended to individual students 58% of the time, with the remainder of the student noticing directed at groups of students. Cohort A attended to individual students less often (46%), with continued attention to both groups of students and student–teacher interactions. The large decrease in percentages of individual student noticing for most PTs from the final to the post-interview data may be attributable to the post-interview's smaller data set or the tendency to use less precise language (i.e., 'they', instead of 'he' or 'she') in spoken as compared to written responses, which would result in more noticing coded as attending to groups.

Specificity A second component of our categorization analysis involved considering the level of specificity with which the PTs described the mathematics in an instance—whether they discussed the mathematics in a specific way, a general way, or did not discuss the mathematics at all. Table 3 shows that members of Cohort A tended to describe the mathematics in a specific way from the beginning, though A2 did so less frequently, and B2 behaved more like the members of Cohort A. This tells us that at least some members of both cohorts were capable of describing the mathematics with specificity without an intervention, although only about half of the PTs did so. However, in the final data, all PTs described the mathematics with specificity the majority of the time, with many doing so at all times. In the post-interview all PTs spoke only about the specific mathematics in the instances they identified.

Focus The focus code includes both attending and interpreting aspects of PTs' noticing of students, in that it indicates what PTs attended to when they were noticing students in the video and, if they were noticing student mathematical thinking, whether they simply reported the thinking (noting) or engaged in making sense of the thinking (analysis). Initially, 96% of Cohort B's student-centered noticing was not focused on noting or analyzing students' mathematics, but on affective or mathematical interactions or making sweeping generalizations about students' understanding (Table 4). However, in the final data, Cohort B noted or analyzed student mathematics in 87% of instances. Cohort A had a stronger focus on noting student mathematics in the initial data, and their noting and analyzing student mathematics rates also increased, from 52 to 96%. Thus, the intervention supported the development of skills in noting and analyzing student mathematics. Cohort B had a far higher rate of analysis than Cohort A in the final data, but B3 had a rate of analysis that was more similar to that of Cohort A. In the post-interview both cohorts had the same high rate of analysis, with two members of each cohort (A1, A2, B2, and B3) analyzing the student mathematics in all of their identified instances. Thus, members of both cohorts were capable of analyzing the student mathematics, but Cohort A was less inclined to do so in their written noticing explanations.

Analysis in Meetings Because we saw that the PTs, particularly Cohort A, engaged in low levels of analysis of student mathematics individually, but did so more in the post-interview, we examined the level of analysis during the weekly meeting discussions to determine whether they did engage in analysis of student mathematics earlier in the intervention, in collaboration with their peers. Our unit of analysis was a complete discussion about a video instance.

Table 5 shows the percentage of the weekly discussions that included analytical comments, included analysis of two or more components of the MOST framework, and included analysis of three or more components of the MOST framework. It also shows what percentage of the analytical comments made in these discussions were offered spontaneously, without prompting by a facilitator question that pushed the PTs to elaborate or consider alternatives to their current thinking. The two cohorts analyzed about the same percentage of instances in the initial as well as in the final meetings. Although not shown in the table, both cohorts engaged in more analytical

Table 4
Participant noticing focus in videos

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Noting student math	Initial	80	25	38	67	52	0	17	4	4
	Final	91	89	88	85	88	47	33	65	48
	Post	0	0	14	17	8	33	0	0	8
Analyzing student math	Initial	0	0	0	0	0	0	0	0	0
	Final	9	0	12	15	8	37	67	12	39
	Post	100	100	86	83	92	67	100	100	92

Table 5
Analysis in weekly group meetings

		Analyzed instances (%)	2+ Components (%)	3+ Components (%)	Unprompted analysis (%)
Cohort A	Initial	50	14	7	29
	Final	91	55	32	38
Cohort B	Initial	50	29	21	42
	Final	87	78	39	58

discussion after the introduction of the MOST framework than they had before; Cohort A also showed an increase in analysis after the PTs developed labels to categorize important moments. Cohort B was more likely to analyze multiple components of the framework and provided more unsolicited analytical comments in each timeframe than Cohort A. This information about the meetings may help explain why members of Cohort B were more likely to analyze the student mathematics in their written comments about the instances they noticed.

Together, these findings may indicate that Cohort B was naturally more analytical than Cohort A. They may also indicate a shift in how the facilitator led the meetings, or be the result of Cohort B being introduced to the MOST Framework earlier in the semester.

Summary The categorization analysis indicated that the noticing of the PTs in both cohorts changed to become more aligned with our goal for their noticing. In fact, a Wilcoxon signed-rank test indicated that changes in primary student noticing, individual student noticing, discussion of specific mathematics, and foci on noting and analyzing from the initial to final data were all significant at a 0.05 significance level. The categorization analysis provided us with a fairly fine-grained picture of how noticing developed in relation to our goals, both for individuals and for cohorts. Meeting data allowed us to understand an aspect of noticing, analyzing, that many PTs did not exhibit individually, but were able to perform with their peers. What this type of measurement does not tell us is whether the PTs were making sense of the "right" instances of student thinking—those that have the most potential to be used during a lesson.

Target Noticing

A way we have tried to synthesize the agent, focus, and specificity analyses is by considering what we have termed *target noticing*—how well PTs engage in noticing consistent with our goals. This target noticing analysis is similar in some ways to analyses that assign levels to teacher noticing (e.g., van Es, 2011), although we are looking only for instances that reach what we would consider the highest level in our categorization. In this analysis, we examined instances in which the PTs simultaneously attended to individual students and discussed the specific mathematics in an instance, while they also either (1) analyzed the student mathematics (narrow target) or (2) noted or analyzed the student mathematics (broad target). This second analysis was prompted by the fact that we saw minimal narrow target noticing for Cohort A, so we were curious if they were engaging in noticing that was even close to our target.

Table 6 shows that no PTs initially engaged in narrow target noticing. All members of Cohort A hit the broad target at least once in the initial data, but no members of Cohort B did so. However, both cohorts had very similar broad target noticing by the end of the intervention, as shown by both the final and post-interview data; both cohorts had approximately 70 and 58% of their noticing hit the broad target in these data sets, respectively. The narrow target data tells a different story though. Cohort B hit the narrow target far more frequently in the final data, especially B2. This PT structured her written responses according to the MOST framework from the time it was introduced, whereas the other Cohort B members structured their responses to address each framework criterion only after being prompted to do so in the last two weeks of the intervention; Cohort A members were never prompted to do so. Additional analysis revealed that the Cohort B PTs all attained a higher rate of analysis when they had a way to structure their responses, suggesting that structured use of the framework increased the PTs' target accuracy.

There was a significant increase in Cohort A's narrow target noticing from the final to the post data. This is no coincidence, since this increase is largely attributable to the increase in analysis of student mathematics that was documented for this cohort. This increase is likely due to the form of data collection in the post-interview, since the oral interview allowed the researcher to probe the PTs' thinking beyond their initial response. Consistent with the analysis finding, this suggests that both cohorts were capable of engaging in narrow target noticing, but did so more often when prompted to elaborate their thinking in the interview.

Changes in both broad and narrow target noticing were found to be significant ($p < 0.05$) from the initial to the final data. This target noticing analysis provides a means of determining the extent to which the PTs exhibited all of the desired

Table 6
Participants' target noticing

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Broad target	Initial	12	4	3	6	5	0	0	0	0
	Final	45	56	94	77	69	57	89	65	70
	Post	67	38	86	50	58	33	67	67	58
Narrow target	Initial	0	0	0	0	0	0	0	0	0
	Final	9	0	12	15	8	25	61	12	33
	Post	67	38	86	33	54	33	67	67	58

aspects of noticing at once, within a particular instance. In some ways, this analysis paints a picture of the PTs not doing as well, since the target is more difficult to achieve than any individual noticing goal, but it also provides a better sense of PTs' overall ability to attend to and reason about students' mathematics. However, like the analysis of the categorization of noticing, the target noticing analysis still does not indicate whether the PTs were noticing instances with which we would want them to engage.

Measuring Teacher Decision Making

Our analysis of the third component of noticing, deciding how to respond, is similar to those that score teacher noticing on each noticing component, but instead of assigning a score, we categorized the deciding component since qualitative categories are more descriptive. In this analysis, we considered whether PTs examined what actually happened in the classroom by describing or evaluating the classroom teacher's response to students, as well as whether PTs put themselves in a teacher's role by proposing responses centered on explaining the mathematics (teacher-centered) or posing a question that would engage the students in making sense of the student mathematics (student-centered). Because PTs will not have someone else's response to examine when teaching, we placed more value on proposed responses than descriptions or evaluations. We also placed more value on student-centered than teacher-centered moves because learning skills to enact student-centered instruction was a goal of the intervention.

Initially both cohorts engaged in a fair amount of describing what the teacher did in the video, and sometimes evaluated whether that response was appropriate (Table 7). Only A4 proposed any teacher responses in the initial data, which was expected given that the PTs were not explicitly prompted to do so. In the final data, all PTs proposed teacher responses in at least 27% of instances and four of the PTs, A2, A4, B1, and B2, proposed teacher responses in more than 60% of instances, indicating that the intervention helped PTs think about how to respond to student mathematics. A major difference between the cohorts is the type of moves they proposed. In the final data, all of Cohort A's proposed responses were teacher-centered, whereas members of Cohort B proposed student-centered responses over half the time. However, when prompted by the facilitator for additional teacher moves in the post-interviews, the data showed that the Cohort A PTs were capable of responding in a student-centered way, doing so almost half the time. Cohort B proposed student-centered responses to all of their post-interview instances.

In Table 8, we examine PTs' consideration of responses in the weekly meetings. Cohort A was initially twice as likely as Cohort B to describe or evaluate the teacher's response. By the end of the intervention, the rates of describing and evaluating teacher decisions were similar and lower for both cohorts. Both cohorts initially proposed teacher- and student-centered moves at the same low rates, but

Table 7
PTs' responses to MOSTs in individual coding

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Described teacher response	Initial	47	44	5	13	24	46	28	23	33
	Final	91	22	0	62	37	25	17	29	24
	Post	33	13	0	17	13	0	17	0	8
Evaluated teacher response	Initial	0	4	0	0	1	11	0	3	6
	Final	9	17	6	0	8	20	6	47	24
	Post	33	13	0	33	17	0	33	0	17
Proposed teacher-centered move	Initial	0	0	0	6	1	0	0	0	0
	Final	27	61	35	69	49	15	0	12	9
	Post	67	63	0	83	50	0	17	0	8
Proposed student-centered move	Initial	0	0	0	0	0	0	0	0	0
	Final	0	0	0	0	0	65	72	29	56
	Post	67	38	57	33	46	100	100	100	100

Table 8
PTs' responses to MOSTs in meetings

		Cohort A (%)	Cohort B (%)
Described teacher response	Initial	79	36
	Final	36	30
Evaluated teacher response	Initial	71	36
	Final	45	39
Proposed teacher-centered move	Initial	14	14
	Final	73	30
Proposed student-centered move	Initial	7	7
	Final	41	78

these rates increased for both cohorts in the final data. A striking difference was that Cohort A proposed teacher-centered moves in almost three-quarters of instance discussions whereas Cohort B proposed student-centered moves in just over three-quarters of discussions. This difference between the two cohorts' meeting discussions likely explains the documented difference in their individual analyses.

The changes from initial to final data in PTs' individual proposing of any move and of teacher-centered moves were found to be significant. Changes in student-centered moves could not be analyzed using the Wilcoxon analysis because many PTs did not exhibit any change. The meeting data allowed us to understand what PTs were able to do with their peers that they did not do individually, particularly proposing student-centered moves. The first two codes in this analysis, describing and evaluating, give us information very similar to the categorization analysis—what teachers notice. The proposing moves give us different information, whether the PTs can think about what a teacher *might* do with an instance that is noticed.

Measuring Noticing Against a Standard

Another way noticing can be measured is by comparing the instances a teacher notices to a standard, such as the instances a group of researchers identify using a defined framework. This analysis is similar to Mitchell and Marin's (2015) measurement of teacher noticing in relation to researcher noticing framed by the MQI. As discussed previously, this type of measurement requires that the standard for noticing be clearly defined. We use the MOST Analytic Framework (Leatham et al., 2015) to define a standard for noticing.

Although we originally thought this analysis would be straightforward, simply comparing the MOSTs identified by PTs to those identified by the researchers, we found it was more complex than this. For example, we found that sometimes an instance a PT noticed aligned with one of our MOSTs but their focus was completely different, or sometimes PTs noticed instances that had elements of MOSTs but that we had decided did not meet some MOST criteria. Thus, to more fully examine PTs' noticing of MOSTs, we developed four categories: consistent MOST, inconsistent MOST, consistent non-MOST, and inconsistent non-MOST. A consistent MOST was when a PT noticed the same event in the video as a MOST and displayed reasoning about the student mathematics that was consistent with what made it a MOST. An inconsistent MOST was when a PT noticed something that occurred at the same time as a MOST, but not for the reason we had identified it as such, instead noticing things such as the teacher's response. Non-MOSTs were PT selected instances that did not align time-wise with a MOST in the video. Consistent non-MOSTs focused on student mathematics but failed to meet other criteria of the MOST Framework, while inconsistent non-MOSTs had no elements of a MOST.

In the initial data, the majority of the instances noticed by members of both cohorts were inconsistent non-MOSTs (Table 9). Nevertheless, one-quarter of A1's instances were consistent MOSTs even at the start of the intervention, indicating that, although unusual, at least some PTs naturally notice certain MOSTs for reasons grounded in student mathematics. In the final data, 79% or more of the instances identified by every PT were consistent MOSTs or non-MOSTs, with A2, A3, A4, and B2 reaching 100% consistency. This indicates the PTs became better able to identify instances that were at least partially consistent with the MOST criteria. The data also showed a striking improvement in the percentage of instances that were consistent MOSTs. Cohort B attained a higher rate of noticing consistent MOSTs than Cohort A, but more participants in Cohort B still identified some inconsistent non-MOSTs in the final data. In the post-interviews, all of the instances identified by the PTs were chosen for consistent reasons, with five of the seven

Table 9
Participants' noticing of MOSTs

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Consistent MOSTs	Initial	25	0	8	0	7	0	6	0	1
	Final	55	56	71	62	61	68	84	71	75
	Post	100	75	71	100	83	100	100	100	100
Inconsistent MOSTs	Initial	19	24	16	33	21	11	28	23	19
	Final	9	0	0	0	2	11	0	12	7
	Post	0	0	0	0	0	0	0	0	0
Consistent non-MOSTs	Initial	19	4	3	0	5	0	6	0	1
	Final	36	39	29	38	36	11	16	12	13
	Post	0	25	29	0	17	0	0	0	0
Inconsistent non-MOSTs	Initial	38	72	74	67	66	89	61	77	78
	Final	0	6	0	0	2	11	0	6	5
	Post	0	0	0	0	0	0	0	0	0

Table 10
Percentage of total MOSTs identified by participants

		Cohort A					Cohort B			
		A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Consistent and inconsistent MOSTs	Initial	47	40	60	33	45	20	30	40	30
	Final	25	36	43	29	33	41	43	38	41
	Post	38	75	63	75	63	75	75	38	63
Consistent MOSTs	Initial	27	0	20	0	12	0	5	0	2
	Final	21	36	43	29	32	35	43	32	37
	Post	38	75	63	75	63	75	75	38	63

participants noticing only consistent MOSTs. This may indicate noticing MOSTs in a short clip of video is easier than in a class-length video.

Another way we compared PTs' noticing to a standard was by examining how many of the researcher-identified MOSTs were noticed by the PTs, both for any reason and for reasons consistent with what made them MOSTs. From the initial to final data, all members of Cohort A and B3 decreased their noticing of MOSTs for any reason (Table 10). At first, this might seem like a cause for alarm, but it is actually not surprising because the PTs marked so many instances in these videos that they were likely to sometimes identify the right instances, even if for wrong reasons. Considering moments chosen for consistent reasons, initially only three PTs identified any such moments, with A1 identifying the highest percentage at 27%. In the final data, Cohorts A and B noticed 32% and 37% of the MOSTs for consistent reasons, respectively, with A1's rate actually decreasing. Even in the post-interview, none of the PTs identified all of the MOSTs, with the highest rate, 75%, displayed by four PTs. This highlights the challenge of identifying all of the MOSTs that occur during a lesson, even in a short video clip.

Our analyses showed significant differences both in the percentage of the instances PTs noticed that were consistent MOSTs, as well as the percentage of the researcher-identified MOSTs that were noticed by the PTs for consistent reasons ($p < 0.05$ for both). The fact that the majority of the instances that the PTs noticed were consistent MOSTs, but that only about one-third of all MOSTs were noticed by the PTs indicates that while more of the PTs' noticing became "correct", they may have become too selective about which instances were important, indicating a need to somehow broaden their noticing of MOSTs. This analysis is different than the categorization analysis in that it tells us whether the PTs were noticing what "should" be noticed—those moments that exhibit the characteristics of MOSTs. This type of measurement loses some of the more fine-grained information about the development of skills associated with our noticing goals, however, such as discussing the mathematics in a specific way.

Measuring What Is Not Noticed

An element of noticing that has not yet been addressed in the literature is measuring what teachers do not notice. Because our participants were prospective teachers, it is not necessarily reasonable to expect them to notice every important instance. Nevertheless, that our data showed the PTs noticed only a fraction of the MOSTs identified by the researchers raised questions about what instances they did not notice and what we might expect prospective teachers to notice.

To attempt to explain why PTs did not identify some MOSTs, we hypothesized what might make certain MOSTs more challenging to notice, developing categories of weak, difficult, and part of a cluster. Each MOST in a video was examined to determine whether it fell into one of these categories. MOSTs were considered weak if they occurred in a one-on-one setting during seatwork (since the teacher would then need to consider how or if to make the thinking public) or if one the MOST criteria might be questionable. Difficult MOSTs were hard to hear, mathematically subtle, or in a different format than was typical (e.g., student work on the board that required close analysis). Multiple MOSTs that occurred in quick succession and related to a similar mathematical point were considered a cluster.

When MOSTs occurred in such a cluster, PTs typically did not notice more than one of the MOSTs, perhaps because they considered selecting one of them to be adequate or because their noticing was not sufficiently refined to distinguish among multiple mathematical statements. Identifying only one MOST in a cluster accounted for 11% of Cohort A's missed MOSTs and 34% of Cohort B's (Table 11), though Cohort B's videos contained more clusters. Weak and difficult MOSTs accounted for 24% and 14% of missed MOSTs for Cohorts A and B,

Table 11
MOSTs researchers coded that participants did not in the final data

	Cohort A					Cohort B			
	A1 (%)	A2 (%)	A3 (%)	A4 (%)	Combined (%)	B1 (%)	B2 (%)	B3 (%)	Combined (%)
Weak MOSTs	10	12	7	11	10	5	5	4	5
Difficult MOSTs	10	12	20	16	14	10	10	9	9
Portions of clusters	10	12	13	11	11	45	24	35	34
Unexplained	70	65	60	63	65	40	62	52	52

respectively. Although these categories explain many of the missed MOSTs, the reason PTs missed the majority of the MOSTs that they did remains unexplained.

We highlight that it is only because we used a framework that clearly defines moments that are important to notice that we were able to engage in an analysis of moments the PTs did not notice. This analysis did not explain a majority of MOSTs that were missed, but it does provide information that could inform future interventions, such as that it might be necessary to focus on decomposing MOST clusters. However, it might be simply that a longer intervention is needed to learn to notice all of the MOSTs in a lesson.

Discussion and Conclusions

The analyses highlight that measuring noticing in multiple ways is important, since different measurements and different units of measure give us different information about teacher noticing. The categorization of noticing and the target noticing analyses that draw on the work of van Es and Sherin (e.g., van Es & Sherin, 2008; van Es, 2011) indicated the extent to which the PTs were able to notice students in the video and analyze their mathematics in desirable ways. The MOST analysis, like the work of Mitchell and Marin (2015), told us whether the PTs were analyzing what we valued, in our case high-leverage instances of student mathematical thinking, MOSTs. Had we only measured noticing in relation to MOSTs, we may have concluded that the intervention was less effective than we now believe it was because the percentage of noticed MOSTs was lower than we had hoped. This analysis alone may have caused us to lose sight of what the intervention did accomplish, laying a foundation for future work with these PTs by developing skills in key areas related to noticing MOSTs.

Unit of measurement also matters. Examining noticing by cohort allowed us to make generalizations about the effects of the intervention, for example, that both iterations of the intervention supported PTs in noticing students and their mathematics, discussing instances with mathematical specificity, proposing next moves, and identifying MOSTs. Examining noticing by individual, however, allowed us to make sense of some of the differences documented between the groups, such as that Cohort B became better at analyzing student mathematics and proposing student-centered moves. Looking across the different measures, one could say that B2 became the "best" at noticing, having the highest percentage of instances in many key noticing categories, including analyzing student mathematics and proposing student-centered moves. A1 was, by many measures, the least refined at noticing, actually decreasing in the percentage of MOSTs with consistent reasoning. With small cohort sizes, individual differences such as these likely accounted for Cohort B's better performance in these and other areas.

Analyzing individual noticing, post-interview noticing, and the meeting discussions allowed us to document how PTs noticed on their own, as well as what they were capable of when prompted or in collaboration with their peers. Had we only analyzed the individual data, we would likely have concluded that the PTs in Cohort A were not able to analyze student mathematics or propose student-centered moves. The interview and meeting data, however, showed that they were able to engage in these aspects of noticing under different conditions. Taken together, this data suggested that changes to the intervention made for Cohort B, such as providing a more structured framework for explaining their reasoning about an instance, were effective in allowing the PTs to exhibit their best noticing in an individual, unsupported context.

Learning to notice within the complexity of the classroom is a challenging endeavor, so we should not expect that measuring that noticing would be any less challenging. Different aspects of noticing may develop at varying rates and through different means, requiring distinct methods of measurement. Measuring noticing in multiple ways has the potential to paint a rich picture of teacher noticing, how noticing develops, and how particular elements of an intervention support that development.

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Using Mathematical Learning Goals to Analyze Teacher Noticing

Sandy M. Spitzer and Christine M. Phelps-Gregory

Abstract Teacher noticing of student mathematical thinking is increasingly seen as an important construct, but challenges remain in operationalizing and assessing teachers' analyses of their classrooms. In this chapter, we present a methodology for analyzing teachers' professional noticing of student mathematical thinking based on its alignment to mathematical learning goals. This process entails first deconstructing a mathematical learning goal into its conceptually important pieces (known as subgoals). Then, researchers can look for references to these subgoals in teachers' attending, interpreting, and deciding (the three skills of noticing). When teachers reference conceptual subgoals of a learning goal in their noticing, it indicates their attention to students' reasoning about the important mathematical ideas of a lesson. This method of data analysis can be used across a variety of contexts and allows for greater precision in understanding teacher noticing by focusing on its mathematical content and attention to relevant student thinking. In this theoretical chapter, we describe this research methodology (and the process of deconstructing learning goals and using subgoals), justify its appropriateness as a measure of teacher noticing, and provide examples from our own and others' work to illuminate its use.

Keywords Assessing teacher noticing · Mathematical learning goals · Student mathematical thinking · Qualitative data analysis · Decimal number concepts

Researchers and teacher educators are increasingly interested in what teachers notice about their classrooms. Teachers who notice productively can target their teaching to students' emerging mathematical ideas, as well as improve their instruction in ways that directly impact student learning (see, e.g., Fennema et al., 1996;

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Hiebert, Morris, Berk & Jansen, 2007). However, as Sherin, Jacobs, & Philipp (2011) note, "The study of noticing poses particularly thorny methodological challenges" (p. 11). One particular challenge is that researchers have operationalized this construct in a variety of different ways. In this chapter, we address one problem of operationalization—analyzing data. In particular, we suggest a method for analyzing teacher noticing data, which uses the mathematical learning goal of the lesson as a yardstick for analysis. We argue that mathematical learning goals are particularly useful as a generalizable method for conceptualizing and analyzing the mathematical nature of teachers' noticing of student thinking across a variety of contexts.

In this chapter, we present a theoretical argument for why and how mathematical learning goals can be used to analyze teacher noticing data, using examples of such data to illustrate the process. Although researchers have used mathematical learning goals and their breakdown into conceptual parts (which we call "subgoals" or "key concepts") to analyze data (e.g., Morris, Hiebert, & Spitzer, 2009), current research contains no methodological guidance on identifying subgoals and using them to analyze data. Most previous work on methodology for studying noticing has focused on the practicality of gathering data in the moment, looking at questions around the use of video cameras or the efficacy of recall (Sherin, Russ, & Colestock, 2011). Here, we do not focus on data collection; instead, we consider how to analyze the data once it has been gathered. We argue for an analysis framework for teacher noticing data that can further our understanding as a field and show how to better help preservice and practicing teachers develop expertise in noticing. In a metaphorical sense, we are arguing that most previous methodological discussions have focused on "attending" to teacher noticing—what researchers should look for in teachers' work and how should we collect this data. In this chapter, instead, we are focusing on the interpretation stage. Once we have this data, how can we interpret it and respond to it in ways that move the field forward?

Definition of Noticing

There are multiple definitions and conceptualizations of noticing (Sherin, Jacobs, & Philipp, 2011). Each of these conceptualizations leads to different methodological choices for studying noticing (Sherin & Star, 2011), so it is important to clarify our definitions and focus. We define noticing as a teacher's ability to identify, understand, and respond to student thinking in the midst of the distractions of the classroom (Sherin, Jacobs, & Philipp, 2011). While not all researchers take this stance, here we focus solely on noticing as a deliberate, individual process (Jacobs, Philipp, & Sherin, 2011). In particular, we define noticing as a discrete teachable skill that can be studied and improved through intervention (as suggested by emerging empirical work, e.g., Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder, 2013).

We focus specifically on teachers' professional noticing of children's mathematical thinking in this chapter (Jacobs, Lamb, & Philipp, 2010). While other researchers have examined noticing more broadly (Jacobs, Philipp, & Sherin, 2011), the analysis framework we present is focused specifically on student mathematical thinking and we are not arguing it will work for broader conceptions of noticing. In addition, in contrast with some researchers, we focus on both what teachers notice and what they miss (Jacobs, Philipp, & Sherin, 2011). We believe researchers need to pay attention to what teachers fail to notice because what they fail to notice tells us as much, if not more, as what they notice. For example, a teacher who fails to notice important student mathematical thinking is unlikely to be able to respond appropriately in the moment or change their practices in beneficial ways (see Schoenfeld, 2011). The analysis framework based on mathematical subgoals that we describe in this chapter is particularly skilled at helping identify what was not noticed.

In this chapter, we focus on three key aspects of noticing: attending, interpreting, and deciding how to respond (Jacobs et al., 2010). The mathematics subgoal framework can help researchers analyze teachers' ability to do all three skills. We take an expansive view of the skill of *deciding*, which various researchers have described in at least three different ways (see Table 1): responding in the moment with an instructional strategy (e.g., Jacobs, Lamb, Philipp, & Schappelle, 2011); seeking additional evidence to clarify students' thinking (e.g., Schack et al., 2013); and reflecting back about how the lesson may have influenced students' thinking and suggesting alternatives (e.g., Santagata, 2011). This third interpretation of deciding links noticing with research on teachers' ability to learn from teaching (Sherin, Jacobs, & Philipp, 2011). The skill of learning from teaching is perhaps less required in the moment, where attending, interpreting, and deciding how to respond can influence immediate instructional decisions. However, we would argue that this skill is key to developing expertise in teaching and that these long-term decisions can be as important as short-term responses. Thus, professional noticing, where deciding is conceived as generating cause–effect hypotheses about the effectiveness of instruction, can help teachers generate new knowledge and improve their teaching over time (Santagata, 2011).

Table 1
Different interpretations of deciding

Conceptualization of deciding	Example
Responding immediately in the moment with an instructional strategy	"I will now give Charlotte this new extension problem, because I want to push her thinking further"
Seeking additional evidence to clarify students' thinking	"I'm not sure what Charlotte means by that mathematical phrase. I am going to ask her a follow-up question to try to better understand"
Reflecting on how teaching caused students' mathematical thinking, with the goal of improving teaching	"I believe Charlotte now thinks this way because I used a problematic example. In the future, I will use an example which illuminates different aspects of the mathematics"

Definition of Mathematical Learning Goals

We propose a methodology for analyzing noticing data focused on mathematical learning goals. Mathematical learning goals are statements of the mathematical content that students should learn in a lesson. They are similar to objectives in that they describe the outcomes of a lesson but different in that they do not need to be directly measurable and describe particular mathematical thinking rather than behavioral or observable student outcomes. Unlike standards, which are quite broad and long term, here we focus on short-term (one or so lessons) mathematical learning goals. These goals should be written in the language of mathematics (Hiebert et al., 2007). For example, a mathematical learning goal is better phrased as "Students will understand the balancing interpretation of the mean" instead of "Students will score 80% or more correct on an exam about the mean." Specifying clear and precise mathematical learning goals has importance for many aspects of teaching, including selecting appropriate tasks (Smith & Stein, 2011), improving the effectiveness of teaching over time (Hiebert et al., 2007), and helping build a knowledge base for mathematics teaching (Jansen, Bartell, & Berk, 2009). We argue in this chapter that clearly specified learning goals can also be useful to researchers as a lens for analyzing teacher noticing data.

Once a learning goal is specified, we can make it more useful by unpacking that learning goal into its component parts, or "subgoals." Subgoals represent the specific, but important, conceptual ideas that are necessary for a student to understand as part of the learning goal. Morris et al. (2009) argue that, "To be clear about learning goals means to identify the learnings required to achieve the goals" (p. 493). Subgoals, or key concepts of the learning goal, are different from pre-requisite knowledge (which describe what a student must know before attempting to learn the goal) and instead attempt to specify as precisely as possible the mathematical ideas inherent in the goal. This skill is closely related to mathematics knowledge for teaching. This construct has been described by Ball, Thames, & Phelps (2008), who state that teachers "must hold unpacked mathematical knowledge because teaching involves making features of particular content visible to and learnable by students" (p. 400). Breaking a learning goal down into its key concepts is the process of stating those particular features and making them explicit.

Several previous studies have used learning goals and their component parts as a method of data analysis (e.g., Meikle, 2016; Morris et al., 2009; Phelps & Spitzer, 2012). For example, in a study of how prospective teachers might decide how to select and sequence students' solution strategies during a class discussion, Meikle (2016) considered a learning goal about the division of fractions, and unpacked it into three component parts:

Learning Goal: Students will understand *why* the invert and multiply algorithm for division of fractions works according to the repeated subtraction meaning of division.

Key Concept A: Division can be interpreted as finding out how many groups of a certain size (the divisor) fit into the dividend.

Key Concept B: The reciprocal of the divisor represents the number of copies of the divisor that fit into one whole.
Key Concept C: The dividend represents the number of wholes we have. If the reciprocal of the divisor represents the number of copies of the divisor that fit into one whole, then we can multiply the reciprocal of the divisor by the dividend to find out how many copies of the divisor fit into the number of wholes (the dividend) we have.

In this example, the key concepts have the following attributes, which allow them to be useful to teachers and researchers. First, they break the learning goal down into constituent parts. These parts cannot be further divided and together make up the mathematics knowledge embedded in the goal. They also unpack important words and ideas in the learning goal (such as *division* and *repeated subtraction*) based on a deep understanding of the underlying mathematics. Although the key concepts are informed by research on developmental patterns of student learning, they are written in wholly mathematical language rather than describing pedagogical strategies or student behaviors.

Helping teachers themselves unpack a learning goal into its component parts has been shown to have the potential to help prospective or practicing teachers learn the skills of noticing (Morris et al., 2009), as well as other classroom skills such as selecting appropriate student responses to share during a whole-class discussion (Meikle, 2014). Collaborative work between teacher educators and teachers engaging in learning goal analysis together also has promise in improving instruction (Phelps, Shore, & Spitzer, 2014). Further research is needed to clarify and strengthen the links between teachers' use of unpacking learning goals and their noticing skills. This chapter does not intend to provide such research, but instead to argue that this same process that teachers can use is helpful for researchers to analyze noticing data.

Why Use Mathematical Learning Goals to Analyze Data?

When teachers reference conceptual subgoals of a learning goal in their noticing, it indicates their attention to students' reasoning about the important mathematical ideas of a lesson. Because the key concepts of a learning goal are unpacked into precise but important details, teachers' attention to these concepts in students' work reveals a mathematically sound and detailed analysis. This approach aligns with many others that have been used to analyze teacher noticing of student mathematical thinking in that it prioritizes detailed analyses which are mathematically relevant and in line with research about how children think and learn (Jacobs et al., 2010). This framework for data analysis has several advantages.

The primary advantage of using subgoals, compared to other analysis methods, is being able to more precisely specify teachers' noticing. Many previous studies have analyzed noticing by focusing on the level of depth or detail in teachers' responses. For example, Walkoe (2015) analyzes the depth of teachers' conversations about students' algebraic thinking using a level 0 code for conversations that discuss student thinking only generally and a level 1 code for those that include more depth and detail. Other researchers use a similar coding scheme focused on depth of analysis or level of detail of evidence (e.g., Choppin, 2011; Fernández, Llinares, & Valls, 2013; Jacobs et al., 2010). The use of mathematical subgoals builds on such an approach and allows a further level of precision about what (and how well) teachers have noticed. A subgoal focus can help researchers delve further into teachers' noticing, sorting out the fine details that can distinguish between different "medium" or "high" depth responses.

This method also keeps researchers' attention on the big mathematical ideas of a lesson, which is important for successfully responding in a way that improves student learning. Hiebert & Grouws (2007) argue that there is no such thing as "effective teaching" considered broadly, but only teaching which is effective at helping students achieve particular learning goals. Similarly, we argue teacher noticing is most effective when it relates to teachers' mathematical goals for the lesson and helps students achieve those goals. For this reason, considering the level of detail alone may be problematic when studying teacher noticing because it is possible to be extremely detailed about unimportant student work or to provide little detail but be focused on important mathematical thinking. Instead, we can distinguish between higher and lower quality teacher noticing using its alignment to the learning goal, and specifically its alignment to the important conceptual ideas underlying that goal (i.e., the subgoals). This method of data analysis prioritizes the mathematical outcomes of a lesson over other pedagogical concerns, and could be used in concert with other coding schemes, which capture those concerns. Because other researchers have explored nonmathematical noticing (e.g., Erickson, 2011), we focus here only on using subgoals as a framework for analysis.

The additional precision granted by a subgoal analysis can help researchers attend to both *what mathematical ideas* teachers notice as well as *how well* they notice. This allows us to analyze both for what teachers notice and what they fail to notice, and helps us identify mathematical ideas that might be more difficult for teachers to notice. The coding schemes in prior research have primarily focused on what teachers notice; however, what teachers fail to notice is just as important because it is often these missed in-the-moment opportunities where growth in student learning could occur. Because a subgoal list attempts to identify all of the most important ideas about the mathematical learning goal, subgoals missing from an analysis often represent important mathematical ideas that teachers have failed to notice. Further, we have often found that some mathematical ideas are noticed by many teachers and others only by a few. For example, in our previous work, we have found that it is easier for PTs to attend to the ideas of "parts" and "wholes" than for them to notice when students are understanding the 10-times relationship between places (Phelps & Spitzer, 2012). Knowing what prospective and practicing teachers fail to notice or find hard to notice can help teacher educators better design professional development.

We also believe a subgoal analysis framework can help researchers recognize growth in teachers' ability to notice. Jacobs, Lamb, and Philipp (2010) argue that several "shifts" in noticing of students' thinking can demonstrate growth in expertise. They identify six particularly important shifts, including "a shift from general strategy descriptions to descriptions that include the mathematically important details" (p. 196). The use of subgoals is ideally suited for identifying such mathematically important details in teachers' noticing that may signal growth shifts. Furthermore, subgoal analyses can be used across a variety of mathematical contexts and student work samples, allowing researchers to better compare teacher noticing across different tasks. By focusing on the mathematical content that we would like to see teachers notice, researchers can rely less on the particular, idiosyncratic features of a single lesson artifact.

Using Subgoals to Analyze Noticing Data

Having argued for the advantages of using subgoals as a method of data analysis, we will now describe how one might conduct such an analysis. We will use sample teacher noticing data to illuminate how subgoals are used and what affordances they allow. The sample data we present here was collected as part of a study of how an online discussion board assignment might help preservice teachers (PSTs) learn to notice, analyze, and learn from student thinking in a sample lesson. We do not share results of this study here (see Spitzer & Phelps, 2011 for further details), but instead to use this data as an illustrative example of the subgoal process. Participants consisted of 16 PSTs enrolled in an elementary mathematics methods course. As part of a course assignment, participants watched the video "Meter Cords" from the Annenberg Collection (WGBH Boston, 1997). In the video, third and fourth grade students are shown being successful at a class activity (hands-on measurement), but reveal serious misconceptions about the learning goal (understanding decimals). The learning goal for the lesson (as stated in the instructions to PSTs) was: "*Students will understand that decimals represent parts of wholes; in particular, the tenths place represents quantities which are ten times smaller than the quantity represented by the ones place.*"

After watching the video, PSTs interacted in an online discussion board that was formatted as a "debate" about the lesson's effectiveness in helping students achieve the learning goal. After the "debate," PSTs wrote a reflective essay (see Spitzer & Phelps, 2011, for results about the effectiveness of the online discussion board intervention). Later in the course, PSTs would be explicitly taught how to unpack a learning goal into its component parts (as well as other skills of teacher noticing), but this data represents their initial, untrained ability. This data provides a rich site to study teacher noticing because the debate and essay prompts related to all three skills of noticing (attending, interpreting, and deciding) and involved a video which provided "windows into student thinking" (Sherin, Linsenmeier, & van Es, 2009, p. 215) as well as a variety of distractors. We will describe how we used the mathematical learning goal to analyze this data through three main stages: creating the subgoal codes, using them to code data, and making claims from the codes.

Defining the Key Concepts and Subgoal Codes

To analyze data based on the learning goal, the first (and often most challenging) step is to define the subgoals of the learning goal. In order to identify and unpack these subgoals, it is useful to begin with a theoretical consideration of what mathematical ideas are inherent in the goal. Empirical research into students' thinking about the particular goal can also be helpful in identifying subgoals. For example, if considering a learning goal related to solving addition and subtraction word problems, the work of Cognitively Guided Instruction (e.g., Fennema et al., 1996) would be a useful resource. Learning trajectories (see, e.g., Clements & Sarama, 2004) can also provide insight into the component parts of a learning goal. Initial subgoal lists can be refined through iterative reading of sample teacher noticing data or through pilot studies and should ideally be checked for validity with an outside expert. Like any qualitative data analysis, valid methods require a back-and-forth iterative process between the data and the codes based on the subgoals of the learning goal (Miles & Huberman, 1994).

For the learning goal associated with our lesson sample, we identified the following component parts (see Figure 1). We feel that this list captures at least a minimum of the important mathematical ideas of the learning goal because we developed it based on the work of previous researchers (see, for example, Morris et al., 2009). In addition, we kept refining the key concepts until we were in agreement, in much the same way that qualitative researchers refine qualitative codes until there is high interrater reliability.

In addition to the specified subgoals, it is usually necessary to record additional features of student thinking noticed by teachers. These primarily fall into four categories: claims related to the learning goal, but not detailed enough to be considered a key concept; claims related only to procedural competence, even though the learning goal is conceptual; claims irrelevant to the learning goal; and claims that are primarily pedagogical (not mathematical) in nature. When using a subgoal analysis, we also use codes for each of these four categories (relevant but vague, procedural only, irrelevant, and nonmathematical). Making a record of what teachers did notice (aside from the important mathematical concepts of student thinking) allows us as researchers to capture shifts in noticing more completely and will aid in quantitative comparisons.

Learning Goal: Students will understand that decimals represent parts of wholes; in particular, the tenths place represents quantities, which are ten times smaller than the quantity represented by the ones place.

Key Concept 1: Digits in different places have a different value (e.g. the "2" in 2 does not mean the same thing as the "2" in 0.2).

Key Concept 2: The size of the places increases or decreases by a factor of 10 as you move to the left or right, respectively. This relationship holds on both sides of the decimal point.

Key Concept 3: In particular, place values to the left of the decimal point represent quantities 1 whole or larger and place values to the right of the decimal point represent quantities smaller than 1 whole.

Key Concept 4: Once you have reached a value of 10 in a particular place value, you must move to the next larger place value; each place can be represented only by a digit from 0-9.

Figure 1. The learning goal and identified key concepts.

Using the Key Concepts to Code Noticing Data

Our conception of teacher noticing of student mathematical thinking follows Jacobs, Lamb, and Philipp (2010), who describe three component skills: attending to student strategies, interpreting those strategies in terms of student thinking, and deciding how to respond on the basis of the noticed thinking. The identified subgoals of a learning goal can be used to code and analyze data from these three skills. We will now describe and justify the use of a subgoal analysis for each of the three skills, using examples from the study described above to illuminate how subgoals can be a useful lens for teacher noticing data.

Attending. According to Jacobs, Lamb and Philipp (2010), the cornerstone of teacher noticing is attending, which focuses on "the extent to which teachers attend to a particular aspect of instructional situations: the mathematical details in children's strategies" (p. 172). As we have argued, the use of mathematical subgoals provides a valid and generalizable way to identify what the important mathematical details are for a given learning goal. The use of mathematical subgoals allows researchers to distinguish between teachers who attend with a high level of detail to irrelevant mathematics from those who attend with a low level of detail to important mathematical ideas.

Consider the following two examples, both written by PSTs in response to the prompt: "Was the lesson successful in helping students achieve the learning goal? Cite evidence of what students do or say that leads you to believe that the lesson either helped or did not help students achieve the learning goal." Beth wrote:

> The children were first asked to create measuring sticks out of string. They then laid the string parallel to a meter stick and placed a red piece of tape at every tenth of a meter. They were then asked to take the string and measure various things throughout the classroom. After collect[ing] all of their data, the students were asked to create charts or graphs that represented the data they found. All groups of students were able to correctly measure the objects and create a chart that accurately represented that data.

In this response, Beth provides a high level of detail about students' actions, but does not address students' understanding of the learning goal (see Figure 1). We coded this as "Irrelevant math (measurement and data analysis)." Contrast this response with Felice, who wrote:

> When the group of students was measuring the width of the desk they thought it was a meter and a twelfth not a meter and two tenths. They looked on the chart for where a meter and a twelfth was but it wasn't there. They saw 1.2 so I believe they thought it was the right answer because without the decimal point it would be the number twelve.

Felice focuses her attending on a conceptual idea inherent in the learning goal, specifically the differences between the numbers 12 and 1.2 (coded as a reference to Key Concept 1; see Figure 1). The presence of this subgoal reference indicates a higher quality analysis than Beth's, even if the level of detail is the same (or less). Attention to students' strategies around multiple subgoals would indicate an even higher quality response.

Interpreting. Once teachers have attended to the details of student strategies, they must decide what those details tell them about students' learning. This skill of noticing is particularly well suited for a subgoal analysis, since its goal is to determine the particular mathematical ideas (e.g., key concepts of the learning goal) that students understand (or do not understand). Additionally, interpreting has been particularly difficult for researchers to operationalize in previous work. For example, Schack et al. (2013) highlight the difficulty of quantifying the depth of analysis for the interpreting case when they write, "Interpreting presented the most difficult set of benchmarks to construct due to [prospective teachers] focusing on different aspects of the child's work and/or different, yet reasonable aspects of the mathematics" (p. 389). The use of subgoals can help both clarify and quantify the aspects of the mathematics that teachers notice.

For example, consider the responses of Haley, Beth, and Alyssa (see Table 2). All three of these examples might be considered "medium depth," but while Beth

Table 2
Interpreting responses of three PSTs

PST	Interpreting response	Subgoal code (see Figure 1)
Haley	"I think that the students also could see that the objects they were measuring were parts [of a whole] because the cords were broken into parts"	Key concept 3
Beth	"Some of the kids were picking up on the concept of breaking the meter stick down into tenths while others were struggling. Some children could say that 20 tenths equaled 2 meters, while other were still unclear about the relationship"	Key concept 2
Alyssa	"The graphs showed that the students understood the process of measuring the length and height of objects, as well as how to write and read decimals"	References no key concepts

and Haley notice details related to student thinking about the learning goal (in different dimensions, but of comparable quality), Alyssa only considers surface features of the lesson and students' procedural competence rather than conceptual understanding. The use of details in Alyssa's quote do not indicate useful insight into student achievement of the learning goal, reflected in the fact that she does not reference any subgoals. The use of subgoals to distinguish between these three examples helps us as researchers tease out what mathematical ideas PSTs are able to interpret in student thinking as well as distinguish between better and worse responses. Thus, a subgoal analysis can help researchers distinguish between different "medium" or "high" depth responses, recognizing depth does not imply relevance and brevity does not imply lack of important mathematical ideas.

Blending the skills of attending and interpreting. One difficulty that we have observed when studying teacher noticing is that, when discussing student thinking, it is natural for teachers to blend together their attending and interpreting. Rather than organizing their thinking as first describing everything that students did, then interpreting it, teachers may instead alternate between describing pieces of students' work and immediately making inferences about student thinking. This is a challenge across the field of teacher noticing, as pointed out by Sherin and Star (2011), who note that "Researchers may be unable—both in practice and in theory—to separate the earlier steps of the intuitive model, that is, to separate noticing from interpreting" (p. 70). Consider the work of Danielle, who organized her response around Key Concept 4 (see Figure 1) and first attends to and then interprets a student response:

> When the students measured the door, they said it was twenty tenths longs. The students did not realize that this measurement really means, 2 meters. The students were unable to grasp the concept of how the method of the "tenths" worked. If they did understand the learning goal of the lesson, they would have known once they got to a full meter, also known as "10 tenths," they would have to start counting over. For example the door was twenty tenths long, this means that the students should have counted 10 tenths on the meter rope, then started over and counted another 10 tenths on the meter rope which would then give them 2 meters in total.

This example illustrates how a teacher can mix attending (lines 1-2) and interpreting (lines 3-4), together with what they notice about what a student does *not* say (lines 4-9), into an argument about what the child knows about a particular key concept of the learning goal (coded as a reference to Key Concept 4: that in a number written in standard form, each place can only "hold" a digit from 0-9). That key concept makes a natural unit of analysis.

Furthermore, just as a subgoal analysis can help us as researchers analyze what teachers do *not* notice, it can also help analyze teachers' responses when they discuss what children do not say, which is an important feature of noticing (Ball, 2011). The hypothetical case that Danielle presents (what a student would have done differently if they had understood this key concept) is a useful act of noticing even if it is not about any specific student work in the classroom. By using the subgoals of a learning goal to analyze responses such as these, researchers can capture the blended skills of noticing.

Deciding. Deciding is the act of using what is noticed about student thinking to make an instructional response. According to Jacobs, Lamb and Philipp (2010), teachers' deciding should be evaluated based on "the extent to which teachers use what they have learned about the children's understandings from the specific situation and whether their reasoning is consistent with the research on children's mathematical development" (p. 173). The use of subgoals can help researchers analyze teachers' decisions along these two dimensions. For example, Beth wrote:

> If I were teaching this lesson, I would not correct the students if they were wrong, rather I would have them explain to me how they came to their conclusion. I would hope by doing this, students could self-correct their mistakes. I do not think it benefits a student to be told that they are wrong. I think it is more beneficial to the students to correct themselves, if possible. This way they will hopefully learn from that mistake.

A focus on subgoals allows us to see that, although Beth's response is aligned with research recommendations for teaching and appropriate pedagogically, it does not address any important ideas of the learning goal (no key concepts are present), and thus does not use the specific situation of the lesson.

Previous research has struggled with analyzing the skill of deciding because often prospective teachers' proposed revisions to lessons are aligned with what they have noticed about student thinking, but only target procedural competence or tangential aspects of the lesson (e.g., Jansen & Spitzer, 2009; Parks, 2008). The use of unpacked learning goals allows researchers to record whether teachers' decisions would address the mathematical ideas of the learning goal. For example, Kaylee addresses her decisions to irrelevant mathematics (measurement and data analysis), writing:

> The only thing I would add to the lesson was more specification on the objects that were being measured. Many of the students were unsure where to start and stop measuring which resulted in different data on the bar graphs. By being more specific the teacher may have been able to avoid this situation and get a better representation of data for the class to look at.

Contrast that response with Marisa, who proposes a response to student thinking based on Key Concept 4 (see Figure 1):

> I think that [it would have been helpful] if the teacher had demonstrated measuring an object that was longer than a meter and counted out loud, "7, 8, 9, 10, ok now once we get to ten that equals 1 whole and then we start counting again to the next whole". Then if she also showed that on a number line on the board I think it would have helped as well. Then she could have shown where the 1, 2, 3, etc. were and then the tenths in between the whole numbers as well.

The use of subgoals can also help researchers determine which ideas teachers tend to target for their revisions. This is one way teacher noticing data could be used to help build a knowledge base for education (see Santagata, 2011).

As mentioned above, we consider that deciding can take several forms, including proposing a next step, seeking more information on student thinking, and using cause–effect hypotheses (which reflect back on the lesson to link teaching moves with student learning outcomes) to suggest lesson revisions. All of these are appropriate ways to respond to student thinking (either in the moment or after the fact). In particular, constructing hypotheses and suggesting lesson revisions allows teachers to use what they have noticed about student thinking to learn from and improve their teaching over time (see, e.g. Hiebert et al., 2007; Santagata, 2011). For example, Felice describes a lesson revision centered around Key Concept 2:

> The teacher should have modified the lesson by having two different color tapes to represents "parts/decimals" and "wholes." While the students were counting tenths they would use the red tape and once they got to a whole/ten tenths, they would mark it with blue tape. I believe this would help them better understand that the tenths place quantities are ten times smaller than the quantity represented by the ones place.

Or consider Corinne's deciding response, shown below, which contains both a thinking-back hypothesis about what features of the lesson might have led to the student thinking she noticed as well as a targeted revision to those lesson features. Her references to Key Concept 4 demonstrate the alignment of these hypotheses and revisions to the learning goal and observed student thinking:

> I also believe that the comparison the instructor made about quarters and tenths as parts of a whole was confusing. She tried to relate money to measurements so the students would understand to use decimals, but this seemed somewhat confusing and the students did not relate this to the topic at hand. It could have been helpful if it was explained more clearly.

As Schoenfeld (2011) states, "What makes noticing consequential, of course, is that people act on what they notice" (p. 230). When teachers' decisions about how to respond to student thinking are aligned with and targeted to key concepts of a learning goal, these decisions are more likely to lead to improved student learning of those goals. Thus, it is important for us as researchers to use a coding scheme that can capture alignment with key concepts of the learning goal.

Using the Codes

Once the data has been analyzed, the task remains of using the codes to make conclusions about teacher noticing (for example, whether teachers have improved their noticing skills from an intervention, or how noticing might compare across different populations of teachers). For small-scale qualitative studies, this might simply entail a close inspection of the subgoals noticed by individual participants. For much research, however, it is useful to envision ways in which subgoal data might be used quantitatively to make judgments about noticing. In order to illustrate some potential avenues for analysis, we will describe some of the methods we have used in our own work; of course, data analysis is a creative process, which must take into account the complexities of each individual context.

Making comparisons across individuals. One primary way in which researchers might want to use subgoals is to make comparisons across individuals, possibly to compare between different teachers (or populations of teachers), or to show whether an intervention was effective at improving noticing skills. In order to do so, it is necessary to be able to use the subgoals to identify areas of strength and weakness in individual teachers.

Throughout this chapter, we have argued that higher quality noticing work references more subgoals of the mathematical learning goal. Thus, the first step of analysis is to look at teachers' coverage of the subgoals, that is, how many of the identified subgoals did they reference in their noticing? Obviously, in this case, noticing responses that reference more subgoals are better responses, so the number of subgoals addressed can represent the quality of noticing. For example, in the study we describe in this chapter, PSTs referenced a mean of 1.73 (SD = 1.4) key concepts (out of four) on the pretest and 2.33 (SD = 1.1) on the posttest ($p < 0.05$). This indicates that the intervention was successful in helping PSTs notice a full range of student thinking around the learning goal.

Another important feature of teachers' noticing is the level of *alignment* of their response across the three skills (attending, interpreting, deciding) in relation to the subgoals. Higher quality responses include attention to all three skills around the same subgoal (as opposed, for example, to making a response decision that is aligned to a different subgoal than was attended and interpreted). In the study described here, on the posttest, 80% of teaching decisions related to a key concept of the learning goal were aligned with an attending or interpreting response around the same key concept, compared to 64% on the pretest (however, due to a low number of such decisions, this difference was not significant).

Recall that in addition to the subgoals, we also code for four other types of noticed student thinking: mathematically relevant but unspecific ideas (e.g. "She understands decimal concepts"); irrelevant mathematical ideas; procedural skills; and nonmathematical behaviors (e.g., "she was highly engaged in the task"). We use these non-subgoal codes to indicate differences between individuals or change over time. These non-subgoal codes provide an overall classification of what a teacher noticed, allowing us to look for differences across categories. For example, a higher percentage of claims that were aligned with a subgoal (as opposed to a higher percentage which were irrelevant) would indicate a higher quality analysis. In the study described in this chapter, we found that among a subgroup of PTs, the percent of claims that referenced a subgoal rose from 29% initially to 52% after the intervention ($p < 0.01$), providing evidence that the intervention was successful for that subgroup (see Spitzer & Phelps, 2011).

Making claims about populations. In addition to these claims about individuals, researchers can also use the subgoal codes to make arguments about populations of teachers. As noted above, the use of mathematical subgoals can help researchers and teacher educators tease out what mathematical ideas are easier or more difficult for teachers to notice, and design interventions accordingly. For example, after the intervention in the study described above, we found that many more PTs (12 out of 16) made claims mentioning Key Concept 3 (wholes vs. parts of wholes on different sides of the decimal point) compared to Key Concept 1 (digits in different places have different value; noticed by only 1 PT). This aligns with previous research (e.g., Morris et al., 2009) which suggests that PTs often attend only to the most prominent features of student thinking, such as a wrong answer to a problem, while failing to notice more subtle clues about the sources of students' misunderstanding. This and similar findings can help teacher educators design interventions to help teachers learn to notice more difficult mathematical ideas as well as look more deeply to uncover hidden misconceptions.

Conclusion

In this chapter, we have argued that when what teachers notice is aligned with specific, important conceptual details of the mathematical learning goal, they will be more likely to respond appropriately in the moment and make productive changes to their teaching over time. Thus, it is possible for researchers to use this alignment to key concepts of a learning goal as a generalizable way to analyze teacher noticing data across different contexts and subjects. The work of analyzing teacher noticing and using this construct to improve teaching is a young but emerging field (Sherin, Jacobs, & Philipp, 2011). We hope that our methodological description will help guide this field in productive ways.

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Measuring Elementary Mathematics Teachers' Noticing: Using Child Study as a Vehicle

Heidi L. Beattie, Lixin Ren, Wendy M. Smith and Ruth M. Heaton

Abstract This study qualitatively examines teacher noticing among 22 primary teachers who conducted child studies, in which they sought to better capture two students' mathematical understanding. Noticing students' mathematical thinking is a key component to better understand student learning and effective teaching. Five levels of teacher noticing were specified for this framework (adapted from the van Es, 2011). The teacher noticing framework ranges from Level 1, general statements or claims about what a student could and could not do, to Level 4, description of detailed evidence, analysis, and discussion of future support that will be provided to students. The authors found teachers on average exhibited Level 2 noticing abilities. This adapted framework may be beneficial in helping teachers notice and understand students' mathematical thinking, as well as provide teacher educators with a tool with which to use to measure teachers' noticing abilities.

Keywords Teacher noticing · Professional development · In-service teachers · Elementary mathematics · Child observation

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Introduction

Mathematics education reform requires teachers to pay particular attention to students' ideas, adapting their teaching accordingly (National Council of Teachers of Mathematics [NCTM], 2000). "This ability to adapt one's teaching in the midst of instruction requires that teachers be able to notice aspects of reform pedagogy and interpret what is happening in their classrooms in new ways" (van Es & Sherin, 2008, p. 244). Research indicates that learning to notice can be challenging for teachers (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). As professional development efforts have been devoted to improving teachers' noticing skills, it becomes necessary to capture teachers' progress in learning to notice in a measurable way. One project, Primarily Math, tried to help K-3 elementary teachers learn skills of noticing through a Child Study project, which is an in-depth observation of a single child. In this study, we adapted a preexisting framework of teacher noticing developed by van Es (2011), and used it to assess the levels of noticing teachers demonstrated in a Child Study project.

This study focused on two research questions: (1) In what ways can we adapt van Es's (2011) framework intended for coding classroom video to instead code written teacher reflections and how can this framework be used to characterize teachers' noticing of student mathematical thinking? (2) To what extent do levels of teacher noticing correlate with teachers' mathematical knowledge for teaching and beliefs about mathematics learning and teaching?

Conceptualizations of Teacher Noticing

Classrooms are complex settings featuring "multidimensionality, simultaneity, and unpredictability" (Doyle, 1977, p. 52), which therefore pose challenges for teachers to notice and manage the "blooming, buzzing confusion of sensory data" (Sherin & Star, 2011, p. 69). Teachers need to (a) decide what particular events to attend to in an instructional setting; (b) reason and make sense of the events; and (c) make informed teaching decisions based on the analysis of observations. These three aspects constitute the three important components of *teacher noticing* (Sherin, Jacobs, & Philipp, 2011; van Es & Sherin, 2002, 2008). In addition, teachers are not passive receivers in the process of noticing but active constructivists. It is worth distinguishing teacher noticing from teacher knowledge. Knowledge is necessary but not sufficient to support effective teacher noticing: teacher noticing is a dynamic process that utilizes teacher knowledge in action (Sherin et al., 2011). Teacher noticing focuses on *how* teachers attend to, analyze, and decide in an educational setting.

Different researchers often focus on different aspects of teacher thinking and practice when defining *noticing*. For instance, some researchers pay particular attention to what teachers initially attend to and what they miss with regard to different aspects of classroom activity (e.g., Star & Strickland, 2008; Star, Lynch, & Perova, 2011). Some researchers are interested in teachers' interpretations of noticed events such as teachers' making sense of a particular student's idea drawing upon knowledge of the student, the mathematics content, and pedagogical knowledge (e.g., Colestock & Sherin, 2009; Sherin & van Es, 2009). Finally, Jacobs, Lamb, and Philipp (2010) included teachers' plans to respond on the basis of children's understanding in their conceptualizations of *professional noticing* along with the components of attending to children's strategies and interpreting children's understandings, based on the assumption that all three components are connected conceptually and temporally. In this study, we leave aside many aspects of mathematics teaching and learning teachers could notice within their own practices and mainly focus on teachers' abilities to: attend to their students' mathematical thinking, reason and make sense of what they notice, and plan next instructional steps based on their analyses.

Assessing Teacher Noticing of Student Thinking

There has been an increasing awareness of and interest in the importance of teacher noticing as a theoretical construct for understanding teaching in mathematics education (Sherin et al., 2011), partly as a result of current mathematics education reform (van Es, 2011). One of the key principles of the National Council of Teachers of Mathematics (NCTM, 2014) is: "Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (p. 10). Thus, effective instruction requires teachers to notice, pay attention to, and respond to students' ideas. Many studies have shown that focusing on students' mathematical thinking improves the quality of teaching, promotes learning for understanding, and leads to better student achievement (Carpenter, Fennema, Franke, Levi, & Empson, 2000; Crespo, 2000; Fennema et al., 1996; Jacobs et al., 2010; Sleep & Boerst, 2010; Swan, 2001; Wilson & Berne, 1999).

Although noticing students' mathematical thinking is a key component to better understanding student learning and teaching effectively (van Es & Sherin, 2008), it is not a simple task. Current teacher education programs mainly focus on teaching teachers how to *act* through providing them with instruction on new pedagogical techniques and activities rather than on helping teachers to *interpret* classroom interactions and student work (Putnam & Borko, 2000; van Es & Sherin, 2002). Teachers face two main challenges in terms of noticing students' mathematical thinking: recognizing interesting and rich mathematical ideas and interpreting these ideas (Cohen, 2004; Ma, 1999; Sherin, Linsenmeier, & van Es, 2009).

Many professional development programs have been designed and implemented to help teachers develop skills of noticing students' mathematical thinking (e.g., Sherin et al., 2009; van Es, 2011; van Es & Sherin, 2002, 2008). However, few have offered a way to measure these noticing abilities in the teachers who participate in such programs. To help teachers progress in their noticing skills, teacher educators need to find appropriate frameworks to accurately measure the noticing abilities of teachers and capture any growth in the development of noticing skills that these teachers may achieve throughout their professional development experience. In this study, we adapted a framework for learning to notice student thinking developed by van Es (2011) in the context of a video-club design (see van Es & Sherin, 2008 for detailed explanations of a video-club design). The current study is situated in the context of a Child Study project, an assignment based on Himley and Carini's (2000) descriptive review of a child, a method used by progressive educators to create deep and rich understandings of children from year-long inquiries by an entire school community. The products of the Child Study projects examined here represent eight-week studies of two children per classroom as math learners by individual classroom teachers. Thus, we first introduce van Es's framework, and then introduce how we adapted the framework to better fit our Child Study project's approach.

van Es (2011) utilized data from seven 4th- and 5th-grade teachers from an urban school. The teachers and the research team met for 60–75 min every meeting for a total of 10 separate times throughout the 2001–2002 school year. At each meeting, teachers shared video clips (5- to 7-min segments) from their classrooms that were videotaped and selected by the research team. The facilitator encouraged the group to examine, interpret, and discuss students' mathematical thinking in the clips using prompts such as "What did you notice?" and "Why do you think she chose that method?" Qualitative methods were used to analyze videotapes and transcripts of the 10 video-club meetings to examine the nature and development of teacher noticing. This analysis resulted in the creation of four main levels of teacher noticing including Level 1: Baseline, Level 2: Mixed, Level 3: Focused, and Level 4: Extended (described later in Table 2). The framework is further described in the Measurement section.

Child Study and Its Use in Promoting Teacher Noticing

The current study is situated in the context of a Child Study project. The process of studying children through observations is an encouraged method for teachers to utilize in order to better understand their students' thinking (Goodman, 1985; Himley & Carini, 2000). Goodman (1985) suggested through a greater understanding of student learning, not only can teachers be better informed about student learning, but other members of the education system, including test and curriculum creators, can make use of this knowledge. Attending to children's participation and educational identity also allows teachers to examine the possible inequalities in the opportunities to learn and participate in quality mathematics they offer their students (Wager, 2014), which can provide teachers with helpful insight in how to improve their mathematical teaching.

The concept and process of "Child Study" is also similar to "documentation" originating from the world-renowned Reggio Emilia early childhood educational approach. Reggio educators strive to make the process of learning visible through documenting the "traces" of children's learning using many forms of media. Teachers routinely take notes and photographs of children in action as well as audio or video record group discussions and children's play. Teachers carefully select transcripts of children's conversations and remarks, photos of ongoing work and activities, and the products that children have produced; they then carefully arrange all these materials to represent children's thinking and learning (Gandini, 1993).

While documentation and observation techniques can provide teachers with a great deal of insight into their students' thinking about a number of different educational topics, very little research has been completed to examine teachers' Child Study projects that are utilized in order to inform teachers of appropriate ways to observe student learning in the classroom. By being better able to understand student thinking in the math classroom through observations, teacher educators can better help teachers plan and implement math instruction that is responsive to the strengths and limits of children's understanding as well as specific mathematical goals. In the current study, we aim to develop and use a measurement framework (adapted from van Es, 2011) to assess teachers' abilities to notice students' mathematical learning.

The Context of the Current Study

Data from this study come from a larger professional development project, Primarily Math. Primarily Math is an elementary mathematics specialist program that includes 18 graduate credit hours of coursework focused on improving K-3 teachers' knowledge about mathematics, pedagogy, and child development. Primarily Math participants complete the program in cohorts across 13 months (two summers and one academic year), in which there is an intentional focus on developing a strong professional community of elementary educators.

Pedagogy Course and the Child Study Project

Across all pedagogy courses, assignments are designed to help participants become more intentional, planful, observant, and reflective with regard to teaching, children, and mathematics. The Child Study project is a major assignment in the first of the three pedagogy courses of Primarily Math. Teachers are required to gather data over an eight-week period about two of their students' mathematical knowledge, skills, and dispositions for the overall purpose of learning how to carefully observe and document students' mathematical understanding. The main focus of the written report is intended to be an in-depth analysis of each student as a learner and doer of mathematics, highlighting strengths, and limitations of each student's development of mathematical knowledge, skills, and dispositions. In addition, teachers are expected to connect their analyses to mathematical learning trajectories as defined by Clements and Sarama (2009), and conclude by stating how they will continue to support the learning of each student in ways that align with lessons learned through observation and documentation of student learning. Each time the project was introduced to a new cohort of teachers, expectations were clarified with greater detail. Furthermore, instructors emphasized the purpose of the project as a vehicle for learning to observe children as learners and doers of mathematics to support teachers instructionally in being responsive to students' mathematical understanding and the curriculum.

The 15-week blended in-person and online pedagogy course, that the Child Study project was a part of, did not have a focus on teacher noticing. Throughout the course, teachers read assigned articles and completed reflection essays; the articles were not specifically about teacher noticing, as they were centered on broad ideas in mathematics education.

Method

Participants

We selected 23 teachers from three cohorts of Primarily Math participants (out of 104 teachers) based on their responses on the Mathematical Knowledge of Teaching assessment (MKT; Hill, Schilling, & Ball, 2004) and Mathematics Belief Scale-short form (MBS; Fennema, Carpenter, & Loef, 1990; Capraro, 2001; Ren & Smith, 2013), and were able to use 22 of their Child Study projects for this research. The MKT assesses teachers' mathematical knowledge unique to the demands of teaching. For the current study, we utilized the Number and Operations subscale as the most relevant subscale for K-3 teaching and the focus of Primarily Math's mathematics courses. The MBS measures teachers' beliefs toward mathematics and mathematics teaching. Higher scores indicate more student-centered beliefs such as beliefs that students construct their own knowledge through active investigation and meaningful exploration (Capraro, 2001), while lower scores indicate more teacher-centered beliefs such as beliefs that students are passive recipients of knowledge.

We first grouped all 104 teachers into six groups based on their baseline Number and Operations scores from the MKT and their MBS scores. Baseline scores provided the best approximation of a teacher's mathematical knowledge and beliefs during the completion of the Child Study project in the first fall semester of coursework. We identified three levels of teachers' mathematical knowledge based on their Number and Operations scores: low (an IRT score of −2.5 to −0.5); medium (−0.5 to 0.5); and high (0.5–2.5). An IRT score of zero represents the national average; the MKT IRT scores have a standard deviation of 1. We also split teachers into a "high beliefs" group and a "low beliefs" group using the median-split of their MBS score (3.66 on a scale from 1 to 5).

We chose three to four teachers from each of the six groups. First, when possible, we aimed to choose teachers who represented all grade levels (K-3) in each group. We next chose teachers from a range of geographic locations in order to represent different school districts. Finally, we chose teachers who represented different cohorts in each group. See Table 1 for descriptive statistics of selected teachers' years of experience, Number and Operations scores from the MKT, and belief scores from the MBS.

Table 1
Descriptive statistics of variables in the study

Variables	*N*	Mean	Std	Min	Max
Years of experience	21	11	8.37	1	31
MKT-number and operations	22	0.14	0.99	−1.64	1.94
Teacher belief	22	3.63	0.43	2.73	4.40
Teacher noticing	22	2.00	0.56	1.14	3.10
Noticing errors	22	1.64	1.29	0	4

Note Information on the "Teacher Noticing" and "Noticing Errors" variables will be provided further in the chapter

Data and Analysis

For the main part of the Child Study project, our first attempt to code instances in which teachers discussed students' mathematical strengths and weaknesses was overly broad and did not adequately reflect the richness of teachers' written work. Thus, we sought a different way to capture what teachers wrote about their students. We determined van Es's (2011) framework for learning to notice student mathematical thinking was particularly well-suited for coding the Child Study project, since teachers were directed to notice students' mathematical strategies and thinking and to reflect on what they noticed. The Child Study project was designed prior to van Es's published work, so her noticing framework was not used in the original design of the Child Study project assignment.

Before we could code teacher noticing, we first delineated the Child Study projects into sections, each focused on a single child and single mathematical topic or concept. All the Child Study projects were analyzed to identify the instances of teachers describing or discussing a mathematics topic, delineating the sections for each child and math concept. Then, these sections of mathematical instances were coded using an adapted teacher noticing framework. We reconciled their codes, having discussions any time the codes did not match. After such reconciliation discussions, we came to unanimous decisions about the codes. From the 23 projects analyzed, we note that one teacher discussed at length (although not in depth) one single mathematical concept for the two chosen students, thus we were only able to assign three codes to her project. Since the rest of the projects had 7–30 codes, we excluded this project from the rest of the analyses.

In the process of coding teachers' mathematical noticing, there were times when teachers made errors. These errors took several forms, including a teacher misidentifying a student's strategy as being misaligned with a story problem. For example, Ms. Frasier[1] wrote the following about a student solving a story problem that involved figuring out $12 - 5$:

Me: Explain your thinking.

Sarah: I took 5 and wrote it there (pointing to the top of the column) then I took 12 and put it there (pointing to the top of the other column). Then drew circles to show the numbers. I crossed out 5 here (pointing to the 5 circles) and crossed out the 5 here (pointing to the 5 crossed out in the 12 column). These not crossed out circles are the answer.

Teachers' Noticing: I didn't think her method was showing the mathematics in the problem and I didn't think she truly understood the story problem.

While it is possible that Mrs. Frasier was correct that Sarah did not understand the story problem, it is likely not the case, as Sarah's method to figure out $12 - 5$

[1]All teachers' names are pseudonyms

resembled comparison, for finding what part added to 5 equaled 12. If Sarah understood that the story problem called for 12 − 5, Mrs. Frasier was not noticing that Sarah may have a deep understanding of the mathematics in that Sarah recognized that addition is the inverse operation of subtraction and comparison is a method for figuring out how many more must be added to 5 to equal 12.

Other possible teacher errors included a teacher accepting incorrect or imprecise answers or reasoning as correct, valid, and complete. For instance, Ms. Jackson wrote

> I asked John to explain what makes a number even or odd. He did this in two ways with the number 27. He said, "Because, if you're counting by odd numbers, it only counts by the ones, if the first number is even, like 2, then if the ones number is odd, then it would still be odd. And the other way I know that is because, if you have 7 and you divide it, you'll have one left over." I liked the way he described this process in two different ways. It seems that he is thinking through the process as he explains it.

While the teacher praised this student's reasoning, the reasoning was incomplete. It is true that having a remainder of one when dividing seven by two is a justification for seven being an odd number, but this was not what the student actually said. This appears to be what Ms. Jackson inferred John meant, which may or may not be the case.

Thus, while teachers did occasionally make errors, once we had coded all of the projects and examined the frequency of errors overall and the frequency per teacher, we determined that the errors were fairly rare, and most could be understood from the teachers' point of view. If a teacher provided an incorrect analysis or incorrect trajectory, we did not credit the teacher with that level of noticing, but instead only coded the level of noticing based on correct teacher noticing. An average teacher noticing score was obtained for each teacher, which will be further explained in the "Measurement" section of the chapter. Not surprisingly, teacher noticing was negatively correlated with the number of errors teachers made, r (21) = −0.59, p = 0.004, which suggests teachers with higher levels of noticing overall made fewer errors.

Table 2
Teacher noticing framework (van Es, 2011)

Noticing level	Description
Level 1: Baseline Noticing	Focus was mainly "on the overall classroom environment, the class's behavior and learning" (p. 141).
Level 2: Mixed Noticing	Focus was mainly general, but "began to attend to students' mathematical thinking" (p. 143).
Level 3: Focused Noticing	Focus "became centrally focused on specific students and their mathematical thinking" (p. 145).
Level 4: Extended Noticing	Focus was on "details of students' mathematical thinking" and extended "to consider the relationship between student thinking and the teachers' pedagogy" (p. 146).

Measurement

Through the analysis of the Child Study projects, we began our coding utilizing the four main levels of noticing developed by van Es (2011), found in Table 2. However, some adjustments had to be made to better fit the Child Study project.

Levels of Teacher Noticing. We organized our coding and analysis around the four main categories of teacher noticing (Table 2). One of the main adjustments we made from the van Es (2011) framework involved accounting for the different types

Table 3
Noticing framework for the child project study

Noticing level	Description
Level 1: Baseline Noticing	Teachers only provided a general statement of what a child can or cannot do without providing any evidence to support claims
Level 1.5: Beginning Noticing	Teachers provided very brief evidence that was not clearly described (e.g., showing student work, but not referring to this work in their text)
Level 2: Mixed Noticing	Teachers provided evidence (e.g., worksheets, dialog, pictures) to support their conclusions about student learning
Level 3: Focused Noticing	Teachers not only provided evidence to support their claims, but also analyzed students' mathematical thinking
Level 4: Extended Noticing	Teachers provided evidence, analysis, and future support for students or reflections of their own teaching practice based on observations

of evidence (student work and quotations, rather than video). Between the baseline and mixed levels of noticing, we found some teachers who did include evidence, but the evidence was not always connected to what they were noticing about children. For instance, some teachers provided photos of the target student's worksheet, but did not refer to the worksheet in text to support statements they made. We decided to call this Level 1.5 (see Table 3).

For Level 1, teachers offered general statements or claims about what a student could or could not do mathematically. These statements seem to be teachers' general impressions of students' performance, and teachers provided no evidence to support their statements. In the first segment (below), the teacher simply stated what the student could and could not do regarding shapes. In the second statement, the teacher first made a general statement about Katie's number sense. Although she continued to talk about Katie's performance in different areas of number sense, she did not provide concrete evidence to show how she decided that Katie had good number sense.

Segment 1: She distinguishes one shape from another, but unable to define attributes of the shape.

> *Segment 2*: Katie has a good idea of number sense. From the beginning of the year, she was able to compare numbers, skip count by twos and fives, and could picture a mental number line in her mind. She scored a proficient grade on the beginning of the year math assessment and could easily count up and down starting at various numbers.

We coded an instance as Level 1.5 if the teacher provided minimal evidence, such as a picture of student work, but did not include any explanation for how the work provides evidence to support an assertion. For example

> Kelly was initially confused about what a pattern was. She did not understand that a pattern needed to have a repeating part, so she lined up her collection of shapes randomly. When I asked her which part of her pattern repeated, she simply named each shape in the entire row. However, when I showed her an a-a-b pattern, by lining up 2 triangles and a circle, followed by another 2 triangles and circle, she was able to extend my pattern. Kelly continued to be somewhat confused when asked to verbalize the repeating section of a pattern, but gained confidence as the lesson continued. She successfully extended 6 patterns during independent practice that same day.

For Level 2, teachers used evidence to support their claims about students' mathematical learning. The evidence was concrete, often with detailed description of what and how a student did in a particular math task. Teachers also often used pictures of students' work samples or of a student completing a math task. Unlike Level 1.5, teachers used text to explicitly explain how those pictures supported their statements about students' mathematical learning. In the following segment, the teacher pulled evidence from her observations of Ryan's performance in a specific lesson, and she provided detailed description of what Ryan did that made her come to the conclusion that Ryan could subitize numbers 1–10.

> From the beginning of school Ryan showed that he could easily subitize numbers 1-10 often seeing the 5 or smaller groups within the larger groups. I remember on one particular lesson the students were being asked to identify and circle groups of objects with a specific number of things in them for example 8. Ryan very quickly without counting circled two separate groups and when I asked how he knew that they were 8 he replied, "I see 6 here and 2 here and I know 6 and 2 are 8." The other group he circled had a group of 5 and 3 when I addressed this, with just as much confidence Ryan said, "I know five plus three is eight."

For Level 3, teachers not only described concrete and detailed evidence, but also analyzed the evidence to show how students' mathematical thinking was embedded in the evidence or connected the evidence with a particular learning trajectory. For example, if a student made an error, the teacher would analyze misunderstandings of a mathematical concept that the student might have that led him to make such an error. In the next segment, two sections are italicized to indicate teachers' analysis of student thinking. In the first section, the teacher analyzed Lauren's mathematical thinking behind the word problem she wrote. The teacher realized Lauren's story did not match the equation in a literal sense, but it showed her conceptual understanding of digits. In the second section, the teacher analyzed Lauren's skills for composing and decomposing numbers.

> Lauren's understanding of place value has really developed during this semester. She was asked to write a word problem using the Eq. 24 + 59 = 83, she wrote "I have 20 dogs, 50 hamsters, 9 lizards and 4 fish. How many pets do I have?" *Although the word problem does*

> *not represent the given equation exactly, she understood the value of each digit was as she was writing the problem. I think that demonstrated a deep understanding of place value and how to break apart numbers.* I asked her to talk to me about her word problem and she said, "Since 24 is really 20 + 4, and 59 is really 50 + 9, I thought that I would just use those numbers instead. It's all the same thing and it all equals eighty three in the end, right?". She did question herself at the end, making me think she was looking for verification on her answer. *I asked her to solve her problem and she drew a number line adding all her tens first, and then adding the ones. Her ability to compose and decompose numbers shows that her developmental progression is late seven to eight years old.*

Level 4 contains all the characteristics of Level 3 noticing, but it also includes description of how the teacher will support the student's progress in the area under discussion, or teachers' reflections on their practices that are related to the mathematical area. In the following segment, the teacher analyzed Jennifer's lack of a strong sense of "Five-ness." In the reflection (italics added for emphasis), the teacher not only discussed how she would help Jennifer to understand the importance of five, but also reflected on her practice.

> I asked Jennifer to tell me the number of dots on three cards that were all organized as all the other number visuals in our curriculum are; in groups of five dots in each row. Since each card represented a teen number, two groups of five dots also ended up making a ten frame, which is something we have not talked about in the curriculum yet.
>
> As stated in the article Developing "Five-ness" in Kindergarten, "Five is an essential benchmark number for young students, and a strong understanding of ten, another significant benchmark number in our number system" (Novakowski, 2007, p. 226). I wanted to see if Jennifer had the understanding of how to use groups of five to help her tell the number of dots on each card quicker. As she was shown the first card, I began to see her mouth move. When asked what she was doing, she quickly said, "I am counting." I asked her what she was counting and she let me know using her finger to continue in her counting process of counting each dot by ones. She had no strategy to make the counting of the dots easier for her other than counting by ones, which she does very well. I then understood that she did not have a strong sense of "Five-ness," and did not see how to use that grouping to her understanding. *Having Jennifer do this task let me know that she is missing a big piece that will be needed all throughout our curriculum and Kindergarten math learning. Knowing that Jennifer is not my lowest math student, yet still does not grasp the concept; I need to become more purposeful in using the group of five dots in my number representations when doing examples with the students. In future lessons, I will be more intentional in asking the students to represent a number using manipulatives by showing a group of five when needed. Jennifer specifically needs more teaching on why five is such an important number in math and how we can use it to help us count. I will do this by relating it to the number ten, which becomes extremely important in our upcoming unit.*

Results

We calculated an average score for teacher noticing by averaging each coded level of teacher's noticing for all mathematical instances. Correlational analysis showed teacher noticing was not significantly correlated with teachers' Number and Operations scores, $r(21) = -0.09$, $p = 0.69$, or teacher beliefs, $r(21) = 0.17$,

p = 0.45. Similarly, the number of errors teachers made was not significantly correlated with teachers' Number and Operations scores, $r(21) = 0.08$, $p = 0.74$, or teacher beliefs, $r(21) = 0.17$, $p = 0.44$. Teacher noticing was negatively correlated with teachers' years of experience, $r(20) = -0.46$, $p = 0.03$, suggesting the newer teachers appeared more attuned to noticing student mathematical thinking.

Teacher Profiles

While most of the focus of this chapter is on the development of the coding framework, we also want to suggest uses of the framework in the form of teacher profiles. Such profiles suggest specific professional development as next steps for moving teachers along the trajectory to higher levels of noticing. On average, teachers in our sample exhibited Level-2 noticing (mean teacher noticing score is 2.00, standard deviation is 0.56). In addition, about a third of the teachers had a noticing score between 2 and 2.5. Therefore, most teachers were able to provide evidence to support their claims about student mathematical learning, but did not extend the evidence to providing analysis, delineating mathematical learning trajectories, or reflecting on how teachers will support and further students' understanding. We note in some cases, shortcomings in teachers' noticing was likely more a product of limitations within the assignment directions and its alignment

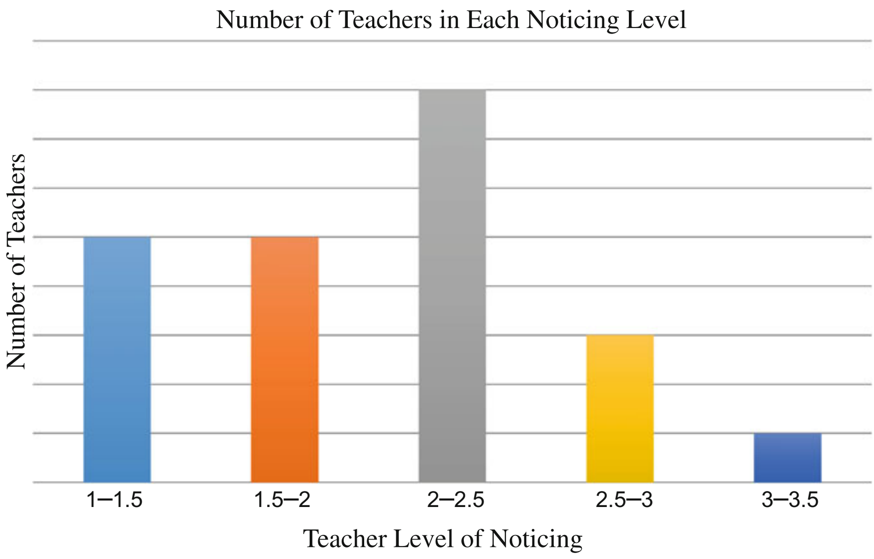

Figure 1. Number of teachers in each noticing level calculated as the average score for each teacher (no teachers had an average score above 3.5).

with van Es's (2011) published work rather than specific weaknesses in teachers' noticing capabilities.

Figure 1 shows the number of teachers in each noticing-level category based upon their average noticing score. We created five categories using a 0.5 interval (approximately half a standard deviation) to present a refined distribution of teacher noticing. We present three teacher profiles to provide a broad description of three groups of teachers–teachers with noticing level of 1–2, 2–3, and 3–4.

Ten teachers obtained a noticing level of 1–2 (average scores greater than one but less than two). These teachers often made a few statements about what a student could or could not do, but did not provide any evidence to support those statements. In some other situations, these teachers provided evidence along with their statements, but the evidence was either very brief or presented without being explicitly referred to in the text. In some other instances, teachers provided clear evidence to support their claims about student learning in some situations, however, did not analyze students' learning trajectories or mathematical thinking embedded in the evidence. Moreover, teachers rarely described how they might support the child to progress on the mathematical topic under discussion or reflected on their own teaching practices based on their observations. In addition, they made a few noticing errors. Teachers at this level could benefit from additional instruction about how to cite and describe evidence to support assertions related to noticing.

Eleven teachers fell within the range of 2–3 (average scores greater than or equal to two and less than three). For most of these teachers, their noticing score of each coded section ranged from one to four, and thus, similarly to the first group of teachers, most of these teachers also made a few statements about students' learning and did not provide any evidence. However, unlike the first group of teachers, in most coded sections, these teachers offered evidence (e.g., student work sample, dialog) to back up their statements. In some instances, these teachers analyzed students' learning trajectories and/or mathematical thinking based on their observations. Furthermore, some of them discussed future support for students to promote students' learning in certain areas, or provided reflections on their own teaching practices, but this only occurred occasionally. Teachers made fewer noticing errors than the level 1–2 teachers. Teachers in this group could benefit from professional development designed to help them analyze their observations and reflect on how to use what they notice to inform their instruction.

Only one teacher had an average score above 3 (3.1). For this teacher, she usually provided evidence to support her claims about students' learning as well as analysis to interpret students' mathematical thinking or students' learning trajectories. In addition, she sometimes discussed future support for students or reflected on her teaching practices. Future support provided was specific, grounded in her observations of students' learning and analysis of students' mathematical thinking. She did not make any noticing errors. Teachers at this level may benefit from professional development to help them think about how to extend the work of a Child Study project focused on two students into the ongoing work of teaching with a whole class of students.

Discussion and Conclusions

While the original purpose of the Child Study project was not specifically to improve teachers' noticing capabilities, we did find the projects attuned teachers to many aspects of teacher noticing. Further, through adapting van Es's (2011) video-club teacher noticing framework, we were able to better understand both how child studies prompt teacher noticing and how to improve future iterations of professional development to accelerate teacher noticing skills. Our main adaptations centered around the differences in types of evidence for noticing afforded by written summaries versus observing videos with a small group of teachers. We also encountered teachers who were at the beginning stages of being able to provide evidence for their thinking, but who did not reach Level 2 noticing, and thus created a Level 1.5 category for the framework.

In Primarily Math, the Child Study project in the first pedagogy course was likely teachers' first experiences with completing an assignment of this nature, in which they needed to provide and analyze evidence of their students' mathematical thinking, and then reflect on how that understanding could inform their teaching. For some teachers, this was the first time they had attempted to understand individual students' mathematical understanding, and to consider where students might fall on a trajectory of mathematical learning. Thus, as novice noticers, the Primarily Math leadership team was very optimistic with teachers' nascent noticing skills.

It is important to note the Child Study project is only one part of the Primarily Math program. Other components of the program may help strengthen the effect of the Child Study project. For instance, teachers took two math content courses during the summer before they started the Child Study project in the fall. The math content courses strengthened teachers' mathematical knowledge: teachers gained a deeper understanding of mathematical concepts and learned new ways to meaningfully teach children through conceptual development and multiple strategies. Teacher noticing is a dynamic process that utilizes teacher knowledge in action. Thus, teacher knowledge may be a prerequisite for good teacher noticing. The lack of significant correlation between teachers' Number and Operations scores and their noticing levels may be due to the sample size and lack of power to detect the relationship. We believe coupling the Child Study project with courses on math content and pedagogy will amplify the positive effect of studies of individual children's understanding on efforts to teach responsively.

One interesting finding is the negative correlation between teacher noticing and their years of experience. Jacobs et al. (2010) found teaching experience alone was not enough to support teachers' growth in their abilities to notice. Specifically, Jacobs et al. (2010) found that teaching experience seemed to help teachers to begin developing expertise in attending to children's strategies and interpreting children's understanding, but it did not help teachers to gain expertise in deciding how to respond on the basis of children's understandings. Additionally, professional development was proved to be vital in supporting teachers' noticing abilities. Teacher noticing is essential to the kind of instruction that recent reform efforts have been endeavoring to promote (NCTM, 2000). However, many teacher preparation programs still fail to cultivate teachers' abilities to attend to and interpret students' mathematical thinking, as well as abilities to build instruction on students' thinking and understandings. Thus, it is not surprising that teaching experience alone did not relate to teachers' noticing levels in this study. Teachers' experience with professional development may be relevant to teacher notching, but unfortunately, such data was not collected in the study.

The measurement framework utilized in this study (an adaption of van Es, 2011) can be utilized in other professional development programs that want to support teachers' learning of noticing skills and assess teachers' growth with regard to their noticing abilities. Through the Child Study project, the teachers in Primarily Math professional development program were able to examine the mathematical learning of two individual children in their classroom in an in-depth way. The project also allowed the teacher educators of the program a way to assess the degree to which teachers in the program were able to utilize quality-noticing skills in their classroom. Other professional development instructors could utilize this rubric in their own program through not only the use of a Child Study project but other written observations. However, as with all quality measurement tools it is important to make sure the measurement framework is reliable. Therefore, further studies should be completed regarding this specific framework in order to examine not only its reliability, but also extending uses beyond mathematics-focused instruction and noticing.

This measurement framework also allows teachers opportunities to closely reflect on their own noticing abilities and identify areas of potential improvement. We recommend sharing this framework with teachers, so they can better reflect on where they may fall on this trajectory, and be able to visualize what the higher levels of noticing would look like for their teaching practices. Through the utilization of quality-noticing skills, teachers may be better able to detect students in their classroom who are either excelling or struggling in different mathematical areas and plan future instruction that is responsive to both mathematical ideas and students' understanding.

In conclusion, the Child Study project was beneficial in helping teachers notice and understand students' mathematical thinking. The framework developed helped the Primarily Math leadership team determine how to improve teacher professional development to focus more on developing teachers' noticing skills. Researchers and practitioners may utilize such projects in order to improve teacher noticing of in-service teachers. Preservice teachers may also use such projects in their teacher preparation programs with some adaptation, taking into account their access to students, opportunities to carry out instructions, and authority to implement what they learn with students.

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Investigating the Relationship Between Professional Noticing and Specialized Content Knowledge

Lara K. Dick

Abstract Professional noticing of children's mathematical thinking as conceptualized by Jacobs et al. (J Res Math Educ 41(2):169–202, 2010) includes attention to and interpretation of children's mathematical thinking, and deciding how to respond instructionally. In order to interpret a child's mathematical thinking, a teacher draws on her or his mathematical knowledge for teaching (MKT) (Ball et al. in J Teach Educ 59(5):389–407, 2008). This research study focuses on the relationship between elementary preservice interns' development of MKT and their engagement with professionally noticing their students' mathematical thinking through analysis of their students' work samples. An integrated professional noticing and MKT framework for simultaneous measurement is applied to the research study. Four preservice interns placed in a first-grade classroom participated in the study. A sequence of three professional learning tasks (PLTs) focused on the preservice interns' analysis of their students' multi-digit addition and subtraction work was developed. Results show specialized content knowledge (SCK), a subset of MKT, as an integral part of professional noticing. The results suggest that in situated contexts focused on developing SCK, preservice interns can increase their engagement with professionally noticing their students' mathematical thinking.

Keywords Professional noticing · Mathematical knowledge for teaching · Specialized content knowledge · Preservice teachers · Student work

The professional noticing of children's mathematical thinking framework (Jacobs, Lamb, & Philipp, 2010) emphasizes attention to and interpretation of children's mathematical thinking, as well as deciding how to respond instructionally. To diagnose children's mathematical thinking, a teacher draws on her or his mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008). Thus, to engage in professional noticing, teachers must rely, at least partially, on their MKT. While others have addressed the intersection between MKT and

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professional noticing either by referring directly to MKT in the context of teacher noticing or by designing noticing interventions focused on developing aspects of MKT (Ball, 2011; Fernandez, Llinares, & Valls, 2013; Flake, 2014; Kazemi et al., 2011; Schack et al., 2013; Vondrova & Žalská, 2013), I have chosen to focus on the integration of the measurement of MKT and teacher noticing. I contend that because MKT is content specific, measurement of teacher noticing is situated both in mathematical content and in the context of interventions designed around professional noticing. For this chapter, one aspect of a research study in which preservice elementary interns' specialized content knowledge (SCK), a subset of MKT, and professional noticing were measured simultaneously will be presented. A discussion of further ideas for integrating the measurement of SCK and professional noticing of children's thinking is included.

Conceptual Frameworks

This study examines the relationship between the development of preservice elementary interns' MKT (Ball et al., 2008) and their engagement with professionally noticing their children's mathematical thinking as conceptualized by Jacobs et al. (2010). For this study, SCK, one component of the MKT framework was integrated into the professional noticing framework. Below, the individual frameworks will be discussed and then the integrated framework guiding the study will be presented.

Mathematical Knowledge for Teaching

For the past 30 years, researchers have sought to determine the types of knowledge necessary for the teaching and learning of mathematics. In Shulman's (1986) seminal paper, he described subject matter knowledge as different domains of knowledge comprised content knowledge, pedagogical content knowledge, and curricular knowledge. Marks (1990) adapted Shulman's concept of pedagogical content knowledge to elementary mathematics. He defined four highly connected subsets of pedagogical content knowledge: subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter (i.e., texts and materials), and instructional processes for the subject matter (p. 4). Manouchehri (1997) proposed having teacher educators infuse content knowledge and pedagogical content knowledge into their preservice teacher preparation programs. She explained that subject specific knowledge should not be taught separately from knowledge of students, and knowledge of teaching and learning.

In 2000, Ball identified three concerns that needed to be addressed for successful merging of mathematics content and pedagogy. She explained, "the [first] problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how much knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice" (p. 244). During the past two decades, Ball and her colleagues have built on Shulman's (1986) work to develop a comprehensive framework for describing mathematical knowledge as it is used in the practice of teaching. Their framework defines MKT and breaks it into two main domains: Pedagogical Content Knowledge is comprised of Knowledge of Content and Students, and Knowledge of Content and Teaching. Subject Matter Knowledge is comprises of Common Content Knowledge along with Specialized Content Knowledge (Hill, Ball, & Schilling, 2008).

Knowledge of Content and Students includes the ability to predict how students will respond to mathematical topics, what they will find interesting and which topics will be the most difficult. Knowledge of Content and Teaching includes making appropriate choices for examples or representations, knowing how to best sequence a topic, and guiding classroom discussions. Common Content Knowledge comprises the ability to solve mathematical problems, provide definitions of mathematical terms, and compute correct answers; this type of knowledge is not specific to teachers. SCK is the knowledge teachers draw upon when evaluating students' invented definitions, interpreting their developed algorithms, and asking questions to press students' thinking. SCK is the MKT subset focus of this study.

Professional Noticing of Children's Mathematical Thinking

Professional noticing in mathematics education is a more recent area of interest for mathematics teacher educators. Noticing student thinking is a teaching practice that requires active real-time engagement from the teacher (Mason, 2002; Sherin, Jacobs, & Philipp, 2011). Past research with preservice and practicing teachers has shown that teachers need support in learning to notice students' mathematical thinking and is therefore a practice to purposefully develop (Santagata, 2011; Star & Strickland, 2008; van Es, 2011). Jacobs et al. (2010) conceptualize professional noticing of children's mathematical thinking as comprised three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings.

Goldsmith and Seago (2011) utilized the professional noticing framework to study practicing teachers' analysis of mathematics student work. They explained that professional noticing of student work "involves attending to both the mathematical content of the task and students' mathematical thinking" (p. 170). Similar to the work of Goldsmith and Seago (2011), for this study, the professional noticing framework was used to study elementary preservice interns' engagement with noticing their students' mathematical thinking via analysis of their students' addition and subtraction work samples.

Integration of SCK and Professional Noticing

For this study, preservice interns were engaged with the teaching practice of analyzing student work. Researchers have found that preservice teachers must possess a special type of content knowledge in order to connect what they learn from student work analysis to their teaching practice (Bartell, Webel, Bowen, & Dyson, 2013; Fernandez et al., 2013; Hiebert, Morris, Berk, & Janson, 2007; Jacobs et al., 2010). Hiebert et al. (2007) explained that making connections between analysis of student work and instructional practice “requires a set of competencies or skills that draw directly on subject matter knowledge combined with knowledge of student thinking” (p. 52). They discussed how teachers must (a) observe and predict types of strategies students will use to solve a problem; and (b) know what a particular response implies about the student’s thinking. Similarly, Jacobs et al. (2010) explained “to interpret children’s understandings, one must not only attend to children’s strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts” (p. 195). I consider these specific types of content knowledge to be in line with Ball and colleagues’ concept of SCK (Ball et al., 2008; Hill et al., 2008).

In 2013, Vondrova and Žalská conjectured that the “ability to notice was possibly a manifestation of mathematical knowledge for teaching” (p. 361). For this study, I directly considered their claim. I looked specifically at how an intervention designed with a focus on the development of SCK related to preservice interns’ engagement with professionally noticing their students’ mathematical thinking. Ball et al.’s (2008) “Mathematical Tasks of Teaching” that draw on SCK (p. 10) as well as their explanation that SCK encompasses teachers’ knowing “features of mathematics that they may never teach to students, such as a range of non-standard methods or the mathematical structure of student errors,” was used to map SCK to each of the components of the professional noticing framework. For example, the mathematical teaching task “using mathematical notation and language and critiquing its use” was considered part of the attend component since it deals with noticing mathematically significant details, while the teaching task “evaluating the plausibility of students’ claims” was considered a part of the interpret component since it is used when interpreting a student’s work sample. Table 1 contains the subset of Ball et al.’s (2008, p. 10) teaching tasks requiring SCK that were mapped to the professional noticing components. This mapping served as the integrated SCK and professional noticing framework for this study.

Table 1
Mathematical tasks requiring SCK as related to professional noticing

Mathematical tasks requiring SCK Ball, Thames and Phelps (2008)	Related professional noticing component
Critique notation and language	Attend
Evaluating plausibility of student claims	Interpret
Evaluate math expressions	
Know non-standard methods and common errors	
Ask productive math questions	Decide

To summarize, for this study, an intervention was designed for preservice elementary interns to develop their SCK regarding multi-digit addition and subtraction, as well as their engagement with professionally noticing their students' mathematical thinking. In what follows, I address the following research question: how does SCK relate to elementary interns' engagement with professionally noticing their students' work in the context of multi-digit addition and subtraction?

Description of Study

Four preservice interns completing their culminating licensure requirement, a semester-long student teaching field experience in first-grade classrooms, participated in the research study. The interns completed a set of three carefully sequenced professional learning tasks (PLTs) facilitated by a college supervisor. The PLTs were focused on analyzing the interns' first-grade students' written work on multi-digit addition and subtraction story problems. Each PLT included a set of directions for the interns and a facilitation guide for the college supervisor. Prior to each of the three PLT sessions, the interns were provided directions on the types of problems to pose to their students as well as how to choose student work samples to bring to the sessions. Figure 1 contains the facilitation guide for PLT #1. See Dick (2016) for the full PLT directions. Each of the PLTs was focused on developing the interns' SCK around a particular aspect of multi-digit addition and subtraction. Table 2 contains information about the PLT sessions and their SCK focus. During the implementation of the PLT sessions, I took on the role of participant observer (Yin, 1998). As a participant observer, I sometimes interjected comments or questions when the conversation veered off course or when my expertise was needed in analyzing a student work sample; my participation was rare.

Begin by having the student teachers share their task and reasons behind their choices as to which of their students' work to bring to the session.
1. What strategies did your students use to solve the task?
2. What did you find surprising or unexpected in your students' work?

Lead a discussion about their initial anticipation of the ways their students would approach the problems

3. What is the mathematics embedded in each of their strategies?

Lead a discussion about different types of addition/subtraction problems.

4. What questions could you ask to help your student reflect on their strategy?

Lead a discussion on how to probe student thinking without guiding their work and on how to describe student work without projecting their knowledge onto the solution. Suggest that the student teachers take notes while monitoring their students as they complete tasks.

Figure 1. PLT #1 facilitation guide.

Table 2
PLT sessions with description of SCK focus

PLT session	PLT specialized content knowledge focus
One	Different types of addition and subtraction story problems (NGACBP, 2010, p. 88)
Two	Multi-digit addition and subtraction problems levels of sophistication: 1. Direct modeling 2. Counting 3. Number fact strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson, 2003; NRC, 2001)
Three	Developing questioning techniques based on student's mathematical thinking: probing versus extending questions (Jacobs & Ambrose, 2008)

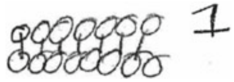
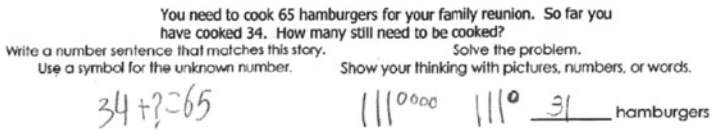
Methodology

The case study presented in this paper is a subset of a larger design research study. Primary data sources were the interns' students' work samples and transcriptions of the three PLT sessions. The interns' discussions during the PLT sessions were divided into "distinct shifts in focus or change in topic" known as "idea units" (Jacobs, Yoshida, Stigler, & Fernandez, 1997). The choice was made to code discourse idea units rather than individual talk turns because collective analysis emerged as an important aspect of the situated learning (Greeno, 1991; Lave & Wenger, 1991) that occurred during the PLT sessions. Each idea unit, when applicable, was coded for the noticing components: attend, interpret, and/or decide. As a means of reliability, the college supervisor was asked to apply the professional noticing codes to the idea units related to student work samples; on the second iteration of analysis, 88% reliability was reached and thereafter, I continued to code alone. After each idea unit was coded for the three noticing components, each component was assigned a level within that component (0: Lacking, 1: Limited, 2: Robust). Table 3 contains the full codebook which was based on Jacob's professional noticing scheme (personal communication, April 3, 2013).

Table 3
Codebook

Level	Description
Attend: mentioning mathematically significant details	
2	Robust evidence • Mentions CORRECT specifics of mathematics they notice – Discussion of strategy type
1	Limited evidence • Mentions some INCORRECT specifics of mathematics they notice • General mention of mathematics including naming strategy
0	Lacking evidence • Missed opportunity for mentioning mathematics
Interpret: what is known about students' mathematical thinking based on student work	
2	Robust evidence • Draws on evidence when giving a plausible interpretation what a student understands – Coherent discussion of students' mathematical thinking
1	Limited evidence • Draws on some evidence when interpreting what a student understands – Making assumptions – Implausible interpretation – Interpretation hard to follow (vague and/or incomplete)
0	Evidence lacking – Missed opportunity for mathematical interpretation – Interpretation lacking mathematical evidence
Decide: next question based on students' mathematical thinking	
2	Robust evidence • Draws on specifics to develop a potentially useful probing or extending question
1	Limited evidence • Develops a probing or extending question – Question not useful or vague – Question potentially useful but not drawing on specifics
0	Decision lacking • No question developed • No mention of students' mathematical thinking

Table 4
Examples of discourse exchanges coded as interpret-2

No evidence of SCK	L: So her counting and marking out the 7 from 17  (Exchange around a correctly answered work sample for 17–7)
No explicit evidence of SCK	CS: 2, 4, 6, 8, 10, 12, 14, 15 circles K: He just didn't finish, it looks like D: 'Cause I've taught them that when you have a certain number, you can make partners to see how many are left over  (Exchange around an incorrectly answered work sample for 18–7) *CS: College Supervisor*
Explicit evidence of SCK	T: Yes, with an unknown number and he used addition so he separated the tens and ones and did a tens stick and one circles and just counted up. So, I guess it's still…it's kind of counting but it's also decomposing in a way K: When I looked at this, I thought it was interesting that he…So he did the 34 here. He knew that he had to get to 65 so instead of counting up to 34–50, he knew…the way he did it was interesting, like he went in and did tens first D: So he just held 34 in his head and did…Like 44, 54, 64  (Exchange around a correctly answered work sample for 65–34)

For each of the three noticing components, within each of the three levels of coding, the designation "with evidence of SCK" was used to highlight instances where the idea unit provided explicit evidence of the interns applying their SCK. The integrated SCK and professional noticing framework presented in Table 1 was used as a guide for coding each of the professional noticing components for evidence of SCK. Ball et al. (2008) described evaluating students' strategies and explanations as teaching tasks that require SCK, which implies that any analysis of student work requires some level of SCK. However, for this study, the decision to code for SCK within the professional noticing components required that an idea unit contain explicit evidence of the interns' drawing on SCK. There most likely were instances where the interns drew on SCK to analyze student work samples, but if they did not explicitly apply SCK to their professional noticing in the discourse exchanges, evidence of SCK was not coded. To illustrate, Table 4 contains examples of excerpts from idea units coded as Interpret 2: Robust Evidence. The first is a plausible interpretation where there is no evidence of the intern drawing on SCK, the second is an exchange where SCK may have been present, but was not explicit and therefore was not coded as evidence of SCK. The third is an exchange where evidence of SCK was explicit.

In the first example, the student colored 17 circles and crossed out 7. The intern developed a plausible interpretation of the student's strategy, but the interpretation lacked evidence that the intern drew on SCK. Interpreting the work sample did not require any expert teacher knowledge; thus, all she needed was common content knowledge for her interpretation. In the second example, the interns interpreted the student's work and determined that the student was employing a familiar strategy, but made a mistake and did not finish drawing the 18 circles. While a plausible interpretation, it is unclear as to whether the interns drew on SCK to interpret the students' thinking. The interns did not explicitly discuss how the student's method made sense with the subtraction problem type or what could have caused the error. Thus, while the interns may have actually drawn on SCK regarding their understanding of non-standard methods to interpret the students' work, there was not explicit evidence of them doing so and therefore this exchange was not coded as exhibiting evidence of SCK. In the final example, the interns interpreted a students' counting up strategy. Their discussion began with one of the interns connecting the strategy to the concept of decomposition. Decomposition was one of the strategies discussed during PLT #2 as part of the SCK focus; an understanding of decomposition is an example of SCK. Because the interns explicitly connected their interpretation of the students' mathematical thinking to SCK regarding decomposition, this exchange was coded as evidence of SCK. To reiterate, explicit evidence was required in order for an idea unit to be coded as "with evidence of SCK."

Results

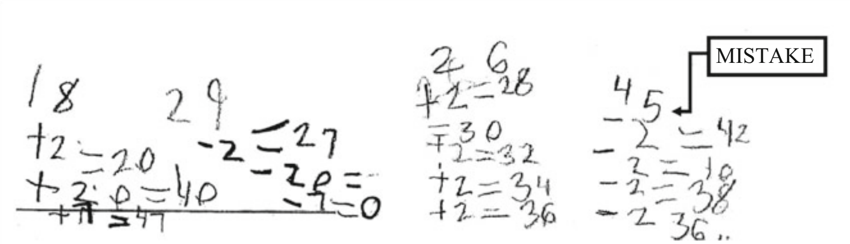
Overall Professional Noticing Results

From the leveled coding that occurred during analysis, it is evident that, for the most part, the interns increased their engagement with professional noticing throughout the PLT sessions. The Appendix contains a table of the professional noticing leveled coding results for all idea units. Blank entries are for idea units that included discussion, but the discussion did not lend itself toward the particular noticing component; only when a noticing component had the potential to occur, was it coded. The interns' growth in engagement with professional noticing can be seen in two directions. The horizontal line represents the change from limited evidence toward robust evidence ($0 \rightarrow 1 \rightarrow 2$) for the three individual noticing components, visible in each row of the table. The diagonal line represents the interns' development in their overall engagement with the three components ($A \rightarrow I \rightarrow D$) as they progressed through the PLT sessions. In general, the interns engaged first with attending to the mathematics found within their students' work samples, then demonstrated an increase of plausible interpretations of their students' mathematical thinking and eventually began to engage with deciding on productive mathematical questions to pose to their students. As the PLT sessions progressed, the intern's analysis of their students' work exhibited increased coherence of the three components of noticing.

Instances with Evidence of SCK

Explicit instances of SCK, as they appeared throughout the PLTs, are discussed before results of the integrated SCK and professional noticing analysis are presented. All names are pseudonyms.

Evidence of SCK attributed to prior knowledge. Specialized content knowledge possessed by some of the interns prior to PLT #1 was exhibited during the first session. During PLT #1, Kelli drew on previously held SCK. One of her students invented a strategy for solving his "put together/total unknown" (NGACBP, 2010, p. 88) problems (Figure 2-sample 1). Kelli recognized the uniqueness of his non-standard method and asked him to explain the strategy while she audio-recorded his explanation. The student explained



Sample 1
(Correctly answered 18+29)

Sample 2
(Incorrectlyanswered 26+25)

Figure 2. Kelli's student work sample from PLT #1.

> I had 18 and 29 so I subtracted 2 from 29 and it equaled 27, then I plussed that 2 for the 18 and it equaled 20 and I kept doing the same thing and the same thing over and over and I finally got the answer and I found out that the strategy was working really good.

Kelli realized that if the student continued to use this strategy, it could be quite inefficient with larger quantities. She stated

> I knew that he was gonna get stuck on the next one 'cause the next answer was 71, so I was like, 'How is he going to do this when we get to the 45 oak leaves and 26 maple leaves?' So I just left him alone for a little bit.

On the subsequent problem, her student made an error subtracting 3 at the beginning when he wrote to subtract 2, which caused him to get an incorrect answer

(Figure 2-sample 2 contains a subset of the student's work on this problem). Kelli did not catch the mistake, and instead of helping her student find it, she pushed him toward a tens-and-ones strategy. It is evident that Kelli attended to the mathematics behind her student's solution by focusing on the language of his explanation and developed an interpretation of his surprising, non-standard strategy. Kelli illustrated her SCK through both her attention to the student's mathematical thinking, and her interpretation and recognition of the inefficiency of his strategy. While Kelli drew on SCK when noticing her students' work, because she did not look for the student's mistake, the exchange was coded as interpret level-1, with evidence of SCK.

Also during PLT #1, Donna and the college supervisor had an exchange about her students' task. Donna had created a worksheet for labeling an appropriate strategy for single-digit addition problems with the choices: "make-a-ten," "counting on," and "doubles plus one" (Fuson, 2009). While not at the highest levels of noticing, Donna's attention to mathematically significant details and interpretations of her students' mathematical thinking showed evidence of previously held SCK. For example, in part of the idea unit, she stated

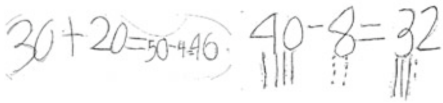
> I found that they really don't have a concept of making a ten or using the double strategy or the doubles plus one, but they all can count on. Like if they have 5 + 4, they go: 1, 2, 3, 4, ***5***, 6, 7, 8, 9 like that, that's how they do it. They don't see that it's 5 + 5 − 1 or that it's 4 + 4 +1 or doubles plus one. They just count on" (emphasis in original).

She then shared individual conferences she had with her students following the assignment. She asked them, "You know, counting on does work but it's not going to work every time for you. What's a more efficient strategy for you to use?" During the exchange, Donna showed explicit evidence of her SCK regarding different strategies for single-digit addition. She was able to evaluate her students' explanations and recognize that she wanted to push her students toward a more efficient strategy. Donna possessed this SCK prior to the PLT session, and she applied it to her interpretation of her students' mathematical thinking. Differently from PLT #1, during PLT #2 and #3, instances of explicit evidence of SCK all related to the SCK foci (see Table 2) and thus, were attributed to the SCK that was being developed through the PLTs.

Evidence of SCK related to the PLT sessions' SCK foci. The only instances of explicit evidence of SCK during PLT #1 were the two instances previously discussed. While PLT #1 focused on developing SCK, the discussions surrounding the development of SCK occurred after the interns' collective analysis of their student work samples. The PLT #1 SCK focus was on different types of multi-digit addition and subtraction problems (NGACBP, 2010) through which the interns were exposed to a variety of different strategies. The interns drew on both of these areas of SCK during PLT #2. At the start of PLT #2, Donna shared the two different types of story problems she posed to her students: an "addend unknown-put together/take apart problem" and a "bigger unknown-compare problem" (NGACBP, 2010, p. 88). She provided evidence of her SCK in her explanation of how finding the missing partner using addition was easier for her students than her traditional view of subtraction as take-away. For the compare problem, when interpreting her students' work, Donna explained how the word "fewer" affected her students' choice of strategy, which drew on the SCK discussion during PLT #2; many of her students were confused as to whether to add or subtract for the "compare problem" (NGACBP, 2010, p. 88). While either operation can be used to solve the problem, the situation equation that would model the story used addition.

In the following idea unit excerpt, two of the interns showed evidence of drawing on SCK while developing an interpretation for one of Kelli's student's approaches to two different subtraction problems (see Figure 3). The interns recognized that the student has "got the concept of tens and ones," then discussed why he directly modeled for one problem and not another problem, though the problems were similar. In the exchange, the interns exhibited SCK regarding different methods and the effect of the actual quantities in the story problem.

K: And there we can see he took away the eight by adding…by drawing out the ones (Sample 2).
D: Um hmm. But it's interesting that he did it there (sample2) are not up there (Sample 1).
K: I know.
D: I mean I guess four is a smaller number so you could probably just count back (Sample 1).



Sample 1 Sample 2

Figure 3. Kelli's student work samples from PLT #2.

Toward the end of PLT #2, the college supervisor introduced the SCK focus for the session: levels of sophistication of different strategies (See Table 2). During their discussion, the group had a conversation about Tammy's students' work on a story problem for 13 + 5. Donna made the comment that she was happy to see Tammy's students "hold the thirteen, and count on from there." The interns were asked if based on the students' work they had analyzed for 13 + 5 they could come up with different strategies the students might employ for 13 + 8. This led to a discussion about making a new 10. Donna shared her experience with students' wanting to work from 10. For example, she discussed a student who, when adding 9 + 8, changed the problem to 10 + 7. She was able to apply her SCK relating to her student's algorithm to this new problem which led to a whole group discussion about different level 3 compensation strategies during which some of the interns exhibited further evidence of SCK.

PLT #3 was designed to develop the interns' ability to decide on appropriate probing and/or extending questions (Jacobs & Ambrose, 2008) to ask their students. Differently from PLT #1 and #2, this SCK focus on asking productive math questions was part of the interns' pre-PLT assignment. Thus, it was discussed toward the beginning of the session with the hope that the interns would draw on their newfound SCK while engaging with noticing the work samples. During PLT #3, the interns exhibited many instances of drawing on different aspects of SCK that they gained throughout the PLT sessions.

For example, one of Donna's students had solved an "add to/change unknown" (NGACBP, 2010, p. 88) problem in a manner that would have been very difficult to interpret without questioning him (Figure 4). During the session, Tammy noted that she was not sure what the student was doing. Donna was able to explain the student's work in light of her asking him probing questions while in class which assisted her interpretation of his mathematical thinking. Throughout the idea unit, the interns collectively discussed the student's solution and drew on their SCK to identify the level of sophistication of the strategy as a level 3 (see Table 3). Kelli explained her reasoning, "I would think so [level 3 strategy]. Since he could explain it. Looking at this I couldn't really tell his work but he could explain it. I mean, I think explaining it is the hardest part. And the fact that he knew he needed less than 50 because he already had over 50." Kelli's comment is focused on the mathematics and illustrates her realization of the importance of asking students questions.

"I didn't ask him about that during but afterwards when I looked at it, I said, 'Why did you draw five on top and on bottom?' and he said, 'Well there were 54 so I drew 54 first' and then he said, 'And then I realized that I needed to get to 100 so I was drawing…I was making partners on top to make sure they added up,' and then he realized that 50 and 50 was 100. So he said, 'And I knew then that I needed less than 50 to make my problem true.' And he can count back and not many of my students can count back, especially with two-digit numbers. So that really surprised me."

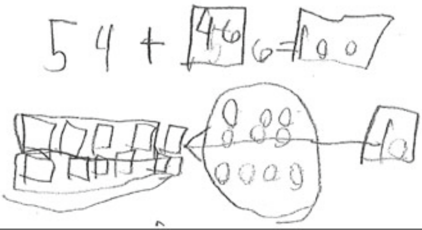


Figure 4. Donna's student work from PLT #3.

As another example, when the interns discussed Kelli's student's work as shown in Figure 5 they realized that while Kelli's student exhibits a higher level of sophistication of strategy for the first problem (sample 1), the student needed help applying her strategy to problems requiring making a new ten (sample 2). In the idea unit exchange, the interns exhibited SCK regarding non-standard algorithms and common student reactions to problems that require making a new ten.

K: It's decomposition (Sample 1) but it's also creating equivalent but easier problems because it's easier to add 20 and 40, and a 6 and 3 together. I mean even I personally use this method when I'm checking their work in my head because I know that it's pretty efficient.

D: The problem that I've seen with my kids is that when they do have to break down their numbers like this, it's great when the ones is the highest that it goes in that line, but when it gets to like 20 + 40 = 60 and they've got like 6 and 7 and that's 13, then you have 60 + 13 and they're like, "Okay…." and then they draw up 13 circles and they don't understand it's a 10 and a 3…

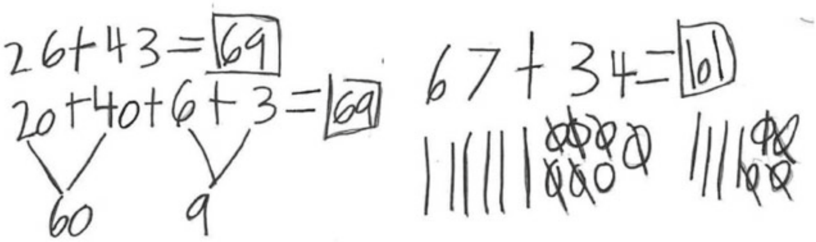
⋮

T: I like how the student used…that's what I try to do with my students, is add the tens first, then add the ones so that it's getting away from the standard algorithm…it's level 3.

⋮

K: I wonder if she knew that she was gonna have to make a new 10 here (sample 2) so she chose to do it this way 'cause she made a new 10.

D: I think I would agree with that too 'cause when my kids know that they have to make a new 10, they'll normally just go ahead and do tens and ones and then they'll circle their new ten rather than doing the decomposing method.



Sample 1 Sample 2

Figure 5. Kelli's first student work example from PLT #3.

SCK Analysis

Overall analysis of explicit evidence of SCK shows that both the number of instances, as well as percentages of the total idea units containing evidence of SCK, increased throughout the PLT sessions (see Table 5). It is evident that the interns drew more on SCK to engage with professionally noticing their students' thinking as the PLT sessions progressed. By PLT #3, the interns provided evidence of their SCK during 10 exchanges which was just under half of the 23 total idea units from the session.

Table 5
Idea unit containing explicit evidence of intern SCK

	# of idea units containing evidence of SCK	% of PLT session's idea units (%)
PLT #1	2	8
PLT #2	4	18
PLT #3	10	43

To delve into the relationship between professional noticing and explicit evidence of SCK, Table 6 includes all of the idea units that contained explicit evidence of SCK together with the coded level of professional noticing. The interns' SCK assisted them throughout their engagement with all three components of the professional noticing framework. For the idea units where the interns exhibited evidence of SCK, their SCK led to greater levels of either attend, interpret and/or decide. Of the 25 documented instances showing evidence of SCK, 20 of them were for the highest level (level 2) of professional noticing.

Table 6
Professional noticing codes for idea units that contain evidence of SCK throughout the PLT sessions

	PLT session #1		PLT session #2				PLT session #3									
Idea unit	2	12	1	7	12	13	1	2	3	4	5	11	12	13	17	22
Attend	1*	2	2*	2	2	2*	2*	2*	2*	2*	2	2	2*	2*	2*	2
Interpret	1*	1*	2*	2*	2*	2*	1	1*			1*	2*	1	2*	2*	2*
Decide		2											2*	1	2*	0

* Indicates the "explicit evidence of SCK" designation

For the individual components of professional noticing, the number of instances of explicit evidence of SCK increased throughout the PLT sessions for both attend and interpret. Refer to Table 1 for the types of SCK which will now be discussed. The interns provided explicit evidence of SCK while attending to mathematically significant details in their students' work samples via the teaching task of critiquing notation and language during 10 exchanges. For interpret, SCK regarding different types of non-standard multi-digit addition and subtraction algorithms was the most prevalent type seen throughout the PLT sessions. The interns also drew upon SCK about different strategies when evaluating their students' claims and when evaluating efficiency of strategies. Furthermore, the interns drew on SCK regarding non-standard methods and common ways students approach different multi-digit addition and subtraction problems when interpreting their students' mathematical thinking. Perhaps expectedly, there was not explicit evidence of the interns drawing on SCK to ask productive mathematical questions for the decide component during the first two PLT sessions. However, during PLT #3, likely due to the SCK focus, the interns exhibited SCK regarding productive math questions in two instances. Overall, the results show a relationship between SCK and professional noticing. Namely, SCK was related to an increase in the interns' engagement with professional noticing for each of the three individual noticing components: attend, interpret, and decide.

Discussion

The results presented above illustrate that there was indeed a relationship between exhibited SCK and increased professional noticing. As part of the design of the PLT sessions, the preservice interns were developing their SCK around multi-digit addition and subtraction. They showed increasing evidence of their SCK as the PLTs progressed, and as their SCK increased, they engaged more with professionally noticing their students' mathematical thinking.

In 2011, Sherin et al. (2011) called for research on how noticing of practicing teachers compares to that of preservice teachers. They asked "What trajectories of development related to noticing expertise exist for prospective and practicing teachers?" (Sherin et al., 2011, p. 11). The results from this study illustrate preservice interns' development of noticing expertise throughout a relatively short intervention. Their growth required support via the situated PLTs facilitated by the college supervisor and discussed with their peers. Like others have shown, support is a necessity when developing preservice teachers' engagement with noticing their students' mathematical thinking (Jacobs et al., 2010; Star & Strickland, 2008; Vondrova & Žalská, 2013), but growth can and does occur with purposefully developed interventions. The SCK focus of this study's intervention assisted the preservice interns as they engaged with professional noticing through the teaching practice of analyzing their students' work.

It is hoped that preservice teachers exposed to professional noticing will take what they learn from their preservice experiences and apply it to their own teaching. Franke, Carpenter, Levi, and Fennema (2001) found that inservice teachers who learn by interpreting their students' mathematical thinking continue to learn after designed interventions. Upon following up on the interns from this study, one intern explained

> As a result of my participation in this study, I began looking deeper at student work and trying to figure out why students did what they did instead of just looking and seeing what strategy they used or the mistakes they made. By thinking about why they did what they did when solving an equation, I was able to help the students better.

The intern's response shows her application of SCK when considering why her students solved problems in different manners. Both the preservice interns' SCK and professional noticing skills assisted her as she continued to grow in her own teaching practice. Thus, working with preservice interns to professionally notice their students' thinking has the potential to better their teaching practice as they continue to engage with noticing in their own classrooms.

Implications

This study provided an example of one method for measuring teachers' professional noticing of children's mathematical thinking and SCK simultaneously within a content and context specific situation. Professional noticing can and should be measured differently depending on the content of focus and the particular context of the designed intervention or teaching practice under study. To develop measures, the integrated SCK and professional noticing framework presented in this paper can be adapted to other situations.

Ball et al.'s (2008, p. 10) list of teaching tasks requiring SCK can serve as a basis for mapping different mathematical concepts and interventions to the components of professional noticing. For example, say a mathematics teacher educator was interested in working with practicing secondary mathematics teachers as they engage with professionally noticing their students' proof writing for non-standard or unfamiliar geometric situations. First, a professional development intervention could be designed that focused on developing secondary teachers' SCK regarding the underlying structure, and connections between proofs about circles that require knowledge about triangles [for example, Euclid's Intersecting Chords Theorem III.35 (Euclid, 2002)]. Referring to Ball et al. (2008, p. 10) list of teaching tasks, in order to engage with professionally noticing their students' proofs, teachers would need to "recognize what is involved in using a particular representation" which could be considered part of the attend component. Teachers might also need to evaluate the ways their students "choose and develop useable definitions" within their proofs, which could be considered part of the interpret component. For making instructional decisions, the teachers might choose to "modify the task to be easier or harder" depending on the needs of their individual students. In order to study the teachers' engagement with professional noticing throughout or as a result of this fabricated intervention, a full mapping of teaching tasks requiring SCK or even extended to teaching tasks that require additional aspects of MKT would be completed. Then analysis of the teachers' professional noticing with evidence of MKT could occur.

The above is just one example of a teaching situation containing different mathematical content and professional development context that could be used to simultaneously measure both teachers' professional noticing and their SCK. The example shows how the integrated SCK and professional noticing framework can be transferred across areas of preservice and inservice teacher education as well as across mathematical topics. Furthermore, the integration of content knowledge for teaching and professional noticing can be measured in other disciplines. Work to expand the MKT and noticing frameworks separately is underway in the field of science education (Barnhart & van Es, 2015; Johnson & Cotterman, 2015). Integrating these frameworks is an area of future work for teacher educators across disciplines.

Regardless of the particular content or context, the practice of teaching requires a set of knowledge specific to teachers. Engaging with professionally noticing how students' think about content is a teaching practice that must be developed. Yet, the practice of noticing cannot occur without discipline specific knowledge for teaching. Thus, the measurement of noticing while simultaneously measuring discipline specific knowledge for teaching is worthwhile and should continue to be addressed throughout different fields of teacher education.

Appendix: Professional Noticing Codes for Idea Units Focused on Student Work Analysis Throughout the PLTs

First Grade Session One																								
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Attend	0	1	1			2		1	1	1	1	2				2			1					
Interpret		1				2		1	0	1	1	1	2						0					
Decide												2												

First Grade Session Two																						
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Attend	2	2	0	1	1	2	2	2	2	1	2	2	2	2	1	2	2	1	1			
Interpret	2	2		1	1	2	2	2	1	0	1	2	2		1	1	1	2				
Decide						2																

First Grade Session Three																							
Idea Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Attend	2	2	2	2	2	1	2	2				2	2	1		1	2	2	2	2	2	2	
Interpret	1	2			1				1	1	2	2	2	1		1	2	2	2	1	2	2	
Decide												2	1	1		2	2	1	0	1	2	0	2

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A Standardized Approach for Measuring Teachers' Professional Vision: The Observer Research Tool

Kathleen Stürmer and Tina Seidel

Abstract In this chapter, we introduce the Observer Research Tool that has been proven to measure teachers' professional vision in a valid and reliable way. Teachers' professional vision is seen as a promising approach to investigate the effectiveness of teachers' professional development in light of the demands of real teaching practice. The Observer Research Tool was developed as the first standardized instrument that combines videotaped classroom situations with rating items. Step by step, we discuss the single components of the instruments in order to illustrate how to best guarantee validity in assessing teachers' professional vision.

Keywords Assessment · Teacher education · Video · Professional vision · Pre-service teacher

Introduction

The effectiveness of teacher education has been the subject of much international discussion in recent years (Bauer & Prenzel, 2012; Brouwer & Korthagen, 2005; Cochran-Smith, 2003; Darling-Hammond, 2006). The cornerstone of the ongoing discourse is the question of how to define and assess the effectiveness of teacher preparation programs in light of the demands of real teaching practice. In this respect, the assessment of teacher education requires the identification of indicators and instruments that provide valid and reliable measures (Seidel, 2012), which thus situate professional development in the context of teachers' work (Sherin, Linsenmeier, & van Es, 2009). In this vein, the assessment of teachers' professional

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vision is seen as a promising approach. In this chapter, we introduce the Observer Research Tool that has been proven to measure teachers' professional vision in a valid and reliable way (Seidel & Stürmer, 2014; Stürmer & Seidel, 2015). The tool is computer-based and was developed as the first standardized instrument that combines videotaped classroom situations with rating items (see Figure 1). By discussing the single components of the instrument in light of the development process of the tool, we aim to illustrate how to best guarantee validity in assessing teachers' professional vision in a standardized way.

As Sherin, Jacobs and Phillip (2011) note, focusing on teachers' professional vision may prove to be a transformative idea in teacher education as "it opens the door to new paradigms and methodologies" (p. 4). In general, the concept of professional vision describes the socially organized way for a professional group to see and interpret the phenomena that are relevant to their work (Goodwin, 1994; Grossman et al., 2009; Sherin, 2007). Sherin and colleagues translated the concept into teaching practice. In their conceptualization they describe teachers' professional vision as the ability to attend to particular events that are relevant for students' learning as well as to make sense of those events within the classroom context (Sherin et al., 2011). It requires the ability to identify what is important within a classroom setting, and to then make a connection between the identified events and the broader principles of teaching and learning (van Es & Sherin, 2002). In other words, teachers' professional vision could be seen as an indicator of the integration of theoretical knowledge concerning effective teaching and learning with the elements of practice (Stürmer & Seidel, 2015), for example, by noticing whether the learning goals are clarified in instruction based on the knowledge that goal-oriented instruction helps students to activate pre-knowledge.

It seems obvious that measurements of professional vision have to be devised that go beyond the traditional paper-and-pencil knowledge tests (Blömeke, Gustafson, & Shavelson, 2015; König et al., 2014; Seidel & Stürmer, 2014; Stürmer & Seidel, 2015). In this vein, videotaped classroom situations are used as representations of practice (Grossman et al., 2009) in order to prompt teachers' professional knowledge in a situated and contextualized way (Kersting, 2008; van Es & Sherin, 2008). In focusing on the observation of videotaped classroom situations, researchers assess different aspects of teachers' professional vision through open questions that are analyzed qualitatively. However, in terms of practical usability, such qualitative approaches are of only limited use when investigating larger samples of persons. Indeed, when evaluating the progress of developments over time, perhaps over the course of initial teacher education at universities, standardized measures that are suitable for formative assessment are more helpful. Although such measures might be less sensitive to the fine-tuned processes of

Start page of the online Tool

Presentation of video clips

Followed by standardized rating items targeting reasoning

Figure 1. The observer research tool.

professional vision such as the capacity and extent to which an individual person is able to attend to a particular event in a classroom, they would still provide a valid and reliable indicator of the achievement of the major objectives of teacher education: (1) applicable knowledge about effective teaching and (2) learning for practice.

Over the past few years, researchers have made progress in developing standardized assessment tools that combine videotaped classroom situations with rating items that target different aspects of teachers' professional vision (König et al., 2014; Seidel & Stürmer, 2014; Steffensky, Gold, Holodynski, & Möller, 2015). However, when developing such tools, researchers face the challenge of designing assessment tasks that demonstrate situations in which professional vision could naturally arise, as well as ensuring a high level of fidelity with regard to the underlying skills being measured (Shavelson, 2012). Regarding the requirement to translate real job demands—seeing and interpreting what is important in classroom interactions—into a representative task for competence assessment within teacher education, we first describe the idea behind the Observer Research Tool and so convey the theoretical construct on which the measurement is based. In the following, we also introduce the development of the individual components of the Observer Research Tool. In the second section we outline the selection and validation of videos as authentic representations of classroom practice, while in the third section we introduce how to develop rating items that target professional vision-related skills based on a theoretically conceptualized model. Furthermore, since research into teaching effectiveness does not provide right or wrong answers with regard to the quality of the events observed in the videos, we present expert judgments as the qualitative norm for the assessment. Finally, we discuss the potential of our approach in terms of formative assessment by presenting a summary of the findings and providing empirical evidence of the validity of the Observer Research Tool.

Professional Vision as an Indicator of the Quality of Knowledge Application to Practice

When thinking about the relationship between teachers' professional development and effective teaching practice, what does it actually mean to focus on and assess their professional vision? In this section, we aim to answer this question by describing the idea behind the Observer Research Tool. We intend to highlight its contribution, alongside more traditional approaches, to investigating professional development within teacher preparation programs. We will also illustrate how we draw on the idea of professional vision in our conceptualization of assessment as the theoretical construct that forms the foundation on which advancement in any field rests and which brings light to the new methodological approach (Sherin et al., 2011).

The Idea of Professional Vision

Central to our work is the understanding that teachers' professional knowledge is essential for high-quality teaching in classrooms (Darling-Hammond & Bransford, 2005; Seidel & Shavelson, 2007). Recent research has made considerable progress in modeling teachers' knowledge in relation to effective teaching practice by drawing on Shulman's (1987) conceptualization of content knowledge, pedagogical content knowledge, and generic pedagogical knowledge. Progress has also been made in providing empirical evidence of the structure of teachers' knowledge by differentiating Shulman's (1987) three facets using paper-and-pencil knowledge tests (Baumert et al., 2010; Blömeke, Kaiser, Lehmann, Felbich, & Müller, 2006; Döhrmann, Kaiser, & Blömeke, 2012; Hill, Rowan, & Ball, 2005; Voss, Kunter, & Baumert, 2011). The three facets have been proved to constitute an important factor in teachers' expertise regarding students' learning processes (Baumert et al., 2010; Voss et al., 2011). Paper-and-pencil tests have been shown to differentiate teachers' declarative–conceptual knowledge ("knowing that") and the explicable aspects of their procedural knowledge ("knowing how") in a standardized, economical and reliable way (König et al., 2014). Both aspects are seen as major types of knowledge, which underlies the value of professional expertise (Anderson, 1983). Furthermore, researchers commonly consider the quality of knowledge representation—as a crucial prerequisite for a successful transfer of knowledge in a certain context (de Jong & Ferguson-Hessler, 1996)—by analyzing the different levels of the target cognitive processes (i.e., remembering, understanding, analyzing).

At this point, however, the traditional tests have reached their limits, although researchers still have to account for the contextualized and situated nature of teachers' knowledge (Borko, 2004; Hammerness, Darling-Hammond, & Shulman, 2002). In comparison to knowledge application in the context of real teaching practice, the contexts provided by written prompts (i.e., questions, case scenarios) could be described as rather rigid and frozen into one fixed situation. In the process of classroom teaching, teachers face a highly varied and amorphous set of phenomena that occur simultaneously and that are constantly in motion (Sherin et al., 2011). The process of accessing and applying knowledge is, therefore, more complex within teaching practice than that which researchers are able to recreate through a written prompt in a paper-and-pencil test.

In this respect, drawing attention to teachers' professional vision could be seen as an innovative and advanced approach (Blömeke et al., 2015; Sherin et al., 2011). By describing, for example, how individuals observe and interpret events and situations specific to their profession (Goodwin, 1994), such an approach indicates how theory-practice-integrated knowledge is represented. Take as an example a minor car accident on a public street. One could imagine that if a medical doctor were to arrive, he or she would probably pay attention to the people involved, focusing particularly on their differing health statuses. Now take the same scenario, but with a police officer arriving. The officer will most likely focus on phenomena that provide an insight into the course of events and so allow him/her to establish safe conditions at the scene of the accident. With this in mind, we can assume that the focus of attention within professional contexts is actively guided by the professional knowledge that a person is able to access and apply in a specific situation (Erickson, 2011; Palmeri, Wong, & Gauthier, 2004; van Es & Sherin, 2002).

It is clear that the measurement of professional vision within the teaching arena goes beyond what standardized formative assessment tools are capable of assessing in a test. However, in using videotaped classroom situations as the context for knowledge application, researchers can account for the process character of real classroom events. Thus, we see in the assessment of professional vision while teachers are observing the videos the opportunity to draw conclusions about the quality of their knowledge representations proximal to the demands of real practice (Seidel & Stürmer, 2014).

The Conceptualization of Professional Vision

van Es and Sherin (2002) translated the concept of professional vision into the context of teaching practice and focused on teachers' ability to notice and interpret the features of classroom events that are relevant for students' learning (see also Sherin, 2007). The target skills are informed by knowledge of what constitutes effective teaching and learning, and so require integrated, flexible knowledge connected to the contexts of the observed situation (Seidel & Stürmer, 2014). Until now, such skills have mainly been studied using qualitative approaches (i.e., Santagata & Angelici, 2010). The findings of this qualitative research have provided a valid basis for describing different levels and processes of teachers' professional vision. However, in order to provide empirical evidence of the construct in terms of the underlying cognitive processes, as well as to provide a standardized tool for formative assessment purposes, we translated the previous findings into a theoretical model of the structure of professional vision.

The qualitative research identifies two interconnected processes that occur in a dynamic interplay within the classroom (i.e., Sherin, 2007): (1) the selective attention paid to classroom events (noticing) and (2) the interpretation of classroom events (reasoning). In the following, we illustrate how we model these processes in relation to each other as the theoretical framework for the assessment approach, which is implemented in the Observer Research Tool.

Noticing: Selective Attention Paid to Important Classroom Events

In this context, noticing involves identifying classroom situations that, from a professional perspective, are decisive in effective instructional practice (Seidel & Stürmer, 2014). Teachers need to develop the skill necessary to recognize the components of effective classroom teaching that support students' learning processes. In classroom teaching, numerous teaching and learning acts occur. Some are particularly important for students' learning, while others are less important. Similarly, the situations to which teachers direct their attention while observing a classroom action serve as the first indicator of their underlying knowledge (Sherin et al., 2011). When it comes to defining situations that are relevant for teaching and learning, different knowledge foci can be used that provide a frame for capturing knowledge application. In our research, we focus on knowledge concerning the principles of teaching and learning as an aspect of generic pedagogical knowledge (Shulman, 1987), which represents a basic component of teacher education (Hammerness et al., 2002). Research on teaching effectiveness is generally based on knowledge about teaching and learning as elements of generic pedagogical knowledge. Over the last decade, a substantial number of empirical studies have investigated the effects of teaching on students' learning. In understanding teaching as a process for creating and fostering learning environments in which students are supported in activities that have a good chance of improving their learning, Seidel and Shavelson (2007) conducted a meta-analysis in which they made the common results of those studies explicit. They integrate the variety of effective teaching variables into the five teaching and learning (TL) components of a cognitive process-oriented teaching and learning model (Bolhuis, 2003). These TL components are goal setting, orientation, execution of learning activities, evaluation of learning processes, and teacher guidance and support (regulation). All of the identified TL components could be regarded as principles of teaching that show positive and differential effects on the cognitive and motivational–affective aspects of students' learning (Fraser, Walberg, Welch, & Hattie, 1987; Hattie, 2009; Seidel & Shavelson, 2007).

Since any attempt to create a standardized yet contextualized measure that empirically captures the assumed structure of professional vision could easily become overburdened if the full model is applied, we restricted our assessment to integrating the three TL components that could be seen as representations of a balanced knowledge base: goal clarity, teacher support, and learning climate (see Figure 2). *Goal clarity* served as an indicator of successful preparation for learning, which includes the aspects of goal setting and orientation. *Teacher support* served as the guiding process involved in the execution and regulation of learning activities, while *learning climate* served as an indicator of the motivational–affective classroom context.

Focusing on just the three TL components means that the interpretations being drawn from the assessment tool have to be restricted to this context (Kane, 1994). However, with respect to the validity of competence assessment tools such as the Observer Research Tool, we see the main challenge as being the development approaches with a high level of fidelity in terms of the measured underlying skills that represent the different levels of the target cognitive processes (Shavelson,

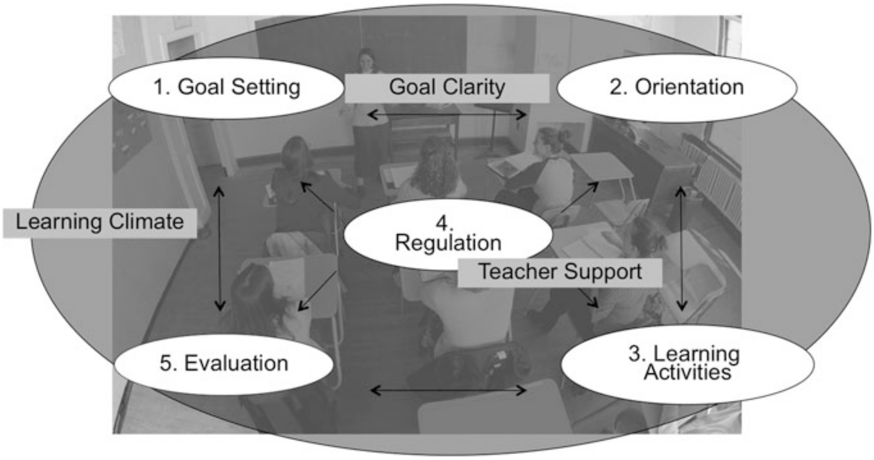

Figure 2. The TL components in a cognitive process-oriented teaching and learning model. Adapted from "Teaching Effectiveness Research in the Past Decade: The Role of Theory and Research Design in Disentangling Meta-Analysis Results," by T. Seidel and R. Shavelson, 2007, Review of Educational Research, 77(4), p. 461.

2012). Such valid approaches could in turn be transferred to different knowledge foci.

Reasoning: Interpretation of Important Classroom Events

The second process of professional vision describes teachers' reasoning concerning classroom events. This aspect captures the ability to process and interpret the situations noticed, based on the individual teacher's knowledge of the principles of teaching and learning (Borko, 2004; van Es & Sherin, 2002). The ability to adopt a reasoned approach to noticed classroom situations provides an insight into the quality of the teachers' mental representations of generic pedagogical knowledge (Borko, Jacobs, Eiteljorg, & Pittman, 2008). In conceptualizing teachers' reasoning, researchers have to distinguish between qualitatively different skills (Berliner, 2001), which we have termed (a) description, (b) explanation, and (c) prediction (Seidel & Stürmer, 2014).

Description reflects the skill necessary to differentiate between the relevant aspects of a noticed teaching and learning component using the terms of the concept but without making any additional judgments. Taking the example of goal clarity, a person observing the first minutes of a lesson might state that the teacher is clarifying (1) the relevant teaching and learning goals, (2) how the lesson is structured, and (3) how the content relates to what the students have previously learned.

Explanation refers to the skill necessary to use conceptual knowledge about effective teaching to reason regarding a particular situation. This involves classifying and accounting for the situations according to the models and principles of learning of the component concept of TL involved. With regard to goal clarity, one would expect a person to link the observation to the concepts, for example by noting that through clarifying teaching and learning goals the teacher is activating students' pre-knowledge.

Prediction refers to the skill necessary to predict the consequences of observed events in terms of students' learning. It draws on broad knowledge about teaching and student learning as well as its application in classroom practice. With regard to goal clarity, a person might use knowledge concerning the effects of goal clarity on students' learning in order to make a prediction about possible consequences. If a teacher, for example, fails to clarify certain learning goals, one potential consequence might be that the students are less likely to direct their learning towards those goals, which could in turn lead to negative consequences for motivation and knowledge acquisition.

Previous research has shown that novice teachers are capable of describing classroom situations. In contrast, their skills in explaining and predicting the consequences and outcomes of those situations lag behind those of experienced in-service teachers (Seidel & Prenzel, 2007). However, only very limited empirical research has systematically investigated the three skills of reasoning. The measurement of description, explanation, and prediction could provide evidence-based knowledge of the structure of teachers' reasoning that could serve to advance the field, especially when it comes to understanding professional learning processes and designing learning environments in teacher education. In this respect, we combine the two processes of professional vision (noticing and reasoning) in assessing teachers' reasoning skills using rating items related to the three TL components of goal clarity, teacher support and learning climate, which are represented in videotaped classroom situations.

Videotaped Classroom Situations as Representations of Practice

The use of video has become a popular tool for studying teachers' learning as well as activating teachers' knowledge (Brophy, 2004; Goldman, Pea, Barron, & Denny, 2007). Videotaped classroom situations are seen as representative examples of practice (Grossman et al., 2009). They are valued for capturing the complexity of classroom interactions in a situated and authentic way and, thus, for providing a lively second-hand experience of teaching (Goldman et al., 2007; Miller & Zhou, 2007). Many approaches to video-based teaching research investigate teachers' reasoning as an indicator of the quality of teachers' knowledge. Most of these studies are embedded in the context of in-service teachers' professional development. Their focus is on describing individual changes in teachers' knowledge or the development of teacher groups reasoning jointly about video, for example in "video clubs" (Borko et al., 2008; Sherin & van Es, 2009; van Es, 2009). Although these studies do measure teachers' reasoning in a contextualized manner, the standardized conditions necessary for assessing individual teachers' knowledge are rarely achieved. The qualitative approaches typically used in such research require considerable time and effort. This leads to the disadvantage that the results can only be used as feedback after a significant amount of time has lapsed. Quantitative instruments, on the other hand, allow for more efficient data analysis. Such quantitative measures can provide a first indicator regarding, for example, the current state of teacher competencies. These indicators can then be used promptly for feedback on teaching and formative assessment. A promising approach developed by Kersting (2008) has responded to this challenge using standardized videos as "item prompts." These prompts are embedded in open questions that tap into teachers' individual interpretations of classroom situations. Kersting's (2008) findings show that the standardized use of videos to measure teachers' skills in interpreting classroom situations represents a valid approach by which to assess their knowledge.

In the development of the Observer Research Tool, we followed Kersting's (2008) approach by integrating videos that prompt teachers' knowledge concerning the target TL components (goal clarity, teacher support, learning climate). With regard to validity, we acknowledge the requirement that the videos used have to constitute a representation of the practice element to which the assessment of the knowledge application refers (Grossman et al., 2009). In this respect, researchers have to follow clear criteria during the video selection process (Sherin et al., 2009; Borko et al., 2008; Brophy, 2004), in addition to proving whether the selection does indeed meet those criteria (Shavelson, 2012). In our work, we established three main criteria that we see as essential to ensuring the context validity of the videos in a standardized assessment approach. In the following, we detail how the selection process was guided by those criteria as well as how we ensured high validity regarding the indented context that the videos should present.

Criterion-Based Selection of Videos

First, given the situated and contextualized nature of teachers' knowledge, the selected videos should be perceived by participants as *authentic examples of classroom practice* (Borko, 2004). To achieve this, we decided to use classroom sequences from the educational system that our target participants (students attending teacher preparation programs) would encounter in German-speaking classrooms. Since the instrument is supposed to capture the generic pedagogical aspects of professional vision, different subjects to which the generic knowledge should be transferred is represented. Based on the first criterion, available video recordings of German-speaking instruction in various subjects (e.g., Reusser, 2005) were screened.

Second, the videos serve as *prompts to activate teachers' knowledge*. Thus, on the one hand, the selection of videos should be perceived by participants as stimulating and activating. On the other hand, the videos should not involve too much complexity, since that could lead to an increased cognitive load. The objective was therefore to strike a balance between the activating nature of the videos and the need to avoid overwhelming observers with a high cognitive load.

Third, the videos constitute *discernible examples of the target practice representation*—in our case, the three TL components of goal clarity, teacher support, and learning climate. We focused on teaching effectiveness research in order to identify sequences that are of particular relevance to students' learning, either in the way that a positive example is represented (positive example) or in the way that a teacher fails to address a relevant component (negative/ambiguous example). Theoretical conceptualizations and video coding schemes from national video studies related to the three components of goal clarity, teacher support, and learning climate were used to identify appropriate video sequences (Seidel, Prenzel, & Kobarg, 2005), resulting in a pre-selection of 86 videos. The pre-selection videos were then given to six external experts in the field of teaching effectiveness research, who were asked to rate the representation of the TL components. During this process, the research team learned that it was nearly impossible to identify videos that only represent one of the three TL components. To account for this, the decision was made to identify videos that represent two of the three components. In addition, as each TL component also includes different content aspects that could arise within a video extract and to which the knowledge assessment has to be specified, we itemized the components with regard to two aspects (goal clarity: (a) the teacher clarifies the learning goals and (b) the requirements of the lesson; teacher support: (a) the teacher asks open questions and (b) gives supportive feedback; and learning climate: (a) the teacher uses humor in his/her instruction and (b) takes the needs of the students seriously). In a final step, based on the external experts' ratings, three experts from the research team independently assigned the videos to the specified content aspects of the components. Then, the videos and their assignments were discussed and validated by the three experts.

Finally, 12 videos (each of two to four minutes in length), covering five different subjects (2 x physics, 2 × mathematics, 4 × history, 1 × French and 1 × English as a foreign language) that met the three main criteria, were selected. All of the videos featured German-speaking Grade 8 and Grade 9 classrooms with students aged between 14 and 16 years.

Criterion-Based Validation of the Selected Videos

To ensure that the intended context of the videos did indeed match their actual context (as perceived by the participants), we conducted a second step involving several validation studies.

The *authenticity* as well as the *activation* potential of the videos was investigated in a pilot study involving the voluntary participation of 40 preservice teachers from the university (Seidel, Blomberg, & Stürmer, 2010a, b). We asked each pilot study participant to think aloud while watching the 12 videos and to evaluate them in a short questionnaire after each clip with regard to their stimulation and required mental effort. The think-aloud protocols were analyzed qualitatively. The results showed that, overall, the videos were perceived as authentic. Acknowledging that the videos should be balanced, we compared them in terms of their stimulation effect and the mental effort required (Seidel & Stürmer, 2014). The results indicated that the videos did not differ in the perception of the participants. In addition, no significant differences between the videos were found with regard to the represented subject and the assigned TL component.

We also investigated the extent to which the 12 selected videos represent the three focal TL components (i.e., goal clarity, teacher support, and learning climate) and so serve as "prompts" to elicit participants' knowledge. In a study of $N = 121$ preservice teachers, two test versions were implemented whereby the videos were systematically rotated and varied with respect to the subject shown and the TL components represented (Seidel & Stürmer, 2014). The mean agreements between the participants and the judgment of the research team were 66.9% for goal clarity, 80.4% for teacher support, and 75.8% for learning climate. Consequently, the 12 videos can be regarded as discernible examples of the three TL components.

Rating Items as Measures of the Quality of Knowledge Representation

In traditional paper-and-pencil tests, rating items are used to measure teachers' declarative–conceptual or procedural knowledge in a standardized and economical way. To account for the quality of knowledge representations, items that target different cognitive processes are formulized. In interpreting the quality of knowledge, participants' responses are compared with a criterion-related norm (resulting, for example, in right or wrong answers) as reference. In this section, we describe how we transferred this approach to the video-based assessment of teachers' knowledge. We explain how to develop rating items that target professional vision skills based on a theoretically conceptualized model. Furthermore, given the fact that teaching effectiveness research does not provide right or wrong answers with regard to the quality of the events observed in the videos, we detail how we use expert judgments as the qualitative norm for the assessment.

Rating Items Targeting Different Cognitive Processes

Based on our theoretical model of the structure of professional vision, we aimed to develop rating items that capture the quality of teachers' knowledge representations. For this reason, we constructed rating items for the combination of declarative–conceptual knowledge about the three TL components (goal clarity, teacher support, and learning climate) and the three reasoning skills (description, explanation, and prediction). Yet, what does it actually mean to describe, explain, or predict a classroom situation with regard to knowledge concerning a certain TL component? In order to provide evidence-based reasoning for the noticed classroom events, models and knowledge regarding these processes are important. In this context, teaching effectiveness researchers refer to self-determination theory (SDT) in order to model the processes involved in the creation of learning environments by teachers as well as the effective use by learners. SDT proposes three basic conditions that a learning environment needs to satisfy in order to make learning processes likely the experience of competence, autonomy, and social relatedness (Deci & Ryan, 2004). A substantial body of research has shown that the perception of these conditions in a learning environment is positively related to both intrinsic motivation and human development. With regard to the three selected teaching and learning components derived from the teaching effectiveness research, it has been shown that *goal clarity and orientation* are important for students to experience competence, autonomy, and social relatedness (Kunter, Baumert, & Köller, 2007; Seidel, Rimmele, & Prenzel, 2005), with positive effects on students' motivation and knowledge development over time. In addition, *teacher support and guidance* in classroom discourse is positively related to the three conditions, with positive effects on intrinsic learning motivation and interest development (Lipowsky et al., 2009; Seidel et al., 2003). Furthermore, a positive learning climate positively affects perceptions of the three conditions, again with positive effects on students' learning (Buff, Reusser, Rakoczy, & Pauli, 2011).

In this vein, the construction of our rating items was based on the framework in Figure 3. Questions measuring *description* targeted the specific observation of the three TL components using knowledge about aspects of each component in naming and differentiating an observed event. Questions tapping into *explanation* focused on the link between an observed event and knowledge about the corresponding TL component, specifically with regard to how a teaching component addresses students' individual perceptions of the supportiveness (e.g., autonomy, competence) of a classroom situation. Questions assessing *prediction* focused on the potential consequences of an observed situation in terms of students' learning, including the consequences for learning motivation, cognitive processing, and affect.

For each TL component, 18 rating items were developed, with nine per content aspect (three for description, three for explanation, and three for prediction). A four-point Likert-scale ranging from 1 (*disagree*) to 4 (*agree*) was used. Participants were asked the extent to which they agree with the items after having watched a video representing the relevant TL component.

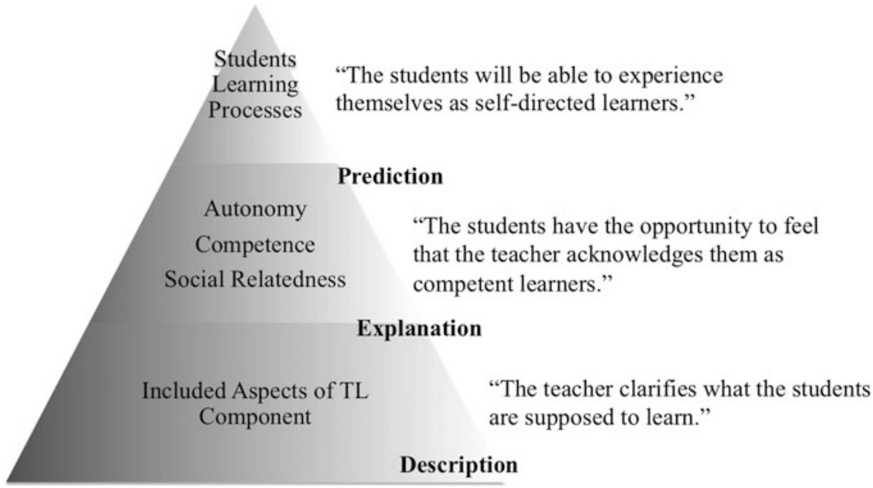

Figure 3. Frame of reference for item construction.

Expert Judgments as the Frame of Reference for the Rating Items

When assessing teachers' professional vision using rating items, it is necessary to identify a suitable frame of reference for comparing participants' responses. In competence assessment, various approaches can be used to define the relevant criteria. In qualitative research concerning professional vision, for example, the individual approach (Fuchs, Benowitz, & Barringer, 1987) has been used to describe the development of individuals' performance over time (Star & Strickland, 2008). According to this approach, growth is measured within individuals over time. However, differences between individuals cannot be compared using an individual norm. To address this shortcoming, the traditional comparative approach has been used in addition to the individual approach to compare a person's performance to a norm based on the performance of other individuals with similar characteristics—a social reference norm (Fuchs et al., 1987). However, given that the significance of a participant's performance is dependent on his/her relative position in comparison to that of the other participants, significant variability is required to apply a norm-based reference to performance (Popham, 1971). Thus, it is essential to utilize representative samples with a great deal of variety.

When it comes to assessing professional vision at the level of teacher education, for example, in initial teacher education programs at universities, only limited variance within the sample can be guaranteed. A potential approach for dealing with this issue is seen in the use of criterion-referenced norms (Goldstein & Hersen, 2000). Criterion-referenced norms use content-related criteria for comparison. One well-established criterion-referenced norm is the expert norm (i.e., Oser, Heinzer, & Salzmann, 2010). This approach is based on the assumption that experts can be characterized as exhibiting a large number of domain-specific organized knowledge structures that they can draw on to successfully deal with the specialized tasks of their profession (Kalyga, 2007; Ericsson, Krampe, & Tesch-Romer 1993).

In our study, we used an expert norm as the criterion norm to measure teachers' reasoning skills. However, a question still arises regarding the most appropriate type of expert to act as a suitable reference for the target competence assessment. With the Observer Research Tool, we aim to assess teachers' knowledge representation regarding effective teaching and learning. With this in mind, we chose as a suitable frame of reference persons with an elaborate base of evidence-based knowledge, such as researchers in the field of teaching effectiveness. Furthermore, the assessment targets the application of knowledge to practice by observing and interpreting classroom videos. Thus, the second criterion for being an expert was a broad treasure trove of experience in classroom observation.

To create our norm, three expert researchers, each with 100–400 h of experience observing classroom situations according to the teaching and learning components under investigation, independently answered all four-point Likert-type items included in the Observer Research Tool (Seidel & Stürmer, 2014). Cohen's Kappa (κ) was calculated to determine the consistency of the expert ratings, with a mean Cohen's κ of.79 across the raters indicating a satisfactory level of consistency (Seidel et al., 2010b). In cases where the experts initially disagreed, agreement was reached by consensus validation. The expert norm was thus established and the participants' responses can be compared in terms of the extent to which they concurred with the expert judgment. With respect to how stringent the comparison to the expert norm should be for a reliable measure, two different strategies for calculating agreement were established and tested: (1) a more strict measure of '0' (miss expert rating) and '1' (hit expert rating); and (2) a less strict measure of '0' (miss expert rating), '1' (correct direction on the scale), and '2' (hit expert rating). The strict recoding proved to be superior to the less strict version, which took tendency into account (Seidel & Stürmer, 2014).

The Observer Research Tool as a Formative Assessment Approach

With the aim of investigating the effectiveness of teacher education in preparing (future) teachers for the real practical demands of teaching, innovative approaches are required that are suitable for formative assessment purposes. We identify three major conditions that such tools must fulfill. First, a high level of usability with regard to an accurate, efficient, and economical assessment. Second, to empirically capture the theoretical model underlying the target skills so as to provide evidence for the assumed cognitive processes involved. Third, the sensitiveness to measure developments in the skills of the target group in the course of educational programs.

The Usability of the Observer Research Tool

For an economical use of our measurement approach, we provided the videotaped classroom situations and rating items as a computer-based online assessment. The Observer Research Tool is presented as a series of HTML pages (Seidel, Stürmer & Blomberg, 2010a). It begins with general instructions as well as short introductions to the three TL components: goal clarity, teacher support, and learning climate. Brief contextual information about the class is provided before each video is presented. Participants have the opportunity to watch the videos for a second time before responding to the rating items (see Figure 1). In order to limit the completion time for the tool and to reach a balanced ratio between the represented subjects and the teaching and learning components, participants are presented with six of the twelve videos showing secondary classroom instruction in physics, math, French, and history. In this form, the completion time for the instrument is about 90 min. In the context of university-based teacher education, the Observer Research Tool was processed under different conditions (“online” versus “on-site” processing and “voluntary” versus “compulsory” participation). Thus, we ensured that the assessment of preservice teachers’ professional vision was not affected by different assessment conditions (Jahn, Prenzel, Stürmer, & Seidel, 2011). Furthermore, with regard to measurement accuracy, we investigated whether the measurement was stable over time. Evidence for this retest reliability could indeed be provided (Seidel & Stürmer, 2014). Regarding the assessment of the generic pedagogical knowledge application, a further study shows no dependencies between the subject background of preservice teachers (i.e., math) and the subject shown in the videos (Blomberg, Stürmer, & Seidel, 2011).

Suitability for Empirically Capturing Reasoning Skills

Regarding the suitability for empirically capturing the theoretical model of describing, explaining, and predicting the noticed TL components, we conducted scaling studies with more than 1000 preservice teachers from German universities and teacher candidates within an induction phase (Jahn, Stürmer, Seidel, & Prenzel, 2014; Seidel & Stürmer, 2014; Stürmer & Seidel, 2015). Analyses of the psychometric properties of the instrument based on item response theory confirmed that the Observer Research Tool provides a valid and reliable assessment of professional vision. In these studies, different measures and models that describe the structure of professional vision were applied, and fit indices were compared. We tested a one-dimensional (reasoning as one overall ability) and a two-dimensional model (describe and integrate, including explain and predict) against the theoretically postulated three-dimensional model (describe, explain, and predict). The results of the scaling studies showed that all three models reliably assessed reasoning, but that the three-dimensional model explained the most variance. Moreover, the three-dimensional model exhibited the best fit with the data. However, bivariate latent correlations of the personal ability scores of the participants showed that the components of "describe," "explain," and "predict" were interrelated and highly correlated with the overall score of reasoning. Moreover, the structure of reasoning proved to be comparable to that of preservice teachers in different teacher education tracks such as primary, secondary, and vocational education (Jahn et al., 2014) as well as between preservice teachers and teacher candidates (Stürmer & Seidel, 2015). Thus, the Observer Research Tool provides a reliable and valid measure of prospective teachers' professional vision with regard to the skills of describing, explaining, and predicting classroom situations during teacher education.

Sensitiveness to Measure Developments

As the Observer Research Tool was developed with a formative assessment purpose, we conducted several studies, regarding the sensitiveness of the tool to measure developments in prospective teachers' professional vision in the course of their educational preparation. Regarding the assumption that professional vision is a knowledge-based process, we investigated whether the measurement captures positive changes in professional vision skills of preservice teachers who acquired different contents of generic pedagogical knowledge. The assessment was used as a pre- and posttest measure within three courses on the topic of teaching and learning principles at university (Stürmer, Könings, & Seidel, 2013). The three courses included (a) a very specific video-based course directly targeting effective TL components, (b) a course focusing on important principles of learning and learner characteristics connected to principles of teaching, and (c) a broad course on "hot topics in instruction," dealing partly with TL components but accompanied by other topics, such as relevance of homework or assessment. For all three courses, positive changes in three professional vision abilities were shown. Regarding the gains in description, explanation, and predication, differential effects occurred. The two content-specific courses on TL components showed the highest increases in explaining and predicting and seem to support the integration of knowledge about TL components and student learning. The general course showed the highest increases in describing. As in addition to formal learning opportunities such as universities, the informal learning through practical experiences in teaching is seen as essential in acquiring integrated knowledge structures (Darling-Hammond &

Bransford, 2005), we further examined the impact of practical experience (in the form of a praxis semester) accompanied by video-based courses at university on preservice teachers' changes in professional vision skills (Stürmer, Seidel, & Schäfer, 2013). The findings revealed overall positive changes, with a special benefit for preservice teachers who started the semester with low professional vision skills. Because the students' practical experiences were guided by video-based courses at university, the study underlined the attempt to support teachers' professional learning processes by theoretical knowledge acquisition that is connected to representations of teaching practice such as videotaped classroom situations (Grossman et al., 2009). Research on the design of learning opportunities has outlined the advantage of videos as a learning tool that guides the acquisition, activation, and application of teachers' knowledge in a meaningful way. However, videos must be implemented with clear objectives in mind (Blomberg, Renkl, Sherin, Borko, & Seidel, 2013). In this, vein we investigated whether the Observer Research Tool is sensitive to capture changes in professional vision skills regarding different video-based designs using different instructional strategies on prospective teachers' learning. Seidel, Blomberg and Renkl (2013) examined the impact of two instructional strategies embedded in video-based courses on preservice teachers' learning at university: rule-example (first, theoretical knowledge about TL components is provided, followed by students analysis of video clips regarding TL components) vs. example-rule (first, students analyze video clips regarding effective teaching, followed by a theoretical summary provided by the lecture referring to TL components). The results revealed that preservice teachers who were taught by the rule-example strategy scored higher on reproducing declarative knowledge about relevant TL components and on professional vision, whereas preservice teachers in the example-rule group scored higher on lesson planning, particularly in identifying possible occurring challenges in a situated way. Furthermore, distinct differences in the capacities of preservice teachers to reflect about teaching were shown. The rule-example approach facilitated reasoning abilities in observing videotaped classroom situations, whereas the example-rule teaching approach fostered preservice teachers' long-term reflection skills about own learning in a learning journal. In sum, the findings indicated that the Observer Research Tool is sensitive to specific learning effects that might occur because of different course objectives and learning goals in teacher education programs.

Conclusion

Choosing indicators and instruments that provide valid and reliable measures of (prospective) teachers' competencies, and that situate their professional development in the context of teachers' work, is seen as the main challenge in developing assessment approaches with a high level of fidelity regarding the measured underlying skills. As stated above, such valid approaches could in turn be transferred to different knowledge foci. The Observer Research Tool was the first standardized yet contextualized approach developed to capture knowledge representation about effective teaching and learning based on the concept of professional vision. Over the last few years, researchers have made progress in developing similar assessment tools that combine videotaped classroom situations as representations of practice with rating items that target different aspects of teachers' professional vision skills (König et al., 2014; Seidel & Stürmer, 2014; Seidel et al., 2010a, b; Steffensky et al., 2015).

To sum up, we consider the Observer Research Tool to be a valuable extension of the concept of professional vision in standardized assessment approaches in comparison to traditional knowledge tests as it provides an approximation of knowledge application in the context of the practical demands of teaching.

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Challenges in Measuring Secondary Mathematics Teachers' Professional Noticing of Students' Mathematical Thinking

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Abstract Our focus is on teachers' professional noticing of children's mathematical thinking which Jacobs et al. (Jacobs, Lamb, & Philipp, 2010) describe as a set of three interrelated skills: (1) attending to students' strategies, (2) interpreting the students' mathematical understandings, (3) deciding how to respond on the basis of students' understandings. We focus on secondary teachers' professional noticing of children's mathematical thinking because we believe teachers with expertise in noticing children's mathematical thinking are better poised to support their students' mathematical performance and understanding. However, studying teacher noticing at the secondary level presents unique methodological challenges. We first consider methodological issues in measuring K–12 teachers' professional noticing of children's mathematical thinking, and then consider three methodological challenges that are particular to secondary mathematics classrooms. These issues center around the challenges of artifact selection, determining the relative sophistication of responses, and the lack of access to experts' responses at the secondary level. Also, we consider the cultures of teaching in the elementary and secondary contexts.

Keywords Professional noticing · Methodology · Inservice · Secondary teachers · Mathematics

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Introduction

Imagine a teacher in a secondary mathematics classroom circulating while her 35 students work in small groups to solve an algebraic-generalization task. Perhaps she makes note of whether all students in a group are engaged and monitors students' affect. She may wish that a particular student's reasoning was visible or more understandable. She may or may not be looking for and may or may not be able to describe connections among the diverse mathematical responses. She likely observes many approaches taken to the task and critiques their sophistication, as well as their alignment with expected mathematical goals and the normative language, notation, and representations of mathematics. She wonders what statements, representations, or questions would support her students' thinking. Furthermore, she must decide whose approaches to highlight: hers alone, one student's approach, or several approaches. Suppose one approach to the task is representative of the majority of thinking. A different approach displays a misconception she feels should be discussed, while yet another approach is novel and unexpectedly touches upon a related mathematical concept. Two approaches may provide an opportunity to compare and contrast. In making the decision about what approach(es) to highlight, she perhaps considers students who need opportunities to share or who will model expertise. She reluctantly recalls the testing calendar. What pedagogical moves and subsequent tasks will best advance her mathematical agenda?

In a typical classroom, teachers must interact with and respond to an overwhelming amount of information. What to do with this information often involves making choices about to what to attend and how to make sense of that information. Before responding to an event in the classroom, a teacher must first *notice* that event. Now imagine *measuring* what the teacher noticed in the classroom scenario described above. We could observe the teacher's practice and infer from the teacher's actions what she is noticing. In the observation though, we cannot access what she noticed unless we see her act upon it. Or we may ask the teacher after class to reflect on what she noticed. Neither approach tells us directly about the in-the-moment, yet hidden, practice of *professional noticing* (Sherin, Russ, & Colestock, 2011). These are just some of the methodological challenges associated with assessing teacher noticing.

In this chapter, we argue that studying teacher noticing at the secondary level presents its own set of unique methodological challenges. Our goals for this chapter are to briefly discuss methodological issues for assessing teachers' professional noticing of children's mathematical thinking for all grade-level teachers (both in-service and preservice) and then to highlight methodological challenges that are particular to secondary mathematics classrooms. To underscore the challenges of studying the noticing of secondary mathematics teachers, we contrast the methods of Jacobs, Lamb, and Philipp (2010) in studying the professional noticing of K-3 teachers and our methods in our recent investigation with 16 secondary mathematics teachers. Because our work focused on secondary mathematics teachers, from this point forward we use the term *students' mathematical thinking*

(SMT) instead of *children's mathematical thinking*. In the next section, we describe an overarching challenge related to data-collection and data-analysis methods for studying teachers' in-the-moment noticing of students' mathematical thinking.

Challenges of Collecting Data on Teachers' Noticing

Sherin and her colleagues (2011) pointed out that challenges in studying teachers' in-the-moment noticing differ from challenges in studying professional noticing in domains other than mathematics education. In other domains, a common approach for collecting noticing data is to conduct think-aloud interviews in which the participant says aloud everything he notices during the event (Ericsson & Simon, 1993). This method is challenging for researchers to enact with teachers given that teachers are interacting with and reflecting on their engagement with as many as 40 individuals. Hence, mathematics education researchers have developed alternative methods for collecting noticing data. The three main ways researchers have collected data on teacher noticing of students' mathematical thinking (Sherin et al., 2011) are through observations of classroom practice and inferring what the teacher noticed (e.g., Choppin, 2011; Hand, 2012), through retrospective reflections on teachers' practice (e.g., Colestock, 2009), and through responses to items in relation to video or student work from others' practices (e.g., Jacobs et al., 2010).

Observations and Inference

As mentioned earlier, one can collect data on a teacher's in-the-moment noticing of students' mathematical thinking by observing the teacher's practice and inferring what teachers are noticing from the teacher's actions (Choppin, 2011; Levin, Hammer, & Coffey, 2009; Sherin et al., 2011). In some cases, these observations are the sole source of data (e.g., Hand, 2012). However, observing a teacher's practice to infer what she noticed may be limiting. That is, a teacher may have noticed something and made a decision *not* to act on what she noticed, and thus the researcher misses an act of noticing by solely observing her practice (Sherin et al., 2011). Therefore, correctly inferring what teachers notice solely from classroom observation can be unreliable. With respect to student thinking, however, researchers often try to address this issue using these observations in conjunction with some type of teacher reflection to triangulate noticing data (Choppin, 2011; Colestock, 2009; Fredenberg, 2015; Levin et al., 2009; Sherin et al., 2011; Walkoe, 2015).

Some have experimented with asking teachers to video record, report, and reflect on their in-the-moment decision-making (Colestock, 2009; Fredenberg, 2015; Sherin et al. 2011; Walkoe, 2015). Sherin and colleagues (2011) provided evidence that interviewing teachers about the clips they chose to record helped teachers relay their in-the-moment reasoning. Fredenberg (2015) utilized a short think-aloud protocol, interrupting elementary school teachers after seeing them modify a task for a child to ask their thoughts about what they had noticed and how they decided how to respond. Fredenberg (2015) found that teachers' noticing in this setting could be further explored than with a single interview.

Reflections on Practice

Researchers can also collect data on teachers' in-the-moment noticing by analyzing teachers' reflections on their own practices (Choppin, 2011; Colestock, 2009; Schifter, 2011; Sherin & van Es, 2009; van Es & Sherin, 2008). These reflections can range from stimulated-recall interviews (Choppin, 2011; Colestock, 2009) to written reflections about video clips (Schifter, 2011) to discussions with other teachers about clips from their own practices (Sherin & van Es, 2005, 2009; van Es & Sherin, 2008). Sherin et al. (2011) commented that through this method of data collection, teachers are not subjected to the same constraints as when they are teaching. Researchers should recognize that what teachers notice retrospectively might differ from what they noticed in the moment.

Video and Student Work from Others' Classrooms

A third common way for researchers to collect data on teachers' in-the-moment noticing is to have teachers reflect on artifacts of student thinking that come *not* from their own practices (Fernández, Llinares, & Valls, 2012, 2013; Goldsmith & Seago, 2011; Jacobs et al., 2010; Sánchez-Matamoros, Fernández, & Llinares, 2014; Schack et al., 2013; Walkoe, 2015; Zapatera & Callejo, 2013). These artifacts could be video clips of other teachers' classrooms, video clips of individual students working on problems, or students' written work for problems. Sherin et al. (2011) noted that because these artifacts come from other teachers' classrooms, teachers, in their reflection, will be unable to call upon the same knowledge they might use in a more familiar setting, such as knowledge of the students and knowledge of what happened during yesterday's lesson. Hence, these data might differ slightly from teachers' authentic in-the-moment noticing.

However, unlike the first two data-collection methods, asking teachers to respond to a video or student artifact enables researchers to compare across participants more easily because teachers reflect on the same artifacts. Consequently, this method of data collection appears to be most appropriate for characterizing different levels of noticing of students' mathematical thinking (Fernández et al., 2013; Jacobs et al., 2010; Sánchez-Matamoros et al., 2014; Schack et al., 2013; Walkoe, 2015; Zapatera & Callejo, 2013).

Across the range of data-collection methods, the challenges vary along multiple dimensions: degree of inference needed, teachers' familiarity with students and tasks, authenticity, the amount of evidence available about student thinking, and ability to compare across teachers' classrooms. These dimensions are important considerations for researchers deciding what type of data to collect and how to collect the data. All researchers who study professional noticing of students' mathematical thinking need to address the methodological challenge identified above—how to collect noticing data. However, we have identified three additional challenges that are particularly acute for researchers seeking to understand the professional noticing of secondary mathematics teachers:

Challenge 1—Availability of artifacts used in measures;
Challenge 2—Determining relative sophistication of responses;
Challenge 3—Access to expert responses.

In the following section, we first briefly describe each challenge in the context of secondary mathematics. We then compare analyses from Jacobs et al.'s (2010) study of 131 K–3 teachers and prospective teachers and our own investigation of 16 specially selected, practicing secondary mathematics teachers to highlight these special challenges.

Challenges Specific to Studying Secondary Teachers

We have identified three methodological challenges that are more acute in studying secondary mathematics teachers' noticing of children's mathematical thinking than elementary school teachers' professional noticing of students' mathematical thinking. First, finding artifacts (video and student work) to use in measures that showcase students' mathematical ideas is more challenging at the secondary level than at the elementary school level because fewer video examples of secondary teaching responsive to students' ideas exist. Second, determining the relative sophistication of teachers' responses to prompts is a challenge given that there are fewer frameworks of student thinking and learning trajectories at the secondary level than at the elementary school level. These frameworks often guide the analysis phase of professional-noticing studies at the elementary school level. Without the frameworks, determining what counts as evidence for interpreting the student's mathematical understanding or evidence that the teacher is basing decisions on the student's mathematical understanding presents a special challenge. Third, we believe that fewer secondary teachers than elementary school teachers are expert at professional noticing of students' mathematical thinking. Thus, researchers have less access to expert responses and fewer resources for determining what a learning trajectory for teachers' noticing might entail. In the following, we compare analyses from Jacobs et al. (2010) study of 131 K–3 teachers and prospective teachers and our own investigation of 16 specially selected, practicing secondary mathematics teachers to highlight these challenges.

Comparison of Analyses: K–3 Teachers Versus Secondary Teachers

Because we used Jacobs et al.'s (2010) characterization of *professional noticing of children's mathematical thinking* (PNCMT) to ground our work with secondary teachers, we are poised to look across both studies to describe methodological similarities and differences. Professional noticing of students' mathematical thinking entails a set of three interrelated skills: attending to students' strategies, interpreting the students' mathematical understandings, and deciding how to respond on the basis of students' understandings (Jacobs et al., 2010).

We chose to focus on professional noticing of students' mathematical thinking because teachers with such noticing expertise are better poised to support their students' mathematical performance and understanding (see also, Franke, Carpenter, Levi, & Fennema, 2001; Gearhart & Saxe, 2004; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sowder, 2007).

Participants

Both studies included one group of experienced, practicing teachers who had yet to begin sustained professional development that was focused on understanding students' mathematical thinking and using students' ideas to inform their instructional practices. A description of the participants follows.

Participants in Study of K–3 Teachers

Jacobs and her colleagues (2010) had as a goal characterizing teachers' levels of noticing students' mathematical thinking. Their 131 participants were comprised of one group of prospective elementary school teachers and three groups of K–3 teachers with varying levels of experience in learning about students' mathematical thinking. Although in many studies of teacher effectiveness teachers are grouped according to years of teaching experience, each of the three teacher-groups in the Jacobs et al. study (2010) had a mean of 14–16 years of teaching experience. Instead of the three groups of in-service teachers being arranged by years of teaching experience, they were grouped according to the number of years they had been engaged in sustained professional development focused on understanding children's mathematical thinking. *Initial Participants* had yet to begin professional development but had already committed to participating in it. *Advancing Participants* had engaged in two years of sustained professional development, and *Emerging Teacher Leaders* had engaged in at least four years of sustained professional development and had begun some informal leadership experiences outside their own classrooms. Jacobs et al. (2010) chose to use common artifacts to allow for comparisons across groups of participants.

Participants in Study of Secondary Teachers

In our study of 16 secondary (eight middle school and eight high school) mathematics teachers, participants were a part of a leadership and professional development program focused on teaching and learning algebra; the teachers were selected because they demonstrated effective teaching practices and held a positive disposition toward learning and growing in their practices. Effective teaching practices included evidence of some ability to analyze examples of student reasoning, quality of student–teacher interactions seen in a 10-minute teaching clip, and strength in content knowledge. These teachers had an average of 13 years of teaching experience, with a range of 2–30 years.[1] They, however, had not had professional development focused on students' thinking and so, although considered exemplary teachers, were most like the *Initial Participants* in the study by Jacobs and her colleagues—experienced teachers but novices with respect to a focus on learning about student thinking.

Challenge 1—Availability of Artifacts Used in Measures

Although identifying classroom artifacts to measure teachers' professional noticing always requires a number of considerations and careful selection, we argue that the challenges of selection for use at the secondary level are greater than at the elementary school level. Below we describe the artifacts used in each study and then share the distinct challenge of artifact selection at the secondary level.

Study of K–3 Teachers

Jacobs et al. (2010) used two artifacts to measure grades K–3 teachers' professional noticing. The artifacts exhibited grades K–3 student responses to whole number operation problems. One artifact was a video (called the Lunch Count

[1] Only one of our participants had 2 years of experience. All other participants had 5 or more years of experience.

video) of three students solving and then sharing their solutions to the problem "We have 19 children, and 7 are hot lunch. How many are cold lunch" (p. 177)? The other artifact was a set of three students' written solutions to the problem "Todd has 6 bags of M&M's candies. Each bag has 43 M&M's candies. How many M&M's candies does Todd have?" (p. 178).

Study of Secondary Teachers

In our study of secondary teachers, we used a single classroom artifact to enable us to compare across participants. Teachers watched an 8 minute video of a seventh-grade classroom. We chose a video with mathematical content that everyone (middle and high school) had experience teaching. The video showcased students working together, creating multiple representations, and solving the task in multiple ways, so that teachers had opportunities to notice several features of students' mathematical thinking. In the Beams task (See Figure 1, Carpenter & Romberg, 2004) students are asked to generalize regarding the change in the number of rods as the length of the base increases and justify that this pattern will continue. Near the end of the clip, two students shared their approaches to solving the problem. (Video source, *Powerful Practices in Mathematics & Science*, 2004).

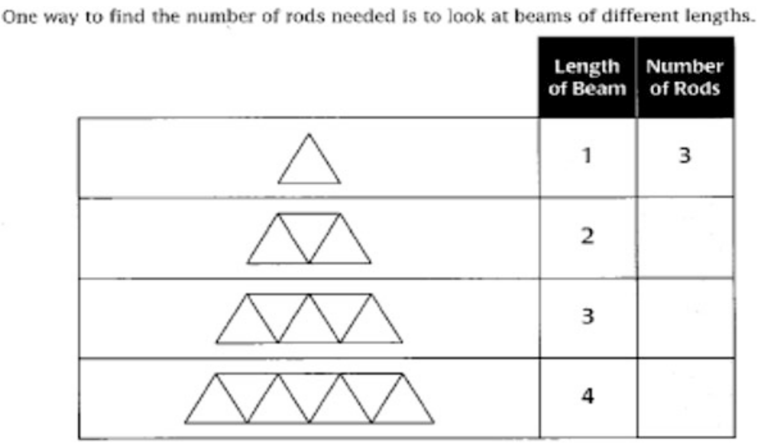

Figure 1. The beams task (Carpenter & Romberg, 2004)

Challenge Selecting Artifacts at the Secondary Level

Researchers seeking to study secondary teachers' professional-noticing encounter the challenge of finding video that showcases students' mathematical thinking. We compare the classroom video clips used in the two studies. Although both clips are approximately the same length and showcase students' mathematical ideas, they differ in important ways. First, the video clip that the K–3 teachers viewed showcased three strategies, whereas the Beams video clip showcased only two, but the Beams video was one of the few we could find that showcased multiple students' mathematical ideas focused on an algebraic topic at the middle or high school level. We believe that the issue of finding video that showcases students' mathematical ideas is a greater challenge at the secondary level than at the elementary school level.

Second, the video shown to K–3 teachers was not widely available; the Lunch Count video was not used in professional development settings or as a resource for professional developers. In contrast, as a resource on the web, the Beams video could be used by professional developers so that participants may have had opportunities to view and discuss the video in other settings. Therefore, high scores on the measure based on the video may be a reflection of the discussions in a professional development setting rather than a measure of the degree of professional noticing the participant was able to provide. Few other examples showcase multiple students' ideas in algebra, and these are widely used (e.g., Boaler & Humphreys, 2005; Seago, Mumme, & Branca, 2004).

Third, the videos one can find at the secondary level do not offer the same variety of students presenting correct and incorrect responses as in the Lunch Count video. Both groups in the Beams video presented relatively sophisticated responses —using the model to justify their formulas.

Challenge 2—Determining Relative Sophistication of Responses

Analyzing levels of sophistication of teacher noticing provides a different set of methodological challenges. Specifically, studies of professional noticing of elementary school teachers have a rich set of developmental trajectories of students' mathematical ideas from which to draw (e.g., Carpenter, Fennema, Franke, Levi, & Empson, 1999; Clarke, Sullivan, Cheeseman, & Clarke, 2000; Clements & Sarama, 2009; Steffe, 1992). Developmental trajectories are far less common at the secondary level (Lamon, 2007; Lobato, Ellis, Charles, & Zbiek, 2010).

Study of K–3 Teachers

Jacobs et al. (2010) coded participant responses on three features: *attending, interpreting, and deciding-how-to-respond.* To score the *attending* responses, Jacobs and her colleagues determined a few key, mathematically important details of each student's solution. These included how students counted and how they may have decomposed numbers to make them easier to manipulate. Then, they looked for evidence in teachers' responses that they were attending to most of the details or only a few. For *interpreting* responses, Jacobs et al. (2010) looked for evidence that teachers were describing individual students' understandings, describing understandings beyond the surface features, and being careful not to over attribute understandings to students. To assess whether teachers interpreted students' understandings beyond a general response, Jacobs and her colleagues used the Cognitively Guided Instruction (CGI) framework to guide analysis (Carpenter et al., 1999). Teachers who provided robust evidence that they were interpreting student understandings through such a framework received higher scores than those who did not (Jacobs et al., 2010).

Similar criteria were used for *deciding how to respond* codes. Codes were determined on the basis of evidence in responses that the participants anticipated students' mathematical ideas or built on the understanding demonstrated in the students' mathematical strategies. Jacobs et al. (2010) wrote,

> We were not seeking a particular next problem or rationale but were instead interested in the extent to which participants based their decisions on what they had learned about the children's understandings from the specific situation and *how consistent their reasoning was with the research on children's mathematical development.*" (p. 189, italics added)

Thus, teachers had freedom when deciding what kinds of problems they might pose next to these children, but to receive high scores, the teachers had to provide evidence that their responses supported the student to build on his or her reasoning in some way, and the teachers' rationales for such a problem likely reflected ideas from the CGI framework (Carpenter et al., 1999). Teachers in the study did not receive high scores if their suggestions were for problems that would be more difficult regardless of who would be solving it. They also were not rewarded for new mathematics without specifying how it would be connected to the child's thinking.

Study of Secondary Teachers

Like Jacobs et al. (2010), we determined, prior to coding, the mathematically important details of each student's solution (e.g., *how* they linked the figure to the formula) and looked for evidence in teachers' responses that they were attending to these details. We categorized responses to each solution on the basis of the number of mathematically significant details to which the teacher attended. When determining *interpreting* codes, we looked for evidence that teachers were describing *individual* students' understandings, describing understandings beyond the general, and being careful not to over attribute understandings to students. For example, a statement attributing a correct solution to students' "good understanding of variable" is general and over attributes. When coding responses for *deciding how to respond,* we accepted various next steps teachers might take. And, consistent with the Jacobs et al. analysis, we analyzed responses for evidence that teachers posed problems and gave rationales for selecting them that built on students' mathematical thinking evident in the video or anticipated students' strategies. Teachers in the study did not receive high scores if they chose a problem on the basis of what mathematics comes next in the curriculum. Nor could they suggest a problem that would provide more practice or one that would be more difficult, as if "more difficult" were the same for all students.

Challenge of Determining Relative Sophistication of Responses at the Secondary Level

In both studies, researchers examined teachers' abilities to attend, interpret, and decide how to respond to students' mathematical thinking. The analytic methods were similar across the three component skills, with one major exception. In the K–3 study, responses with the highest codes for *interpreting* and *deciding how to respond* had to align with the research base. In particular, the researchers looked for evidence that teachers' responses and rationale were consistent with the extensive research base related to children's problem-solving—both the problem-types and problem-solving frameworks shared within Cognitively Guided Instruction (Carpenter et al., 1999), a research program that has existed for more than 30 years. In contrast, in our coding of secondary teachers' responses, the research base on algebraic thinking was a guide, but we lacked a framework as clearly articulated as that for K–3 students' thinking. Coding was based on the three researchers' extensive knowledge of the research in algebra and was facilitated by the fact that few participants focused on the students' mathematical strategies and the understanding conveyed in the use of the strategies. That is, teachers tended *not* to build on the specifics of students' responses when determining next steps, and so the need for a framework was not as critical for this analysis as it would have been had the teachers attended to the specifics of students' ideas. An example of such a response is a general rationale like the following:

> Students can gain a sharp sense of linear relationships when given opportunity to discover constant rates of increase and decrease in one quantity with respect to another. They can then apply this to slopes of lines when working with abstract functions.

This finding leads us to Challenge 3, described in the next section.

Challenge 3—Access to Expert Responses

Access to robust *interpreting* and *deciding how to respond* responses provides researchers with a guide for analysis so that they can refine and describe the characteristics of expert responses and provide examples to coders and readers. The secondary teachers provided far fewer examples of robust responses for interpreting and deciding how to respond than elementary school teachers provided in the earlier study.

Study of K–3 Teachers

Jacobs et al. (2010) analyzed responses of 131 participants, more than 30 of whom (called Emerging Teacher Leaders) had engaged in sustained professional development focused on students' mathematical ideas for four or more years. Because they analyzed responses of a group of prospective teachers as well as three teacher groups, they had access to a range of responses that reflected knowledge of children's mathematical thinking. Among Emerging Teacher Leaders, about three-fourths and two-thirds, respectively, had robust responses for *interpreting* and *deciding how to respond,* providing the researchers with extensive opportunities to characterize and understand expert responses; among Initial Participants, 16% and 3%, respectively, had robust responses for *interpreting* and *deciding how to respond.*

Study of Secondary Teachers

In contrast, none of the 16 participants in the secondary study had robust responses for either *interpreting* or *deciding how to respond,* providing researchers with no opportunity to characterize high-level responses. In an attempt to understand what might characterize robust responses and provide validity, we administered the instrument to four experts with a broad range of experience in research on students' algebraic thinking at the secondary level: two university faculty (one full professor and one with a few years as an assistant professor), one school-district professional development provider, and one doctoral mathematics-education student. These four had extensive teaching experience in high school (including algebra), ranging 6–23 years and extensive knowledge of research on students' algebraic thinking. Drawing heavily on their research of students' algebraic reasoning, all four gave responses deemed robust for *interpreting.* The one expert response and rationale deemed robust for *deciding how to respond* framed the next steps in terms of building on specific students' ideas and anticipating students' strategies for the next problem posed. We note that this expert has the most experience at the primary level and with the noticing literature.

Challenge of Access to Expert Responses at the Secondary Level

As mentioned earlier, access to robust *interpreting* and *deciding how to respond* responses provides researchers with a guide for analysis so that they can refine and describe the characteristics of expert responses and provide examples to coders. The lack of access to expert responses in the area of *deciding how to respond* is problematic not only methodologically but also in considering how to support secondary teachers' abilities to robustly notice students' mathematical ideas. In the next section, we address these concerns and their influence on not only methods but also support for teachers.

Discussion and Conclusion

Artifacts, both video clips and student work, can make visible both common misconceptions and well-conceptualized student thinking, but finding artifacts that showcase students' mathematical ideas at the secondary level can be particularly challenging. Additionally, meaningfully analyzing responses of *interpreting* and *deciding how to respond* to students' mathematical ideas requires knowledge of students' possible learning trajectories, trajectories not as available for secondary mathematical topics as for elementary school topics. Consequently, access to expert responses to noticing prompts is much less common at the secondary level than at the elementary school level. We believe that these issues transcend methodological issues and are also important for professional developers working to support secondary mathematics teachers. We offer our thoughts about the reasons for these differences and next steps to be taken to address these challenges.

Lack of a Research Base of Students' Conceptions of Secondary Mathematics Topics

First, we believe that the three challenges noted above may be rooted in the fact that, in general, the research base for students' conceptions at the secondary level is less robust than at the elementary school level. Although critical research is available for important topics such as algebraic expressions, equations, and functions in middle school (Lloyd, Herbel-Eisenmann, Star, & Zbiek, 2011); functions in high school (Cooney, Beckmann, Lloyd, Wilson & Zbiek 2010); and middle school measurement and geometry (Sinclair, Pimm, Skelin, & Zbiek, 2012), the research at the secondary level tends to be more focused on content and curricular issues and less focused on students' conceptions and how they develop. Additionally, although exceptions exist, examples of exemplary secondary teaching (especially at the high school level) and research related to secondary students' conceptions of mathematical topics are far fewer than those at the elementary school level (Daro, Mosher, & Corcoran, 2011). This relative dearth has consequences not only for studying professional noticing of students' mathematical thinking but also for supporting secondary teachers in noticing their students' mathematical ideas.

In contrast to secondary topics, there exists a rich articulation of students' ideas and essential understandings in many elementary school content areas: early number, whole number reasoning, place value, rational number, geometry, and algebraic reasoning. In coding responses, frameworks like CGI (Carpenter et al., 1999) and Early Numeracy (Clarke et al., 2000) provide support for researchers to determine levels of sophistication. In contrast, these same frameworks do not exist at the secondary level. One notable exception is the work on proportional reasoning (Lamon, 2007; Lobato et al., 2010). Although we provided only one example of a *professional-noticing* study at the elementary school level, many others exist. As another example, Schack et al. (2013) employed a framework to measure how well prospective elementary school teachers could map videos of children's mathematical thinking to the Stages of Elementary Arithmetic Learning (SEAL) trajectory (Steffe, 1992, as cited in Schack et al., 2013). They found that over a five-session module designed to improve professional-noticing skills, preservice teachers (PSTs) significantly increased their abilities to properly attend to identified nuances in children's thinking and place the children's understanding within the correlating SEAL developmental level. Further, the researchers determined that most participants also improved their skills in deciding how to respond relative to the children's developmental progression in early numeracy.

Additionally, the knowledge base students bring to secondary school may vary widely—given the experiences and opportunities to learn afforded them in the previous seven years of schooling—making a somewhat standardized developmental learning trajectory for any particular mathematical topic challenging to articulate at the secondary level. By secondary school, the mathematical roots of elementary school are sprouting branches, and those trajectories are not always straightforward or as consistent as one might expect. These avenues add to multiple considerations for secondary teachers. For example, research on teaching algebra shows that teachers must be aware of an immense number of subtleties and nuances (Chazan & Yerushalmy, 2003; Dreher & Kuntze, 2015).

In secondary mathematics, researchers lack frameworks upon which to draw. In particular, we sought a framework that supported coding for levels of sophistication for students' generalizing and writing a linear function to describe a rate of change in a series of geometric figures. Although developmental learning trajectories may be challenging to formulate, we call on the field to intensify its efforts to better understand and articulate students' conceptions of mathematics topics at the secondary level. Additionally, we call on the field to develop more video of secondary classrooms that highlights those conceptions and how teachers build on them.

Culture of Secondary Mathematics Classrooms

Next, we recognize that secondary teachers have different opportunities from elementary school teachers to professionally notice students' mathematical ideas. Most secondary schools are departmentally organized, so a secondary mathematics teacher may teach mathematics to five different classes of approximately 35 students (Blatchford, Bassett, & Brown, 2011; Ferguson & Fraser, 1998). In contrast, most primary schools designate one teacher to the same class of approximately 30 students, and that teacher will teach all subjects to the same group of students.

Professional noticing of student's mathematical thinking involves interpreting the student's mathematical understanding (Jacobs et al., 2010). Interpreting the student's mathematical understanding is most likely supported by having more knowledge about the student (Sherin et al., 2011). Hence, this component skill may in part be supported by interacting with the same group of students all day, so primary teachers may have an advantage in developing their professional-noticing expertise.

Subject domains are an important context for teaching. Mathematics teachers tend to see the content as relatively constant, with much instruction predetermined. Mathematics has high "sequential dependence." High school mathematics teachers, more than other subjects, feel more constrained by subject-matter culture. McLaughlin & Talbert (2001) suggest that in high school mathematics classrooms "… one would expect to see the 'same' general content and pedagogy, regardless of students or organizational context …" (p. 57).

Although we recognize that cultural differences exist between primary and secondary schools, we acknowledge that classrooms vary significantly with respect to state, district, school, and teacher. We recognize that the organization of secondary schools may influence the culture in certain ways, but many other factors also influence the classroom culture. *What is important for developing this expertise is to have opportunities to interact with student thinking* (Fernandez et al., 2012; Jacobs et al., 2010; Schack et al., 2013). The TIMSS videos portrayed a picture of teacher-centered secondary classrooms. We are encouraged that an awareness of this condition may lead to a change toward more student-centered classrooms.

We note that some studies have already demonstrated that when teachers take up opportunities to interact with student thinking on deeper levels, they can develop more nuanced orientations toward student thinking (e.g., Choppin, 2011). Although Choppin did not relate his teachers' noticing expertise to the levels of evidence used by Jacobs et al. (2010) (No evidence, Limited evidence, Robust evidence), these nuanced orientations may indicate that the teachers in his study were basing more pedagogical decisions on their students' mathematical thinking than those teachers who did not take up such opportunities.

The field of professional noticing of students' mathematical thinking is itself relatively new. Researchers lack an historical, institutional knowledge base from which to draw. We are hopeful that the current interest in learning trajectories will contribute to teachers' and researchers' noticing and abilities to assess noticing, respectively. Significantly, secondary teachers do not have many resources regarding student thinking. We must develop video and materials to help them attend to students' mathematical thinking. At the elementary school level, frameworks and learning trajectories are much more detailed than at the secondary level, including ideas about how to extend student thinking. At the secondary level, the notion of extending student thinking may be more complex. More research articulating development and sophistication of secondary students' mathematical thinking is needed to help secondary teachers interpret and decide how to respond. The field of professional noticing of students' mathematical thinking will continue to mature while we address these methodological challenges for secondary professional noticing of students' mathematical thinking (PNSMT).

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Part V
Exploring the Boundaries of Teacher Noticing

Exploring the Boundaries of Teacher Noticing: Commentary

Miriam Gamoran Sherin

Abstract This chapter explores the construct of teacher noticing and asks "What belongs in a theory of teacher noticing?" To begin, the chapter reviews recent conceptualizations of teaching noticing. The chapter then explores how contributions to this monograph push the boundaries of what constitutes teacher noticing and how it operates. In particular, the chapters in this section suggest ways to expand outwards, extending the construct to new applications, and push inwards, introducing distinctions, and proposing additional sub-substructure.

Keywords Teacher noticing · Teacher cognition · Theory of teacher noticing · Teacher expertise · Perception

What belongs in a theory of teacher noticing? This is the question that I believe we must consider as we reflect on the chapters in this section. As researchers who seek to develop the theoretical constructs that we have for describing teaching, we are routinely faced with an important tension. On the one hand, when we have found a theoretical construct to be productive, and to newly illuminate aspects of teaching, there is a tendency for us to want to push that theoretical construct as far as it will take us. On the other hand, if a theoretical construct is pushed *too* far, then it might become so diluted that it loses the very power that makes it attractive.

The chapters in this section bring us face to face with this tension. In my view, they each push the boundaries of what heretofore has been presumed to be encompassed by a theory of teacher noticing. This raises real dilemmas—but these are interesting and healthy dilemmas. In what follows, I first discuss a bit of the evolution of the construct of teacher noticing in my own work and the work of my collaborators. While many have contributed vital ideas to the development of this construct (in particular see Erickson, 2011; Mason, 2002; Schoenfeld, 2010; van Es, 2011), I focus primarily on my own experiences simply as a way to illustrate

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tensions that we face as we seek to develop the construct of teacher noticing. I then suggest three dimensions along which the construct of teacher noticing might be further developed, each of which is taken up by the chapters here.

Teacher Noticing: A Construct Under Development

I trace the origins of my interest in teacher noticing to experiences I had as a graduate student at the University of California, Berkeley. At the time, I was working with John Frederiksen on an early prototype of a video portfolio for mathematics teaching (Frederiksen, Sipusic, Sherin, & Wolfe, 1998). John's idea was that in order to score the portfolio, one needed to identify noteworthy moments in the video, what he referred to as "callouts" (Frederiksen, 1992). I had been a mathematics teacher prior to graduate school, and had used video extensively while a graduate student. The idea of "callouts" resonated with me. There was too much going on in a classroom at any one moment to be able pay attention to everything—whether one was a researcher or a teacher (Sherin, 2001). Instead, to understand what was taking place in a classroom, one needed to focus on those moments of interaction that stood out for one reason or another.

As I became more and more involved in the project I found myself watching video with an eye towards "making callouts." Others involved in the project experienced a similar development. In fact, the idea of "callouts" had shifted the way we looked at classroom interactions and we found ourselves "seeing" events that had not stood out before. Several members of the project were currently teaching high school mathematics. They found that the notion of callouts spilled over to their instructional practices as well, and that during teaching they found themselves paying particular attention to key events as they took place (Frederiksen et al., 1998). Teacher noticing as *identifying significant interactions* seemed a tractable and important idea.

Over the following decade, as my work on teacher noticing evolved (Sherin, 2007), I found that thinking of teacher noticing only as identifying significant moments of instruction was limiting. First, significant moments were not noticed independent of one another. Instead, what a teacher noticed at one point in time seemed to influence what they noticed in the future. In addition, teachers did not simply identify significant moments of instruction; they also interpreted these events (Sherin & Russ, 2014). For example, when I asked Mark what he noticed in a short video of a group of students using algebra tiles he commented, "She knew the answer was 10 and … that she needed to subtract 8 from 18 to get 10, but she couldn't figure out [how to show it]… And then her friend showed, the guy showed her how to do it… I don't think she'd be able to solve [another] problem, yeah, and I'm not too sure where they're going with it, whether they're solving equations or doing area." What is important to note here is that Mark did not simply state that he noticed that the student in the video knew the answer and that a second student in the group showed the answer with the tiles. In addition, Mark explains what he understands about what he noticed—that he suspected the student would not be able to solve another problem and that he was not sure what the purpose of the task was.

Here we see a first example of the sort of tension that we face in refining the construct of teacher noticing. It does seem that, in some manner, the sort of interpretation evidenced by Mark is very closely related to what he "sees." But does that mean that it should be folded into the construct of teacher noticing, or should it be a separate construct, linked to, but not incorporated within, that construct?

In this case, with my collaborators, I decided that it made sense to fold interpreting into noticing. Here we were influenced both by our own empirical work, as well as existing theory. On the theoretical side, we were influenced by the substantial work on basic perceptual processes in human cognition (e.g., Rumelhart, 1980). In particular, we know that perception is as much a top-down as a bottom-up process. On the empirical side, we found that distinguishing, in our data, noticing from interpretation, was frequently not possible; noticings often came with interpretations.

We were thus led to describing teacher noticing as consisting of two key processes: (1) *identifying*, which we referred to as selective attention, and (2) *interpreting*, which we referred to as knowledge-based reasoning (Sherin, 2007; van Es & Sherin, 2002). *Selective attention* reflected the idea that teaching involves attending to some interactions while not attending to others, in other words, identifying key events. *Knowledge-based reasoning* concerned the ways in which teachers interpret what they notice, and in particular, that they likely draw on a range of prior knowledge and experience to do so. Furthermore, like all perceptual processes, we argued that selective attention and knowledge-based reasoning interact in a dynamic manner. In some cases, selective attention may drive a teacher's knowledge-based reasoning. For example, noticing an unexpected student error might prompt the teacher to reason about why that error arose. In other instances, a teacher's knowledge and experiences will influence what the teacher notices. Thus, if a teacher suspects that logarithms are particularly difficult for her students, this might prompt the teacher to be on the lookout for specific types of errors.

This discussion points to heuristics that we should apply as we seek to further expand the concept of noticing. If, in our data, there are universal concomitants of noticing events (such as interpreting), then this suggests that perhaps there is something about these concomitants that should be folded into the noticing construct. However, we should not only be driven by data; theoretical constructs get their meaning from how they participate in larger models and theories. Thus we should also have in mind larger theoretical concerns, and we should in particular be careful not to create too many free-floating constructs.

It is worth noticing that our expansion of teacher noticing to include interpreting has not been entirely non-controversial. While many researchers have adopted a view of noticing as involving both identifying and interpreting classroom interactions, some researchers (e.g., Star & Strickland, 2008) continue to focus primarily on *identifying* as the central component of teacher noticing (see Sherin, Jacobs, & Philipp, 2011 for a further discussion of this issue).

Castro-Superfine, Fisher, Bragelman, and Amador claim that it can be difficult to distinguish between identifying and interpreting when asking teachers to comment on classroom interactions. Sherin and Star (2011) make a similar argument. For example, when asked what he noticed in a video of a whole class discussion of ratios Dan commented, "If I were the teacher I would have wanted to gather the class's attention. I have a thing that I do to get everybody focused on me with a hand signal" (Sherin & Russ, 2014). Here, Dan is not explicit about what he noticed in the video. It might be the case that Dan noticed something about a lack of attention on the part of the students, but where *identifying* stops and *interpreting* begins is unclear. Furthermore, Castro-Superfine et al. explain that, as discussed above, we must not mistake these processes as happening chronologically, in that first a teacher attends to an event and then interprets that event. Rather, the relationship between these processes is dynamic and complex.

There have been recent attempts to push the construct of noticing still farther, beyond selecting and interpreting. In 2010, Vicki Jacobs, Lisa Lamb, and Randy Philipp published an article in which they argued that teacher noticing (more specifically, what they refer to as *professional noticing of children's mathematical thinking*) consisted of three central processes: (a) attending to students' ideas, (b) interpreting these ideas, and (c) deciding how to respond. The inclusion of *deciding how to respond* is, of course, a very significant move. Jacobs and her colleagues argued that deciding how to respond should not be separated from identifying and interpreting because the three processes happen at the same time. As they explained, "We suggest that…the three component skills of professional noticing of children's mathematical thinking—attending, interpreting, and deciding how to respond—happen … almost simultaneously, as if constituting a single, integrated teaching move. Thus, our conceptualization of the construct of professional noticing of children's mathematical thinking makes explicit the three component skills but also identifies them as an integrated set" (Jacobs, Lamb, & Philipp, 2010, pp. 173–174).

In some respects, this argument is very similar to the one described above, for incorporating interpreting into the noticing construct. In particular, it draws on a similar empirical heuristic: If there is, empirically, a universal concomitant of noticing, then it might make sense to fold this concomitant into our theory. But, in some ways, this case is less clear-cut. It is not clear to me that there is the same theoretical force beyond this move; I am unaware of models of perception, which argue that deciding what to do is tightly integrated into perception. Furthermore, there can be features of our models that occur together, while still being treated as separate elements. So concomitance is not the only criterion we should consider.

It happens that all five chapters in this section draw on Jacobs et al.'s (2010) perspective on teacher noticing. It is clear to me that this perspective has proven quite valuable in advancing our understanding of what it means for teachers to learn to productively notice students' mathematical ideas. In particular, Jacobs, Lamb, Philipp, and Schappelle (2011) documented that being able to identify children's mathematical thinking is foundational to being able to decide how to respond to children based on their thinking, and yet this skill develops only over extended time and with a great deal of experience attending to and interpreting children's mathematical thinking.

New Approaches to Extending Teacher Noticing

The five chapters in this section further push the boundaries of what constitutes teacher noticing and how it operates. They do so in three main ways that both expand outwards, extending the construct to new applications, and push inwards, introducing distinctions, and proposing additional sub-substructure. First, the chapters by Amador, Males, Earnest and Dietiker and Choy, Thomas and Yoon extend the construct of teacher noticing by expanding the instructional activities in which we understand teacher noticing to take place.

Typically, research has focused on understanding teacher noticing as a process that occurs during instruction. To be clear, efforts to examine teacher noticing have sometimes used a proxy for noticing during teaching—asking teachers to comment on videos of instruction, for example (e.g., Borko et al., 2015; Goldsmith & Seago, 2012; van Es, 2011). But this has primarily been done because of the difficulty in accessing teacher noticing in the moments of instruction (Sherin, Russ, & Colestock, 2011). In contrast, these chapters suggest that we should not restrict our study of teacher noticing to moments in which teachers interact with students and that, instead, we should think of noticing as something that occurs when teachers prepare for and reflect on their teaching experiences. This is a dramatic—and extremely interesting—proposal.

Specifically, Amador et al. contend that research on teacher noticing should be applied to the study of how teachers use curriculum materials. They introduce a new type of noticing, "curricular noticing," a set of interrelated skills related to attending to, interpreting, and deciding how to respond to curriculum materials. Furthermore, they use this framework to examine the development of preservice teachers' ability to productively adapt curriculum materials. With respect to the larger theory of teacher noticing, I understand their proposal as a start toward listing multiple species of noticing, which must include both curricular noticing and something we might call classroom noticing. Presumably these different species of noticing would share some attributes, such as both involving both selective attention and knowledge-based reasoning. But they must also be distinct in some ways.

Choy et al. take a related approach and illustrate the ways in which teacher noticing aligns with instructional practices that occur before, during and after instruction. For example, Choy et al. argue that prior to instruction, teachers can be thought of as preparing to notice (Mason, 2002) as they design lessons in order to reveal students' thinking. Similarly, following a lesson, teachers can engage in "post-noticing" as they continue to analyze the students' thinking that was revealed in the lesson (Mason, 2011). In doing so, Choy et al. demonstrate that the three key noticing processes of attending, interpreting and deciding can figure in noticing-like processes that occur in contexts other than live practice.

A second direction taken in extending our understanding of teacher noticing is illustrated by the chapters by Stockero, Leatham, Van Zoest, and Peterson and Tran and Wright. Rather than broadening the construct of teacher noticing to new contexts, these authors enhance our understanding of this construct by looking inside the construct and dividing it up in new ways. For example, Stockero et al. claim that we need to go beyond simply *identifying* students' ideas and should emphasize the need for teachers to *discriminate* among the ideas that students raise in order to select those that would be most productive for the class to examine. In addition, Stockero et al. further develop the processes involved in identifying and interpreting student thinking by contrasting *noticing among* students' ideas and *noticing within* students' ideas. Teachers notice *among* students' ideas when they look across a set of ideas raised by students to understand the mathematical thinking at play; teachers notice *within* students' ideas when they focus on the particulars of a specific student's thinking.

The chapter by Tran and Wright can also be understood as an attempt to deconstruct teacher noticing further. In particular, Tran and Wright examine the relationship between Jacobs et al.'s framework for teacher noticing and a number of key instructional practices. For example, the authors suggest that the act of addressing an incorrect response from a student can be decomposed into smaller moments in which the teacher attends to the student's strategy, interprets the student's idea, and decides how to respond. In doing so, Tran and Wright enrich our models of how teacher noticing operates, in the very moments of teaching.

A third way that the papers in this section extend the notion of teacher noticing is by expanding the theory to include ideas about how teacher noticing develops. If we believe that teacher noticing is a key component of teaching expertise, then working towards a theory of how teachers learn to notice is a critical step. Furthermore, any theory of teacher noticing must be consistent with a reasonable model of how the ability to notice might develop. If we cannot explicate possible origins of teacher noticing, and a theory of how it might develop, then that would be a serious problem for our models of noticing. Thus, I believe that as we think about teacher noticing, it makes sense to always have some ideas about development in mind.

All of the chapters in this section touch upon this goal, and it is a central focus of the chapter by Castro-Superfine et al. Of particular interest is their prescriptive claim that developing preservice teachers' ability to notice might be best served by working first to hone their ability to interpret students' ideas and following this, to develop their skills in the area of identifying significant interactions on which to focus. In making this claim, Castro-Superfine et al. extend our understanding of the nature of these processes in development, that is, how teachers learn to integrate them into their practice as they develop noticing expertise.

What belongs in a theory of teacher noticing? And what belongs outside of that theory? The chapters in this section make valuable contributions by pushing the boundaries of what teacher noticing is, what processes are involved, and how it develops among teachers. Given the increased attention that teacher noticing has received in the field, continuing to expand this construct is an important goal. As these chapters illustrate, doing so can open up productive new avenues for understanding the expertise underlying and driving mathematics teaching practices.

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Shifting Perspectives on Preservice Teachers' Noticing of Children's Mathematical Thinking

Alison Castro Superfine, Amanda Fisher, John Bragelman and Julie M. Amador

Abstract Noticing children's mathematical thinking is an important aspect of what teachers need to know. Researchers generally agree that noticing involves two main processes, namely attending to and making sense of particular events in an instructional setting. We report on our work involving preservice teacher noticing and our efforts to scaffold their noticing. We argue for a shift in perspective on preservice teacher noticing, a perspective that considers interpreting classroom events as an important first step for preservice teachers in their development of noticing, which then positions preservice teachers to attend to important and noteworthy events.

Keywords Preservice · Elementary · Mathematics · Videocases · Noticing

Introduction

The use of video to support teacher learning, in both inservice and preservice contexts, has grown considerably in the field of teacher education (see Brophy, 2008). Video provides a medium in which teachers can critically analyze teaching practice in ways that are safely distanced from their own teaching experiences. In addition, such a medium affords more time for teachers to respond to and reflect on what they are

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observing, and also provides a narrower view of classroom interactions and thus a more focused investigation of children's thinking (Ball & Cohen, 1999; Sherin, 2001). To understand what and how teachers learn from such uses of video, researchers have focused on what events and interactions teachers attend to in video and how they make sense of those events and interactions, using the construct of *teacher noticing* to describe such questions (c.f., Sherin, Jacobs, & Philipp, 2011). Teacher noticing is generally defined as attending to and making sense of particular events in an instructional setting (Sherin, 2011). Indeed, "[effective] teaching involves observing students, listening carefully to their ideas and explanations…and using the information to make instructional decisions" (NCTM, 2000, p. 19). If the expectation for teachers is to attend to and understand children's thinking, then preservice teachers (PSTs) need to learn to productively notice children's thinking (Erickson, 2011).

Much of the extant research focused on PST noticing presents a mixed picture of effectiveness. For example, some research studies indicate that PSTs can develop the ability to notice with structured support (Fernandez, Llinares, & Valls, 2012; Star, Lynch, & Perova, 2011; Star & Strickland, 2008). In contrast, in our own work, we often found no significant improvements in PSTs' noticing in courses involving a videocase curriculum (Castro Superfine & Li, 2011; Castro Superfine, Li, Bragelman, & Fisher, 2015). We conceptualized our earlier research studies, however, without considering how intrinsically interrelated the components of teacher noticing are, and how the relationship among components may look different for PSTs who typically do not have the same wealth of teaching experiences and knowledge that inservice teachers draw on when viewing video. Expert teachers, for example, are able to recognize which features of classroom practice warrant attention and generate hypotheses about children based on available information (Carter, Cushing, Sabers, Stein, & Berliner, 1988). Novices, on the other hand, describe what they see but not with the same depth or accuracy as experts (Carter et al., 1988; Jacobs, Lamb, & Philipp, 2010). Thus, it seems reasonable to expect that novices, such as PSTs, may struggle to notice children's thinking in video.

In this chapter, we report on our work involving PST noticing and our efforts to scaffold their noticing. We begin by arguing for a shift in perspective on PST noticing, a perspective that considers interpreting classroom events as an important first step for PSTs in their development of noticing, which then positions PSTs to attend to important and noteworthy events. We focus on the following research questions: (1) To what extent can PSTs' interpret children's mathematical thinking after it is explicitly identified for them? and (2) To what extent can PSTs attend to CMT following scaffolding to support their interpretations?

Defining Teacher Noticing

While researchers generally agree that noticing involves two main processes, namely attending to and making sense of particular events in an instructional setting, researchers have conceptualized noticing in quite different ways (Sherin, Jacobs, & Philipp, 2011). For example, van Es and Sherin (2008) and Sherin and van Es (2005) define noticing as comprised of three components: identifying what is important in a teaching situation, interpreting what is noticed, and linking noticed events with broader principles of teaching and learning. In their work, they are concerned with teacher noticing of a broad range of events and interactions, including teacher instructional moves and children's thinking, among others. In contrast, other researchers focus on noticing a particular feature of teaching, and are concerned with a more narrow focus of what is noticed. Jacobs et al. (2010), for example, focus on the professional noticing of children's mathematical thinking (CMT), which is comprised of three components: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understanding. Still, other researchers narrowly define noticing as what is and is not attended to in an instructional setting, where a broad range of classroom events and interactions are noticed (e.g., Sherin, Russ, & Colestock, 2011; Star & Strickland, 2008; Star et al., 2011). All of these conceptualizations of noticing not only have methodological implications but were derived to support the researchers' goals and particular settings in which researchers were working with teachers (e.g., inservice teacher professional development, teacher education coursework). This is not to say that one conceptualization or foci of teacher noticing is better suited or more mutually agreed upon than another, but rather different conceptualizations of noticing make visible different aspects of teacher learning and decision-making.

In our own work with PST noticing, we define noticing as attending to and interpreting particular features of classroom practice. Drawing from Star and Strickland (2008) and Star et al. (2011), we argue that PSTs need to learn to productively attend to pertinent features (e.g., students working collaboratively, teacher asking students certain types of questions) of an instructional setting. Furthermore, PSTs need to be able to make sense of and understand the features to which they attend. At the same time, we are concerned with a narrow noticing of particular features of classroom practice. Similar to Jacobs et al. (2010), we focus on PSTs' noticing of CMT, as the noticing of such features of classroom practice are part of what PSTs will be called upon to do in their future work as teachers of mathematics. In short, we conceptualize PST noticing as attending to and interpreting CMT.

Expert–Novice Differences in Noticing

There are considerable differences between what experts and novices notice about teaching practice in video. One difference between experts and novices is an awareness of what is important to react to and what to ignore (Miller, 2011). Experts, for example, are able to recognize which features of classroom practice warrant attention and generate hypotheses about children based on available information, often drawing on their past experiences in, and knowledge of, the classroom (Carter et al., 1988). In their study of teacher professional noticing of CMT, Jacobs et al.

(2010) found that teachers with more expertise were better able to recall details of children's strategies, a finding that is consistent with expertise research on how experts hold knowledge (Donovan, Bransford, & Pellegrino, 2000).

Novices, on the other hand, describe what they see, but not with the same depth or accuracy as experts (Carter et al., 1988; Jacobs et al., 2010) and struggle in determining which features of classroom practice warrant attention (Lampert & Ball, 1998). For example, in their study of the use of video conferencing with classroom lesson clips, Sharpe et al. (2003) found that PSTs struggled to make sense of a three minute video clip and thus had to watch the clip several times in order to understand what was being viewed. Novice observers, particularly PSTs, also focus on the more ritualistic aspects of teaching practice [e.g., teacher moves, classroom management (Star & Strickland, 2008)] and less on understanding children's thinking. Indeed, despite having spent years in the classroom as learners, PSTs have limited conceptions of what the work of teaching entails and have not been privy to decisions and actions teachers make to support children's learning (Lampert & Ball, 1998).

Preservice Teachers' Noticing of CMT

Research specific to noticing of CMT indicates PSTs have difficulty attending to salient moments of CMT and often struggle to interpret these events (Morris, 2006; Star & Strickland, 2008; Star et al., 2011). Morris (2006), for example, focused on PSTs' abilities to analyze CMT and support their analysis with evidence. Findings indicated that PSTs could analyze cause and effect of instructional strategies related to learning, but lacked the ability to collect evidence to support their interpretations about children's thinking. In related work focused on PSTs noticing CMT, Fernandez et al. (2012) studied the noticing of PSTs to understand how the professional practice could be developed. At the onset of the study, PSTs had a difficult time attending to or interpreting CMT. They commonly described CMT without mentioning significant aspects about the situation or children's strategies, further illuminating the importance of developing these skills among PSTs. Related studies also support the notion that PSTs often have difficulty noticing the mathematical content and children's mathematical understandings within lessons (Star & Strickland, 2008; Star et al., 2011). For example, Amador and Weiland (2015) engaged PSTs in a structured lesson study process to support their noticing of CMT and noted that of all utterances considered to be noticing (i.e., attending and interpreting), only 6% were specific to CMT. These findings support the notion that PSTs struggle in attending to and interpreting CMT.

Similarly, in the first phase of our work, we examined the effectiveness of a videocase curriculum for supporting PST noticing. We conducted a quasi-experimental study of changes in PSTs' knowledge of, beliefs about, and ability to notice CMT to understand the overall effectiveness of the videocases. In one study, we found no significant differences in PSTs' knowledge of, beliefs about, or noticing of CMT between control and treatment groups (Castro Superfine

& Li, 2011; Castro Superfine et al., 2015). These findings suggested that the use of videocases to support PSTs' ability to notice children's thinking, in particular, seemed ineffective. Other findings from our work also suggested minimal changes in PSTs' noticing children's thinking over time with the videocases (Castro Superfine & Groza, 2012; Li & Castro Superfine, 2011, 2012).

In short, our work with PST noticing builds on more narrow conceptualizations of teacher noticing, and draws from expert–novice research and extant research on PST noticing. An assumption underlying our work is that, like novices, PSTs need support in noticing. Because PSTs struggle to make sense of children's thinking (e.g., Castro Superfine et al., 2015; Fernandez et al., 2012), we scaffold PSTs' interpretations by asking PSTs questions that are targeted to the substantive mathematics underlying children's thinking. As we scaffold their interpretations of children's thinking, we are implicitly, and simultaneously, scaffolding their attending by directing their attention to the noteworthy aspects of CMT. Indeed, developing expertise in noticing is as much about what is noticed as it is about what is not noticed. As PSTs develop their interpretation skills, and the scaffolding is removed, they are able to attend not just to CMT, but are able to attend to substantive aspects of CMT. Our focus in this chapter is to provide empirical evidence for such a theoretical shift in our understanding of PST noticing.

Situating Our Perspective on Noticing

Our data suggests that when scaffolded in their interpretations first and then in their attending to what is in video, PSTs are able to notice CMT in more robust ways. We first situate our perspective on noticing by describing the evolution of our work on PST noticing, and then elaborate on these findings and discuss the implications of our work for research on PST noticing.

The VPEM Project

The goal of the Videocases for Preservice Elementary Mathematics (VPEM) Project is to support PSTs noticing through the implementation of a collection of videocases designed to highlight aspects of CMT. The VPEM Project began with a collection of videocases focused on CMT, initially designed for use with PSTs in mathematics content courses. Our Phase 1 research suggested that the use of videocases to support PSTs' ability to notice children's thinking seemed ineffective. However, we hypothesized that with more support in viewing the videocases, they could still be effective in supporting PSTs' noticing, including their skills at both attending to and interpreting CMT. Following this initial phase of our work, we developed the VPEM online platform whereby PSTs view the videocases online, and their viewing of the videocases is scaffolded with different features.

VPEM Online Platform

Developed with funds from the National Science Foundation and the University of Illinois at Chicago, the VPEM platform is uniquely designed to scaffold PSTs' noticing of CMT in video format. The design framework for the VPEM online platform is presented in Figure 1.

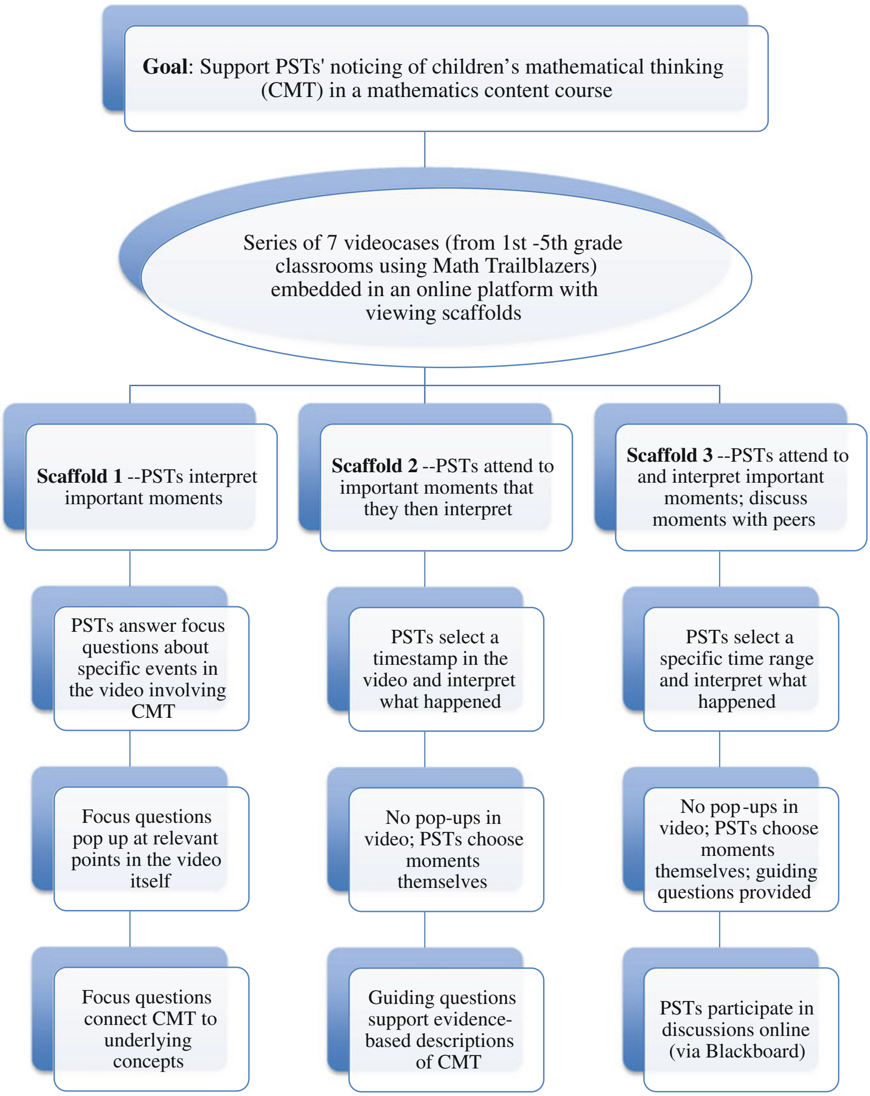

Figure 1. VPEM online platform framework.

Drawing from Vygotsky's (1978) notion of zone of proximal development and Wood, Bruner, and Ross's (1976) work on scaffolding, the VPEM online platform includes a series of scaffold levels that support a shift across noticing levels, moving from providing non-evidence-based descriptive comments to highlighting noteworthy events that attend to CMT as described by van Es (2011). In this way, we draw from van Es (2011) to inform the design of the different scaffold levels. Further, in accordance with van Es (2011) we recognize that the development of noticing may not always be linear and PSTs may shift among different levels of noticing as they develop the component skills (Figure 2).

The first scaffold level is designed to support PSTs' transition from baseline level of noticing, in which they form general impressions, to a mixed level of noticing (i.e., descriptive comments with some interpretation). The videos in this scaffold include pop-up questions that appear when a notable moment is happening, prompting PSTs to respond to those questions in the platform. This is typically an out-of-class assignment. The instructor then downloads the responses and uses those responses to structure the related in-class discussions. This format allows opportunities for PSTs to discuss important moments in the videos without expecting PSTs to highlight noteworthy events on their own. Highlighting noteworthy events is a hallmark of mixed level noticing and higher (van Es, 2011), and thus we do not expect PSTs to be able to do so in the first scaffold level. Also during the first scaffold, the instructor can focus the in-class discussion on comments that are most related to the mathematical goals of the lesson featured in the video, offering PSTs opportunities to move away from the baseline noticing of the classroom environment (Figure 3).

Figure 2. Example of pop-up question in the video.

Figure 3. Platform comment section for PST responses for first scaffold level.

The second scaffold level requires PSTs to take over the selection of important moments, rather than having important moments predetermined by the pop-ups in the video. By the time they encounter the second scaffold, PSTs have had multiple opportunities to view important moments that are preselected and have responded to multiple focus questions that are targeted to CMT. When the instructor downloads the comments on PSTs' selections, it is possible to sort the comments by start time and arrange the in-class discussion around particular moments (Figure 4).

Figure 4. Platform comment section for PST responses for second scaffold level.

Finally, in the third scaffold level, the in-class discussion portion is shifted to an online discussion, thus promoting interactions among PSTs in the course and removing the instructor as an intermediary in the discussion of the videocase (Figure 5).

Figure 5. Platform comment section for PST responses for third scaffold level.

Evidence of Noticing

A two-stage coding scheme was followed to explore whether PST responses to videos from scaffold levels two and three attended to CMT. The first stage assessed whether PST responses described some aspect of what the children were doing or saying about mathematics. This was an initial attempt to see if PSTs could focus in on important moments without being directed to them via pop-ups. Two coders were assigned to each videocase, and for each case there was greater than 90% reliability. Final codes were agreed upon by the two coders before moving on to the next coding stage. Results for this stage are in Figure 6. In scaffold two, 93.63, 90.00, and 97.42% of PST responses discussed CMT in some way. In scaffold 3, 94.94% discussed CMT. Recall that the mathematical content in the videocases for the scaffold levels were intentionally different.

Video	**Number of Comments**	**CMT**
Scaffold 2		
Representations	144	97.92%
Remainders	200	90.00%
Fractions	204	93.63%
Scaffold 3		
Patterns	99	94.64%

Figure 6. Percentage of PST responses discussing CMT.

The second stage of coding explored the degree of evidence in the PST response for attending to CMT. Following from the coding scheme developed by Jacobs et al. (2010), we identified three categories that describe the depth of attending to CMT: no evidence of attending, limited evidence of attending, and robust evidence of attending. We purposely drew from this coding scheme because it was specific to CMT and fit with our definition of noticing, whereas other coding schemes for noticing (e.g., van Es, 2011) were not specific to CMT. A PST response with no evidence of attending did not describe any details about the children's solution strategies or how children solved the problem. A PST response with limited evidence of attending included some aspect of the children's solution strategy or how the children solved the problem. Finally, a PST response with robust evidence of attending included children's solution strategies and some interpretation or inference that attempted to explain why the children did or did not understand the mathematical concept or factors that contributed to the children's understanding. In other words, a response with limited evidence included evidence of only attending while a response with robust evidence included evidence of both attending and interpreting. Doing so allowed us to examine the relationship between these component skills of noticing, and specifically, to examine whether an interpretation always followed attention to CMT or vice versa. A distribution of PST responses for one videocase can be seen in Figure 7, and examples of PST responses coded at each level of attending follow.

Videocase & Evidence Level	**Percentage**
Fractions	
No Evidence	15.18%
Limited Evidence	38.74%
Robust Evidence	46.07%

Figure 7. Distribution of PST attending responses displaying evidence of CMT in one videocase.

The following, Figure 8, represents a series of PST response categories by the evidence of attending in the response. The responses are separated by scaffold and videocase.

	No Evidence of Attending to CMT	Limited Evidence of Attending to CMT	Robust Evidence of Attending to CMT
Scaffold 2--Fractions	Ali did not show the correct solution the teacher asked for, but came up with another solution - 1 1/2. Then changes his mind when he is told his answer is correct.	Rebecca wrote out her word sentence the expanded form. She wrote out 1/3 + 1/3 + 1/3 which she knew added up to 3/3 and then added 8/12 because she knew that simplified down to 2/3 and when you added it all up you got 5/3.	In minute 2:20 it seems that Alex has a good understanding of what pieces can make a whole and what is the relationship/proportion of those pieces to the whole. Alex also shows to know how to write the representation in a mathematical sentence and adds the fractions having as a guide the representation. In the other hand the girl that went up and tried to do another representation in minute 4:30 shows to have more difficult time when making a representation with smaller pieces like twelfths. This can show that to the students its easier to make a representation and add smaller denominator fractions like 1/3 than bigger ones like 1/12.
Scaffold 2--Remainders	The boy had a clear picture of how many tables were needed to fit the people in each table. He then kept insisting that there were 3 remainders. He also tried to ask his friend what	Shanna kept trying to make Anthony understand that the remainder was 1 and contradicted herself a couple of times. At times Shanna did explain correctly	The girl is using what she knows about multiplication [to] elaborate a different strategy to solve the problem. She considers 15 as 16-1 because she can easily figure out that 16:4=4, but she calls the extra person a reminder of 1. Her strategy is effective; it is the concept of reminder

Figure 8. PST response categories by the evidence of attending in the response.

	they had to do with the 3 remainders. They are basically looking for the number of people in each table, but they are supposed to see how many total tables they need not how many people in each table. They basically sort of misinterpreted the problem.	how there were 15 people that needed to be seated and kept saying that there is 1 remainder which she stated as being one person to be taken out and then as being one seat left. (8:20) Anthony and Shanna were more focused on finding which was the remainder that they went off a little from what the question was asking which was how many tables did Tina need if she were to use tables that seat four.	that is not clear. Both students find the same answer, which is four tables. For example, the boy finds is four tables by using the method of counting up since there are three people standing. However, the girl find the number of tables by using multiplication. She knows that 16 people fit on 4 tables, and she knows that she only needs 15 spots; therefore, she knows that 4 tables are needed. It shows that children have their own way of thinking depending on how advanced they are on their knowledge on math
Scaffold 2--Representations	I think that the boy is very smart when he puts it in terms of 100. I believe that it is easier to understand in terms of 100 because 100 is the whole when figuring out percents. The students are using their previous knowledge of halves in order to	So the students are given a task to show using the centiwheel how 1/20 is written as a decimal. At the time 00.59 the students yells "cinco, cinco, cinco" he knew the answer, but the other students continued on doing what they thought was the correct method. The other student ignores the	The boy was explaining to the other boy that if you looked at 1/20 as a decimal it would be 5. He pointed out the equation on the board (1/10) and said that if you replaced the 1 with a 2, you would have 2/10, which would give you 5. Even though that is technically not true, you would actually get 1/5, which as a percent would be 20%. But we are trying to represent 1/20 as a decimal and percent. I think the boy thinks that 1/20 is equal to 0.2 or 20% because he sees that 2/10 is 0.2,

Figure 8. (continued)

	solve the problem and explain. It helps that the numbers are easy to divide.	student who is clearly giving them the answer saying, "1/20 is 5". They continue to discuss why 1/20 as a decimal is 0.2 or 20 and the percent is 20% which is incorrect.	which is incorrect.

Figure 8. (continued)

Our first stage in the coding showed the PSTs were indeed attending to CMT but it was clear that not all PSTs were attending to the same degree and that some PSTs provided more evidence of their attending than others. Some PSTs were attending to CMT by providing a restatement of actions while others attended to and interpreted CMT. Thus, the second stage of coding attempted to clarify how robustly the PSTs were attending to CMT. We found that in certain videocases such as Fractions, the PSTs were able to provide robust evidence that they are attending to CMT more often than they provided limited or no evidence. Overall, our findings suggest that the theoretical shift in perspective described above can be supportive for PSTs. In other words, when PSTs are first asked to interpret without attending and then asked to attend on their own, they are able to identify and describe important moments for understanding CMT and are also able to interpret those moments.

Issues Emerging from Our Research

Throughout the evolution of our work on PST noticing, several issues have emerged that we argue are important for researchers to consider as the field continues to refine the construct of teacher noticing. We discuss each of these issues in the following sections, and suggest directions for researchers moving forward.

Interrelationship Between Attending and Interpreting

One issue emerging from our research relates to the difficulty in distinguishing attending from interpreting and identifying the role of evidence as either a support of interpretation or as evidence of attending. Many definitions of noticing (e.g., Jacobs et al., 2010; van Es & Sherin, 2008) have distinguished between these two components of noticing, attending and interpreting, and research studies have measured attending and interpreting skills independently of each other. However, in our work, identifying parts of PST responses as one or the other has proved problematic, as these two components of noticing are closely related, meaning at what point does attending to CMT involve interpretation and at what point does interpretation of CMT result in further attending. Given research that suggests novices do not identify important events and that they struggle to provide evidence-based descriptions of events, we posit that PSTs' attending and interpreting are inextricably linked.

To further understand the interrelatedness of attending and interpreting, consider the following excerpt from a PST, "The students are using their previous knowledge of halves in order to solve the problem and explain. It helps that the numbers are easy to divide." We determined that there was an interpretation made by the PST without explicit attending. This PST neglected to include evidence to support the claim that children were using previous knowledge and it is unclear how this PST arrived at that interpretation. It is possible that the PST attended to the children's thinking about halves, but was not explicit in making such a statement.

In contrast, this example of a PST response includes evidence of both attending to and interpreting CMT:

> At this point, the students are trying to show 1/20 on their centiwheels. It seems like the student closest to the camera is having trouble interpreting what fractions would look like. In the video he tries to show 1/30 and 1/40 as well but instead of making those pieces smaller he makes them larger. From this, it seems as though this student doesn't understand that the larger the denominator in a fraction the smaller that piece is … When he sees 1/30 and 1/40 he assumes 1/40 is larger because he already knows 40 is larger than 30, but when they are put into the denominators of fractions the 1/40 becomes smaller than 1/30.

In this response, we determined the PST included an interpretation with evidence related to that which had been attended to, noting what the child did in the video (i.e., attending) and forming conclusions about what the child knew (i.e., interpreting).

Finally, PSTs are able to attend to important events without necessarily making interpretations about what they attended to. The following PST response serves as an example: "In this part of the video, the students took the example from the board to make the fraction (1/20) into a decimal. The example on the board was a fraction that was out of 100, so when he got the decimal 0.2, his answer was wrong." In this response, the PST described what occurred, but did not make interpretations about what the children knew or did not know. Thus, PSTs expressions of attending and interpreting are complex, with some responses including evidence of attending to children's thinking and others void of any evidence of attending to CMT.

As researchers, the interrelatedness of attending and interpreting presented a unique challenge as we examined PSTs' development of these component skills of noticing. Initially, we were interested in determining whether or not PSTs were interpreting and then whether or not they were interpreting and supporting their claims with evidence. We were careful to note whether or not the evidence provided was directly linked to the interpretations. Some PSTs made interpretations and included descriptions of what students said or did, but the two components were not related. Thus, we concluded that they attended to CMT, but did not interpret that thinking on which they had explicitly attended. This process became problematic as we considered the extent to which their evidence of attending was lacking or robust. Recall that we parsed this into three categories: no evidence of attending, limited evidence of attending, and robust evidence of attending, similar to Jacobs et al. (2010); however, this became difficult in the instances when interpretations occurred without any evidence of attending to children's thinking or in cases when the PSTs were evaluative instead of objective. We situate this within our aforementioned definition of noticing as attending and interpreting to make sense of how PSTs were noticing, but argue that existing frameworks, and our modified frameworks for analysis may not fully capture all intricacies of noticing. Specifically, the van Es (2011) framework for learning to notice situates attending and interpreting on a continuum, which was not fully supported with our data (i.e., instances when interpreting occurred without explicit evidence of attending). This raises further questions about the chronology of components of noticing and the order in which attending and interpreting occur for PSTs, and how we analyze noticing as a research field.

From our research, we have concluded that Jacobs et al.'s (2010) definition of the components of noticing as interrelated skills may suggest difficulty for researchers attempting to disaggregate these two components of noticing to determine order of occurrence when analyzing PSTs' noticing of children's thinking. In other words, evidence from PSTs that include attending or interpreting do not necessarily suggest chronological order, rather it is possible that interpretations occur before attending in some cases and in others, attending likely occurs before interpretations are made. However, we argue that if PSTs are provided with scaffolds that support their attending and provide for a clear focus on CMT, they can in fact begin to interpret children's thinking. They do not need to explicitly attend to CMT in order to interpret, as evidenced by the examples presented. The interpretation can take place prior to any attending with the proper supports. Following this, PSTs can learn to support their interpretations with evidence through attending to children's thinking and later attend and interpret on their own. Thus, being able to interpret classroom events is an important first step in being able to later attend to and interpret CMT.

Complexity of Video Representations

Another issue emerging from our research is that not all video is created equal, which has considerable implications for scaffolding PSTs' noticing. Indeed, there are a variety of types of video that are used in teacher education, including commercially produced videos, videos of teachers' own classrooms, and videos of other teachers' classrooms. When used in teacher education, these different types of videos are often edited for different purposes and foci, thus highlighting certain aspects of teaching and learning in the captured events while masking others.

Incorporating video clips that are more or less complex in terms of the nature of the teaching and learning events is particularly important for novices, such as PSTs, who often struggle to pay attention to children's thinking in video (Jacobs et al., 2010), and tend to focus on aspects of pedagogy or classroom management rather than CMT (Star & Strickland, 2008).

We have started to examine the nature of the captured events in the VPEM videocases. Drawn from a large database of elementary classroom footage, our videocases were initially developed and edited to focus explicitly on classroom events where CMT was the focus of the scene. Yet, our prior research indicated that, despite various revisions to the video clips, accompanying focus questions, and viewing scaffolds built into the video clips, PSTs still struggled to attend to children's thinking in robust ways (Castro Superfine et al., 2015). These results pointed us to consider the nature of the teaching and learning events captured in the video clips used in the project. Video is a type of representation of complex teaching and learning practices, which highlights the salient teaching and learning events and at the same time fails to capture other events related to the represented events (Hatch & Grossman, 2009). We define this simultaneous highlighting and masking as the *complexity* of the video clips.

We developed a framework for analyzing the complexity of the salient teaching and learning events captured in video clips. While other researchers have proposed frameworks for understanding the nature of the captured teaching and learning events (e.g., Sherin & Es van, 2009), such frameworks are used to characterize video clips that focus on both teaching and learning events. The VPEM video clips were edited to focus explicitly on CMT. In addition, such frameworks do not account for the presence of nonmathematical or non-pedagogical aspects of captured events, what we define as *noise*. Considering the noisiness of a video clip is particularly important for novices, such as PSTs, as they often do not know what features warrant attention. For these reasons, we needed a framework that was applicable to a particular type of video clip (i.e., of other teacher's teaching), that made salient a particular foci of video clips (i.e., CMT), and that accounted for the *noisiness* of video clips (i.e., presence of nonimportant events).

While our research on the complexity of video representations is only in its initial stages, our findings thus far suggest that the videocases in scaffold level 2 are more complex with respect to the mathematical thinking displayed by children in the clip (i.e., several strategies are being discussed, children's thinking is not transparent) and are less noisy (i.e., the presence of nonmathematical and non-pedagogical events is minimal). Thus, it may not be surprising that PSTs attended to substantive aspects of CMT in scaffold level 2 because there were not many other events in the video clips to which to attend. Perhaps more importantly, our initial findings suggest that scaffolding PST noticing of children's thinking may be more effective when the nature of the video representations used do not include many distracting events (e.g., teacher discussion) or are less noisy. As research on PST noticing continues to evolve, researchers should consider the nature of the video representations used in their work, and the relationship between what PSTs are attending to and how they are interpreting the captured events.

Concluding Thoughts

Though the use of video in teacher preparation appears to have multiple advantages, as a field we must work to understand this rather complex tool for effectively supporting teacher learning. As we have discussed in this chapter, PSTs may not be able to effectively notice CMT in video unless appropriate supports are implemented, such as the type of scaffolding we implemented in the VPEM online platform or effective instructor facilitation. How best to design the scaffolding will likely depend on the complexity of the video, as we are only just starting to explore, as well as careful consideration of how to order PSTs' introduction to the components of professional noticing.

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Curricular Noticing: Theory on and Practice of Teachers' Curricular Use

Julie M. Amador, Lorraine M. Males, Darrell Earnest and Leslie Dietiker

Abstract This chapter presents a new theoretical construct, curricular noticing, used to understand how teachers interact with curriculum materials, and shares findings from four coordinated research studies. Curricular noticing draws from work on professional noticing of children's mathematical thinking and is defined as how teachers make sense of the complexity of content and pedagogical opportunities in written or digital curricular materials. This construct is situated within existing literature on curriculum use, documenting growing concerns about teachers' curricular reasoning and decision-making. Taken together, these studies explore the curricular noticing of 62 preservice teachers (PSTs) in elementary and secondary mathematics methods courses at four institutions. Participants engaged in one of two aspects of noticing: (a) attending and interpreting in the context of particular tasks; or (b) noticing (attend, interpret and respond) in the context of multiple published curricula. Data include pre/post measures, video of interventions, and written assignments that were collected and qualitatively analyzed. Dimensions of curricular noticing highlight the importance of noticing at the task level and raise questions around curricular sequencing and comparisons of curricula. Findings imply that PSTs would benefit from evaluating curriculum materials using specifically designed analysis tools, and highlight the importance of curricular noticing as a framing for understanding PSTs practices with curricula.

Keywords Curriculum · Adapt · Task · Lesson plan · Resources

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This chapter presents a framework called *Curricular Noticing* and explains how this framework is a useful mechanism for the analysis of teachers' reading, critiquing, and making decisions with curriculum materials. We frame curricular noticing as a set of interrelated skills including attending, interpreting, and deciding how to respond to curriculum materials. This construct closely mirrors that of professional noticing of children's mathematical thinking (Jacobs, Lamb, & Philipp, 2010), with an emphasis on noticing curricular components rather than children's mathematical thinking. The work of noticing in the context of curriculum materials is both highly pedagogical and highly mathematical, thus illuminating its importance. Further, curriculum materials are tools that support both teachers and students in a variety of contexts, and curricula are a key mediator through which teachers analyze and make decisions regarding content and pedagogy in their classroom. With so much variance and importance, we argue for the necessity of the process by which teachers learn to use curriculum materials. Therefore, the purpose of this chapter is to explore how preservice teachers (PSTs) attended to mathematics curriculum materials, interpreted these materials through analysis and evaluation, and made decisions about responding to enact lessons. Drawing upon four exploratory studies across different university settings, we consider the construct of curricular noticing and how this may be supported in mathematics methods coursework. Additionally, we explore how the curricular noticing framework supports the development of the dimensions of curricular noticing in four studies across various research institutions.

Research on Teachers' Curriculum Use and Noticing

Recent research shows mounting evidence that teachers, including PSTs, need to develop curricular reasoning, which refers to "the thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction" (Breyfogle, Roth-McDuffie, & Wohlhuter, 2010, p. 308). Research on teachers' use of curriculum provides a foundation for describing how teachers interact with materials that are often provided—and at times mandated—for use in their classrooms. While planning and enacting instruction, teachers engage in a variety of activities involving varied tools and artifacts (Remillard, 2005). This sociocultural framing of this relationship emphasizes the interplay between teaching moves and curriculum that manifests as enactment.

Using this conception, researchers have outlined ways in which teachers participate with curriculum materials. This includes the activities teachers engage in such as reading, evaluating, and adapting (Drake & Sherin, 2009) and what Brown (2009) describes as offloading, adapting, and improvising. This research has offered a description of what teachers do with curriculum materials, but little is known about *how* teachers make curricular decisions. Furthermore, less is known about how to understand the decision-making process and how to support teachers, both novice and experts, in using curriculum materials. Specifically, understanding first how PSTs interact with materials is important in order to consider supporting how they make use of those curriculum materials. For the purposes of this chapter, curriculum materials include printed or digital textbooks, written lessons, and single tasks.

Researchers have proposed that mathematics reform efforts should include assisting teachers in examining unfamiliar curriculum resources and developing new ways to use these materials. Remillard and Bryans (2004) argued that "teacher education programs should provide aspiring teachers with opportunities to critically analyze curriculum materials, to examine the mathematical and pedagogical assumptions implicit in their design, and to consider how curriculum materials might be read, used, and adapted" (p. 386). We contend that teacher educators cannot expect PSTs to independently develop strategic and effective ways to read and enact mathematics curriculum materials and, as a result, we need to target curricular reasoning in the context of mathematics methods courses.

Noticing

Beyond research focused on teachers' use of curriculum materials, recent studies have proposed the construct of professional noticing, a core instructional activity that is integral to ambitious teaching (Philipp, 2014). Researchers have conceptualized noticing in ways that have some important differences, yet most include two main components: attending and making sense (Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2008). During the instructional process, teachers must manage the complexity of the classroom and must pay attention to some things and not to others, thus attending (Sherin, Jacobs, & Philipp, 2011). Additionally, teachers are not passive observers for those features to which they attend. Instead, teachers necessarily interpret what they see, relating observed events to abstract categories and characterizing what they notice in terms of familiar instructional episodes. In this process, they are able to attend to certain features of students' understanding and make sense of how students are reasoning.

To further expand this definition of noticing, Jacobs et al. (2010) proposed the idea of *making decisions to respond* as an additional component of noticing. Their work included the central idea that teachers should decide how to respond on the basis of attending and making sense, and termed the process *professional noticing of children's mathematical thinking*. They describe professional noticing as: *attending to*, *interpreting*, and *deciding* how to respond to children's thinking (Jacobs et al., 2010). This framework, which describes how teaching can be responsive to the mathematical thinking of students, has brought clarity to the teacher–student dimension of classroom instruction. We posit that this lens of noticing is similarly useful as a mechanism for the analysis of teachers' interactions with and use of curriculum materials. In terms of curricular noticing, *attending* refers to reading and recognizing aspects of curricular materials, *interpreting* refers to the process by which teachers make sense of that to which they attend, and *deciding* how to respond refers to making curricular decisions as a result of attending and interpreting. This chapter brings together two lines of research: studies to improve PST practice with respect to noticing and studies to enhance teachers' work with curriculum materials.

Transferability of the Noticing Construct

This chapter is situated around the transferability of the noticing construct from the analysis of children's mathematical thinking to the analysis of curriculum materials. The findings of this chapter are situated around transferability, as the notion of professional noticing is transferred from the analysis of children's mathematical thinking to the analysis of curriculum materials. Curricular noticing applies the constructs from the work on noticing children's mathematical thinking (Jacobs et al., 2010) in order to illuminate the work involved using curriculum materials. First, both frameworks treat task selection as a necessary and critical component of ambitious teaching. While there have been varied empirical techniques in research on noticing, much of this underscores the role of teachers' attending to the mathematics of the present task and interpreting how students interact with the mathematics of that task. Second, both constructs allow the field to consider methods to support PSTs. Cultivating PSTs' practices of noticing of children's mathematical thinking has been identified as a mechanism to provide PSTs with opportunities to understand student-centered teaching and develop the pedagogical content knowledge necessary for high-leverage instruction (Hill, Ball, & Schilling, 2008; Jacobs et al., 2010). Similarly, we conceptualize curricular noticing as inextricably linked to these efforts to support PSTs. Although we cannot know what curriculum materials PSTs will use, PSTs will encounter some form of materials as they begin their careers and these materials will influence their teaching decisions (Banilower et al., 2013; Brown & Edelson, 2003; Remillard, 2005). Therefore, learning to notice in the context of curriculum materials is an integral part of learning to teach and, just as with noticing students' thinking, is necessary in developing the pedagogical content knowledge needed to teach. We focus our work with PSTs as a way to support the development of curricular noticing by this population. We focus on the following research question: How do PSTs attend to mathematics curriculum materials, interpret those through analysis and evaluation, and decide to plan and enact lessons?

Studies Aimed at Examining and Developing Curricular Noticing

To understand the applicability of the curricular noticing framework in the teacher education context, we conducted studies on various components of the framework at four universities in the United States. The intent was to understand how the framework could be applied to provide mathematics teacher educators with an understanding of PSTs' curricular noticing capabilities and how we might develop these abilities. Here we present the methods and findings from these four independent and exploratory studies. We consider these studies in relation to the dimension to which the study aimed to explore (i.e., attending, interpreting, deciding to respond) and by curricular grain size (i.e., individual tasks, tasks/lessons across curriculum materials), moving from studies that focused on single tasks to studies that focused on a collections of tasks or lessons. This section provides an overview of the four studies, followed by detailed subsections for each study.

Study 1 aimed to illuminate *attending* by engaging PSTs with variations of a single mathematics task in order to highlight how task characteristics could impact students' experience and opportunities to learn. The second and third studies aimed to illuminate *interpreting*. Study 2 focused on an area model fractions task design typically featured in curriculum materials, and how PSTs' interpretations of mathematical opportunities available in this routine task shifted after explicitly discussing a nonroutine task. Although Study 3 also explored interpreting, it expanded the grain size from single task to PSTs working with multiple tasks/lessons in units across multiple curricular programs in order to evaluate the quality of curricular programs on various dimensions. Finally, we conclude with Study 4, another study that has PSTs working with a collection of tasks/lessons across multiple curricular programs. This final study aimed to illuminate *deciding* how to respond by describing the decisions PSTs made when producing a lesson plan given multiple resources to choose from.

Although we connect each study to particular dimensions of curricular noticing, we admit that this categorization was somewhat difficult. We make no claims that each of these studies addresses only the dimension that is highlighted, nor do we feel that these dimensions should or could be mutually exclusive. For example, although Study 4 explored PSTs' responses, there is no doubt that these responses were based on some kind of interpretation of what was attended to by PSTs.

Study 1

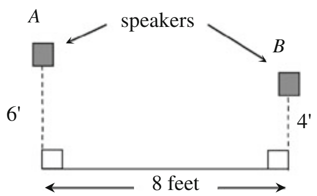
Our first study identified curricular opportunities to which PSTs *attended* within written curriculum materials by focusing on two versions of one task. In particular, if we aim to support PSTs in the development of the ability to recognize and critically evaluate written curriculum materials in order to recognize potentially rich mathematical opportunities and overcome limitations of materials, this raises the questions: (a) What task qualities are distinguishable by PSTs? and (b) What activities can support shifts in attending to curricular opportunities of mathematical tasks? The following reports on an exploratory study of these questions.

Study 1 focused on eight PSTs (Grades 9–12) enrolled in a Masters level licensure program at a private university. In coursework, they were prompted to solve and compare two versions of the same mathematical task, one open strategy and the other closed strategy, in order to compare the affordances and constraints for student mathematical experiences (see Figure 1 for the two versions). The PSTs were given 10 minutes to work together in groups of two or three to solve their version of the task while unaware that not all groups had the same version. As they worked to solve their version of the task, PSTs were asked to record any questions they asked, observations that were made, and strategies that were used. After each group completed the task, the whole class discussed what mathematical challenges they encountered and what strategies they had used, at which point the difference in the types of experiences afforded by the two versions became evident. When the different versions were revealed, the PSTs had another five minutes with their group to read through the other version and consider the differences it would have had on their experience solving the tasks. A second discussion focused on the comparison of their experiences while solving the two versions of the task. During this discussion, the PSTs were asked to identify the features of each version of the mathematical task that enabled or constrained their experience.

Laura needs your help. She needs to order expensive wire to connect her stereo to her built-in speakers and would like your help to save her money.

She plans to place her stereo somewhere on a cabinet that is 8 feet wide. Speaker A is located 6 feet above one end of the cabinet, while speaker B is located 4 feet above the other end. She will need wire to connect the stereo to speaker A, and additional wire to connect the stereo to speaker B.

Where should she place her stereo so that she needs the least amount of wire?



Open-Strategy Version

- Before you discuss this with your team, make your own guess. What does your intuition tell you?
- Then work with your team to determine where on the cabinet the stereo should be placed. How can you be sure that you found the best answer? In other words, how do you know that the amount of wire you found is the least amount possible?

Closed-Strategy Version

a. Calculate the total length of wire needed if the stereo is placed 2 feet from the left edge of the cabinet (the edge below Speaker A).

b. Now calculate the total length of wire needed if the stereo is placed 3 feet from the same edge. Does this placement require more or less wire than that from part (a)?

c. Continue testing placements for the stereo and create a table with your results. Where should the stereo be placed to minimize the amount of wire?

Figure 1. An open strategy (**a**) and closed strategy (**b**) version of the same task, adapted from *Geometry Connections* (Dietiker, Kysh, Sallee, & Hoey, 2007). Reprinted with permission of the publisher

Data included audio recordings of each group's discussion and of the whole-class discussion and responses to a pre- and post-assessment. The audio recordings were transcribed and themes of the observations were coded. The assessment provided PSTs with a lesson on areas of polygons from a geometry textbook and prompted PSTs to describe the central mathematical ideas (including concepts and processes) that could be developed with these materials. The PSTs were also asked to explain how they would design a lesson using this textbook material including how a lesson would begin and end. Finally, this assessment asked for details about: (a) what specific parts of the textbook materials students would plan to use, and (b) how these parts of the text materials would be used. The post-assessment was administered seven weeks after the pre-assessment and two weeks after the in-class activity.

Overall, the PSTs noted five features of task design that could enable them to distinguish between the affordances and constraints of the two tasks. These dimensions were: (a) the focus on mathematical ideas, (b) the assumed role of the student or teacher, (c) the accountability of students for justification, (d) the purpose of mathematical work, and (e) the potential for mathematical discourse.

To learn what changed in how the PSTs attended to the written textbook lessons, the pre- and post-assessments for each PST were analyzed for differences. That is, when looking at the same lesson from a textbook a second time many weeks later, PSTs then noted new features and limitations and described new ways they would use parts of the textbook lesson. Changes were different in scope across a dimension. Changes might have been as minor as changing questions that were asked for a task to as large as changing the entire lesson format to enable different forms of student participation. For example, the excerpt below demonstrates how one PST edited the pre-assessment document. This PST used strikethrough to indicate text the students wished to remove and brackets were used to indicate additions:

> Around this time, I would ~~put~~ [have the students generate] the formulas for area of a triangle and area of a rectangle/parallelogram. [A student would come up to the board and explain how they got these formulas. Even if the students had the formulas memorized from prior mathematics classes, I would want them to be able to tell me how they got one formula from the other. This is so they would be practicing their justification skills and getting more creative with mathematics.] These will also be in the students' notes and/or glossaries.

The changes PSTs made to their assessment were analyzed to learn whether any of the five dimensions of attending to mathematical tasks were evident in the explanations or justifications for the changes. In the post-assessment, every PST described ways to adjust their plan in order to change the student engagement with mathematics, although this looked different for each plan. For example, two PSTs decided to change a lecture to a student-centered group activity, while a third altered the design of a task to require collaboration with other students and a fourth changed the design of his activity to include physical manipulatives instead of drawing in the textbook. All but one PST made explicit reference to at least one of the dimensions as reasons for including, excluding, or adapting portions of tasks. Half of the PSTs attended to four or five of the dimensions.

In conclusion, the comparison of multiple versions of the same task enabled these PSTs to not only identify differences in task design that led to different student mathematical experiences and learning outcomes, but also provided the PSTs dimensions of attending to curriculum materials that enabled them to recognize new opportunities within curriculum materials, specifically mathematical tasks.

Study 2 and Study 3

Study 2 and Study 3 both aimed to document and support how PSTs *interpret* curriculum materials. Study 2 focused on how PSTs interpreted the mathematical features of routine tasks commonly featured in curriculum materials while Study 3 focused on how PSTs interpreted how three curriculum programs aligned with the Common Core Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Many teacher education programs preparing PSTs to teach mathematics across K-12 support inquiry-oriented approaches that are consistent with the CCSSM Standards for Mathematical Practice and the National Council of Teacher's mathematical processes (National Council of Teachers of Mathematics, 2000). Nonetheless, many of the textbooks teachers will encounter once they have jobs will likely not be inquiry-oriented (Banilower et al., 2013), and tasks and lessons that teachers encounter, whether in their district-adopted text or online resources, may in fact hide key mathematical properties and discourage inquiry-oriented instruction. Therefore, learning to interpret curriculum materials, whether they be single tasks or collections of tasks, is critical in a teachers' use of the materials as they take in what they see and relate this to the kinds of practices that they wish to enact. This raises the following questions: (a) What interpretations do PSTs make with regard to mathematical and pedagogical opportunities in tasks and lessons within curriculum materials? and (b) What activities can support shifts in what and how PSTs interpret curricular opportunities in tasks and lessons? The following two studies examined these questions.

Study 2

The second study focused on how PSTs interpret the mathematical properties of standard or routine tasks and how discussion in the methods course might further support their noticing of such properties. With participants that included 18 elementary (Grades 1–6) PSTs at a state university, the study focused on fractions, an area of elementary mathematics that is notoriously hard-to-learn and hard-to-teach (Lamon, 1996; Saxe, Taylor, McIntosh, & Gearhart, 2005), with a particular focus on the role of equal parts in determining fractional quantities of area models.

Routine problem design often hides important mathematical ideas (Earnest, 2015; Schliemann, 2002). For example, the canonical representation for ¼ features a rectangle or circle divided into four equal sections with one of the four equal parts shaded. However, this common curricular treatment of areas partitioned into equal parts may actually hide the importance of equal parts in determining fractional quantities of an area. Consider the area models featured in Figure 2, each which features a shaded region that is ¼ of the whole area. Children who successfully identify the shaded region in Figure 2a as ¼ may also identify the shaded area in Figure 2b as 1/3, indicating set model rather than an area model treatment. In order for instruction to highlight for children the idea of equal parts for the routine task, teachers must first interpret this to be a property of equally partitioned area models. The goal of Study 2 was to investigate how to support such noticing.

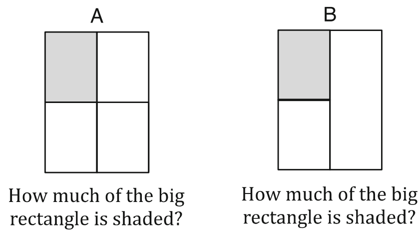


Figure 2. Area models for ¼ that feature **a** equal partitioning and **b** unequal partitioning

Data included video recordings (two cameras) of a target session during the methods course. The pre-assessment was administered at the beginning of the 13-week semester (September), with the intervention taking place approximately four weeks later (October) and the post-assessment four weeks after that (November). The pre- and post-assessments each included analogous tasks featuring equally and unequally partitioned area models on which PSTs were asked to identify the big mathematical ideas they saw as important for children to understand in order to identify the shaded area.

On the pre-assessment, only one PST identified *equal parts* as an important mathematical idea of the equally partitioned area model task (6%). For example, one student described the big mathematical ideas in the problem as “Fractions—knowing what number is the numerator and denominator.” This result indicated that equal parts was not a big mathematical idea for PSTs in the context of a routine area model task.

In the intervention that came four weeks after the pre-assessment, PSTs engaged in an analysis of student work in which a student provided a response of 1/3 for the area model shown in Figure 2b and were asked to describe what they thought that particular child understood about fractions and where that child would need further support, building on a theme of the course that children are sophisticated problem solvers even when reaching incorrect responses (Bray & Santagata, 2014). A goal was to investigate if sustained discussion involving a nonroutine task would support PSTs’ noticing—in particular, interpreting—of key mathematical properties underlying routine representations of fractions. Such properties are critical for PSTs to notice in order to decide to respond on the basis of children's mathematical thinking. During the intervention, the role of equal parts in determining fractional quantities of area models came out as an important mathematical idea.

A post-assessment was administered four weeks after the intervention, between which the course did not focus on fractions. In responses on the post-assessment, PSTs shifted from their pre-assessment responses in how they interpreted the important mathematical ideas featured in a routine area model task. For a task analogous to the equally partitioned area model task in Figure 2a, more than half of PSTs (56%) identified equal parts as an important mathematical property. For example, the same PST quoted above wrote on the post-assessment, "This problem looks at the idea of fractions and knowing what equal parts are." Since the equally partitioned task is one that they are quite likely to see across grades in curriculum materials, results suggest an important shift in how PSTs notice (i.e., interpret) mathematical properties of routine area model fractions tasks.

To summarize, Study 2 was concerned with how PSTs interpret the mathematical properties of routine tasks they are likely to encounter in any curriculum. While this study was exploratory in nature, results suggest that discussion involving student work on nonroutine tasks may in turn support PSTs' interpretations of ubiquitous routine tasks and, as a result, their curricular noticing involving such tasks. While subsequent research must be conducted to understand how such interpretations may carry over to PSTs' lesson design, the noticing framework was a useful mechanism for analyzing PSTs' interpretations of the routine tasks they will encounter in curriculum materials.

Study 3

The purpose of Study 3 was to examine and support secondary (Grades 6-12) PSTs' interpretations of curriculum materials by having them use an analytic tool to decide which of three curriculum programs was aligned to the CCSSM and describe which they would choose to adopt and why. Participants in this study were 17 PSTs enrolled in the second of two secondary mathematics teaching methods courses at a state university. The two methods courses broadly focused on issues of mathematical thinking and learning with a focus on access and equity, lesson and unit planning, classroom discourse (i.e., interaction patterns, questioning, discourse moves), and working with curriculum materials. In the first methods course, PSTs were exposed to a variety of curriculum materials through small tasks completed in class and through a microteaching assignment that included teaching from The Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006).

This study employed a pre/post design. In the first four weeks of a 15-week semester, PSTs were asked to examine the quadratics content in the student and teachers' guides for three curricular series: *Prentice Hall Mathematics Algebra 1* (Bellman et al., 2007), *CME Project Algebra 1* (CME Project, 2009) and *CPM*

Algebra Connections (Dietiker, Kysh, Sallee, & Hoey, 2006). PSTs were asked first to determine what was similar and different between the three sets of curriculum materials and then to determine which text, if given the option, they would choose to use in their classroom and why. Each PST turned in a written response to these questions. For the next eight weeks PSTs used the Common Core Curriculum Analysis Tool (Common Core State Standards Mathematics Curriculum Materials Analysis Project, 2011) on the quadratics content within each set of materials. The Common Core Curriculum Analysis Tool (CCCAT) was designed to provide guidance to assist in the selection of curriculum materials that support implementation of the Common Core Standards and includes three sub-tools: (1) Content, (2) Practices and (3) Equity, Assessment and Technology. Following this analysis using the CCCAT, PSTs were asked to respond to the same questions from the beginning of the semester.

Data included the pre-tool and post-tool written responses. Each response was read three times to generate initial codes using a grounded theory approach (Corbin & Strauss, 2008). In the first reading of each sentence, the researcher asked the question "What is this sentence about?" in order to generate codes. As each response was read, notes were recorded on all codes. Each code was then reread and the set of codes generated in the initial pass were used to capture additional aspects in the responses. When the responses were read a third time, themes were generated for the pre-tool and post-tool responses.

The post-tool responses indicated that, if given the chance, 76% would choose to adopt CPM (compared to 72% before using the CCCAT), 6% would choose CME (no change), and 18% would choose PH (compared to 22%). 72% of PSTs had already chosen to adopt one of the reform-oriented texts, CPM, before using the tool. As described earlier, PSTs engaged in this assignment in the second methods course after a first methods course in which they used reform-oriented materials frequently. This prior experience with reform-oriented materials may have impacted the ways in which PSTs interpreted the materials in this assignment, resulting in the lack of change in adoption.

Although there was not much of a difference regarding which text PSTs chose to adopt after engaging with the CCCAT, there was a shift in the reasoning used by PSTs when discussing their choice. This indicated that the CCCAT provided scaffolding for PSTs to interpret what they read in each text. PSTs' pre-tool responses were quite generic and included the general approach of the materials, whether the materials had good or bad teacher resources, the tools included in the materials such as calculator and manipulatives, and the clarity of layout for students. After using the CCCAT, their responses were more detailed and they described different aspects of the materials. On average, PSTs wrote 32% more (as measured by number of sentences) in their post-tool response and included more examples from the materials (mostly to illustrate features that they liked). Six out of the 10 most frequent reasons were explicitly aligned to aspects that PSTs were asked to use when evaluating texts using the CCCAT. PSTs made reference to the CCSSM Mathematical Practices and the balance between procedural and conceptual opportunities and when referring to the teacher resources described in detail the supports for assessment, differentiated instruction, and working with English Language Learners. They also commented more on the ways in which technology was integrated, meaning whether it seemed to be an integral part of the text rather than just naming what tools were used in the text. All of these aspects were explicitly addressed by the CCCAT. In addition, PSTs also discussed aspects that were not an explicit object of analysis in the CCCAT. PSTs discussed the types of participation structures that were emphasized in the materials, whether detailed lesson plans or suggestions were provided to teachers, the cognitive demand or richness of the tasks, the flexibility (or often lack of flexibility) of the text and the level of planning needed in order to be successful in using the textbook.

PSTs interpretations were also more nuanced. Most notable was the way in which PSTs were able to discuss the opportunities in the text beyond just naming the surface features. For example, before using the tool, many PSTs mentioned that two of the texts included visual representations and manipulatives [Prentice Hall (PH) and College Preparatory Mathematics (CPM)], particularly for the lesson on Completing the Square. In the post-tool response, a number of PSTs ($n = 6$) articulated that the ways in which manipulatives were embedded within the two texts might impact how effective they were. One PST stated:

> I agree with my previous statement that both PH and CPM use visual representations, but now I'm wondering how effective it would be for PH to use these algebra tile representations. After looking through the entire chapter, PH never uses Algebra Tiles or manipulatives. I wonder if the students will actually be able to relate to these representations since they are never used throughout the chapter except in this one instance. With CPM, however, they use Algebra tiles and visual representations as a part of the *normal routine* in the classroom. The students become familiar with the expectations of the manipulatives and they understand what each block means. That makes the completing the square section very attainable for students' understanding in CPM. I don't think that PH can get that same reaction because they are introducing blocks for the first time in this lesson.

Although potentially helpful in being able to apply the CCCAT, these aspects were not explicitly addressed, meaning that PSTs were not asked to attend to these aspects in the same ways they were the others, yet they did. These results indicate that the CCCAT may have aided in shifting how PSTs interpreted the content in these three texts, even beyond what the tool explicitly asked PSTs to attend to.

Study 4

Our final study aimed to explore how PSTs decided to respond. Research on the professional noticing of children's mathematical thinking has stressed the necessary role of *deciding* how to respond to children's thinking, and that responding involves selecting a next problem. In this case, the next problem is not just a problem with more complicated numbers or an analogous version of what a student had just been struggling with; rather, the next problem supports children in engaging with mathematical ideas strategically identified by the teacher in order to support particular students. Likewise, responding could extend beyond the next problem and encompass larger considerations. The next study describes approaches of PSTs as they consider how they would respond when planning a lesson given multiple curriculum programs to choose from. This study specifically addresses the following question: What responses do PSTs make when planning lessons given multiple curriculum resources and what reasons do they provide for these responses?

The purpose of Study 4 was to understand how 19 elementary (Grades K-8) PSTs at a state university made decisions about intended teaching actions as they interacted with multiple curricular resources to further understand the reasons behind their instructional decisions. PSTs were provided with Grade 6 teacher materials for a lesson on the division of fractions from each of the following curricular resources: CPM *Core Connections Course 1*, Everyday Mathematics, enVisionMATH, and Saxon Mathematics (Charles et al., 2009; Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013; Larson, 2010; The University of Chicago School Mathematics Project, 2007). PSTs were tasked with using components of any of the resources to write out a detailed lesson plan that would address the following standard: "Apply and extend previous understanding of multiplication and division to divide fractions by fractions" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, 6.NS.1). PSTs were asked to provide rationale for their decisions to include or exclude particular curricular resources. Following the design of their lesson plan, they were prompted to respond to questions about their use of the resources, causes for using particular materials, causes for not including particular materials, and an overall rationale for their decision making with respect to curricular materials. Data included the written lesson plan with rationale from each of the PSTs, their responses to the follow-up questions, and any notes they had written. Data were analyzed using constant comparative methods (Corbin & Strauss, 2008) to identify incidents in the data, compare them with other incidents for similarities and differences, and then group them into higher level descriptive themes. Results from the analysis indicated: (a) PSTs considered their own experiences and perspectives on effective teaching when deciding to respond, and (b) PSTs found value in lesson components that were authentic, meaning relatable to students often with real-world contexts, or components that incorporated tangible manipulatives.

First, findings suggest that PSTs decided how to respond by considering their personal conceptualizations about effective mathematics teaching and the content of dividing fractions. They commonly reflected on their own experiences with learning to divide fractions as they made decisions about how to respond. They considered their experiences in relation to effective teaching to formulate interpretations. For example, one PST commented about preconceived ideas about dividing fractions, "I already had an idea for the lesson once I read the standard and none of the other [curriculum materials] had anything that worked into my lesson idea." This pattern was identified in data from multiple PSTs who also related their own experiences to teaching fractions. One PST wrote, "This is all from personal experience. Fractions scared me and it took me a long time to realize that they're just numbers." This PST built her lesson around supporting students in ways she considered beneficial for them. She decided to make a connection between whole numbers and fractions for the students to clarify the relationship—something she would have benefitted from when learning fractions. She considered this to be an effective approach because she thought it would have helped her when she was learning to divide fractions.

Second, the PSTs deemed problems with authentic contexts and those incorporating tangible manipulatives to be exemplary components that aligned with effective pedagogical practices. For example, one PST relied on the enVisionMATH materials because of how the curriculum incorporated fraction strips in the lesson, "I thought Envision had the best visual aid/manipulative, so I used that. The fractions strips are a good, easy-to-make, tool that students can use as a visual aid for dividing fractions." A majority of PSTs purposely included some type of physical visual support for students or centered their lesson on an authentic context, such as pouring a fractional amount of lemonade out of a pitcher that was partially full, which was mentioned in enVisionMATH.

PSTs' decisions to respond were filtered through the PSTs' understanding and considerations of effective mathematics teaching and their interpretations of the curriculum materials alignment with those considerations. Given this, it is plausible that the PSTs' perceptions of effective teaching influenced their curricular noticing. This raises questions about how PSTs learn what is effective and how this may influence their interactions with curriculum materials. Additionally, many of the PSTs based their decisions on their preconceived ideas about what the lesson should entail, suggesting teacher educators support the development of research-based pedagogical and content decisions among PSTs.

Discussion

The construct of curricular noticing provides a mechanism for decomposing the professional practice of teaching, thereby complementing existing work on ambitious teaching (Philipp, 2014). The exploratory studies presented above illustrate how research on the professional noticing of children's mathematical thinking (Jacobs et al., 2010) may be transferred to the context of teachers' use of curriculum materials. Specifically, the work explored how PSTs attend to mathematics curriculum materials (Study 1), interpret those through analysis and evaluation (Study 2 and Study 3), and decide to plan and enact lessons (Study 4).

First, knowing that PSTs need to develop curricular reasoning (Breyfogle et al., 2010) and that teachers participate with curriculum materials (Remillard, 2005), the curricular noticing framework affords opportunities for the analysis of PSTs' participation with curriculum materials in the process of making instructional decisions. Application of this framework in four different contexts provided insight about the decision-making process of PSTs as they interacted with materials and resulted in an answer to the research question: *How do PSTs attend to mathematics curriculum materials, interpret those through analysis and evaluation, and decide to plan and enact lessons?* From these studies, we concluded: (a) comparison of multiple versions of the same task enabled PSTs to identify, or attend to, differences in task design and attend to new opportunities within materials, (b) interpretation of task design is not straightforward and PSTs may benefit from task exploration that problematizes underlying mathematical ideas, (c) the use of analytic tools may support PSTs in interpreting materials and considering future use, and (d) PSTs' decisions to respond are often linked to their conceptualizations about effective mathematics teaching and related content. Collectively, the curricular noticing framework afforded opportunities for the analysis of PSTs' abilities to attend, interpret, and decide how to respond to curricular materials revealing PSTs attend differently to different materials, may need support with interpreting materials, and often based their decisions to respond on their own understanding or knowledge.

Together the findings of the four studies suggest that the curricular noticing construct may serve as a tool for mathematics teacher educators to decompose and then highlight particular aspects of using curriculum materials: attending, interpreting, and responding. Across the institutions, curricular noticing provided a mechanism for understanding how PSTs were interacting with specific tasks as well as lessons and units, thereby revealing the ways in which PSTs notice as well as provide data for mathematics teacher educators to make decisions about next steps. The exploratory studies suggest that the curricular noticing framework may be applied in multiple contexts and at various levels of scrutiny when it comes to teaching (task level, lesson level, unit level).

Finally, the curricular noticing framework complements the existing framework for the professional noticing of children's mathematical thinking (Jacobs et al., 2010). In such, the curricular noticing framework transfers the analytic power of the framework for noticing children's mathematical thinking to the context of curricular materials by providing a mechanism for analysis of how PSTs are using materials. Rather than suggesting that more work is required of a teacher, we see each of the frameworks as highlighting the complex decision-making process in which teachers constantly engage. The application and transfer of the subconstructs—attending, interpreting, and deciding how to respond—to the context of curriculum materials highlights consistencies in the work of teaching.

Conclusion

These findings and implications for teacher educators are the result of the application of the curricular noticing framework, which details what and how PSTs attended, interpreted, and decided to respond to curriculum materials after being presented with tasks, reading multiple resources, or engaging in experiences around such resources. Use of the curricular noticing framework with these four studies led to increased understanding about the role of teacher educators in providing opportunities for PSTs to learn to interact with curricular materials. Essentially, the framework provided a mechanism for understanding the basis upon which PSTs attend, interpret, and make decisions about what they do and how they would do it when designing and enacting tasks, lessons, or units.

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The FOCUS Framework: Characterising Productive Noticing During Lesson Planning, Delivery and Review

Ban Heng Choy, Michael O.J. Thomas and Caroline Yoon

Abstract Enacting the work of diagnostic teaching is challenging and demands that teachers pay attention to mathematical details when designing tasks, orchestrating discussions and reflecting on their lessons. This chapter presents the FOCUS Framework on teacher noticing, which can be used to characterise teachers' efforts to notice productively during all three phases of diagnostic teaching: lesson planning, delivery and review. Using the two key components of the framework, the focus and its focusing, we provide snapshots of a teacher's mathematical noticing in each of the phases. The findings from this research suggest that productive noticing in all the three phases is highly consequential, and illustrates how the FOCUS Framework can be used to analyse a teacher's mathematical noticing.

Keywords Productive teacher noticing · Lesson planning · Orchestrating discussions · Lesson review · Fractions

Introduction

Teaching mathematics well does not just depend on what you teach but on what and how you notice. Mathematics teacher noticing—what mathematics teachers see and how they understand instructional events or details in classrooms (Mason, 2002; Sherin, Jacobs, & Philipp, 2011a)—is central to mathematics teaching practices and is considered necessary for improving teaching (Mason, 2002;

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Schoenfeld, 2011). The processes of noticing help teachers break down and analyse their practice in order to learn from their teaching (Mason, 2002; Sherin, Jacobs, & Philipp, 2011b). Placing noticing in the context of developing students' mathematical thinking, there are three productive classroom practices that are of interest in this chapter: designing a task that reveals students' thinking; listening and responding to students' thinking during the lesson; and reflecting about students' thinking after the lesson. If noticing is considered to be productive when teachers respond with instructional decisions that promote student thinking, then although all teachers may notice, it can be argued that not all noticing is productive. For example, it can be difficult for teachers to notice the mathematical features of learning tasks (Star, Lynch, & Perova, 2011; Vondrová & Žalská, 2013), or teachers may be distracted by noticing features that are not useful for enhancing mathematical thinking (Ball, 2011; Star & Strickland, 2008). Furthermore, it is possible for teachers to describe the specific strategies that students use to solve problems but have difficulties relating these strategies to important characteristics of the problems (Fernandez, Llinares, & Valls, 2012). Therefore, the crux of enhancing instruction to promote mathematical thinking lies in what teachers attend to, and how they think about instructional events (Ball, 2011).

Despite the apparent simplicity of the construct of teacher noticing, the ability to notice productively during mathematics teaching can be both difficult to master and complex to study (Jacobs, Philipp, & Sherin, 2011, p. xxvii). Moreover, what teachers deem productive may be highly subjective and dependent on one's views about teaching and learning of mathematics (Clarke, 2001). Nevertheless, if teachers want to teach in a way that enhances students' reasoning, they may need to attend to relevant aspects of student thinking evidenced in classroom artefacts and students' explanations, and interpret them using a mathematical perspective before, during and after a lesson.

Most researchers who study and support mathematics teachers' noticing do so by examining what teachers observe from video clips of lessons (Star et al., 2011; van Es, 2011), while others (Sherin, Russ, & Colestock, 2011) try to capture what teachers notice in the moment during lessons. One limitation of these approaches is the lack of focus on preparation to notice. As Mason (2002) put it, 'noticing is an act of attention, and as such is not something you can decide to do all of a sudden. It has to happen to you, through the exercise of some internal or external impulse or trigger' (p. 61). More specifically, Mason (2002) highlights the importance of advanced preparation to notice, and the use of prior experience to enhance noticing in order to have a different act in mind in the moment. Therefore, it is critical for researchers to examine the role of noticing during lesson planning.

However, examining what teachers notice is non-trivial. Most research generally focuses on developing teachers' ability to notice a wide range of classroom features—classroom environment; classroom management; tasks; mathematical content; communication; mathematical thinking, and so forth—without specifying what teachers should notice (Jacobs, Lamb, & Philipp, 2010; Star et al., 2011). A study by Star and Strickland (2008), as well as a replication study by Star et al. (2011), found that teachers seemed to notice more instructional events, *both* mundane and important, after participating in professional development that involved viewing video clips of actual teaching. But, neither study provided a focus for noticing, nor tested the usefulness of an explicit focus. Moreover, even when teachers are given a focus it can still be challenging for them to sieve out and reflect upon critical incidents amongst the 'buzz' in the classroom.

On the other hand, the ability to describe specific details when planning, teaching and reviewing mathematics lesson is seen as the distinguishing mark of a proficient teacher in China (Yang & Ricks, 2012). They detail how Chinese teachers think about teaching using the Three Points Framework: the 'Key Point', the 'Difficult Point', and the 'Critical Point' (p. 54). The Key Point of a lesson is the mathematical concept to be learned during the lesson. The Difficult Point refers to the difficulty or confusion students have when learning the Key Point. By having a strong grasp of these two points (the concept and its associated confusion), teachers can design tasks that address specific difficulties that students may have when learning the concept. The teaching approach or the main considerations used by teachers when designing the task is then the Critical Point, which forms the 'heart of the lesson' (Yang & Ricks, 2012, p. 43). Noticing that the Critical Point is targeted at the Difficult Point related to the Key Point is essential if teachers want to promote students' reasoning.

In addition, how teachers notice also matters. Many researchers focus on the specificity of what teachers have noticed as an indicator of noticing expertise, but specificity is not sufficient for noticing to be productive. In a study involving seven prospective secondary school mathematics teachers, Fernandez et al. (2012) found that most were unable to relate the strategies used by students to the characteristics of the problem, even though they were all able to describe the specific strategies at the beginning of the study. Choy (2014b) also highlights the role of pedagogical reasoning, beyond giving teachers an explicit focus, as a means to promote more productive noticing when they plan their lessons.

The research presented here addresses the challenge of noticing student thinking, building upon previous research to bring task design into the realm of teacher noticing. The research was guided mainly by the following question:

What makes teachers' mathematical noticing, during planning, teaching and reviewing of lessons, productive for enhancing students' mathematical reasoning?

This question reflects the importance of preparation in noticing, and draws attention to the ways teachers can plan to anticipate student thinking as they engage with the tasks (Smith & Stein, 2011). In this chapter, we describe the FOCUS Framework, developed from part of a larger doctoral study (Choy, 2015), which pinpoints specific focal points and actions teachers can take to attend to, make sense of and respond to students' thinking when planning, teaching and reviewing a mathematics lesson. More importantly, we demonstrate how the FOCUS Framework can be used to characterise, analyse and support teacher noticing.

Research Design

Design Research Paradigm

The FOCUS Framework was developed from a design-based research project (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), which addressed the twin challenges of theoretical development and practical application (Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008). Using an iterative and highly interventionist approach (Cobb et al., 2003), a design-based research project aims to generate usable knowledge (Design-Based Research Collective, 2003) that is grounded in complex real-world settings (McKenney & Reeves, 2012). Data collection for the doctoral study, which consisted of three phases, took place in Singapore over a period of eight months in 2012 and 2013. A total of 36 teachers from three schools, a primary school and two secondary schools, participated in the study. The three schools volunteered for the study when their principals responded to our advertisement seeking research participants. All three schools had processes in place to support learning communities and the teacher participants had used Lesson Study as a professional development activity. Hence, they were familiar with the Lesson Study protocol.

We engaged teachers in a systematic investigation of their teaching (Hiebert, Morris, & Glass, 2003) as they participated in the five key tasks of Lesson Study (Lewis, Friedkin, Baker, & Perry, 2011). First, teachers began by clarifying pedagogical research questions (Hiebert et al., 2003), whereby they articulated their own hypotheses that connect the task design with the intended learning goals. Next, teachers reasoned about their choice of instructional strategies, and specified how these tasks can help change students' thinking before they designed the lesson. This shift from 'spontaneous' decision-making to one in which teachers plan and consider possibilities is the essence of the discipline of noticing—'to be methodical without being mechanical' in order to be more sensitised to notice in the moment (Mason, 2002, p. 61). Teachers then collected data on students' thinking, which helped to inform future revisions to the lesson design. Finally, teachers interpreted the data, and drew conclusions about the effectiveness of the task on student learning.

This systematic investigation of teaching via Lesson Study was useful since it provided a theoretical justification for, and an operationalisation of, the design study methodology adopted in this research. Lesson Study, which situates the systematic investigation of teaching within a cycle of activities to make teachers' thinking visible, focuses on improving teaching, instead of improving teachers. Hence, Lesson Study was adopted for this study because it not only encapsulates the essence of the design research paradigm, but also provides a lens, both to examine the noticing of groups of teachers, and to zoom in on a single teacher.

Participants and Setting

This chapter recounts vignettes of what, and how, six teachers from Greenhill Primary School (a pseudonym) in Singapore noticed as they collaboratively designed a lesson on Fraction of a Set for Primary Four students (age 10), given three explicit focal points: mathematical concept, students' confusion, and teachers' instructional decisions. Six teachers, who volunteered for this research, were involved in this Lesson Study group: Kirsty (facilitator); Cindy; Flora; Anthony; Rani; and James (research teacher). All teachers had at least five years of experience teaching mathematics.

In our study, we incorporated Mason's (2002, p. 95) practices of noticing—systematic reflection; recognising; preparing and noticing; validating with others—into the Lesson Study protocol. This modified protocol provided a way for teachers to discuss the mathematical aspects of teaching and learning. The first author primarily took on a participant observer role, shifting between observational and participatory roles during the seven lesson study sessions. During the discussions, he used questions to prompt and direct the teachers' attention to explicit focal points and provided necessary mathematical content knowledge when needed.

Data Collection, Condensation and Analysis

Data were collected and generated through voice recordings of the lesson study discussions and video recording of the lessons observed. A key challenge during data analysis was to deal with the huge amount of data generated from the recordings. In order to condense the data to a level that was manageable, the following procedure was followed:

1. All recordings were reviewed with the field notes taken;
2. The voice recordings were marked for discussion segments that dealt with the five key tasks of Lesson Study. Segments that were focused on logistical issues, administrative matters, and other unrelated incidents were not marked for further analysis;
3. These selected segments were then reviewed again, and initially classified using the framework for noticing (van Es, 2011) student thinking, which is shown in Table 1;
4. Mathematically noteworthy segments were then selected for transcription. Care was taken to ensure a wide spread of segments ranging from baseline noticing to extended noticing (van Es, 2011).

The classification of noticing segments as productive or otherwise, and the selection of noteworthy segments were potentially biased, but this issue was negotiated partially through the use of the five key tasks in the Lesson Study (Lewis et al., 2011), and the aims related to enhancing student reasoning (Hiebert et al., 2003). Segments were characterised as productive using our defining characteristic of

Table 1
Framework for noticing students' thinking adapted from van Es (2011, p. 139)

	What teachers notice	How teachers notice
Level 1 Baseline	Attend to generic aspects of teaching and learning, e.g. seating arrangement, student behaviour, etc.	Provide general descriptive comments with little or no evidence from observations
Level 2 Mixed	Begin to attend to particular instances of students' mathematical thinking and behaviours	Provide mostly evaluative comments with a few references to specific instances or interactions as evidence
Level 3 Focused	Attend to particular students' mathematical thinking	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence
Level 4 Extended	Attend to the relationships between particular students' mathematical thinking, mathematical concepts and teaching approaches	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence, make connections to principles of teaching and learning, and propose alternative pedagogical solutions

whether teachers responded with instructional decisions that promote student thinking. The selected segments were of mathematical or pedagogical interest, and were characterised mainly by discussions surrounding issues related to the Three Points (Cohen, Raudenbush, & Ball, 2003; Yang & Ricks, 2012).

After we selected and condensed the huge amount of data, we began the process of transcribing the selected segments to facilitate further analysis. The selected episodes were transcribed word for word, including pauses (…), and ungrammatical or colloquial language, which were not edited. Words added into the transcript to enhance clarity were given in angled parentheses [ ], and actions, if any, were indicated within round parentheses ( ). Findings related to teachers' noticing were developed through identifying categories, codes and themes related to the elements of productive mathematical noticing. To aid analysis, the Three-Point Framework (Yang & Ricks, 2012) and the processes of noticing (Jacobs et al., 2010) were used to code instances in the selected episodes. A 'thematic approach' was used to develop patterns within the instances of these selected episodes (Bryman, 2012, p. 578).

The FOCUS Framework

The FOCUS framework characterises two important components of noticing by teachers who engage in productive classroom practices:

1. An explicit focus: The three focal points, and their alignment;

2. Focusing: The active process of pedagogical reasoning that aligns the instructional decisions to the observations made.

An explicit focus reflects the notion that noticing is more likely to be productive when teachers use a frame to guide what they attend to (Levin, Hammer, & Coffey, 2009). The second component of the FOCUS framework stems from the idea that it is not trivial to direct one's noticing, but this may be realised through teachers' pedagogical reasoning, which connects what they observe to how they respond to classroom situations. Together, attention to these two components of the FOCUS Framework can support a teacher's efforts to enact productive classroom practices that can enhance students' reasoning.

An Explicit Focus

The FOCUS framework uses three specific mathematically significant aspects of learning and teaching as explicit foci for noticing. These three focal points are (1) Concept; (2) Confusion; and (3) Course of action. These points parallel the Three Point Framework suggested by Yang and Ricks (2012). The teaching of *fraction of a set* at Primary 4 (age 10) can be used to illustrate these focal points: a teacher may identify the key concept as the fact that the relationship between the number of elements (items) in a subset and the set can be represented as a fraction (Concept); recognise students' confusion with this concept in terms of their inability to see a set of objects as the whole (Confusion); and propose to create tasks where students can partition a set of items and explain how their partitions relate to fractions (Course of action). The three focal points also provide a *language* for teachers to describe and analyse the relationships between specific aspects of the concept (Concept and Confusion) to the design of the task (Course of action).

Besides these three focal points, the FOCUS Framework also highlights the crucial notion that aligning these three points is challenging. A teacher, for instance, may be able to identify the concept and students' confusion around the concept, but may not be able to respond appropriately during the planning, delivery or review of a lesson (Choy, 2014a, b). Ensuring that the teacher's response targets the confusion associated with the concept can increase the likelihood of a more productive stance in noticing. Therefore, the alignment of the three focal points forms part of the explicit focus for noticing.

Focusing Noticing

The process of focusing attention in order to bring the three focal points into alignment may not come naturally to teachers. This highlights the critical role of pedagogical reasoning as a mechanism to connect the process of attending to the process of responding in noticing. The alignment of the three focal points thus depends on how teachers connect their responses to what they see or attend to. The FOCUS framework proposes that teachers' responses can be better aligned with the other two focal points when they base their instructional decisions on the interpretation of what they attend to. This can be achieved by justifying responses using specific details from observations, and by considering other possible courses of action. To a large extent, this component of the FOCUS Framework resonates with what van Es (2011) termed as focused or extended noticing.

Together, the explicit focus (the Three Points and their alignment) and the pedagogical reasoning (focusing), not only provide a way to examine at the macro level what makes noticing productive, but more importantly, can capture a micro view of what happens during the planning, teaching and reflection of a lesson. These perspectives can be combined to build a theoretical or ideal model of the noticing process, which describes and decomposes noticing at a more fine-grained level, as demonstrated in Figure 1.

The theoretical model from the FOCUS Framework (see Figure 1) describes what, and how, a teacher can notice productively when learning from practice. It maps a teacher's noticing processes (attending, making sense and responding) through three stages of learning from practice (planning, teaching, and reviewing) to the three key productive practices for mathematical reasoning (designing lessons to reveal thinking; listening and responding to student thinking; and analysing student thinking). In other words, the model describes an idealised process of productive noticing, where teachers make instructional decisions that promote student thinking. The model explicitly highlights the three crucial focal points, and how the alignment between these three points can be achieved. Referring to the planning portion in Figure 1 as an example, a teacher is more likely to design a task that targets and reveals student thinking when he or she:

1. Identifies specifics of the mathematical concept(s) for the lesson;
2. Recognises what students may find difficult or confusing about the concept;
3. Analyses why students might find the concept difficult or confusing;
4. Analyses possible ways to address students' confusion about the concept; and
5. Develops and implements a high-level cognitive demand task (Smith & Stein, 1998) to target students' potential confusion about a concept.

The explicit focus (Steps 1 and 2) helps support teachers in their systematic reflection of student thinking. Teachers' analysis of students' confusion (Step 3) and possible ways to address the identified sources of confusion (Step 4) prepares the teachers to consider possibilities so that they can respond with a better designed task (Step 5), which targets students' confusion to support them in their learning of the concept.

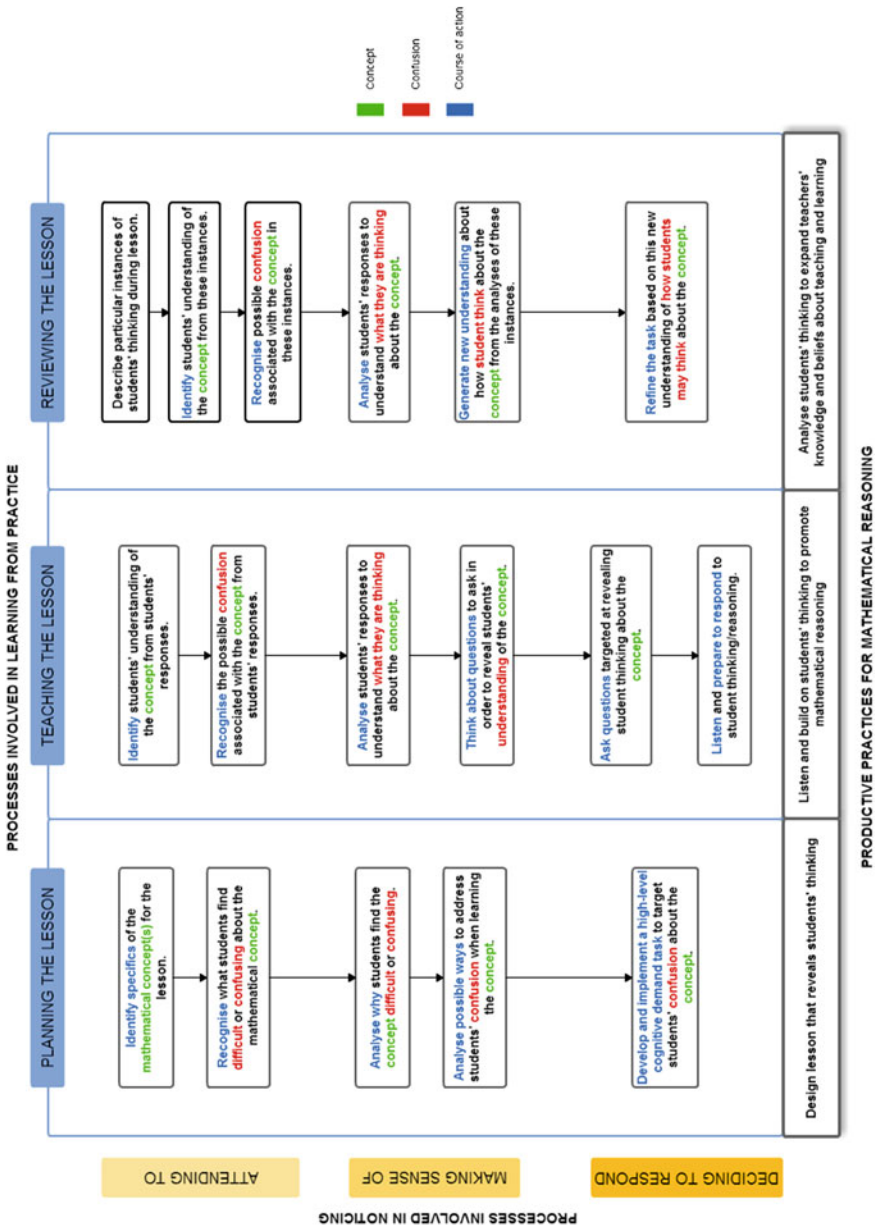


Figure 1. Theoretical model for productive noticing.

Snapshot of Noticing: From Planning to Reviewing

This section illustrates how the two components of the FOCUS Framework—an explicit focus and focusing noticing—can be used to provide snapshots of teacher noticing by analysing and characterising teachers' noticing. Three vignettes, which centre on James (the teacher who taught the lesson) are presented and analysed: The first focuses on a few discussion episodes that happened during the planning; the second highlights what James noticed in the moment during the lesson; and the third recounts what teachers noticed during the post-lesson discussion. Each vignette is then followed by a discussion on how the FOCUS Framework can be used to support teacher noticing before, during and after the lesson.

Vignette 1: Analysing James' Noticing During Task Design

The role of analysing and justifying in aligning the Three Points can be seen in James' explanation of how a *met-before* (McGowen & Tall, 2010) of 'fraction as part of a whole' may hinder students' understanding of 'fraction of a set'. During the first Lesson Study discussion, James highlighted the targeted concept and possible student confusion:

> I think the objective for fraction of a set is for students to see, to interpret fraction as part of a set of objects. Previously, the fraction [concept] they learnt is more of part of a whole. They are very used to thinking about part out of a whole. Now that we give them a lot of whole things, they cannot link that actually these fractional parts can refer to a set of whole things also. So I think, to me, I feel that the connection that is missing, is that, how this fraction concept—which is part of one whole, which they have learnt so far—can be linked to [a set of] whole things. For example, previously we used to teach fractions as parts of a cake or pizza. From that, how can it be that we have many pizzas, we don't cut out the pizza, there is a fraction of the pizzas. I think they cannot make a link there.

In this episode, James not only described specific details about the Concept ('… to interpret fraction as part of a set of objects'.) and the Confusion ('They are very used to thinking about part out of a whole'.), but he was also able to relate these aspects to his knowledge and experience. James then suggested that students may only possess an image of fractions as 'part of a whole' (see Figure 2); and highlighted how the type of examples used by teachers to teach fractions ('…previously we used to teach fractions as parts of a cake or pizza…') may have been stuck in the students' minds. Thus, according to James, students' notion of fraction as 'part of a whole' might have conflicted with the notion of fractions as part of a set of objects:

> For me, the main difficulty is to relate part of a whole into items that are "whole" but you take a fraction out of it. So, I think that's where the confusion comes… [After some time] For example, if you say ¾ of the cats are… [Imitating the students] Ah… you cut the cat into three quarters? [Laughter] Cut each cat into four parts. So, yeah, but based on what they learnt so far, that may be the first thought they might have. To them, fraction could still be cutting up into parts. Whereas, fractions of a set, we leave the things as a whole entity but we look it as a collection of things.

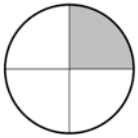


Figure 2. Concept image of fraction as 'part of a whole'.

The link between students' image of fraction as 'part of a whole' and their difficulty grasping the idea of 'fraction of a set' was further elaborated by James with the use of two examples—the pizza and the cat. Particularly, he drew teachers' attention to students' ways of thinking about fractions with his vivid example of 'cutting up the cat' to illustrate how students might be thinking of fractions as 'cutting up into parts'. In the next session, James highlighted an example from the textbook to reiterate what students were confused about:

> I think that the difficulty is putting the things into the sets, and imagining that each of this set is one part. The textbook makes it look like a very good way to teach this, they arrange the items very neatly into visible lines like this, for example, like this one, 2 fifths of the circles are yellow [See Figure 3]. It is very clear and you can see two sections. But without the pictures, the children cannot imagine neatly like that.

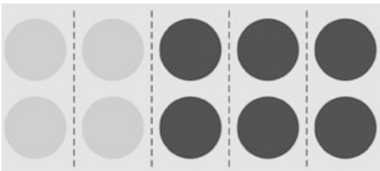


Figure 3. '*Visible lines* 'to show equal partitions of a set of items.

As seen from these instances of noticing, James was able to direct his colleagues to consider possible reasons for students' difficulties by maintaining a focus on, and reasoning about, the Concept and Confusion. He stressed the diagrams might have made it obvious for the students to see the partition, and students possibly find it difficult when the diagrams were removed. James's noticing prompted Flora to suggest getting students to 'arrange' the items into the partitions and explain why they arranged it that way. James then suggested a possible teaching approach that made explicit links between the three focal points:

> I think the confusion part also comes when… we tell [them] that … ¼ of the cups are yellow and then the answer is 4 cups. Huh… ¼ and then why got 4 in the 1/4? They cannot link between the… the ¼ in their mind is still ¼ of a whole… and then there is these four cups, four whole things… and so they cannot link… I was thinking whether we can put it into… something more familiar because… eh… they have learnt models [referring to the Singapore Model Method], how to represent questions in model also, so, I was just looking at this… could we box the whole thing up instead… These lines can be the partitioning of the whole model… they [students] can still see that the 4 items are still inside the parts.

James' suggested approach was directly linked to students' image of 1/4 as 'part of a whole'. He attempted to use the part-whole model (see Figure 4), which the students were familiar with, as a scaffold to help them see that there could be 'whole items' inside a 'part'. This provided a bridge for students to extend their notion of fractions by emphasising fraction as a way to express the relationship between a part and its whole.

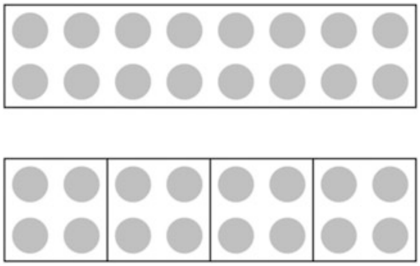


Figure 4. James' use of the part-whole model.

By directing students' attention to the number of discrete items in a partition of the whole, James hoped to create a way for students to see that fractions can be used to refer to 'whole things'. James' noticing would be characterised as productive in this case because he directed his noticing to the three focal points and justified how the suggested approach might target students' confusion about the notion of fraction of a set. Hence, what James attended to and analysed provided some design considerations for the task. What distinguished James' noticing as more productive was not the workability of the approach suggested, but rather the justification that reinforces the alignment between the three points. Justification based on what was noticed not only helped the teachers maintain their attention on specific concepts and students' confusion, but also lessened the likelihood of generating a course of action that does not provide opportunities to enhance students' reasoning.

Productive Noticing in Task Design. James' noticing, as analysed by the FOCUS framework, illustrates that both the focus and the focusing are crucial for designing a task that reveal student thinking. Engaging students with appropriate tasks is critical for developing students' mathematical reasoning (Brodie, 2010; Sullivan, Clarke, & Clarke, 2013) and, hence, the design of mathematics tasks plays a key role in facilitating and encouraging student thinking (Ball & Bass, 2003; Mason & Johnston-Wilder, 2006; Smith & Stein, 1998). Therefore, teachers need to design, select and adapt tasks thoughtfully so that they can provide ample opportunities for students to generalise, explain and justify their mathematical ideas (Ball & Bass, 2003; Smith & Stein, 1998; Sullivan et al., 2013). The FOCUS Framework can support teachers to do this work by offering them a language to explain their task design with regard to the three focal points. This helps to direct teachers' attention to how students think about the concept so they can prepare to notice when they teach the lesson.

Vignette 2: Analysing James' Noticing During Lesson Delivery

After the initial warm-up activity, James then went on to explain the proposed task using 12 physical cubes with a colour configuration of 2 green, 4 blue, 3 red and 3 yellow. In the following interaction, James engaged Student S5, perceived as competent in mathematics, in an interesting conversation (Figure 5):

In this episode, James attended to S5's use of the cubes to reveal how S5 thought about the partitioning. He realised that S5's idea of partition was different from what he had in mind (Line 15). James then tried to ask S5 some questions to understand what S5 was thinking with regard to the six groups (Lines 9 and 11). S5 seemed to have understood about the 'six parts' and counted each cube (Line 9) in one of the rows he created. James could see that S5 understood that 1/6 of the total number of cubes in the first row is green ('And the green is what? 1 out of? 6, is it?'). S5's answer of 'still the same' in Line 12 indicated that he perceived the grouping as two equal groups of 6 cubes, with 1 green cube in each group or possibly a different partition. James's expected answer—that the two green cubes form one out of the six equal partitions—was thus different from S5's. Therefore, James tried to get Student S5 to see his expected answer by putting the two rows of cubes together (Line 13).

James' question (Line 13 and 15) indicated he was trying to get students see his expected arrangement of the cubes. His use of the cubes as a way to hint at the intended arrangement did not seem to convince S5 (Line 13). S5's hesitation pointed to a possible confusion and showed he did not attend to the same structural features (e.g. imaginary lines) as his teacher. This was evident from S5's arrangement of the cubes that did not show the six partitions clearly (Line 15).

Sensing that S5 might not have caught his expected answer, James then asked another student, S7, to arrange the cubes and he came up with a configuration meeting James' expectations. It appears that James noted and interpreted specifically what S5 was thinking with regard to the partitioning, but his response was limited in revealing explicitly what S5 was thinking. James tried to direct S5 to see the intended arrangement through a series of questions to funnel his thinking. This approach did not seem to work and S5 was confused at the end of this episode. An alternative approach would have been for James to ask S5 to explain his own reasoning for his arrangement, so that James could then make sense of what S5 was thinking (Burns, 2005; Davis & Renert, 2014). His response during the interaction (Critical Point) did not help S5 to overcome his difficulty in seeing the proposed partition, and James missed an opportunity to find out what S5 was thinking. It appears that James did not have, at his disposal, other ways of responding when the

1. James: What fraction of my cubes is green? OK, [S5]?

2. S5: 1 out of 6.

3. James: 1 out of 6.... 1 sixth. Let me shift it up a bit (James shifts the cubes on the table so that everyone can see on the projector). Anybody disagree with [S5]? He said it's 1/6. Hey... [S6]? No? Do you agree or disagree with [S5]?

4. S6: No.

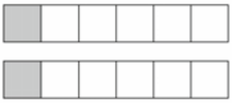
5. James: Don't agree. Then what would be your answer then?

6. S6: 2 out of 12.

7. James: Ok. We have two answers here. 2 out of 12 and S5 said 1 out of 6. (Writes the fractions on the white board) Do you think they are related?

8. Students: [Chorus] Yes…

9. James: Ok. First, [S5]. Can you come and show us how you got 1 part out of 6 when there are so many cubes here. (S5 comes out and arranges the cubes.)



Ok. [S5], stay there... stay there. Where's your six parts? (S5 points to the cubes and counts 1, 2, 3, 4, 5, 6...)

And the green is what? 1 out of? 6, is it?

10. S5: Yeah.

11. James: Then what about the remaining cubes?

12. S5: Still the same.

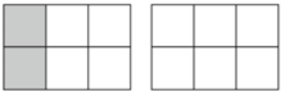
13. James: Still the same, ok? If I put it this way? (James puts the two groups of cubes together.)

Would you all be able to see the six parts?

Figure 5. Transcript for Vignette 2.

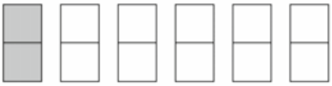
14. Students: [Chorus] Yes...

15. James: Yes... So, [S5], where are the six parts? (S5 points to the cubes again, and shrugs his shoulders.) Ok. Can you imagine the imaginary lines between the cubes? OK. How can you have put this better? (S5 rearranges the cubes.)



How many parts can you see now? Anybody wants to give [S5] a hand?

Yes, [S7]. Ok. Thank you, [S5]. (S7 comes out to do another arrangement.) Mmm ... Something different from what [S5] did. (S7 rearranges the cubes to be 6 groups of 2. See Figure 5.)



Ok. Let's shift this a bit. Ok. Do you see 6 parts now?

16. Students: [Chorus] Yes…

17. James: A bit clearer?

18. Students: [Chorus] Yes…

19. James: Thank you, [S7]. I was asking for the fraction of…

20. Students: One out of six…

21. James: Green cubes right? So, it's one part out of…

22. Students: Six.

23. James: Six parts. Same thing, yeah? Has my number of cubes changed?

24. Students: No…

25. James: So, actually, is [S6] right to say that it's actually 2 parts of 12 also?

26. Students: Yes.

27. James: Actually, he's correct also? But how did I get from 2/12 to 1/6?

28. Students: Divide… Simplify…

29. James: Yes… we could have simplified it, right? They are equivalent fractions, right?

Figure 5. (continued)

student gave an unexpected explanation. The lack of alignment between his response and S5's confusion reflects a lapse in James' awareness of the student's thinking. Hence, his noticing would be classified as non-productive, according to the FOCUS Framework, even though his attention was focused and his interpretation might be accurate.

Noticing in responding to critical incidents. James' encounter with Student S5 is an example of a critical incident. Critical incidents are events that occur during a lesson, and which have the potential to deepen our understanding of students' mathematical thinking (Goodell, 2006; Yang & Ricks, 2012). These incidents can involve students' unexpected responses to teachers' questions (Yang & Ricks, 2012); those that raise questions about teaching approaches or students' understanding (Goodell, 2006); or events that change the direction of the lesson from what was planned (Fernandez, Cannon, & Chokshi, 2003). Reflecting on critical incidents is important for developing teaching practices that enhance students' mathematical thinking (Fernandez et al., 2003; Goodell, 2006).

The ability to see and interpret these incidents in the moment can impact how teachers decide to respond to these events. The FOCUS Framework highlights that the key to respond productively to enhance student reasoning lies in the ability of the teacher to adopt a more interpretive stance in listening, and allow students' responses or answers to modify the flow of the lesson (Davis & Renert, 2014). Moreover, the teacher has to think on the spot to attend selectively to the myriad responses from the students. The alignment between the three focal points is helpful for directing teachers' attention to the mathematically significant details in the midst of a lesson. By being more sensitive to students' Confusion, and maintaining a focus on the Concept, teachers might be able to raise their own awareness of how they listen to students' responses that make aspects of their thinking visible. In so doing, they might have a better chance of generating a Course of action that enhances students' mathematical thinking. On the contrary, as illustrated by James during the critical incident, when teachers fail to maintain a keen awareness of student thinking, they are more likely to think about their own thinking, instead of the students'.

Vignette 3: Analysing James' Noticing During Post-lesson Discussion

In the post-lesson discussion, the first thing that James brought to the attention of the teachers was students' inability to partition, which he hypothesised was because they counted and simplified the fraction. For example, students might have counted 2 green cubes out of 12 and written the answer as 2/12 before they simplified to 1/6:

> The glaring thing that I noticed about my pupils is that too many of them, they didn't get their fraction by partitioning … they got it more by counting and then simplifying… so that was the easy option to them. Which was why later when I got them to explain, "How did you get this fraction for example?"… "one sixth of the cubes were red" or something like that. Some of them were not able to show the six parts or to group the objects into six parts.

> So they were a bit lost. Because how they did it was, count the number of red cubes over the total number of cubes, then simplify. When they cannot put it in parts, right … it was very clear what their thought process was – simplify …

James highlighted that the students ‘didn’t get their fraction by partitioning’, but instead by ‘counting and then simplifying’. He explained how that prompted him to try asking students to reason how they arrived at the fraction. James was able to give very specific details about students’ difficulty in showing the partitioning of the cubes (‘they were not able to show the six parts…’), and interpreted that as a manifestation of their ‘thought processes’. James’ noticing was not only specific and focused on the three focal points, but also more importantly, it set the stage for the teachers to learn about another possible student confusion not previously discussed. James attended to the Course of action—getting students to show their understanding by representing the fractions through partitioning of the cubes—and realised that students had difficulties doing that (a new Confusion), and supported his claim using his observations from the critical incident. He reasoned that there could be a gap in students’ understanding even though they might give the correct answers.

Even though the teachers did not decide precisely how to respond, they suggested different possible interpretations that could potentially generate new understanding of how students think. The teachers attended to specific instances, and made connections between their observations, and that of others to their own knowledge and experience. The process of detailed interpretation further encouraged teachers to examine these observations more deeply. For example, the teachers argued that getting students to explain their partitioning, even when they were able to give the correct answers, could have given teachers insight into students’ thinking. Flora articulated the need to listen and referred to Student S5 as an example:

> [Student S5] is very complex when he does maths. I’ve had him to explain to me. He can get an answer just like that – without workings or anything. The boy is very complex up here (pointing to her head). And I don’t fault him for doing things a bit differently – as long as I understand what he is trying to say like, I can imagine how he does things. I think it’s okay. Like for him, he may arrange it that way, but he may mean it like the second way…

James agreed and also highlighted that it is important to be more specific in the questioning with regard to the three focal points. The emphasis on getting students to explain more specifically in order to reveal their reasoning suggests a shift from explaining to listening as a result of teachers’ noticing. As Mason (2002) suggests, the purpose of noticing is to bring to mind the possibility of a different decision. James’ noticing, throughout the post-lesson discussion, sensitised his awareness and helped him think more deeply about students’ thinking beyond giving the right answers:

> I was just thinking the danger of – during the design of this lesson, we didn’t see that maybe they may skip the partitioning part of it … they didn’t show how the answer is found. It is something we need to recognise. It is good that we now know that if they missed the partitioning part… this may cause a problem later. Missing the partitioning part will be fine

> until we show them they have a problem. Even though they can do a fraction of a set, and they can solve fraction of a set problems – it will pose learning problems in future when they move on… I think we need to look at it more carefully.

James' noticing can be largely characterised as productive because his suggestion was targeted at what he saw and understood about students' confusion when learning the concept. What he noticed about the critical incident helped other teachers to gain insights into students' thinking: students' difficulties in partitioning and how that is related to understanding fractions. More importantly, he recognised the 'blind spot'—that students might skip the partitioning—for their initial lesson plan and suggested that they should look at the task design more carefully. James was able to see how this gap in students' understanding could have implications beyond the lesson to find 'the general meaning of such incidents' (Yang & Ricks, 2012, p. 46). The use of specific instances to support his claims or suggestions also indicates that James has begun to gain a heightened awareness of student thinking when viewing the critical incidents that happened during the lesson.

Noticing to zero in on student thinking during reflection. Although James' in-the-moment noticing in Vignette 2 was less productive, his noticing during reflection was productive as described in the preceding paragraphs. Fruitful post-lesson discussion occurs when the points raised help teachers to refine their ideas about students' thinking or lesson design. They should go beyond vague or broad statements to focus on supporting or refuting claims made by teachers about students' learning. In this way, the discussions can move towards a more generative position when these claims are supported or refuted based on teachers' observations of specific instances. For this study, we supported teachers' systematic investigation of their practice using the three focal points and their alignment to frame the post-lesson discussions.

As seen from James' responses, his focus on the three focal points seemed to help him zero in on the mathematical features of the critical incident. In particular, James and other teachers were able to draw on specific instances from the lesson to analyse and explain whether the planned Course of action targeted students' confusion. As a result, the discussion, as described in Vignette 3, generally centred on students' strategies related to the incident, and the implications for the design of the task. More importantly, the two components of the FOCUS Framework enable teachers to think about what they observed from the lesson, which led the teachers to gain new insights about students' thinking as seen in Vignette 3.

Concluding Remarks

The FOCUS framework highlights that teachers' noticing is more productive when they direct their attention to the *mathematically significant* aspects of engaging in all three phases of diagnostic teaching—the planning, delivery and review. The three focal points offer a focus for teachers to attend to, and make sense of, in order to respond with an instructional decision that can potentially enhance student reasoning. Our findings support the use of an explicit focus to frame noticing (Goldsmith & Seago, 2013), rather than not directly specifying a focus for teacher noticing (Star et al., 2011).

The snapshots of noticing, presented here, demonstrate how the FOCUS Framework is useful for researchers when analysing what, and how, teachers notice during the planning, teaching and reflecting of mathematics lessons. The three focal points and their alignment, together with the pedagogical reasoning processes to align the three points, provide a means to describe and characterise both more productive and less productive noticing in terms of the instructional decisions undertaken by the teachers. These snapshots paint a detailed portrait of a teacher's noticing, and can be used to point out the strengths and areas for improvement to promote more productive noticing. Such characterisation can provide researchers with a language to decompose and analyse complex interactions between the processes of noticing before, during and after a lesson.

With regard to its practical implications, the framework provides a means to support teachers in reflecting systematically, suspending one's habitual reactions to classroom events, in order to have a different act in mind (Mason, 2002). By emphasising both specificity (van Es, 2011) and *alignment* of the three focal points, the framework can be used to create opportunities for teachers to focus on mathematically relevant details when planning to teach a lesson. In this chapter, the two components—explicit focus and pedagogical reasoning—of the FOCUS Framework were used to support teachers in their planning and reasoning about the evidence from observations, in order to target their instructional decisions at enhancing student reasoning.

While this research aimed to characterise the notion of productive mathematical noticing, it is important to acknowledge that our study was limited to investigating the mathematical and pedagogical aspects of enhancing students' reasoning. This study, for example, did not investigate what teachers notice about classroom management (van den Bogert, van Bruggen, Kostons, & Jochems, 2014) even though it may play a role in carrying out mathematical activities to enhance reasoning. Moreover, what students notice mathematically about a task, and how, was not examined. Since teacher noticing and student noticing are 'two sides of the same coin', it could be fruitful for future researchers to explore the relationships between teacher and student noticing. Finally, despite the measures taken to reduce researcher's bias, our interpretation of the data only constitutes one possible emerging narrative about productive teacher noticing. It remains to be seen whether the FOCUS Framework is robust enough to be applied, and adapted, for other contexts and in other studies.

Notwithstanding the limitations of this study, our findings demonstrate how the framework can support teachers in the 'practices' of noticing, so as to enhance their 'sensitivity to notice opportunities to act freshly in the future' (Mason, 2002, p. 59). Therefore, this study suggests the potential of incorporating the framework into the design of professional development activities. In conclusion, the FOCUS Framework, as a research and practical tool, can afford opportunities for both researchers and teachers to investigate the high-leverage practice of teacher noticing.

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Noticing Distinctions Among and Within Instances of Student Mathematical Thinking

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Abstract In this chapter, we argue that there are two critical aspects of noticing student mathematical thinking: noticing within an instance of student thinking and noticing among instances of student thinking. We use the noticing literature to illustrate these distinctions. We then discuss how the MOST Analytic Framework analysis provides structure and guidance for noticing both within and among instances, and illustrate the complex interaction of these two types of noticing through the analysis of an excerpt of classroom dialogue. We conclude by offering the perspective that studies of noticing must go beyond placing value on student mathematical thinking to discriminating among instances of student thinking based on their potential to be used to support students' understanding of important mathematics.

Keywords Teacher noticing · Student mathematical thinking · Teachable moments · Ambitious instruction · Building on student thinking

Introduction

Effective mathematics teaching is a complex endeavor, particularly when it is viewed as necessarily being grounded in the mathematics of students. Such teaching is sometimes referred to as *ambitious instruction* (Kazemi, Franke, &

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Lampert, 2009; National Research Council [NRC], 2001), and one way to unpack and begin to better understand it is to understand its core or high-leverage practices (Grossman, Hammerness, & McDonald, 2009). One such high-leverage practice is that of productively using student mathematical thinking during whole group instruction. As have others (e.g., Sherin, Jacobs, & Philipp, 2011), we see noticing as a critical skill related to this practice. Our ongoing work investigates instances of student mathematical thinking made public during whole class interactions that, if made the object of consideration, have the potential to foster learners' understanding of important mathematical ideas—instances of student thinking that we call Mathematically Significant Pedagogical Opportunities to Build on Student Thinking [MOSTs] (Leatham, Peterson, Stockero, & Van Zoest, 2015). In essence, our work focuses on developing tools to identify instances of student mathematical thinking that are particularly worth noticing—worth it because they provide opportunities to enact the high-leverage practice of productively using student mathematical thinking to further mathematical understanding.

As we have considered the relationship between our MOST work and the teacher noticing literature, we have come to realize that there are different aspects of noticing that teachers need to learn to become skillful at noticing. Teachers need to be able to notice important features of particular classroom instances, while at the same time being able to notice which classroom instances have more or less potential to support student learning. In this chapter, we first highlight two different categories of noticing that we have identified in the literature. We then discuss how our MOST research attends to each of these categories of noticing. In doing so, we theorize that learning to notice in different ways and to move back and forth between these ways of noticing is necessary to fully make sense of and discriminate among the student thinking that emerges during a classroom lesson.

Noticing Within and Among

We adopt a definition of noticing as being comprised of three interrelated skills—attending, interpreting, and deciding how to respond (Jacobs, Lamb, & Philipp, 2010). As we have reviewed the noticing literature, we have come to see that noticing interventions focus on each of these component skills to different degrees and through different means. Some interventions develop noticing skills by engaging teachers in learning to make sense of and respond to a few specific classroom instances (e.g., Fernández, Llinares, & Valls, 2013; Schack et al., 2013); others do so by having teachers learn to notice important instances in larger artifacts of practice, such as classroom video (e.g., Sherin & van Es, 2009; Stockero, Rupnow, & Pascoe, 2017). Before we take a more detailed look at these differences in the literature, we emphasize that our intent is not to place more value on one type of noticing study than another, but to highlight important differences in the foci of work related to teacher noticing.

Although the literature on noticing has become quite diverse, we limit our discussion here in two important ways. First, we focus only on studies that relate to in-the-moment noticing—those in which there is some sense of immediacy to teachers' noticing in that participants need to offer an analysis or response within a relatively short time frame after viewing a classroom instance. Thus, we do not include work that is more broadly related to teachers' reflection on a lesson after it has taken place (e.g., Santagata, Zannoni, & Stigler, 2007). Second, although studies have focused on a variety of noticing that takes place in a classroom—such as teachers' noticing of children's equitable participation (Wager, 2014), "salient features of classroom instruction" (Star, Lynch, & Perova, 2011, p. 117), and students' noticing of the mathematics in a lesson (Lobato, Hohensee, & Rhodehamel, 2013)—we narrow our discussion to studies that are grounded in teachers' noticing of student mathematical thinking.

We see studies focused on the professional noticing of student mathematical thinking as generally falling into two categories: (a) noticing *within* an instance, and (b) noticing *among* instances. The first category includes interventions in which teachers (or prospective teachers) are given a specific instance of student thinking that they are asked to analyze. In such studies, the task of the teacher is not to identify instances to be analyzed, but instead, to notice what is happening *within* the instance of student thinking they are provided. Some studies of this type use one-on-one student interviews to prompt teachers to notice a student's thinking related to a particular mathematical focus—for example, early arithmetic reasoning (Jacobs, Lamb, Philipp, & Schapelle, 2011; Schack et al., 2013)—and then propose a response to that thinking. Other such studies ground teachers' noticing in student written work (e.g., Fernández, Llinares, & Valls, 2013; Haltiwanger & Simpson, 2014; Llinares, 2013), but are similar in nature to the interview studies in that they focus on participants' ability to notice individual student's reasoning in a particular mathematical context. An interesting aspect of the Llinares' studies is that although the primary focus of the prospective teachers' noticing seemed to be *within* the given instances of student thinking (i.e., noticing and interpreting each student's strategy), the analysis of the data had an element of noticing *among* instances in that the analysis also focused on whether participants were able to discriminate among proportional and additive reasoning strategies in the student work.

Another variation of noticing *within* studies is illustrated by Jacobs et al. (2010), who used both a set of written student written work and a video excerpt of a lesson to prompt teachers' noticing of "the mathematical details of children's strategies" (p. 172) related to whole-number operation. Although many studies that use video excerpts depicting multiple students' strategies fall within the noticing *among* category (as we discuss below), we see this study as noticing *within* because each students' strategy shown in the video clip was provided to teachers in a list to ensure that they discussed each strategy in their response. To summarize, noticing *within* studies narrow the noticing focus in some way by limiting the "number of salient features" (Schack et al., 2013, p. 395) to which teachers might attend, typically by asking participants to analyze particular instances of mathematical thinking of individual students in isolation from the complexity of classroom interactions.

The second category of studies focuses on noticing *among* instances, typically in classroom video. In this collection of work, teachers are asked to select the instances they deem important in videos of classroom instruction, and write about or discuss what they noticed in these instances (i.e., explain why those particular instances were important or interesting). Much of Sherin and van Es' noticing work around video clubs (e.g., Sherin & van Es, 2005, 2009; van Es, 2011; van Es & Sherin, 2002) falls into this category. In these studies, a general "What did you notice?" prompt was used to incite teachers to identify instances in excerpts of classroom video that they found interesting or compelling in some way, with a goal of pushing them to become more attuned to noticing students' thinking. A more recent study by Roth McDuffie et al. (2014) is similar in that general prompts related to four different viewing lenses were used to ground small and whole group discussion of video excerpts. A difference between this and the Sherin and van Es studies is that here the prompts pushed participants to make generalizations based on their viewing of the entire video clip (i.e., "What specific math understandings and/or confusions are indicated in students' work, talk, and/or behavior?", p. 250), rather than select particular instances to discuss. We still see this as noticing *among*, however, since the participants are asked to make generalizations drawn from among all the instances of student thinking they observe in the video, rather than being asked to engage in analysis of a given instance of student thinking.

Other noticing *among* studies have required teachers to select instances from complete classroom video. These studies include those that have teachers document in-the-moment noticing during their own instruction by using a self-mounted wearable camera to archive interesting instances while they are teaching a lesson (e.g., Russ & Luna, 2013; Sherin, Russ, & Colestock, 2011; Sherin, Russ, Sherin, & Colestock, 2008) and that have teachers select instances to analyze from videos of complete lessons they have taught (e.g., Barnhart & van Es, 2015). As with the video excerpt studies, participant noticing in these studies is typically analyzed by categorizing the instances that participants notice in terms of factors such as who and what is noticed, the level of detail of the analysis, use of evidence, and relationships among what was observed—measurements that are typical for noticing work that falls within the *among* category.

It is important to note that in all of these studies focused on noticing *among*, at least some level of noticing *within* was required of participants as they engaged in writing about the instances they selected and in small and whole group discussion of their noticing. We see the studies falling primarily in the *among* noticing category, however, because no clear structure was provided to support the participants' selection or analysis of particular instances and no value was placed on whether participants noticed some instances over others. There are, however, some studies in which both aspects of noticing are prevalent. Walkoe (2015), for example, had teachers tag in a video excerpt instances of "interesting student algebraic thinking" (p. 529) and then choose the three instances they found most interesting and explain why. An Algebraic Thinking Framework developed for the study was used to help participants make sense of the type of student thinking contained in the instances they identified. Similarly, Stockero (2014), Stockero et al. (2017) had teachers identify instances of student thinking in whole class video that met the criteria of the MOST Analytic Framework (Leatham et al., 2015). In both of these studies, the participants had to select instances of student thinking from *among* those in the video, and then reason *within* those instances to determine whether they met specified criteria. In the Walkoe (2015) study, the participants then stepped back to determine which instances were most important from *among* those they had selected.

We reiterate that we do not claim that either of these aspects of noticing is more or less important than the other. Rather, we claim that both aspects of noticing—*within* and *among*—are critical to the work that teachers do in the classroom. Teachers need to be able to key in on instances of student thinking that surface during a lesson, and then engage in analysis of these instances to determine whether they would be productive to incorporate into a lesson and if so, how this might best be done. In the following section, we use the MOST Analytic Framework to discuss the potential benefits of attending simultaneously to both noticing *among* and noticing *within* instances of student thinking. To set up that discussion, we first describe the MOST Analytic Framework that we have developed to support researchers and teachers in noticing important instances of student mathematical thinking within the complexity of a classroom lesson.

The MOST Analytic Framework

We define MOSTs as occurring in the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunity (Leatham et al., 2015). Although in practice a teacher would consider these characteristics almost simultaneously during a lesson, we have found it useful for our purposes to define them such that they build on one another. With a goal of understanding the practice of productively using student mathematical thinking during a lesson, we consider the foundational characteristic of a MOST to be student mathematical thinking. We then focus on whether the student thinking is mathematically significant—whether it is likely that incorporating the student mathematical thinking into the lesson would advance students' understanding of important mathematical ideas. Finally, we consider whether there is a pedagogical opportunity—whether the student mathematical thinking can and should be incorporated into instruction at the moment it becomes public.

The MOST Analytic Framework (Leatham et al., 2015) uses two criteria per characteristic to determine whether an instance of student thinking embodies the three characteristics of a MOST. To be characterized as student mathematical thinking, an instance must meet the two criteria of having *student mathematics* and having a *mathematical point*. To meet the student mathematics (SM) criterion, the observable student action (typically a statement, gesture, or written work) must provide sufficient evidence to allow one to make a reasonable inference about the mathematics the student has expressed. For an instance to have a mathematical point (MP), one must be able to articulate the mathematical idea “closest” to the student mathematics of the instance, in the sense that the idea is “one that learners could better understand by considering the student mathematics” (p. 95). The criteria for significant mathematics are that the instance is *appropriate* and *central*. For an instance to be appropriate, the MP must be at a suitable developmental level for the students in the classroom, neither an idea that is likely already understood by most students in the class nor an idea that is too advanced to be accessible to the students. Centrality is determined by considering whether the MP is related to a mathematical goal for the students’ learning, ranging from an immediate goal of the lesson to broader goals related to the unit, course, or the discipline of mathematics. Finally, for an instance to embody the third characteristic of a MOST, pedagogical opportunity, it must meet the *opening* and *timing* criteria. The opening criterion is met when “the expression of a student’s mathematical thinking seems to create, or has the potential to create, an intellectual need for students to make sense of the student mathematics, thus providing an opportunity to understand the mathematical point” (p. 101). Another way to think about determining whether there is an opening is by considering whether the instance has created a cognitive conflict for students in the class that needs to be resolved. To meet the timing criterion, one must determine whether the time is pedagogically right, at the moment the instance occurs, to take advantage of the opening to further students’ understanding of the MP of the instance. When an instance satisfies all six criteria, it embodies the three requisite characteristics of student mathematical thinking, mathematically significant, and pedagogical opportunity, and thus is a MOST. For further elaboration on the MOST Analytic Framework see Leatham et al. (2015) and Stockero, Peterson, Leatham, and Van Zoest (2014).

By definition, MOSTs are high potential opportunities for a teacher to use an instance of student thinking in a particular way to enhance student learning—by *building* on the student thinking (Leatham et al., 2015). We define building as the practice of making the student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea. Thus, the most productive teacher decision in response to a MOST is predefined to be a series of moves that engages the whole class in making sense of the SM in order to gain an understanding of the MP of the instance. We see the teaching practice of building on student thinking as aligned with core ideas about effective teaching and learning of mathematics in that this practice engages students in constructing knowledge socially by participating in meaningful discourse aimed at developing a shared understanding of mathematical ideas (National Council of Teachers of Mathematics, 2014). Thus, we see building as a particularly productive way for teachers to capitalize on the MOSTs that surface in their classroom.

Noticing *Within* and *Among* Using the MOST Analytic Framework

We view the three main activities of teacher noticing (attending, interpreting and deciding) as requiring a complex combination of both *within* and *among* noticing. In our work, we are trying to unpack the complex practice of productively using student mathematical thinking, a practice that requires skill in moving back and forth between noticing *among* and noticing *within* instances of student mathematical thinking.

Attending seems to be primarily an *among* activity. Our vision of productive use of student mathematical thinking values first and foremost student mathematical thinking—of all the things teachers could attend to in a mathematics classroom, we place student mathematical thinking at the top of the list. Yes, teachers should attune themselves to students, which is critical, but they specifically need to attune themselves to what students are saying (or trying to say) mathematically. It requires effort and training to allow student mathematical thinking to become the primary focus of a teachers' attention. Although this focus on finding the mathematics in what students are saying includes an element of noticing *within,* attending primarily requires noticing *among* classroom instances.

We see interpreting as primarily a *within* activity. Applying the MOST Analytic Framework to an instance of student thinking provides guidance to help one engage in this interpreting activity. In applying the framework, once one has identified student thinking (by attending to it), one analyzes it to determine if it is possible to make a reasonable inference about what the student is saying mathematically—akin to critical aspects of what Confrey (1993) referred to as "close listening" (p. 311). If such an inference is possible, the SM is articulated and the MP must then be determined. One then analyzes the MP with respect to the significance of the mathematics, determining whether the mathematics is appropriate and, if so, whether it is central to student learning for these particular students. These multiple levels of interpretation go beyond the typical analysis of what a student is saying, to also determine whether what the student is saying has the potential to enhance students' mathematical understanding. This interpretation all happens with respect to a given instance, although certainly it is informed by the context leading up to the instance.

Instances of student mathematical thinking that qualify as mathematically significant are then analyzed pedagogically to evaluate the extent to which they provide an opening to build on student thinking and, if so, whether the timing is right for the teacher to make a building move. The pedagogical opportunity portion of the framework fits into the decision aspect of noticing—as one translates the interpretation of the student action into a teacher action. Thus, these last two MOST criteria provide the means for a teacher to decide whether a given instance of student mathematical thinking is worth pursuing in a particular way—being made the object of consideration for the class—so that, together, the class can move toward better understanding the MP. This decision stage is using the result of *within* noticing to influence *among* noticing. By engaging in detailed analysis of a given instance of student thinking, one can make decisions about which instances from among all those that surface during a lesson are the ones that have the most potential to be built upon. Thus, the MOST Analytic Framework unpacks the practice of productively using student mathematical thinking into a back-and-forth process of attending *among* to identify student mathematical thinking, interpreting *within* to determine the nature of that thinking, and deciding first *within* as to whether an instance qualifies as a MOST and then *among* by privileging MOSTs as instances most worth building on.

Illustration of the *Within* and *Among* Interaction

We now illustrate how the MOST Analytic Framework supports noticing by analyzing a brief classroom excerpt to highlight the *within* and *among* work—the back and forth—one must do to try to enact this framework "in practice." We give the caveat, however, that we are positing this back-and-forth noticing theoretically, not empirically. At this point, we do not know how teachers actually do this work while they are enacting a lesson, and there is likely great variation in how teachers engage in the type of noticing we describe. We are simply proposing a theoretical way to navigate this noticing, one that privileges students, their mathematics, and students actively making sense of each others' mathematical thinking in the service of developing mathematical understanding.

The lesson from which this excerpt is taken occurred in a junior high pre-algebra class using the Connected Mathematics Project (CMP) curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). In this particular lesson, students had been given the Day 2 Progress graph (Figure 1a), a graph that represents progress on a bike ride—in this case, the distance the bikers are from their starting place every half hour for 8 h. One of the questions students were asked to consider is whether it makes sense to connect the dots in this graph. The class decides that it does makes sense to connect these dots. The teacher then asks the class whether there are graphs where it does not make sense to connect the dots. He then reminds them of a graph

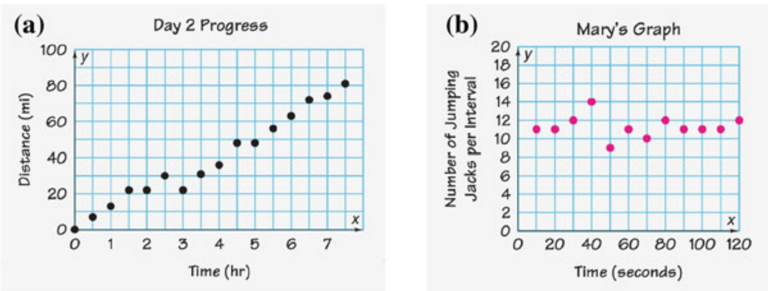

Figure 1. The two graphs under consideration in the lesson excerpt.

they had looked at several days earlier, Mary's Graph (Figure 1b), which is then projected. This graph represents data from a jumping jack counting activity and represents the number of jumping jacks per 10-s interval. When the class had conducted this jumping jack activity themselves, however, they had collected the data cumulatively rather than by interval. Thus their graphs had looked much more like the Day 2 Progress graph than Mary's Graph. Our transcript excerpt begins just after the teacher displays Mary's Graph.

Teacher: What is this graph [referring to "Mary's Graph"—Figure 1b] for?

Student: Jumping jacks.

Teacher: Did the jumping jack graphs we made look like this one?

Student: No.

Teacher: What did the graphs look like?

Student: They were different.

Teacher: What did they look like?

Student: They turned diagonally.

Teacher: How is this different? Ours were, you know, how is this different? Number of jumping jacks?

Student: Isn't this one counted by intervals?

Teacher: What does that mean? It says it on there—'per interval'—but what does that mean?

Student: Like um… I don't know but, right here, in 50 s she had only done 9 jumping jacks.

Teacher: Does that mean she had only done 9 total jumping jacks?

[several student say "no" and or shake their heads]

We now illustrate how applying the MOST Analytic Framework to this brief excerpt of classroom mathematics discourse requires and facilitates both *among* and *within* noticing. First, although we certainly must take into account what the teacher is saying, we do so primarily for context sake, focusing in on instances of student mathematical thinking. Among the types of instances here, then, we attend to (notice) those involving student mathematical thinking. Having attended to the first instance ("Jumping Jacks") we infer the SM—what the student is saying mathematically—to be "Mary's graph is for jumping jacks." Given the nature of this observation as it relates to the graph under consideration, the related MP is "The labels on a graph tell you the independent and dependent variables that are represented in the graph." Students in this class, however, likely already understand this MP given the previous work they have done on constructing and interpreting graphs, so the MOST analysis would stop—this instance is not a MOST. The work of inferring the SM, articulating the MP and evaluating whether that MP is appropriate is within noticing—interpreting within the instance of student mathematical thinking. The decision to "not build" on this particular instance is *among* noticing. Given the goal of the MOST

Analytic Framework of identifying opportunities to build on student thinking, this instance does not seem to be one that should be chosen from among the instances of student thinking that surface during the lesson to be made the object of consideration for the class.

With the next three instances of student mathematical thinking ("No," "They were different," and "They turned diagonally") one can infer the SM, but these statements are too vague to determine an MP toward which one could build by making those statements objects of consideration. In each of these instances, the teacher would have to elicit more information from the student (which the teacher here did) in order to determine whether the student thinking could be used to move toward an MP. In unclear instances such as these, brief *within* noticing leads directly to the *among* noticing not to build on the instance.

Next we attend to a student saying, "Isn't this one counted by intervals?" Based on the question this student was answering, we infer the SM to be, "The graphs are different because the graph on the board [1b] was counted by intervals." We articulate the related MP to be "Interpreting a graph requires that you understand the nature of the quantity the vertical axis represents." Although this MP is also one with which the students have been working, it seems likely that students are still in the process of developing a full understanding of this concept, so the instance is appropriate. Given the nature of the lesson itself, this MP seems to be central to the lesson. The statement, however, is true and relatively straightforward. It seems unlikely that such a statement would create intellectual need for students to make sense of it—they would likely simply agree. Thus, this instance does not create an opening for building toward the MP (important as it may be), so the *within* noticing stops and one notices that *among* the instances on the table thus far, this is still not one to make the object of a class discussion. Nevertheless, the teacher probes for further explanation, not because students would see intellectual need for it, but because the teacher recognizes the value in fleshing out the underlying idea.

In response to this teacher probing, a student says, "Like um… I don't know but, right here, in 50 s she had only done 9 jumping jacks." We infer the SM to be, "The graph shows that in 50 s she had only done 9 jumping jacks." Now, in practice a teacher might decide to pay special attention to this instance because what the student has said is incorrect—Mary had done nine jumping jacks in the 10-s interval leading up to 50 s, not in the first 50 s. We can use the MOST Analytic Framework to help us unpack why this instance might be worth focusing in on from among the several instances that have emerged thus far. The MP related to this SM is, "When measuring a quantity 'per interval' the dependent variable tells how many units per interval (a rate) and not the total number of units." This MP is a critical characteristic of Mary's graph and a key distinction between the types of graphs being compared. The idea is often difficult for students and is likely one that these students do not fully understand yet, so this MP is both appropriate and central and can thus be considered to be mathematically significant. Now, the nature of the student thinking also creates an opening, as it is incorrect and there are likely at least a few students in the class who would recognize it as such. There is thus an opening that could allow the teacher to engage other students in making sense of this students' idea and to orchestrate that discussion toward better understanding (and articulating explicitly) the MP. That the task at hand is to make sense of the distinction between per interval and cumulative graphs seems to indicate that the timing is right for taking advantage of the opening. We thus have a pedagogical opportunity. Because all of the criteria are satisfied, this instance is a MOST. The *within* analysis has revealed an instance that, when compared *among* others, is one we would like to take note of—to notice in the sense of valuing it for a particular purpose. This is an instance worth building on.

Discussion and Conclusion

Both *within* and *among* noticing are important to ambitious instruction. To productively use student mathematical thinking during instruction, teachers need to select from *among* all of the possible instances of student mathematical thinking that are available by analyzing *within* each instance to determine whether it is worth building upon. The MOST Analytic Framework (Leatham et al., 2015) supports this process by providing a tool to help teachers move back and forth between *among* and *within* noticing to key in on the most valuable instances of student thinking that surface during a lesson.

We highlight two important ways that noticing using the MOST Analytic Framework is different than the noticing we see in most other studies. First, the framework provides a targeted way of analyzing *within* instances without narrowing the focus to any particular mathematics topic. That is, the criteria of the framework were defined to be applicable to any mathematical topic at any grade level. When articulating the student mathematics in an instance and a related mathematical point, we attend to the specifics of the mathematics on the table without being limited to any specific piece or area of mathematics. This is in contrast to studies whose frameworks focus on one particular area of mathematics (e.g., Fernández et al., 2013; Schack et al., 2013). Additionally, the mathematically significant analysis, which focuses on whether the mathematical point of the instance is appropriate and central to the students *in the class in which the instance has occurred,* allows one to draw different conclusions about the value of an instance depending on the context in which it has occurred. Thus, the framework allows one to see why an instance of student mathematics that may be worth building on in one class, may not be in another class.

The MOST Analytic Framework also supports noticing that is different from that in many *among* noticing studies. While others have focused on helping teachers learn to key in on student mathematical thinking and to analyze it well (such as by using evidence) (e.g., Jacobs et al., 2010; Sherin & van Es, 2009), the MOST Analytic Framework provides a means for prioritizing some instances of student thinking over others. It goes beyond the work of identifying and analyzing to include the work of discriminating; such discrimination is critical to ambitious instruction since it is neither possible nor desirable to build on every student contribution during a lesson. Although teachers often try to incorporate as much student thinking as possible with the well-meaning intent of honoring student contributions, we believe that discriminating among instances of student thinking actually better honors student mathematics because it leads to more productive use of that thinking. When their ideas are productively built upon, students can see that their ideas matter in a meaningful way and can contribute to everyone's learning.

We conclude by offering the perspective that in order to improve mathematics education, studies of noticing must place value on something, and we concur with others (e.g., Jacobs et al., 2010) that that something should be student mathematical thinking. Moreover, we offer the perspective that we must go even further, beyond just valuing student thinking, to discriminating among instances of student thinking to determine which are more valuable to notice because of their potential to be built upon to support students' understanding of important mathematics. The MOST Analytic Framework is a tool for doing just that. Helping teachers learn to notice both *within* and *among* instances of student mathematical thinking, and to fluidly move between these forms of noticing during the act of teaching, has great potential to support teachers in enacting ambitious instruction.

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Teachers' Professional Noticing from a Perspective of Key Elements of Intensive, One-to-One Intervention

Thi L. Tran and Robert J. Wright

Abstract Teaching practice, which uses student mathematical thinking to develop mathematical concepts, is valued by the mathematics education community, but the nuances of this practice are relatively unexplored (Leatham, Peterson, Stockero, & Van Zoest, 2015). We observed about 33 hours of video recordings of one-to-one instruction in the Mathematics Intervention Specialist Program (MISP), involving four teachers and six students, in order to identify patterns relating to how the teachers act in particular situations to achieve particular pedagogical goals. We conceptualized a set of instances of such patterns in one-to-one teaching sessions, called Key Elements (KEs)—micro-instructional strategies used by a teacher when interacting with a student solving an arithmetic task. Twenty-five KEs are described and incorporated into a framework consisting of four categories: before posing a task, during posing a task, during solving a task, and after solving a task. A scenario of one-to-one instruction is described and the teacher's use of the following nine KEs is highlighted: Post-posing wait-time, post-responding wait-time, rephrasing the task, giving encouragement to a partly or nearly correct response, changing the setting during solving, scaffolding during, recapitulating, linking settings, and affirming. The three skills of professional noticing—attending, interpreting and deciding—are used to categorize the KEs of teaching occurring in the scenario. This highlights the linking of the KEs of instruction and the three skills of professional noticing. Thus, the study supports the notion that teacher development focusing on professional noticing can enhance teachers' learning to use the KEs of one-to-one instruction.

Keywords Key elements · One-to-one instruction · Intervention · Learning difficulties in arithmetic · Teacher–student interaction

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The role of teacher–student interactions in developing students' conceptual understanding and knowledge construction has been emphasized recently (Grandi & Rowland, 2013). Further, teaching practice that builds on students' mathematical thinking to develop mathematical concepts is valued by the mathematics education community (e.g., Leatham, Peterson, Stockero, & Van Zoest, 2015; Lester, 2007). Collectively, we have observed about 33 hours of video recordings of one-to-one instruction in the Mathematics Intervention Specialist Program (MISP) (Wright, Ellemor-Collins, & Lewis, 2011), involving four teachers and six students, in order to identify patterns relating to how the teachers act in particular instructional situations to achieve particular pedagogical goals. We conceptualized a set of instances of such patterns in one-to-one teaching sessions, called Key Elements (KEs) of one-to-one instruction (Tran & Wright, 2014a). A KE of one-to-one instruction is a micro-instructional strategy used by a teacher when interacting with a student in solving an arithmetical task.

The use of a KE refers to when, why, and how the KE is used. It seems that the successful use of each KE, for most of the KEs, requires a particular expertise. This expertise involves teacher professional noticing (Jacobs, Lamb, & Philipp, 2010). This is not to say that professional noticing is the only expertise required in the use of KEs, but the expertise related to professional noticing was a central feature of our examination of KEs and their use.

In this chapter, we report the findings of our research on KEs and describe links that can be observed between teachers' use of the KEs and the interrelated skills of professional noticing—attending, interpreting, and deciding (Jacobs et al., 2010). Thus the focus of this chapter is not what teachers notice, but what we, as researchers, notice when we study KEs in relation to teacher professional noticing.

The effectiveness of one-to-one teaching has been well documented in English speaking countries (e.g., Bloom, 1984; Cohen, Kulik, & Kulik, 1982; Graesser, Person, & Magliano, 1995). In addition, research on the effectiveness of one-to-one instruction has found that, in supporting student learning, expert tutors perform more effectively than non-expert ones (e.g., Bloom, 1984; Chae, Kim, & Glass, 2005; Lu, Eugenio, Kershaw, Ohlsson, & Corrigan-Halpern, 2007). However, the effectiveness of expert tutors is largely unexplored (Lu et al., 2007), and there is a scarcity of empirical research focused on learning gains from instruction by expert tutors. As well, decades of research on one-to-one instruction or tutoring have shown that tutor–student interactions are complex, and a common set of expert tutoring strategies has not yet emerged (e.g., Graesser et al., 1995; Lu et al., 2007; McMahon, 1998; Person, Lehman, & Ozbun, 2007; Wright, 2010; Wright, Martland, Stafford, & Stanger, 2002). Therefore, there is a need to study, in depth, the nature of the tutoring strategies that expert tutors use during interactive one-to-one instruction with their students. Such studies might illuminate the efficacy and effectiveness of particular numeracy intervention programs in terms of improving students' mathematical performance.

When studying the KEs, we are interested in how the expertise of professional noticing is evident in teachers' use of the KEs of one-to-one instruction. Jacobs et al. (2010) defined teacher professional noticing as three interrelated skills. The first of these—attending to students' strategies—refers to a teacher attending to a particular aspect of an instructional situation such as the mathematical details in students' strategies. The second skill—interpreting students' understanding—refers to teachers interpreting students' understanding as reflected in their strategies. The third skill—deciding—refers to the teacher's reasoning when deciding how to respond on the basis of a student's understanding.

Researchers have highlighted the need for teachers to notice the teacher–student interactions in order to determine the implications the interactions might have for student learning (Miller, 2011; Van Es, 2011). Similarly, teachers who develop a deep knowledge of the framework of KEs of intensive one-to-one instruction can observe and reflect on their students' responses to instruction for the purpose of determining subsequent teacher moves. Thus, knowledge of the skills of professional noticing has the potential to significantly support and strengthen one-to-one intensive instruction.

Based on research on professional noticing and our research on KEs, we envisage a reflexive relationship between KEs and professional noticing: understanding of KEs has the potential to enhance teacher professional noticing. Likewise, enhancing teachers' professional noticing has the potential to support their learning to use KEs. Thus, these two processes are mutually supportive. The study addresses the following research question: To what extent are the three interrelated skills of professional noticing evident in teachers' use of KEs of one-to-one instruction related to whole-number arithmetic?

Method

The primary data set for this study was drawn from the MISP (Wright, Ellemor-Collins, & Lewis, 2011) in which teachers provided intensive, one-to-one instruction to six low-attaining 3rd and 4th grade students. The participants consisted of four teachers and six students. The four teachers were selected from a pool of approximately 50 teachers in the MISP and were regarded by MISP leaders as being particularly competent in intervention teaching. They are regarded as expert tutors as they have completed MISP professional development in order to be specialist teachers at their schools. Two teachers each taught two students singly and the other two each taught one student only. Thus, the data involves six sets of video recordings of teaching sessions. Each set consists of up to nine teaching sessions, each of 25–45 min duration, with supplementary video recordings of approximately 90 min of pre- and post-assessment interviews for each teacher–student dyad. This resulted in approximately 33 hours of video for analysis. A methodological approach for analyzing large sets of video recordings (Cobb & Whitenack, 1996) and a model for analysis of video data (Powell, Francisco, &

Maher, 2003) were adopted. The videos were transcribed and then coded with respect to the KEs using the NVivo 10 software program.

Each teacher–student dyad is observed in a context of one-to-one instruction in order to identify significant events, likely to be regarded as KEs of one-to-one instruction. Consideration of these events enabled description and naming of relevant emergent concepts. The process of identifying KEs was conducted in two subprocesses, occurring concurrently during data analysis. First, emergent concepts similar to those occurring in the literature (e.g., Wright et al., 2002; Wright, 2010) were compared and developed in terms of their properties and dimensions (Strauss & Corbin, 1998). Second, emergent concepts that appear over and over again in the data and seem significant in one-to-one instruction, but have not been reviewed in the literature, were considered in light of the definition of a KE established earlier.

Findings of Research on KEs

Three sets of findings are described. These are (i) KEs of one-to-one instruction and their descriptions; (ii) problematic teacher behaviors; and (iii) a framework of KEs for analyzing one-to-one instruction.

A Collection of KEs of One-to-One Instruction

In this section, we examine the findings, resulting from each of the sub-processes of identifying KEs described in the method section. A collection of 25 KEs is presented in two sets in Table 1. Set A consists of 12 KEs and results from the first

Table 1
A collection of KEs of one-to-one instruction

Set A: a revision of KEs in relation to the research literature	Set B: KEs arising during the current study
Directing to check	Recapitulating
Affirming	Giving a meta-explanation
Changing the setting during solving	Confirming, highlighting and privileging a correct response
Post-task wait-time: Post-posing wait-time and post-responding wait-time	Reposing the task
Introducing a setting	Rephrasing the task
Preformulating a task	Stating a goal
Reformulating a task	Querying an incorrect response
Screening, color-coding and flashing	Focused prompting
Querying a correct response	Giving encouragement to a partly or nearly correct response
Explaining	Referring to an unseen setting
Scaffolding before	Linking settings
Scaffolding during	Directly demonstrating
	Directly correcting a response

sub-process of identifying KEs. Set B consists of 13 KEs and results from the second sub-process of identifying KEs. Collectively, Sets A and B constitute a cluster of KEs likely to be useful for analysis of one-to-one instruction.

In the following section, each of the 12 KEs in Set A is described in detail, drawing on the data analyzed in this study and taking account of relevant research literature. The 13 KEs in Set B arose during the current study and are also described in detail.

Set A of KEs: Descriptions

Directing to check (KE1) refers to a situation where a teacher assists a student indirectly by asking or allowing the student to check their last response. Directing to check is used by the teacher to respond to either a correct or incorrect answer. In the case where the student answers correctly, directing to check has the purpose of allowing the student to confirm their assuredness about the correctness of their solution. In the case that the student answers incorrectly, directing to check has the purpose of indirectly assisting the student to solve a task. Student checking in this way typically involves a resort to an easier or simpler strategy.

Affirming (KE2), (Wright et al., 2002), refers to statements or actions by a teacher, having the purpose of affirming effort or achievement on the part of the student and acknowledging that the student's answer is correct.

Changing the setting during solving (KE3), (Wright et al., 2002), refers to a deliberate action on the teacher's part in changing a material setting during the period when the student is attempting to solve a task. This KE often occurs when the student apparently reaches an impasse, that is, when the teacher perceives that the student is unable to solve the task that they are currently attempting. When using changing the setting during solving, the teacher deliberately introduces new elements. From the teacher's perspective, these new elements can be linked to elements in the original setting. Thus, the intention on the teacher's part is that the new elements enable the student to reconceptualize the task and arrive at a solution unavailable to the student before the change of setting.

Post-task wait-time (KE4) refers to a teacher behavior of providing sufficient time for a student to think about and solve the task. In this chapter, the term post-task wait-time is categorized into two terms, called *post-posing wait-time* and *post-responding wait-time*. Post-posing wait-time refers to wait-time that occurs after the teacher poses the task and before the student answers. Post-responding wait-time refers to wait-time that occurs after the student has responded to the task. Post-responding wait-time might occur when the student answers correctly, but also shows a lack of certitude. In this case, the given wait-time might enable the student to self-check or self-confirm their answer. Post-responding wait-time also was observed more frequently when the student answers incorrectly. In this case, after wait-time, the student might be able to self-correct and thus solve the task.

Introducing a setting (KE5) refers to a situation where a teacher introduces a setting new to the student. When a new setting is introduced, it is important to undertake preliminary explanations and activities in order for the student to become familiar with the setting. Wright et al. (2002) suggested a procedure to introduce a new setting as follows. The teacher places the setting on the table and tells the student what it is called. The teacher then proceeds with a series of questions having the purpose of revealing the student's initial sense of, and idea about, the setting. In this way, the teacher can gain insight into the ways the student is likely to construe and think about tasks presented using the setting.

Preformulating a task (KE6) refers to statements and actions by a teacher, prior to presenting a task, that has the purpose of orienting the student's thinking to the coming task (Cazden, 1986; McMahon, 1998). Preformulating might draw the student's attention to the setting or direct the student's thinking to related tasks solved earlier in the teaching session or in an earlier session. Preformulating also has the purpose of providing a cognitive basis for a new task or sequence of tasks that the teacher intends to pose (Wright et al., 2002).

Reformulating a task (KE7) refers to statements or actions by a teacher after presenting a task and before the student commences solving the task (Cazden, 1986; McMahon, 1998). Reformulating has the purpose of refreshing the student's memory of some or all of the details of the task or providing the student with additional information about the task thought to be useful to the student.

Screening, color-coding and flashing (KE8). Screening refers to a technique used in presenting tasks where the teacher conceals the material setting from the student (Wright et al., 2002). Screening has the purpose of developing student thinking in the sense of moving from using concrete materials to more formal arithmetic. *Color-coding* refers to a technique used in presenting tasks where, for example, the teacher intends to highlight the partitions of a number such as 5 or 10, by using two contrasting colors, for example, red and green (Wright et al., 2002). Color-coding has the purpose of highlighting the additive structure of numbers. Partitioned five frames (see Figure 1) and partitioned ten frames (see Figure 2) are well-known settings involving color-coding.

Flashing refers to a technique used in presenting tasks involving settings where spatial arrangement or color-coding is particularly significant (Wright et al., 2002).

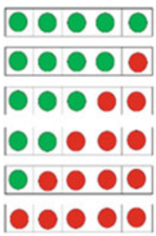

Figure 1. Partitioned five frames.

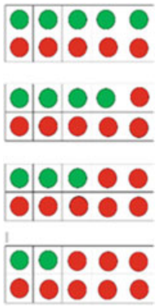

Figure 2. Partitioned ten frames.

The term *flashing* is used in the sense of displaying briefly, typically for about half a second.

Querying a correct response (KE9) refers to a situation where the student has answered correctly and the teacher questions the student about their answer (Wright, 2010). Typically this has the purpose of either helping to determine the student's solution method or gauging the student's certitude.

Explaining (KE10) refers to a situation where a teacher intends to engage a student in a conversation for the purpose of explaining some mathematical aspect or aspects relevant to the current instruction (Wright, 2010). In this study, the teacher, when providing an explanation to the student after solving the task, sometimes comments with the purpose of evoking a different strategy for solving the task.

Scaffolding refers to statements or actions on the part of the teacher to provide support for a student in an interactive teaching session (Wood, Bruner, & Ross, 1976). For intensive, one-to-one instruction in particular, Wright (2010) categorized scaffolding into two main forms: s*caffolding befor*e and s*caffolding during*.

Scaffolding before (KE11) refers to a situation where a teacher provides support prior to presenting the task or in the act of presenting the task. Thus scaffolding before occurs in cases where the scaffolding is integral to the presentation of the task.

Scaffolding during (KE12) refers to a situation where a teacher provides support in response to a student's unsuccessful attempt to solve a task. Thus this refers to scaffolding that is not provided during the presentation of the task.

Set B of KEs: Descriptions

Recapitulating (KE13) refers to a situation where a teacher reviews one or more strategies used during solving a task. This usually involves providing a brief summary of how the task is solved. This typically occurs after the student has solved the task. Recapitulating allows the teacher to emphasize crucial features of the student's strategy or solution.

Giving a meta-explanation (KE14) refers to an explanation that is of a general nature rather than specifically related to tasks that the student is currently solving. Giving a meta-explanation typically takes the form of clarifying the meaning of a mathematical term or describing the topic they are currently learning and where the learning can progress to.

Confirming, highlighting, and privileging a correct response (KE15) refers to statements and actions by a teacher after a student answers correctly. This has the purpose of either (i) confirming the correctness of the answer, particularly in cases where the student appears to lack certitude or (ii) highlighting and privileging the correctness of the answer in order to have the student reflect on their solution and thereby potentially increase their learning.

Re-posing the task (KE16) refers to a situation where the teacher restates the task in order to help the student fully understand the task or to remind the student of some details of the task. In this situation, the student typically indicates that they cannot solve the task because they have lost track of some of the details of the task. In some cases, the student explicitly requests a re-posing of the task.

Rephrasing the task (KE17) refers to a situation where the teacher expresses the task in an alternative way with the purpose of making the meaning clearer to the student. For example, the teacher rephrases the task by changing the language from "counting backwards by tens" to "take away ten."

Stating a goal (KE18) refers to a situation where the teacher summarizes a student's recent progress and describes what needs to be practised more or what needs to be done next. This has the purpose of motivating and guiding the student toward a goal.

Querying an incorrect response (KE19) refers to a situation where the student answers incorrectly and the teacher questions the student about their response. Typically this has the purpose of helping the student to realize the errors in their solution method, so that the student might find a way to solve the problem. Then, if the student still cannot self-correct, the teacher provides scaffolding to help the student to solve the task.

Focused prompting (KE20) has the purpose of asking in an open-ended way, what the student is aware of or thinking of, for example, is the student aware of an arithmetical pattern in a setting such as a sequence of partitions of 10 (9 + 1, 8 + 2, etc.) (see Figure 3).

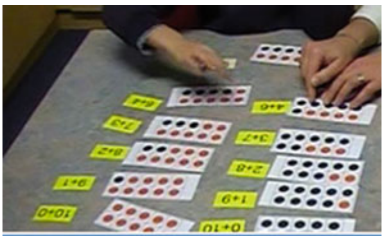

Figure 3. Matching expression cards and partitioned ten frames.

Giving encouragement to a partly or nearly correct response (KE21) refers to a situation where a student gives an incomplete or partly correct answer. The teacher usually responds to indicate that the student is on track. This involves confirming the correct part and then providing scaffolding. Concurrently, the teacher encourages the student to continue, without being overly concerned about their inadequate response. This typically has a purpose of keeping the student on track and giving them more motivation and confidence to continue solving the task.

Referring to an unseen setting (KE22) refers to a situation where, when posing a task, the teacher reminds the student about a setting that has been distanced, that is, the setting was used at an earlier time in the teaching segment but is currently not being used. Referring to an unseen setting has the purpose of focusing the student's thinking on how the teacher uses the setting when posing a task.

Linking settings (KE23) refers to a situation where the teacher makes a connection between two or more settings. Linking settings has the purpose of enabling the student to regard an arithmetical problem from two or more perspectives. For example, *base ten dot material* could be linked to *bundling sticks*; or a *partitioned ten frame* could be linked to an *arithmetic rack*. Figures 4 and 5 show linking of the base ten dot material and bundling sticks to display the number 145.

Directly demonstrating (KE24) refers to a situation where, when commencing a new sequence of tasks, the teacher demonstrates how a task can be solved. This is similar to the practice in literacy instruction, of using a sequence of modeled, guided, and independent modes (e.g., Clay, 1979). Thus, directly demonstrating corresponds to modeling in literacy instruction. This KE does not occur frequently in the data of this investigation because the teachers were not encouraged to directly demonstrate. Nevertheless, this KE shows its usefulness in some particular task sequences. One case of this is when a task involves a physical action by the student and the teacher models the action.

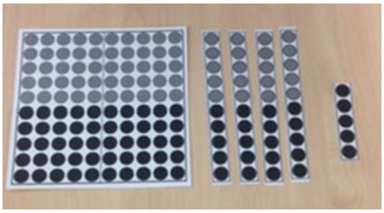

Figure 4. Base ten dot material.

Figure 5. Bundling sticks.

Directly correcting a response (KE25) refers to a situation where the student responds incorrectly to a task. The teacher either (i) directly corrects the student's answer or (ii) directly indicates to the student that they are incorrect and then directly corrects the student's answer. This can be useful in particular kinds of tasks such as *answer-focused* tasks.

Problematic Teacher Behaviors

Observing and analyzing 48 teaching sessions in the data set not only provides significant insight into the KEs, but also provides some insights into problematic behaviors associated with one-to-one instruction, for example, a teacher provides unnecessary support or is unduly hasty. We categorize problematic teacher behaviors according to the instructional situations where the teacher is: (i) presenting a task; (ii) providing support; (iii) giving an explanation; or (iv) giving feedback. Table 2 presents the set of problematic behaviors in relation to the instructional situations where they occurred and then follows with descriptions of each behavior in turn.

Problematic Teacher Behaviors: Descriptions

Flagging a task as being difficult refers to a situation where, before presenting a task, the teacher advises the student that the coming task will be difficult or tricky. For example, the teacher says "Are you ready for a super, super, super tricky one?". For some students, particularly those with a lack of confidence, such statements might make the students think they are not going to be able to solve the task and this can hinder the student's attempt to solve the task and reduce the student's motivation.

Table 2
The ten problematic teacher behaviors

Instructional context	Problematic teacher behaviors
Presenting a task	Flagging a task as being difficult Flagging a task as being easy Simultaneously making more than one request
Providing support	Interrupting the student Inappropriately reposing Rushing or indecent haste Miscuing Red-herring
Giving an explanation	Non sequitur
Giving feedback	Giving a "back-handed" compliment

Flagging a task as being easy refers to a situation where, before presenting a task, the teacher advises the student that the coming task will be easy. For example, she says "Okay, now here is an easy one.". Such a statement is likely to put additional pressure on the student to solve the task. In particular, if the student gives an incorrect answer, the student might feel uncomfortable about their ability and might lose confidence in solving tasks.

S*imultaneously making more than one request* refers to a situation where the teacher poses a task but in doing so, asks the student an additional question. This might confuse the student in that they do not know whether to respond to the task posed or to the additional question.

Interrupting the student refers to a situation where the teacher distracts the student when they have already commenced solving a task. In the case of using of reformulating, for example, when the student seems genuinely to be unaware of critical information relating to the task, or to require additional information, reformulating is likely to be productive. However, when the student does not require a restatement of critical information or does not require additional information about the task, reformulating may be counterproductive. A counterproductive reformulation might distract the student and hinder their attempt to solve the task.

Inappropriately reposing, refers to a situation where the teacher unnecessarily reposes a task, apparently to ensure that the student fully understands the task, but in fact the student has already commenced solving the task.

Rushing or indecent haste refers to a situation where the teacher proceeds unnecessarily quickly in providing support to the student. Thus, in this instructional situation, the teacher seems to be speaking and acting too quickly and in some cases, this haste is transferred to the student. In any event, the haste on the part of the teacher is likely to be counterproductive.

Miscuing refers to a situation where, after the student has commenced to solve the task, the teacher provides assistance to the student in the form of a hint or a suggested strategy, but in fact, the teacher's comment serves to mislead the student.

Non sequitur refers to a situation where, from the student's perspective, a statement by the teacher does not seem to logically follow from or connect to the immediately prior discussion.

G*iving a "back-handed" compliment* refers to a situation where the teacher compliments a student but in a way that tends to understate or underestimate the student's ability. An example is when, after the student has solved a task, the teacher says with a surprised tone, "That's great. I didn't think you would be able to do that."

Framework of KEs for Analyzing One-to-One Instruction

Figure 6 sets out a framework for analyzing one-to-one instruction that resulted from analysis of the teacher–student interactions in the data. This framework

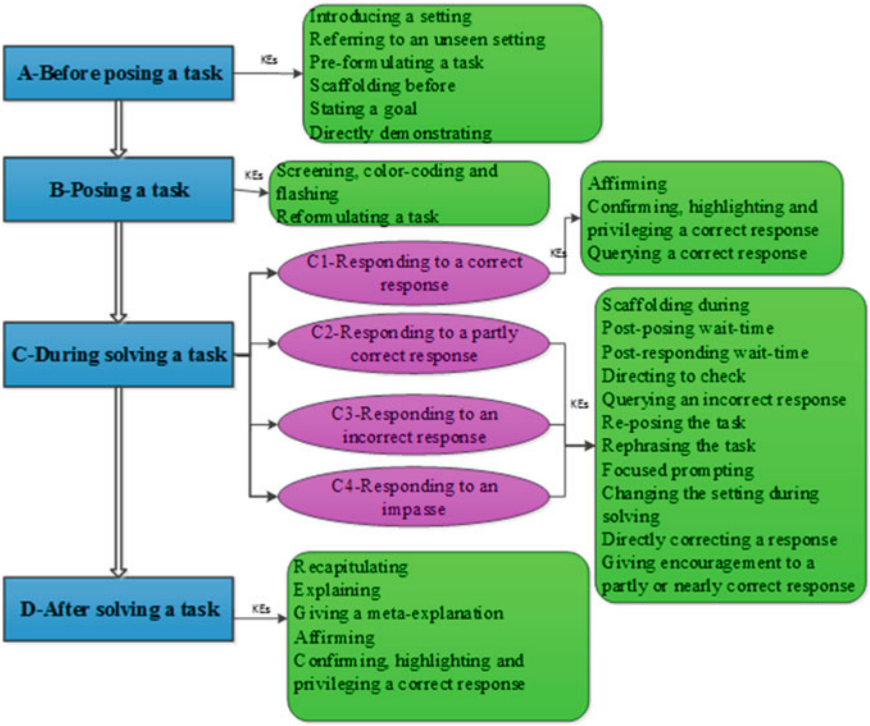

Figure 6. A framework for analyzing one-to-one instruction.

provides the context necessary for understanding how a teacher uses a specific cluster of KEs to achieve particular pedagogical goals.

The framework is layered into four stages of the teacher dealing with a task: A—Before posing a task; B—Posing a task; C—During solving a task; and D—After solving a task. Collectively, these constitute the first or highest level of analysis. As well, the stage of C—During solving a task, is construed as four categories of teacher responses: C1—Responding to a correct response; C2—Responding to a partly correct response; C3—Responding to an incorrect response; and C4—Responding to an impasse. For each stage or category, there are specific KEs that teachers usually use to respond to the student's answers. The following section provides a detailed description of how the framework can be used.

A—Before Posing a Task

Teachers typically intend to create a supportive environment for students before posing a task. It is important to undertake preliminary preparation of material settings and perhaps to review mathematical knowledge in order to prepare for the students to be ready for the coming task. The statements and actions taken by the teacher before posing a task have the purpose of orienting the student's thinking to the coming task and draws the student's attention to key features relating to the task setting. The KEs that are typically used at the stage of before posing a task involve introducing a setting, referring to an unseen setting, preformulating a task, scaffolding before, stating a goal, and directly demonstrating.

B—Posing a Task

Teachers can present tasks involving material settings in several different ways. For example, when presenting tasks involving conceptual place value, the teacher might choose to display base ten materials. At a later point, the teacher might only momentarily display the material. Later still the teaching might choose to screen the material without displaying it. Varying how much the teacher screens or displays the material exemplifies a particular dimension of mathematizing called *distancing the setting* (Ellemor-Collins & Wright, 2011). Necessarily reformulating a task is also a typical action taken by the teacher when posing a task. This involves the teacher realizing that the student has not understood, has misunderstood or has misconstrued a task. The teacher's responses could involve presenting again all or part of the task. Necessarily reformulating can involve simply reposing the task or rephrasing the task. The KEs that are typically used at the stage of posing a task involve screening, color-coding and flashing, and reformulating a task.

C—During Solving a Task

After a student initially responds to a task, the teacher's response is categorized as follows.

C1—Responding to a correct response

This refers to the teacher's response to a student's correct answer. The teacher's response takes account of the student's answer and typically has the purpose of extending and consolidating the student's understanding of the task. The KEs that are typically used in this case involve affirming; confirming, highlighting, and privileging a correct response; and querying a correct response. This results in actions by the teacher relevant to the task. We categorize C1 as follows.

C1.1 The teacher gives affirmation and moves on to another task. This case occurs typically for *answer-focused* tasks. In particular, for some sequences of answer-focused tasks, after the student answers correctly the teacher moves quickly on to the next task, then gives affirmation at the end of a sequence of tasks.
C1.2 The teacher confirms, highlights, and privileges the correct answer, and then gives affirmation. This case occurs typically for *linked-tasks* that is, tasks linking with the immediately prior task, in the sense that the answer for one task is used directly in the next task. For example, a sequence of tasks involving incrementing or decrementing a number by 1s, 10s, or 100s and using bundling sticks or dot materials, unscreened or screened. For these tasks, after each increment or decrement, the student says the number. Therefore, confirming and highlighting a correct answer after each task helps the student to solve the next task.

C1.3 The teacher solicits the student's answer by asking the student to explain their strategy or thinking in solving the task. The teacher may ask the student to solve the task in a different way, for example, by using a different strategy. Also, the teacher might encourage the student to examine the mathematical similarities and differences among two or more strategies. This case occurs typically for *strategy-focused* tasks referring to tasks where the teacher is interested in a particular strategy that the student uses to solve the task (Munter, 2010).

C2—Responding to a partly correct response

This refers to a situation where the student gives an incomplete or partly correct answer, then, the teacher responds to indicate that the student is on track by confirming the correct part and then follows up by providing scaffolding. Concurrently, the teacher encourages the student to continue without being overly concerned about their inadequate response.

C3—Responding to an incorrect response

This refers to a teacher's response to an incorrect answer on the part of the student. This results in actions by the teacher relevant to the task and typically has the purpose of helping the student to solve the task. We categorize C3 as follows.

C3.1 The teacher responds by directly correcting the student's answer. This typically applies to answer-focused tasks.
C3.2 The teacher assists the student indirectly by asking or allowing the student to check their answer. Student checking in this way typically involves a resort to a simpler strategy. Checking, therefore, might involve counting a collection previously screened or using a device such as a hundreds chart or a numeral roll that was not available at the time of initially solving the task.
C3.3 The teacher provides assistance resulting in a less-challenging task. In this situation, the teacher typically uses one or more KEs such as scaffolding during, post-task wait-time, querying an incorrect response, rephrasing the task, reposing the task, and changing the setting during solving. This typically applies to strategy-focused tasks.

C4—Responding to an impasse

This refers to a situation where the student appears unable to solve a task at hand. In such situations, the teacher is likely to provide an appropriate adjustment or a scaffold for the student's learning. An impasse is usually resolved in one of four ways as follows.

C4.1 The teacher directly releases the student from the obligation to solve the task.
C4.2 The teacher tells the student the answer then moves on.
C4.3 The teacher provides sufficient time for the student to be engaged in sustained and focused thinking to solve the task. The student arrives at a method to solve the task. In this situation, the teacher typically uses the KE of post-task wait-time.
C4.4 The teacher micro-adjusts or provides scaffolding to such an extent that the student can now solve the task. In this situation, when necessary, the teacher uses a

KE such as scaffolding during, focused prompting, re-posing the task, rephrasing the task, or changing the setting during solving.

D—After Solving a Task

After the task is solved, the teacher typically provides an opportunity for review and reflection. The student is engaged in a conversation for the purpose of explaining some mathematical aspect or aspects relevant to the current instruction. The teacher draws together what has been learned and summarizes the key features of the student's strategies and insights. Eventually, success is celebrated. The KEs typically used at this stage involve recapitulating, explaining, giving a meta-explanation, confirming, highlighting, and privileging a correct response and affirming.

Linking KEs and Professional Noticing

This section describes the linking of the use of KEs and professional noticing focusing on two instructional situations: (1) the student is engaged in solving a challenging task; and (2) the student answering incorrectly. Table 3 describes the links that can be observed between the three interrelated skills of professional noticing and the teacher's use of KEs of one-to-one instruction.

In task-solving situations, students' signals are often tacit. These can challenge the teacher in interpreting the student's understanding, in order to make an appropriate decision about how to respond. The teacher, therefore, might fail in their attempt to support the student by using particular KEs, because the teacher has misinterpreted the student's response. Such supports might interfere with the student's thinking. Five of the problematic teacher behaviors described earlier—unnecessarily reformulating a task, interrupting the student, inappropriately re-posing, rushing or indecent haste, and miscuing are cases of unsuccessful use of KEs in attempting to support the student. Therefore, the expertise developed in relation to professional noticing is essential for teachers to use the KEs effectively, as well, avoiding problematic behaviors.

An Example of Linking Professional Noticing and the Use of KEs

Due to the limited space for this chapter, we discuss one scenario only. We chose the following scenario because it contains a rich diversity of KEs and involves two categories in C—During solving a task. These are: responding to an incorrect response and responding to an impasse (Figure 6).

Figure 7 describes a scenario involving the teacher, Sophia, and her student, Ben. This scenario focuses on decrementing by 100s. Sophia initially posed a task verbally. Subsequently, she uses the setting of arrow cards and then the setting of dot materials.

Table 3
Linking professional noticing and KEs in teacher's decision-making

Attending to the student's strategies	Interpreting the student's understanding	Deciding how to respond on the basis of the student's understanding
It is observed that, when using any KE, the teacher initially attends to the student's strategies carefully. This allows the teacher to capture observable, noteworthy aspects of the student's mathematical strategies. The information that the teacher perceives through attending to the student's strategies might be used to interpret the student's mathematical understanding and to decide how to respond to the student	**Instructional situation 1: student is engaged in solving a challenging task**	
	Student: Challenged in solving the task because they have lost track of some details of the task	Teacher's response could involve using the KE of re-posing the task to help the student fully understand the task or to remind the student of some details of the task
	Student: Challenged in solving the task because they do not understand the task clearly in terms of its mathematical aspects or verbal expression	Teacher's response could involve using the KE of reformulating the task or rephrasing the task. Thus, the teacher expresses the task differently in order to make the meaning clearer for the student without changing the task
	The student commences solving the task and indicates a desire to proceed with the task, but still has difficulty figuring out an appropriate method to solve the task	The teacher's initial response could involve using the KE of post-posing wait-time or post-responding wait-time rather than giving them any support. If providing wait-time is not successful the KE of scaffolding during or focused prompting could be used
	In the case where the student apparently reaches an impasse, that is, the student is unable to solve the task that they are currently attempting	Depending on the specific circumstances, the teacher's response could involve using one or several of the following KEs in turn; scaffolding during, focussed prompting, re-posing the task, rephrasing the task, or changing the setting during solving

(continued)

Table 3 (continued)

Attending to the student's strategies	Interpreting the student's understanding	Deciding how to respond on the basis of the student's understanding
	Instructional situation 2: the student answered incorrectly	
	In the case where the student made an error in one or more steps	The teacher's response could involve using the KE of querying an incorrect response, that is, the teacher questions the student about their answer with the purpose of helping them to realize their error and solve the problem. In some cases, the teacher's response could involve directing to check with the purpose of indirectly assisting the student
	The student gave an incomplete answer, but from the teacher's perspective the student is on the right track and they might be able to solve the task with reasonable support	The teacher's response could involve using the KE of giving encouragement to a partly or nearly correct response. This could involve indicating that the student is on track, confirming the correct part, and then providing scaffolding. Concurrently, the teacher would encourage the student to continue, without being overly concerned about their inadequate response

Scenario	Key Elements
S: What's a hundred less than a thousand and fifty? (Looks intently at Ben)	
B: .... (After 10 seconds) One hundred and fifty.	Post-posing wait-time
S: (Continues to look at Ben)	
B: (Looks ahead and counts subvocally for 16 seconds) No... What did you say again?	Post-responding wait-time
S: One thousand and fifty. Then a hundred less.	Rephrasing the task
B: (After 9 seconds). Three hundred and fifty? No. Ninety fi-, ninety f-, one hundred and, no, nine hundred and five. No. One hundred and five.	
S: (Looks at Ben and smiles encouragingly) Nearly, I think you're nearly there.	Giving encouragement to a partly or nearly correct response
B: What did you say it was...	
S: So, it's one thousand. (Places arrow card sheet in front of Ben). Can you make one thousand and fifty? See what it looks like.	Changing the setting during solving
B: (Builds 1050 with arrow cards)	
S: Now, a hundred less.	
B: (Looks ahead for five seconds then removes the 50 arrow card from the 1000 card)... no hundreds in this	Scaffolding during
S: Yes, so where could you take the hundred from?	
B: Oh, the fifty? No. (Taps the 100 card) You take, you taking the hundred from a thousand?	
S: Mm hmm. So how many is that? How many would I have left of that thousand if I took a hundred away from it?	Scaffolding during
B: (After 7 seconds) Fif-, no f-, five hundred. No.	
S: Do you want to make it with the dots and see?	
B: Mmm.	Changing the setting during solving
S: Yep. (Places plastics on the desk). One thousand and fifty, so you've got to make a thousand and fifty.	
B: (Lays out ten 100-squares on table in two rows of five)	
S: Mm hmm. (Hands the ten-dot strips to Ben)	
B: (Lays out five 10-dot strips)	
S: Right, so how many have you got? How many dots?	Scaffolding during
B: (Looks at the dot cards on the desk) One thousand and fifty.	
S: Mm hmm. So you want a hundred less.	Scaffolding during
B: (Takes one 100-square card away) Nine hundred and fifty.	
S: Good, Ben. Well done.	Affirming
S: (Places the 1000 arrow card adjacent to the ten 100-squares. Then places the 50 arrow card adjacent to the five 10-strips). So, you had one thousand and fifty. Yeah?	Recapitulating
B: Mmm.	

Figure 7. Scenario Sophia–Ben.

S: Where did you take the hundred from?	Linking settings
B: From the one thousand.	
S: Mm hmm. (Removes one 100-square from the desk) And when you took that one hundred away what did you have left?	
B: Nine thousand and fifty.	
S: Mmm…	
B: No, one hundred and fifty, no nine hundred and fifty.	
S: Nine hundred and fifty. What does nine hundred look like?	
B: Umm	
S: (Gets out arrow card sheet). There's nine hundred (points to it).	
B: (Takes 900 from sheet and starts to build a number)	
S: Then you ...	
B: Oh, fifty. (Grabs 50 arrow card).	
S: That's it. Good on you. That's it. Well done. That's good.	Affirming

Figure 7. (continued)

Table 4 describes the linking of KEs occurring in the scenario above (see Figure 7), and the interrelated skills of professional noticing.

Discussion

The study provided insight into the essence (Van Manen, 1997) of KEs of intensive, one-to-one intervention focusing on whole-number arithmetic instruction involving 3rd and 4th grade students. The 25 KEs identified constitute a cluster of KEs likely to be useful for analysis of one-to-one instruction. Each KE was comprehensively described in order to provide a deep and richly layered understanding of the nature of the KEs and how the KEs arise during instruction. This understanding of the KEs allows for extension and refinement of the research relevant to intensive intervention in the learning of whole-number arithmetic.

While the 25 identified KEs can be regarded as good teaching practices, the 10 identified problematic teacher behaviors can be regarded as teaching practices to be discouraged. The identification of the problematic teacher behaviors, therefore, complements the collection of KEs in helping teachers to refine their teaching practices. Developing teachers' expertise in professional noticing has the potential to enable teachers to judiciously incorporate particular KEs into their intensive one-to-one instruction. As well, this expertise has the potential to reduce the use of problematic teacher behaviors. Making teachers explicitly aware of behaviors regarded as problematic could lead to a change in behavior on the part of the teacher. Researchers can apply the questions raised by Sherin, Jacobs, and Philipp (2011, p. 3) about teacher noticing—where do teachers look; what do they see;

Table 4
Linking KEs and the interrelated skills of professional noticing

Attend to the student's strategy	Interpret the student's understanding	Decide the KE to use
Sophia initially posed the task of "What is a hundred less than a thousand and fifty?" verbally and looked intently at Ben and waited for 10s	Ben engaged with the task	Post-posing wait-time
Ben answered incorrectly "One hundred and fifty"	Ben was unsure with his answer and wanted to keep attempting to solve the task	Rather than comment on Ben's answer, Sophia continued to look at Ben and waited for 16 s—using the KE of post-responding wait-time
Ben then asked "What did you say again?"	Ben lost track of the task	Sophia responded to Ben's request by rephrasing the task and waited for 9 seconds
Ben answered "Three hundred and fifty," but immediately changed his answer to "Ninety fi-, ninety f-, one hundred and, no, nine hundred and five." Thus, on two occasions, Ben immediately changed his answer	Ben might have a strategy to solve the task, but he still seems to struggle	Sophia's response was to attempt to keep Ben on track by giving encouragement to a partly or nearly correct response—"Nearly, I think you're nearly there"
Ben again asked Sophia to repeat the task	After several attempts to help Ben solve the task Sophia perceived that Ben apparently reached an impasse	Sophia used the KE of changing the setting during solving by bringing out a sheet of arrow cards
Sophia asked Ben to make the number 1050 using the arrow cards and solve the task by using that setting	It seemed that Ben was struggling to take way 100 from 1050 which he built with arrow cards	Sophia provided scaffolding during to help Ben solve the task
	Ben attempted to solve the task using arrow cards. It seems that Ben reached an impasse again	Sophia again use the KE of changing the setting during solving by presenting another setting—dot materials involving 100-squares and 10-strips

(continued)

Table 4 (continued)

Attend to the student's strategy	Interpret the student's understanding	Decide the KE to use
Sophia asked Ben to make the number 1050 using the dot materials	The setting of dot materials seemed to support visualization related to the number 1050 and this enabled Ben to reconceptualize the task by removing a 100-square	Sophia kept scaffolding to help Ben to solve the task
	Ben was engaged in a sustained period of highly interactive, one-to-one instruction. This culminated in him solving the task of 100 less than 1050	Sophia briefly summarized the process of how the task was solved by using the KE of recapitulating. As well, she linked the two settings of arrow cards and dot materials by using the KE of linking settings in order to emphasize crucial features of Ben's strategy and consolidate his learning

and what sense do they make of what they see—to analyzing teachers' use of the KEs of one-to-one instruction.

The framework of KEs for analyzing one-to-one instruction illuminates how teachers use a specific cluster of KEs to achieve particular pedagogical goals. The framework of KEs could serve as a guide to leaders in mathematical instruction in their analysis of one-to-one instruction. Further, the framework could provide useful information for teachers working with low-attaining students about their interaction with their students. This, in turn, may illuminate how particular practices influence student learning outcomes.

Conclusion and Recommendations

The findings in this study suggest that success in intensive, one-to-one instruction can depend on how quickly and accurately the teacher can understand the student's mathematical thinking and engagement. In this chapter, we have exemplified how linking teacher professional noticing and the use of KEs of one-to-one instruction may serve as a basis for teacher's professional learning. We believe that professional learning involving the introduction and practice of the KEs can benefit teachers' professional learning related to teaching. Teachers who engaged with the framework of KEs, stated that the set of the KEs of one-to-one instruction is a useful self-reflection tool (Tran & Wright, 2014b). Further, teacher development focusing on professional noticing has the potential to enhance teachers' learning to use the KEs of one-to-one instruction. We believe that teachers with more expertise in professional noticing will be better able to use multifarious KEs appropriately. Finally, problematic teacher behaviors described in this chapter can be considered as examples of a teacher failing to notice and they could serve as counter examples for teacher's professional learning.

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Part VI
Conclusion

The Ascendance of Noticing: Connections, Challenges, and Questions

Jonathan Norris Thomas

Abstract This chapter serves as a reflection upon the work presented in this volume. Toward that end, I will examine key themes, persistent issues, and lingering questions in the area of teacher noticing.

Keywords Teacher noticing · Theory · Measurement · Teaching practice · Teacher development

Introduction

In Schoenfeld's concluding chapter of the seminal book by Sherin, Jacobs and Philipp (2011), *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes*, he wrote that "noticing matters" (2011, p. 223). The variety of scholarship contained in this volume suggests that teacher noticing continues to matter and is ascending in prominence on the landscape of educational research. As a means to coherently explore the directions of teacher noticing inquiries, the editors have organized this volume into broad thematic sections (e.g., Noticing in Various Grade Bands and Contexts, Exploring Teacher Noticing and Equitable Teaching, etc.). These sections form a useful structure for navigating current scholarship in this area, and provide the occasion to revisit Schoenfeld's titular question, "Now what?" (p. 223). Toward this end, I will examine some lingering issues and emerging challenges. I will conclude with an unanswered question regarding the practical articulation of teacher noticing.

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The Fundamental Nature of Teacher Noticing

Looking across the literature, Sherin et al. (2011) characterized teacher noticing as consisting of two primary processes, “attending to particular events in an instructional setting” and “making sense of events in an instructional setting” (p. 5). These two processes correspond with the interrelated, component skills of “attending,” and “interpreting” put forth by Jacobs, Lamb, and Philipp (2010); however, Jacobs et al. (2010), identified a third component skill, deciding, which described teachers’ responses that flowed from interpretations (derived from events and behaviors to which teachers had attended). As one might expect, there is much consensus regarding the enactment of teacher noticing across the chapters in this volume. Whether noticing is described as identifying “what is noteworthy about a particular situation … [and] making connections between specific events and broader principles of teaching and learning” (van Es & Sherin, 2002, pp. 573–574) or a fluid enactment of attending, interpreting, and deciding (Jacobs et al., 2010), the presented research reflects a relatively shared understanding of what it means *to notice*. However, there still seems to be some differences in perspectives regarding *how one should notice*. Specifically, is noticing more appropriately focused on capturing and interpreting as much of the instructional landscape as possible including individual movements and postures? Or, should noticing processes be used as a filter to identify only the most impactful moments of a particular block of instruction? Indeed, one finds each of these perspectives in this volume. The former perspective is typified by Wells’ chapter on noticing of gesture as well as prior research by Schack et al. (2013). The latter perspective is represented by the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) analytic framework described in multiple chapters (see Stockero, Leatham, Van Zoest, & Peterson; Teuscher, Leatham, & Peterson; and Stockero & Rupnow). In her commentary for this volume, Sherin, describes teacher noticing as a “construct under development” and examines these differing perspectives regarding enactment.

Digging even more deeply, the research community continues to grapple with the essence of noticing itself and the manner in which such noticing is situated within the social landscape of teaching and learning. Specifically, is teacher noticing a practice? Practice within complex environments (such as classrooms) is described as involving “the orchestration of understanding, skill, relationship, and identity to accomplish particular activities with others … [and] practice can be understood in terms of its goals, its activities, and its historical tradition” (Grossman et al., 2009, p. 2059). Teacher noticing, for some, would seem to exist as a socially situated practice. Schoenfeld (2011) argued that, “teachers’ noticing is intimately tied to their orientations (including beliefs) and resources (including knowledge)” (p. 231). Similarly, when describing professional vision, Goodwin (1994) remarked, “the ability to see a meaningful event is not a transparent, psychological process but instead a socially situated activity accomplished through the deployment of a range of historically constituted discursive practices” (p. 606).

In this volume, chapters from the section on equitable teaching (see Baldinger; Kalinec-Craig; van Es, Hand, & Mercado) characterize or operationalize noticing in such socially situated terms. Conversely, Spitzer and Phelps, in their examination of noticing in the context of learning goals, define such noticing as a “discrete teachable skill” (p. 304) suggesting a construct organized around learning and carrying out a specific task. Similarly, Sturmer and Seidel’s development of a standardized measure to assess teachers’ professional vision implies a perspective oriented more toward skill than a social practice while other chapters appear more agnostic on this front. Given these dissimilarities regarding the fundamental nature of teacher noticing, perhaps some further consideration is in order. Certainly, the manner in which noticing is theoretically constructed by the research community will necessarily influence how it is studied and enacted.

The Relationship Between Teacher Noticing Components

Returning to the notion of consensus regarding the enactment of teacher noticing, all of the chapters espouse a component perspective. That is, noticing is typically described as consisting of multiple processes or skills (e.g., attending, identifying, reasoning, interpreting, connecting, deciding, responding, etc.). Jacobs et al. (2010) referred to such components (i.e., attending, interpreting, and deciding) as “interrelated skills” that must be executed in “an integrated way” (p. 169, 192); however, some researchers tend to isolate particular components for examination. For example, Males focused primarily on the component process of attending in her examination of middle and secondary teachers’ noticing in video-based contexts. Similarly, Amador, Carter, Hudson, and Galindo also concentrated their analysis on specific components (attending, interpreting, and deciding) and as they tracked changes in component performance over time.

Conversely, others have adopted a more integrated view of noticing components. To evaluate teacher noticing in the context of equality-oriented tasks, van den Kieboom, Magiera, and Moyer developed a rubric that synthesized the components of attending and interpreting. This perspective furthers a central thesis put forth by Castro-Superfine, Fisher, and Bragelman—that the component processes of attending and interpreting are deeply and reflexively related. Spitzer and Phelps also echoed this notion of attending and interpreting being deeply related. Additionally, in their chapter on influencing preservice teacher noticing, Teuscher, Leatham, and Peterson state that noticing components are nested and should be considered in concert. Moreover, several chapters seem to functionally blur the component processes of noticing in their analysis and/or reporting of results (see chapters by Lee & Choy, Baldinger, Kalinec-Craig, for examples). In such instances, the researchers describe noticing in component terms; however, these components tend to recede or disappear as one delves more deeply into the study itself. Sherin, in her commentary for this volume, succinctly describes such competing perspectives and argues that, “there can be features of our models that occur together while still being treated as separate elements"; however, she acknowledges that empirical study could, possibly lead to theoretically construed concomitant noticing components (p. 404).

From my vantage, advancing our understanding of the relationship among the component processes of teacher noticing is key area of growth for the field. Can we isolate and examine individual components? Or, do the components of teacher noticing only have meaning when considered in concert with one another? As with the theoretical nature of teacher noticing, the manner in which the research community conceives the relationship between noticing components will necessarily influence the very nature of teacher noticing.

The Measurement of Teacher Noticing

To some extent, each of the chapters in this volume focused on the measurement or evaluation of noticing performance. Nickerson, Lamb, and LaRochelle, in their examination of noticing measurement in secondary contexts, identify three primary means through which noticing data has been collected: (1) "observations of classroom practices and inferring" what was noticed, (2) "retrospective reflections on teachers' practice," and (3) "responses to items in relation to video or student work from others' practices" (p. 383). Building on these varied data collection methods, Stockero and Rupnow describe three approaches to the measurement of teacher noticing, (1) "measurement using categorization of instances", (2) "measurement using point or ranking systems," and (3) "measurement in relation to a standard" (pp. 283–284). While these organizational structures are quite useful for considering similarities and differences in measurement perspectives among studies, several tensions still exist within this aspect of teacher noticing inquiry.

First, there remains a vexing problem of *generalizing the specific*. Measures of teacher noticing are increasingly useful to the extent that they may be enacted across differing contexts and projects. Thus, the creation of more generalized measures of noticing performance would be a positive development. However, such generalized measures appear to be in some fundamental conflict with the highly situated nature of noticing enactment. That is, teacher noticing is, by its very nature, inseparable from a particular context, community, and time. This connection to context is highlighted in the chapter by Nickerson et al. as they entail measurement challenges that are specific to secondary settings, and this uniqueness of the secondary context is echoed in the chapter by Krupa, Huey, Lesseig, Casey, and Monson. Nevertheless, researchers have attempted to negotiate the challenge of generalizing the specific in various ways. Notable among these attempts is the MOST analytic framework described in several chapters. This framework provides researchers with a general perspective for considering the extent to which individuals identify and capitalize upon key mathematical opportunities. Like nearly all of the other studies detailed in this book, though, the evaluation of noticing performance rests upon some manner of inductive analyses (e.g., coding, thematic organization, etc.). Perhaps the only exception is Sturmer and Seidel's standardized approach to measuring professional vision; however, participation in this measure is distant, to some extent, from the enactment of teacher noticing in the mathematics classroom. This is not to suggest that Sturmer and Seidel's approach is not a viable proxy for the measurement of noticing; rather, simply that responding to video-anchored Likert-type prompts is dissimilar from enacting the practice of noticing within a classroom. Yet, as a measurement tool in research, the standardized approach may yield valuable understanding of the construct of professional vision.

This brings me to another measurement tension—the use of proximal instrumentation for the study of teacher noticing. Returning to the three methods for data collection put forth by Nickerson et al., each method is distinct from actual teacher noticing. Likely the closest in proximity, inferring noticing from observed instruction, relies on post hoc researcher interpretations of an instructional act, part of which (interpretation, reasoning, etc.) is inherently veiled to everyone but the actual teacher. Many other researchers rely on video-recordings to measure or evaluate noticing performances, which constitutes, arguably, an even more remote approach. In some cases, teachers view videos of their own practice, while other studies task participants with noticing aspects of another teacher. While both forms of video use are contextually removed from in-the-moment teacher noticing, this latter use of video gives rise to important questions regarding its proximal viability. For example, if teacher noticing is characterized as a socially situated practice organized around not just knowledge and skill, but also goals and identity, to what extent can teachers assume the role of another? Can teachers "step into the video" such that their interpretations and decisions reflect what would actually transpire in their own teaching? Can teachers ably superimpose their own knowledge, goals, and identity upon a video-recorded instance of strangers from another time and place? My own research reflects a belief that such proximal measures can be valuable indicators of noticing practice; however, investigations focused on the distance between measure and practice would be quite useful.

Development of Teacher Noticing

Schoenfeld (2011) and van Es (2011) explored possible developmental progressions for teacher noticing, and their work has been extended in this volume by several authors (see chapters by Beattie and Lee & Choy for examples). Moreover, new findings by other authors may further illuminate such developmental pathways. For example, Males noted a shift from teacher-focused to student-focused comments in her study of middle and secondary teachers' noticing. Such empirical findings could be used to increase the authenticity of noticing developmental progressions. One caveat drawn from the chapter by Krupa et al., though, is that contextual affordances and constraints likely influence how teacher noticing is practiced which, in turn, would affect one's progression towards more sophisticated enactment. One such developmental context likely worthy of examination is the asynchronous online or technology-mediated learning environment. Typically (but not always), researchers, teacher educators, and professional developers build and implement experiences organized around face-to-face interactions. Given the directions of many post-secondary institutions (and professional development designers), it may be wise to further explore noticing development in technology-centered contexts. Such explorations may yield potential pedagogical possibilities regarding the practice of noticing. It would seem that most researchers, including myself, primarily rely upon various types of video analyses, interviews, and examination of artifacts within structured and supportive settings often to positive effect. Nevertheless, studies of teacher noticing in asynchronous technological contexts could result in the creation of new pedagogical designs for the development of this practice.

An Unanswered Question—What Do Teachers Think About All of This?

Schoenfeld (2011) concluded that teachers are able to develop noticing capacities, and the research in this volume further strengthens that conclusion. Many of the presented inquiries demonstrate a substantive positive change in some aspect of participants' noticing abilities, which, in turn, leads to more responsive instruction. Thus, mathematics education professionals seem primed to broaden the impact of teacher noticing among practitioners. In addition to structured contexts for development (e.g., mathematics methods classrooms, professional learning events, etc.), practitioner articles on this topic have been published for both elementary (Thomas et al., 2014) and middle grades teachers (Thomas et al., 2015). Moreover, at least one statewide mathematics professional develop center has incorporated teacher noticing into its professional learning frameworks (KCM, 2015). Given this deliberate and appropriate linking of research to practice, I find it quite interesting that limited inquiry has been conducted regarding practitioners' enactment and perceptions of the practice of noticing. Certainly, we have learned a great deal about how teachers, in various contexts, come to develop noticing capacities; however, we seem to know very little about how the structures and processes of schools and school mathematics (Steffe & Wiegal, 1992) influence how teacher noticing is perceived and implemented by practitioners. On this front, an introductory paragraph from the chapter by Nickerson et al. sets the stage quite nicely for the consideration of some key questions.

> Imagine a teacher in a secondary mathematics classroom circulating while her 35 students work in small groups to solve an algebraic-generalization task. Perhaps she makes note of whether all students in a group are engaged and monitors students' affect. She may wish that a particular student's reasoning was visible or more understandable. She may or may not be looking for and may or may not be able to describe connections among the diverse

> mathematical responses. She likely observes many approaches taken to the task and critiques their sophistication, as well as their alignment with expected mathematical goals and the normative language, notation, and representations of mathematics. She wonders what statements, representations, or questions would support her students' thinking (Nickerson et al., p. 382).

With this in mind, do teachers perceive trajectory-oriented noticing focused on tailoring responses to individual students (Thomas et al., 2014) to be a viable practice in their classrooms? Might special education teachers and intervention specialists feel differently regarding such viability? Per suggestions by Thomas et al. (2015), do middle grades teachers feel comfortable interpreting mathematics and implementing teaching strategies aimed at learning goals well outside of their grade level—and would their school principal support such actions? Facing contextual or logistical constraints, how might teachers and other educational professionals adapt teacher noticing for their particular classroom? How do they claim ownership of this practice? The more I interact with teachers and witness the evolving structures of the contemporary classroom and school, the more convinced I become that exploration in this area is worthy and perhaps even necessary to facilitate stronger connections between this important research and our practitioner communities.

Concluding Remarks

As a researcher thoroughly enamored with teacher noticing, I find it thrilling to see a rising interest in this topic among members of the scholarly community. I feel there is something quite intuitive about the component structure (e.g., attending, interpreting, deciding) and find the inherent responsivity of teacher noticing desirable in a very fundamental way. My goal with this brief chapter is not to suggest that researchers must converge on a particular understanding of noticing, its components, or how noticing should be measured or developed among teachers. Rather, we in the research community might explore the impact of different ways that noticing is characterized or operationalized. For example, what are the differences (or similarities) in teachers' developmental pathways when teacher noticing is treated as a socially situated practice or, alternately, an assemblage of skills? Indeed, my hope is that we might capitalize on this growing concentration of scholarly energy to further our understanding of teacher noticing on a variety of fronts. I look forward to learning more from my fellow travelers as we continue to cultivate this exciting research field.

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