# Geometry Regularized Joint Dictionary Learning for Cross-Modality Image Synthesis in Magnetic Resonance Imaging

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Abstract. Multi-sequence MRI protocols are used in comprehensive examinations of various pathologies in both clinical diagnosis and medical research. Various MRI techniques provide complementary information about living tissue. However, a comprehensive examination covering all modalities is rarely achieved due to considerations of cost, patient comfort, and scanner time availability. This may lead to incomplete records owing to image artifacts or corrupted or lost data. In this paper, we explore the problem of synthesizing images for one MRI modality from an image of another MRI modality of the same subject using a novel geometry regularized joint dictionary learning framework for non-local patch reconstruction. Firstly, we learn a cross-modality joint dictionary from a multi-modality image database. Training image pairs are first co-registered. A cross-modality dictionary pair is then jointly learned by minimizing the cross-modality divergence via a Maximum Mean Discrepancy term in the objective function of the learning scheme. This guarantees that the distribution of both image modalities is taken jointly into account when building the resulting sparse representation. In addition, in order to preserve intrinsic geometrical structure of the synthesized image patches, we further introduced a graph Laplacian regularization term into the objective function. Finally, we present a patch-based non-local reconstruction scheme, providing further fidelity of the synthesized images. Experimental results demonstrate that our method achieves significant performance gains over previously published techniques.

### 1 Introduction

Magnetic Resonance Imaging (MRI) is a versatile and noninvasive imaging technique extensively used in neuroimaging studies. MRI comes in many different flavors (viz. MRI sequences, or henceforth also referred as MRI modalities<sup>1</sup>), each providing diverse and complementary image contrast mechanisms unraveling structural and

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<sup>&</sup>lt;sup>1</sup> Here, we use the word modality in the sense of a specific kind of MRI sequence. Note that the proposed technique would equally be applicable when the protocol involves different imaging modalities in a more classical sense (e.g. MRI, CT, US, SPECT, and PET).

functional information about brain tissue. Multi-modality MRI are nowadays very common in many pharmaceutical clinical trials, in research studies of neurosciences, or in population imaging cohorts targeted to understand neurodegeneration and cognitive decline. The acquisitions of a full battery of all these MR images can face constraints associated with their cost, limited availability of scanning time, patient comfort or safety considerations. Moreover, in large scale studies it is not uncommon to face incomplete datasets due to the presence of imaging artifacts, acquisition errors or corrupted data. While many such studies use imputation techniques to compensate for these latter issues, this is usually only at the level of the derived imaging biomarkers and not of the data itself. Finally, in longitudinal imaging studies where image data is collected over several years, evolution of imaging technology may lead to the appearance of new MRI sequences that are added to an existing imaging protocol at some point in time but for which are not available as part of the imaging battery acquired at earlier time points. In these and other applications, it would be desirable to have a methodology that is able to synthesize the unavailable data from the available MRI studies. The assumption here is that the synthesis ability comes from the cross-modality correspondences of sparse codes obtained during training, and can be used to encoding missing MRI. The degree to which this hypothesis is valid will have to be scrutinized in each application but is worth exploring.

To cope with this problem, several methods were proposed through either transforming MRI intensities or reconstructing tissue contrasts to obtain the missing MRI data. Histogram matching is the most common approach within this group. Although this technique is widely used in neuroimaging, it has been pointed out its inefficacy for multi-modality image synthesis due to the lack of specificity for certain ratios of tissue types [1]. On the other hand, techniques based on sparse representations have been presented, which separately learn two corresponding dictionaries from co-registered image pairs and synthesize a desired MRI modality data from the patches of the available MRI modality [1]. These approaches, however, boil down to an example-based synthesis strategy, which does not fully exploit the available training data to its fullest. In contrast, here, we establish fundamental connections with transfer learning (a.k.a. domain adaptation) used in many fields, e.g. [2, 3]. Such methods can successfully solve the above problem by learning a paired dictionary from both modalities while assuming each co-registered image pair with a nearly identical distribution [1]. However, this assumption cannot be fully satisfied in practice since cross-modality data may have very different feature distributions in different spaces.

In this paper, we propose a novel geometry regularized joint dictionary learning method for synthesizing any unavailable MRI from available MRI data. This paper offers the following three contributions: (1) We address cross-modality MRI synthesis by jointly learning a cross-modality dictionary that penalizes differences in the statistical distribution of the sparse codes in both domains rather than directly imposing the same code to both domains as done before. This is achieved by incorporating a new term in the computation of the joint sparse codes using the Maximum Mean Discrepancy measure; (2) We exploit the geometrical information underlying the input data and incorporate this new term into the cross-modality joint dictionary learning optimization; (3) A non-local reconstruction framework that provides a more expressive and compact patch representation is adopted to synthesize the corresponding patch

from a different MRI protocol. To the best of our knowledge, this is the first time that joint dictionary learning is computed by minimizing the discrepancy between the statistical distributions of the codes of the involved MRI modalities while preserving the intrinsic geometrical structure of the image. In the remainder of this paper, we first define the cross-modality synthesis problem, and then introduce our proposed method in Sect. 2. The experimental results are demonstrated in Sect. 3. Finally, we discuss the results and conclude the paper in Sect. 4.

# 2 Method

In this section, we propose cross-modality image synthesis via geometry regularized joint dictionary learning for effectively minimizing the cross-modality discrepancy. This consists in an extension of the conventional dictionary learning by jointly learning from the data of two modalities at the same time while minimizing the sparse codes divergence between the different modalities.

### 2.1 Problem Definition

Let  $L^{M_k} = \{I_i^{M_k}\}_{i=1}^m$  be a library of *m* subjects imaged with *k* modalities each (k = 1 or 2), with  $I_i$  being the training image of the *i*-th sample. Each pair of images in both libraries, i.e.  $\{I_i^{M_1}, I_i^{M_2}\}$  is assumed co-registered. Further, images are treated as the combination of many patches and denoted as  $X^{M_k} = \{x_i^{M_k}\}_{i=1}^n \in \mathbb{R}^{s \times n}$  where *s* is the size of a vectorized patch, and *n* represents the number of training patches for both modalities. We denote the test image in the same way by a matrix  $Y = \{y_1\}_{l=1}^c \in \mathbb{R}^{s \times c}$ , where *c* is the number of patches in the test image. All of the elements in *Y* are considered with either modality  $M_1$  or modality  $M_2$ . A summary of the notation used throughout this paper is presented in Table 1.

Notation	Description	Notation	Description
$L_1, L_2, L_t$	Training library of modality 1 or 2, testing library	M, G	MMD matrix, graph Laplacian matrix
$X^{M_1}, X^{M_2}, Y$	Training matrix of modality 1 or 2, testing matrix	$\Phi, W, N$	Diagonal degree/weight matrix, nearest-neighbor graph
$D^{M_1}, D^{M_2}$	Dictionary matrix of modality 1 or 2	$\lambda, \beta, \gamma$	Sparsity, balance parameter of MMD/graph Laplacian
$\alpha^{M_1}, \alpha^{M_2}$	Sparse codes matrix of modality 1 or 2 in training set	$\Omega, \mu$	Similar patch set/weight matrix in testing domain
$\alpha^t, \hat{\alpha^t}, \alpha^u,$	(Optimal) sparse codes matrix in testing/unified space	C,h	Normalization constant, scalar parameter

Table 1. Summary of notations and their meanings as used in this paper

**Problem:** Given  $X^{M_1}$  and  $X^{M_2}$ , our goal is to learn a pair of dictionaries  $\{D^{M_1}, D^{M_2}\}$  and the unified sparse codes  $\alpha^u$  minimizing the cross-modality discrepancy of  $\alpha^{M_1}$  and  $\alpha^{M_2}$ , where  $\alpha^{M_k}$  is the sparse codes matrix of  $X^{M_k}$ 

#### 2.2 Dictionary Learning

Let  $X = \{x_i\}_{i=1}^n \in \mathbb{R}^{s \times n}$  be a training data matrix with *n* input items sampled in the *s*-dimensional space,  $D = \{d_i\}_{i=1}^K \in \mathbb{R}^{s \times K}$  be a projection dictionary with *K* atoms, where K > s to make the dictionary overcomplete. Learning *D* from a sparse representation of *X* can be formulated as:

$$\min_{D,\alpha} \left\| X - D\alpha \right\|_{F}^{2} + \lambda \left\| \alpha \right\|_{0}, \tag{1}$$

where  $\alpha = {\alpha_i}_{i=1}^n \in \mathbb{R}^{K \times n}$  is a set of *n K*-dimensional sparse codes of *X*,  $\|\cdot\|_F$  is the Frobenius norm,  $\|\cdot\|_0$  is  $l_0$ -norm, which fixes the number of non-zero elements of  $\alpha$ , and  $\lambda$  denotes a regularization parameter to trade off the sparsity and the reconstruction error. As shown in [4], the minimization problem as stated in (1) is an NP-hard problem under the  $l_0$ -norm constraint. An alternative solution is to relax the  $l_0$ -norm constraint with the  $l_1$ -norm constraint to obtain a near-optimum result [5].

#### 2.3 Geometry Regularized Joint Dictionary Learning

Following the dictionary learning procedure described in Sect. 2.2, instead of transferring the estimated sparse codes from the first domain to the other [1, 6], we can learn the dictionaries of both domains independently:

$$\min_{D^{M_1}, \alpha^{M_1}} \left\| X^{M_1} - D^{M_1} \alpha^{M_1} \right\|_F^2 + \lambda_1 \left\| \alpha^{M_1} \right\|_1,$$

$$\min_{D^{M_2}, \alpha^{M_2}} \left\| X^{M_2} - D^{M_2} \alpha^{M_2} \right\|_F^2 + \lambda_2 \left\| \alpha^{M_2} \right\|_1.$$

$$(2)$$

However, such a strategy is time-consuming and results in two sets of independent sparse codes that do not necessarily satisfy the assumption of high-correlation between both modalities to reconstruct  $M_2$ -like images from  $M_1$ -like ones. To solve a similar problem, Yang et al. [6] proposed an image super-resolution approach that uses coupled dictionary learning. Their method maps image pairs (e.g. low and high resolution or, here, two different modalities) into a common space, which enforces the sparse codes of paired data possess the same values. Instead of directly imposing the same sparse codes across each pair, our work allows the codes to be different for each modality, and fosters the most similar distributions across them. This is done by measuring the distribution divergence for the co-registered image pairs over the empirical Maximum Mean Discrepancy (MMD), which is then minimized and incorporated into the dictionary learning problem.

**Maximum Mean Discrepancy Regularization:** We seek that the probability distributions of the codes associated to cross-modality patch pairs is identical when computing the optimal sparse representation. To this effect, the MMD [7] is used. The MMD is a nonparametric statistic utilized to assess whether two samples are drawn from the same distribution. In our case, the two samples correspond to the sparse codes of the training set for the two modalities involved. The MMD is calculated as the largest difference in the expected mean value of the K-dimensional codes for both modalities. To compute the MMD, we follow [7–9] to estimate the largest difference in expectations over functions in the unit ball of a reproducing kernel Hilbert space:

$$\mathbf{MMD} = \left\| \frac{1}{n} \sum_{i=1}^{n} \alpha_i - \frac{1}{n} \sum_{j=n+1}^{2n} \alpha_j \right\|_{H}^{2} = Tr\left(\alpha^{u} M \alpha^{u^{T}}\right), \tag{3}$$

where  $\alpha^{u}$  represents the unified sparse codes,  $\alpha^{u^{T}}$  is the transposed matrix of  $\alpha^{u}$ , and *M* denotes the MMD matrix defined as:

$$M_{i,j} = \begin{cases} 1/n^2, & x_i, x_j \in X^{M_1} \text{ or } x_i, x_j \in X^{M_2} \\ -1/n^2, & otherwise \end{cases}$$
(4)

**Graph Laplacian Regularization:** During dictionary learning, high-level patch semantics are captured in each dictionary atom. However, this process fails to introduce any prior knowledge on the geometrical structure within patches. Instead, by introducing a graph Laplacian (GL) term [10], we can preserve the local manifold structure of the sparse graph and better capture the intrinsic geometrical properties of the entire data space. Given  $\{X^{M_1}, X^{M_2}\} \in \mathbb{R}^{s \times 2n}$ , a *q*-nearest neighbor graph  $\mathcal{G}$  with 2n vertices can be constructed. The weight matrix of  $\mathcal{G}$  is  $W \in \mathbb{R}^{2n \times 2n}$ , defined as the matrix with elements  $W_{i,j} = 1$  if and only if for any two data points  $x_i, x_j, x_i$ ,  $x_i$  is among the *q*-nearest neighbors of  $x_j$  or *vice versa* ( $w_{i,j} = 0$ , otherwise). Let  $\phi = diag(\phi_1, \dots, \phi_{2n})$  be the diagonal degree matrix with elements  $\phi_i = \sum_{j=1}^{2n} W_{i,j}$ . The GL term, incorporated into the sparse representation as a regularization criterion [10], imposes that the obtained sparse codes vary smoothly along the geodesics of the manifold that is captured by the graph. The GL matrix is then defined as  $G = \phi - W$ . In order to preserve the geometrical structure in  $\mathcal{G}$ , we map  $\mathcal{G}$  to the unified coefficients  $\alpha^u$  by:

$$\frac{1}{2} \sum_{i,j=1}^{2n} \|\alpha_i - \alpha_j\|_2^2 W_{i,j} = \sum_{i=1}^{2n} \alpha_i \alpha_i^T \phi_{ii} - \sum_{i,j=1}^{2n} \alpha_j \alpha_i^T W_{ii} = \operatorname{Tr}\left(\alpha^u G \alpha^{u^T}\right).$$
(5)

**Objective Function:** To maximize the correlation between patch pairs in both modalities, we map them into a common higher-dimensional space that meets two complementary objectives to those of Eq. (2), viz. the MMD and GL terms. Therefore, our geometry regularized joint dictionary learning objective function becomes:

$$\min_{D^{M_{1},\alpha^{M_{1},\alpha^{u}}}} \frac{1}{2} \left\| X^{M_{1}} - D^{M_{1}}\alpha^{u} \right\|_{F}^{2} + \frac{1}{2}\lambda_{2} \left\| X^{M_{2}} - D^{M_{2}}\alpha^{u} \right\|_{F}^{2} + \operatorname{Tr} \left( \alpha^{u} (\gamma M + \delta G) \alpha^{u^{T}} \right) + \lambda \|\alpha^{u}\|_{1}.$$

$$(6)$$

where  $\gamma$  and  $\delta$  are the regularization parameters for trading off the effect of the MMD and GL terms, respectively.

#### 2.4 Image Synthesis via Nonlocal Reconstruction

Once the cross-modality dictionary pairs have been computed by solving Eq. 6, we seek to reconstruct a test image  $Y \in \mathbb{R}^{s \times c}$  by, first, sparsely representing Y with respect to  $D^{M_1} \in \mathbb{R}^{s \times K}$  by solving Eq. (1) with  $l_1$ -norm as:

$$\alpha^{t} = \arg \min_{\alpha^{t}} \left\| Y - D^{M_{1}} \alpha^{t} \right\|_{F}^{2} + \lambda \left\| \alpha^{t} \right\|_{1,}$$

$$\tag{7}$$

where  $\alpha^t \in \mathbb{R}^{K \times c}$  denotes the sparse codes of *Y*. The estimated coefficients can be directly used (or "transferred") to synthesize the image  $\hat{Y}$  of our target modality  $M_2$  by a linear combination of elements in the dictionary  $D^{M_2}$ , namely,  $\hat{Y} = D^{M_2} \alpha^t$ .

To achieve richer synthesis ability, in this paper, we improve the sparse representation performance through an optimized nonlocal reconstruction model. To faithfully synthesize the desired image, we enforce the sparse coefficients  $\alpha^t$  as close as possible to the target codes. That is, by groups of similar patches being encoded onto subsets of the dictionary that are similar, the estimated sparse codes vary smoothly as the patches themselves vary. This makes the whole reconstruction scheme more robust to the influence of patch noise and more accurate. To this end, we adopt the representative non-local means [11] in the sparse representation model by modifying Eq. (7) as

$$\hat{\alpha}^{t} = \arg\min_{\beta^{t}} \left\| Y - D^{M_{1}} \beta^{t} \right\|_{F}^{2} + \lambda \|\beta^{t}\|_{1},$$
(8)

where  $\beta^t = \sum_{p \in \Omega_i} \mu_{i,p} \alpha^t_{i,p}$ , and  $\alpha^t_i$  indicates the sparse codes of  $y_i$ . For each  $y_i$ , we express its similar patch set as  $\Omega_i$ , and define p as a random element within  $\Omega_i$ . Also, we define  $\mu_{i,p}$  as the weight for computing the level of similarity between  $y_i$  and  $y_q$ , where  $\mu_{i,p} = \frac{1}{C} \exp\left\{-\frac{\|y_i - y_{i,p}\|_2^2}{h^2}\right\}$ , with C being the normalization constant and h being a scalar (note that  $\mu_{i,p}$  satisfies  $0 \le \mu_{i,p} \le 1$  and  $\sum_{p \in \Omega_i} \mu_{i,p} = 1$ ). Then, we can update the synthesized image via  $\tilde{Y} = D^{M_2} \hat{\alpha^t}$ .

## **3** Experiments

In this section, we show extensive experiments for the MRI cross-modality synthesis problem to verify the effectiveness of our proposed method.

Experiment Setup: We evaluated our method in two different scenarios. Firstly, we used the IXI dataset [12] for synthesizing the  $T_2$ -w image considering the proton density (PD) acquisition from the same subject. We randomly selected 12 subjects from IXI containing both T2-w and corresponding PD-w images. We trained the dictionaries from 5 subjects including both modalities, and the other 7 subjects were used for In the second experiment, we considered the generation testing. of magnetization-prepared rapid gradient-echo (MPRAGE) images based on spoiled gradient recalled (SPGR) acquisitions, allowing us to compare our method with an existing approach [1]. In each experiment, for each co-registered image pair in the training set, we randomly selected 100,000 patch pairs of  $5 \times 5 \times 5$  voxels size to train our dictionaries. We also took the factor of dictionary size and sparsity into consideration, and fixed the dictionary size as 1024 and  $\lambda = 0.15$  based on our experiments trading off cost and synthesis quality. For other parameters, we used the following settings according to our extensive experiments: q = 5,  $\gamma = 10^5$ ,  $\delta = 1$ , and the searching window for nonlocal reconstruction equals 10. Finally, we adopted Root Mean Square Error (RMSE), Peak Signal to Noise Ratio (PSNR) in decibels (dB), Structural Similarity Index (SSIM) and voxelwise relative error (RE) as evaluation metrics.

**Compared Methods:** To show the performance of our approach, we compared our results of the following state-of-the-art methods: (a) Joint Dictionary Learning (JDL); (b) MRI example-based contrast synthesis (MIMECS) [1]; (c) Geometry Regularized Joint Dictionary Learning (GRiDLE) with only MMD term; (d) The proposed GRiDLE. Note that JDL is a special case of GRiDLE with  $\gamma = \delta = 0$ , and GRiDLE with only MMD term is another special case with  $\delta = 0$ .

**Experimental Results:** Table 2 shows the error measures of the synthesized  $T_2$  images using JDL, GRiDLE ( $\delta = 0$ ) and GRiDLE. We did not compare our GRiDLE with MIMECS in this case, because there is no available dictionary within this algorithm to generate arbitrary results. We can see that the proposed method outperforms the other two, obtaining the lowest RMSEs and the highest PSNRs and SSIMs for all 7 subjects. In the second example we compared the performance of the proposed method with that of the state-of-the-art MIMECS. The clear advantage of our approach over the MIMECS and JDL is shown in Fig. 1, which can be seen in overall tissue contrast, as well as in the lowest voxelwise RE. Table 3 compares the average error measures of all the methods for MPRAGE synthesizing from SPGR images. As shown, the proposed method achieves the best results.

	RMSE			PSNR (dB)		SSIM			
	JDL	GRiDLE	GRiDLE	JDL	GRiDLE	GRiDLE	JDL	GRiDLE	GRiDLE
		$(\delta = 0)$			$(\delta = 0)$			$(\delta = 0)$	
Sub .1	9.43	8.53	8.29	36.72	39.93	41.73	0.9025	0.9069	0.9075
Sub .2	9.42	8.53	8.27	37.15	39.92	42.05	0.9021	0.9054	0.9062
Sub .3	10.42	9.73	9.49	39.35	38.23	40.35	0.8997	0.9018	0.9029
Sub .4	10.53	9.26	9.01	36.17	37.61	41.34	0.8669	0.8999	0.9016
Sub .5	12.03	11.07	10.94	34.12	36.01	39.17	0.8990	0.8962	0.8970
Sub .6	10.21	9.30	9.06	36.73	38.66	41.02	0.9002	0.9049	0.9062
Sub .7	10.98	9.87	9.63	36.18	38.18	41.01	0.8964	0.9028	0.9034
Avg.	10.43	9.47	9.24	36.63	38.36	40.95	0.8953	0.9026	0.9035

Table 2. Error measures of the synthetic images using JDL, GRiDLE, and GRiDLE.



Fig. 1. Comparison of the synthesized results with ground truth.

Table 3. Comparison of methods used for synthesizing MPRAGE based on SPGR.

	MIMECS [1]	JDL	GRiDLE ( $\delta = 0$ )	GRiDLE
RMSE	14.55	12.58	11.03	10.89
PSNR (dB)	32.76	34.51	35.52	39.35
SSIM	0.9303	0.9368	0.9403	0.9500

### 4 Conclusion

In this paper, we proposed a novel geometry regularized joint dictionary learning (GRiDLE) approach for MRI cross-modality synthesis. The distribution divergence is effectively reduced by including the MMD term for both modalities and a mapping function in the sparse domain. The learned dictionary pair can not only minimize the distance between each coupled coefficients but also preserve the geometrical structure in the data while spanning both spaces for stable mapping of image details. Extensive experiments have demonstrated that GRiDLE can achieve superior performance over the state-of-the-art methods. Future work will focus on the simultaneous generation of multimodality images.

### References

- Roy, S., Carass, A., Prince, J.L.: Magnetic resonance image example-based contrast synthesis. IEEE TMI 32(12), 2348–2363 (2013)
- Shao, L., Zhu, F., Li, X.: Transfer learning for visual categorization: a survey. IEEE TNNLS 26(5), 1019–1034 (2015)
- 3. Pan, S.J., Yang, Q.: A survey on transfer learning. IEEE TKDE 26(5), 1345-1359 (2010)
- Davis, G., Mallat, S., Avellaneda, M.: Adaptive greedy approximations. Constr. Approx. 13(1), 57–98 (1997)
- Chen, S.S., Donoho, L.D., Saunders, A.M.: Atomic decomposition by basis pursuit. SIAM Rev. 43(1), 129–159 (2001)
- Yang, J., Wright, J., Huang, T.S., Ma, Y.: Image super-resolution via sparse representation. IEEE TIP 19(11), 2861–2873 (2010)
- Gretton, A., Borgwardt, K., Rasch, M.J., Scholkopf, B., Smola, A.J.: A kernel two-sample test. JMLR 13, 723–773 (2012)
- Long, M., Wang, J., Ding, G., Shen, D., Yang, Q.: Transfer learning with graph co-regularization. IEEE TKDE 26(7), 1805–1818 (2014)
- Steinwart, I.: On the influence of the kernel on the consistency of support vector machines. JMLR 2, 67–93 (2002)
- Zheng, M., Bu, J., Chen, C., Wang, C., Zhang, L., Qiu, G., Cai, D.: Graph regularized sparse coding for image representation. IEEE TIP 20(5), 1327–1336 (2011)
- Buades, A., Coll, B., Morel, J.M.: Image denoising methods. A new nonlocal principle. SIAM Rev. 52(1), 113–147 (2010)
- 12. Rowland, A.L., Burns, M., Hartkens, T., Hajnal, J. V., Rueckert, D., Hill, D.L.G.: Information eXtraction from Images (IXI). In: DiDaMIC, pp. 55–6 (2004)