

Transient Processing Analysis in a Finite-Buffer Queueing Model with Setup Times

Wojciech M. Kempa and Dariusz Kurzyk

Abstract A finite-buffer queueing model with Poisson arrivals and generally distributed processing times is investigated. Every time when the service station restarts the operation after the idle period, a random-length setup time is needed to achieve full readiness for the work, during which the service process is suspended. A system of integral equations for time-dependent departure process, conditioned by the initial buffer state, is built. The solution of the corresponding system written for double transforms is obtained in a compact form. Hence the mean number of packets completely processed up to fixed time epoch can be easily found. The analytical approach is based on the idea of embedded Markov chain, total probability law and integral equations. The considered queueing system can be successfully used in cellular networks or WSNs modelling, where the setup time corresponds to leaving the sleep mode in energy saving mechanism. Numerical utility of analytical formulae is shown in a network-motivated computational example.

Keywords Departure process · Finite-buffer queue · Integral equation · Setup time · Transient state

1 Introduction

Evidently, queueing models with finite buffer capacities have wide network applications, especially in modelling some processes occurring in network nodes, like IP routers or nodes of WSNs. As it seems, systems with different-type limi-

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tations in the access to the service station are of particular importance, due to their potential adoptions as models of energy saving mechanisms. Typically, each busy period of the queueing system may correspond to the active mode and any idle period—to the sleep mode. In practice, when a packet (job, call, etc.) arrives into the empty system, it is impossible to start its operation immediately. Predominantly, it takes some time (called a setup time) for the server to achieve the full readiness for processing.

The steady-state behavior of the M/M/1-type queue with “queued” waking up and setup times is investigated in [2]. A Markovian system with server setups preceding the first service in a new busy period is analyzed in [13]. In [15] (see also [1]) an equilibrium threshold strategy for customers’ behavior in a queue with setup times is derived. Applications of queueing systems with setup periods in WSNs can be found, e.g. in [16], where a sleep/wakeup protocol in the IEEE 802.15.4 standard is modelled. In [3] a queueing system in modelling the IMS session re-setup delay in WiMAX/LTE heterogeneous networks is applied. A BS sleeping mode in cellular networks is in [14] modelled via the M/G/1-type queueing system with server vacations and setup times. In [4] a model of data center with servers having independent setup times after idle periods is considered. The system is mathematically described by a two-dimensional Markov chain and some performance measures are found using a generating function approach.

As one can observe, analytical results obtained for different-type queueing models with server setup times relate, mainly, to the stable queues, i.e. to the stochastic characteristics in the case of $t \rightarrow \infty$. However, quite often time-dependent analysis of the system behavior seems to be more desired, in particular due to the high variability network traffic, e.g. in TCP/IP connections. Moreover, in rare traffic (like in some WSNs) the system stabilizes longer, so the investigation of its performance shortly after the opening or the application of a new control mechanism requires transient analysis (at fixed time t). Transient analysis of the queue-size distribution in the M/G/1-type queueing model with arrivals in random batches, N-policy and server setup times can be found in [7]. In [5] a similar model is considered with additional multiple vacation policy. Time-dependent solution for the queue-size distribution in a model with Poisson arrivals and setup/closedown times is obtained in [11].

In the paper we deal with the finite-buffer queueing model of the M/G/1 type in which the first processing in each new busy period (after finishing the idle time) is preceded by a generally-distributed setup time, during which the service process is still suspended (the arriving packets accumulate in the buffer queue). Using the idea of embedded Markov chain, a system of integral equations for the distribution of the number of packets completely processed up to the fixed time t (departure process), conditioned by the initial buffer state, is built. The solution of the corresponding system written for double transforms is obtained via linear algebraic approach and presented in a compact form by using a certain recursively defined sequence.

In [9, 10] new results for departure process in the [9] model with “queued” waking up (N-policy) can be found. The same characteristic in the finite-buffer system with

auto-correlated input stream modelled by the BMAP process is studied in [8]. In [6] time-dependent results for departure process can be found for the model with multiple vacation policy.

2 Description of the Model and Auxiliary Results

In the paper we investigate a single-server finite-buffer queuing model in which packets arrive according to a Poisson process with rate λ and are being served individually with a general-type CDF (=cumulative distribution function) $F(\cdot)$ of the processing time, according to the FIFO service discipline. The maximal number of packets present in the system simultaneously equals K , i.e. we have $K - 1$ places in the buffer queue and one place “in service station”. Obviously, packets arriving during the buffer overflow period (when the server is occupied and the buffer is saturated) are being lost. We assume that the buffer may contain a number of packets being accumulated before the start of the system at time $t = 0$. Each busy period, which is initialized together with the first arrival after the idle time, is preceded by a setup time, which is a random variable with a general-type CDF $G(\cdot)$. The server needs a setup time to achieve full operational readiness. If the system is empty before the opening, then it also starts a setup time at the moment of the first arrival. We assume independence of all inter-arrival, processing and setup times in the evolution of the system.

Let us denote by $h(t)$ the (random) number of packets completely processed up to the fixed time t , and define the distribution function of $h(t)$, conditioned by the initial level of buffer saturation, as follows:

$$H_n(t, m) \stackrel{\text{def}}{=} \mathbf{P}\{h(t) = m | X(0) = n\}, \tag{1}$$

where $t > 0, m \geq 0, 0 \leq n \leq K$ and $X(0)$ stands for the number of packets present in the system at the opening (at time $t = 0$). The (conditional) departure process defined in (1) is one of the most important operating characteristics of each queuing system, illustrating its performance. Moreover, in network applications, the output stream of packets transmitted from one node of the network becomes the arriving stream of packets into another node.

We are interested in finding the explicit representation for the PGF (=probability generating function) of the LT (=Laplace transform) of $H_n(t, m)$, i.e. for the functional

$$\tilde{h}_n(s, z) \stackrel{\text{def}}{=} \sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st} H_n(t, m) dt, \tag{2}$$

where $|z| < 1$ and $\Re(s) > 0$. In further analysis we use the following result from linear algebra which can be found in [12]:

Lemma 1 *Introduce two number sequences $(\alpha_k), k \geq 0$, and $(\psi_k), k \geq 1$, with the assumption $\alpha_0 \neq 0$. Each solution of the following system of linear equations with respect to $x_n, n \geq 0$:*

$$\sum_{k=-1}^n \alpha_{k+1} x_{n-k} - x_n = \psi_n, n \geq 0, \tag{3}$$

can be written in the form

$$x_n = CR_{n+1} + \sum_{k=0}^n R_{n-k} \psi_k, n \geq 0, \tag{4}$$

where C is a constant independent on n , and (R_k) is the sequence (called a potential in [12]) connected with the (α_k) in the following way:

$$\sum_{k=0}^{\infty} \theta^k R_k = \frac{1}{P_z(\theta) - 1}, \tag{5}$$

where

$$P_z(\theta) \stackrel{\text{def}}{=} \sum_{k=-1}^{\infty} \theta^k \alpha_{k+1}, |\theta| < 1. \tag{6}$$

Moreover, in [12] it is proved that successive terms of the sequence (R_n) can be found recursively as follows:

$$R_0 = 0, R_1 = \alpha_0^{-1}, R_{k+1} = R_1 \left(R_k - \sum_{i=0}^k \alpha_{i+1} R_{k-i} \right), k \geq 1. \tag{7}$$

In the further analysis we use the nomenclature $\bar{L}(x) \stackrel{\text{def}}{=} 1 - L(x)$, where $L(\cdot)$ stands for arbitrary CDF, and the notation $I\{A\}$ for the indicator of the random event A . Moreover, let us define

$$f(s) \stackrel{\text{def}}{=} \int_0^{\infty} e^{-st} dF(t), \tag{8}$$

$$g(s) \stackrel{\text{def}}{=} \int_0^{\infty} e^{-st} dG(t), \Re(s) > 0. \tag{9}$$

3 Integral Equations for Conditional Departure Process

In this section, by using the idea of embedded Markov chain and the continuous version of the formula of total probability, we find a system of integral equations for $H_n(t, m)$ ($t > 0, m \geq 0, 0 \leq n \leq K$), defined in (1). Next, we build the corresponding system written for double transforms of conditional departure process, i.e. for functionals $\tilde{h}_n(s, z)$ ($0 \leq n \leq K, \mathcal{R}(s) > 0, |z| < 1$), given in (2).

Assume, firstly, that the system is empty before the opening, so its evolution begins with idle period and the setup time begins simultaneously with the arrival epoch of the first entering packet. We can, in fact, distinguish three mutually excluding random events:

1. the first packet occurs before the moment t and the setup time also completes before t (we denote this event by $E_1(t)$);
2. the first packet arrives before t but the setup time ends after t ($E_2(t)$);
3. the first arrival occurs after time t ($E_3(t)$).

Let us introduce the following additional notation:

$$H_0^{(i)}(t, m) \stackrel{\text{def}}{=} \mathbf{P}\{(h(t) = m) \cap E_i(t) | X(0) = 0\}, \tag{10}$$

where $t > 0, m \geq 0$ and $i = 1, 2, 3$. Obviously

$$\mathbf{P}\{h(t) = m | X(0) = 0\} = H_0(t, m) = \sum_{i=1}^3 H_0^{(i)}(t, m) \tag{11}$$

and

$$\tilde{h}_0(s, z) = \sum_{i=1}^3 \sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st} H_0^{(i)}(t, m) dt. \tag{12}$$

According to the random event $E_1(t)$, we obtain the following representation:

$$H_0^{(1)}(t, m) = \int_{x=0}^t \lambda e^{-\lambda x} dx \int_{y=0}^{t-x} \left[\sum_{i=0}^{K-2} \frac{(\lambda y)^i}{i!} e^{-\lambda y} H_{i+1}(t-x-y, m) \right. \\ \left. + H_K(t-x-y, m) \sum_{i=K-1}^{\infty} \frac{(\lambda y)^i}{i!} e^{-\lambda y} \right] dG(y) \tag{13}$$

Indeed, the first summand on the right side of (13) relates to the situation in which the buffer does not become saturated during the setup time, while the second one—to the case of the buffer overflow occurring during this time. Similarly, considering $E_2(t)$, we obtain

$$H_0^{(2)}(t, m) = I\{m = 0\} \int_0^t \lambda e^{-\lambda x} \bar{G}(t-x) dx. \tag{14}$$

Evidently, if the setup time completes after t , the service at time t is still blocked, so the only possibility is $m = 0$. Similar argumentation explains the last case, namely

$$H_0^{(3)}(t, m) = I\{m = 0\} e^{-\lambda t}. \tag{15}$$

From (13)–(15) we get, referring to (11),

$$\begin{aligned} H_0(t, m) = & \int_{x=0}^t \lambda e^{-\lambda x} dx \int_{y=0}^{t-x} \left[\sum_{i=0}^{K-2} \frac{(\lambda y)^i}{i!} e^{-\lambda y} H_{i+1}(t-x-y, m) \right. \\ & \left. + H_K(t-x-y, m) \sum_{i=K-1}^{\infty} \frac{(\lambda y)^i}{i!} e^{-\lambda y} \right] dG(y) \\ & + I\{m = 0\} \left(\int_0^t \lambda e^{-\lambda x} \bar{G}(t-x) dx + e^{-\lambda t} \right). \end{aligned} \tag{16}$$

Let us consider now the case of the system being non-empty at the opening (i.e. $1 \leq n \leq K$). Since successive departure epochs are Markov (renewal) moments in the evolution of the M/G/1-type system, then, applying the continuous version of the formula of total probability with respect to the first departure epoch after $t = 0$, we obtain the following system of integral equations:

$$\begin{aligned} H_n(t, m) = & I\{m \geq 1\} \int_0^t \left[\sum_{i=0}^{K-n-1} \frac{(\lambda x)^i}{i!} e^{-\lambda x} H_{n+i-1}(t-x, m-1) \right. \\ & \left. + H_{K-1}(t-x, m-1) \sum_{i=K-n}^{\infty} \frac{(\lambda x)^i}{i!} e^{-\lambda x} \right] dF(x) + I\{m = 0\} \bar{F}(t), \end{aligned} \tag{17}$$

where $1 \leq n \leq K$. Let us explain in a short form successive summands on the right side of (17). The first summand under the integral describes the situation in which there are some free places in the buffer before the first departure occurring at time $0 < x < t$, while the second one corresponds to the case of the buffer saturation occurring before time t . In the last summand the first packet leaves the system after t . Introducing the double transforms $\tilde{h}_n(s, z)$, defined in (2), and utilizing the following identities:

$$\sum_{m=0}^{\infty} z^m \int_{t=0}^{\infty} e^{-st} dt \int_{x=0}^t \lambda e^{-\lambda x} dx \int_{y=0}^{t-x} \frac{(\lambda y)^i}{i!} e^{-\lambda y} H_j(t-x-y, m) dG(y) = a_i(s) \tilde{h}_j(s, z), \tag{18}$$

where we define

$$a_i(s) \stackrel{\text{def}}{=} \frac{\lambda}{\lambda + s} \int_0^{\infty} \frac{(\lambda y)^i}{i!} e^{-(\lambda + s)y} dG(y), \tag{19}$$

and

$$\begin{aligned} & \sum_{m=0}^{\infty} z^m I\{m = 0\} \int_{t=0}^{\infty} e^{-st} \left[\int_{x=0}^t \lambda e^{-\lambda x} \bar{G}(t-x) dx + e^{-\lambda t} \right] dt \\ &= \frac{\lambda[1 - g(s)] + s}{s(\lambda + s)} \stackrel{\text{def}}{=} \beta(s, z) = \beta(s), \end{aligned} \tag{20}$$

we transform the Eq. (16) to the following one:

$$\tilde{h}_0(s, z) = \sum_{i=0}^{K-2} a_i(s) \tilde{h}_{i+1}(s, z) + \tilde{h}_K(s, z) \sum_{i=K-1}^{\infty} a_i(s) + \beta(s). \tag{21}$$

Putting

$$\alpha_i(s, z) \stackrel{\text{def}}{=} z \int_0^{\infty} e^{-(\lambda + s)x} \frac{(\lambda x)^i}{i!} dF(x), \tag{22}$$

where $\Re(s) > 0$ and $|z| < 1$, we rewrite now (17) in the form

$$\tilde{h}_n(s, z) = \sum_{i=0}^{K-n-1} \alpha_i(s, z) \tilde{h}_{n+i-1}(s, z) + \tilde{h}_{K-1}(s, z) \sum_{i=K-n}^{\infty} \alpha_i(s, z) + \frac{1 - f(s)}{s}, \tag{23}$$

where $1 \leq n \leq K$.

Let us apply to equations of the system (21) and (23) the following substitution:

$$\tilde{d}_n(s, z) \stackrel{\text{def}}{=} \tilde{h}_{K-n}(s, z), 0 \leq n \leq K. \tag{24}$$

Now we obtain from (23) the following equations:

$$\sum_{i=-1}^n \alpha_{i+1}(s, z) \tilde{d}_{n-i}(s, z) - \tilde{d}_n(s, z) = \phi_n(s, z), \quad (25)$$

where $0 \leq n \leq K-1$ and functionals $\phi_n(s, z)$ are defined as follows:

$$\phi_n(s, z) \stackrel{\text{def}}{=} \alpha_{n+1}(s, z) \tilde{d}_0(s, z) - \tilde{d}_1(s, z) \sum_{i=n+1}^{\infty} \alpha_i(s, z) - \frac{1-f(s)}{s}. \quad (26)$$

Similarly, introducing (24) into (21), leads to the following representation:

$$\tilde{d}_K(s, z) = \sum_{i=0}^{K-2} a_i(s) \tilde{d}_{K-i-1}(s, z) + \tilde{d}_0(s, z) \sum_{i=K-1}^{\infty} a_i(s) + \beta(s). \quad (27)$$

4 Compact-Form Solution for Transforms

In this section, utilizing Lemma 1, we find the solution of the system (25) and (27) in the compact form. Let us note that the system (25) has the same form as (3) with unknowns $\tilde{d}_n(s, z), n \geq 0$, but with functional coefficients, namely $\alpha_{i+1}(s, z)$ and $\phi_i(s, z), i \geq 0$. Hence, the solution of (25) can be found by applying (4). Moreover, observe that the number of equations in (25) is finite comparing to (3). This fact can be used to find the value of $C = C(s, z)$ explicitly, treating the Eq. (27) (written for $n = K$) as a specific-type boundary condition.

Thus, for $0 \leq n \leq K$ the following formula holds true [compare (4)]:

$$\tilde{d}_n(s, z) = C(s, z) R_{n+1}(s, z) + \sum_{i=0}^n R_{n-i}(s, z) \phi_i(s, z), \quad (28)$$

where $n \geq 0$, and [see (7)]

$$\begin{aligned} R_0(s, z) &= 0, R_1(s, z) = \alpha_0^{-1}(s, z), R_{k+1}(s, z) \\ &= R_1(s, z) \left[R_k(s, z) - \sum_{i=0}^k \alpha_{i+1}(s, z) R_{k-i}(s, z) \right], \quad k \geq 1, \end{aligned} \quad (29)$$

and the functional sequence $\alpha_{i+1}(s, z)$ was defined in (22). Substituting $n = 0$ into (28), we get

$$\tilde{d}_0(s, z) = C(s, z) R_1(s, z). \quad (30)$$

Next, taking $n = 1$ in (28) and applying (26) and (30), we obtain

$$\begin{aligned} \tilde{d}_1(s, z) &= C(s, z)R_2(s, z) + R_1(s, z)\phi_0(s, z) \\ &= C(s, z)R_2(s, z) + R_1(s, z) \left[\alpha_1(s, z)R_1(s, z)C(s, z) \right. \\ &\quad \left. - \tilde{d}_1(s, z) \sum_{i=1}^{\infty} \alpha_i(s, z) - \frac{1-f(s)}{s} \right] \end{aligned} \tag{31}$$

and hence

$$\tilde{d}_1(s, z) = \theta(s, z) \left[C(s, z) (R_2(s, z) + \alpha_1(s, z)R_1^2(s, z)) - R_1(s, z) \frac{1-f(s)}{s} \right], \tag{32}$$

where

$$\theta(s, z) \stackrel{\text{def}}{=} \left[1 + R_1(s, z) \sum_{i=1}^{\infty} \alpha_i(s, z) \right]^{-1} = \frac{f(\lambda + s)}{f(s)}. \tag{33}$$

From representations (30)–(33) follows that $\phi_i(s, z)$ [defined in (26)] for any fixed $i \geq 0$ can be written as a function of $C(s, z)$, which is the only unknown functional. To find the formula for $C(s, z)$, let us, firstly, write (27), applying on the right side of identities (26), (28), (30) and (32), and utilizing the fact that

$$\sum_{i=0}^{K-2} a_i(s) \tilde{d}_{K-i-1}(s, z) = \sum_{i=1}^{K-1} a_{K-i-1}(s) \tilde{d}_i(s, z). \tag{34}$$

We obtain

$$\begin{aligned} \tilde{d}_K(s, z) &= \sum_{i=1}^{K-1} a_{K-i-1}(s) \left[C(s, z)R_{i+1}(s, z) + \sum_{j=0}^i R_{i-j}(s, z)\phi_j(s, z) \right] \\ &\quad + C(s, z)R_1(s, z) \sum_{i=K-1}^{\infty} a_i(s) + \beta(s) \\ &= \sum_{i=1}^{K-1} a_{K-i-1}(s) \left[C(s, z)R_{i+1}(s, z) \right. \\ &\quad \left. + \sum_{j=0}^i R_{i-j}(s, z) \left(\alpha_{j+1}(s, z)\tilde{d}_0(s, z) - \tilde{d}_1(s, z) \sum_{r=j+1}^{\infty} \alpha_r(s, z) - \frac{1-f(s)}{s} \right) \right] \\ &\quad + C(s, z)R_1(s, z) = \Psi_1(s, z)C(s, z) + \chi_1(s, z), \end{aligned} \tag{35}$$

where we denote

$$\begin{aligned} \Psi_1(s, z) \stackrel{\text{def}}{=} \sum_{i=1}^{K-1} a_{K-i-1}(s) & \left[R_{i+1}(s, z) + \sum_{j=0}^i R_{i-j}(s, z) (R_1(s, z) \alpha_{j+1}(s, z) \right. \\ & \left. - \theta(s) (R_2(s, z) + \alpha_1(s, z) R_1^2(s, z)) \sum_{r=j+1}^{\infty} \alpha_r(s, z) \right] + R_1(s, z) \sum_{i=K-1}^{\infty} a_i(s) \end{aligned} \quad (36)$$

and

$$\chi_1(s, z) \stackrel{\text{def}}{=} \sum_{i=1}^{K-1} a_{K-i-1}(s) \sum_{j=0}^i R_{i-j}(s, z) \left[R_1(s, z) \frac{1-f(s)}{s} \theta(s) \sum_{r=j+1}^{\infty} \alpha_r(s, z) - \frac{1-f(s)}{s} \right] + \beta(s). \quad (37)$$

Now let us substitute $n = K$ in (28) and utilize representations (26), (30) and (32). We obtain

$$\begin{aligned} \tilde{d}_K(s, z) &= C(s, z) R_{K+1}(s, z) \\ &+ \sum_{i=0}^K R_{K-i}(s, z) \left\{ \alpha_{i+1}(s, z) R_1(s, z) C(s, z) \right. \\ &\left. - \theta(s) \left[C(s, z) (R_2(s, z) + \alpha_1(s, z) R_1^2(s, z)) - R_1(s, z) \frac{1-f(s)}{s} \right] \sum_{j=i+1}^{\infty} \alpha_j(s, z) - \frac{1-f(s)}{s} \right\} \\ &= \Psi_2(s, z) C(s, z) + \chi_2(s, z), \end{aligned} \quad (38)$$

where

$$\begin{aligned} \Psi_2(s, z) \stackrel{\text{def}}{=} R_{K+1}(s, z) \\ + \sum_{i=0}^K R_{K-i}(s, z) \left[\alpha_{i+1}(s, z) R_1(s, z) - \theta(s) (R_2(s, z) + \alpha_1(s, z) R_1^2(s, z)) \sum_{j=i+1}^{\infty} \alpha_j(s, z) \right] \end{aligned} \quad (39)$$

and

$$\chi_2(s, z) \stackrel{\text{def}}{=} \sum_{i=0}^K R_{K-i}(s, z) \left(\theta(s) R_1(s, z) \frac{1-f(s)}{s} \sum_{j=i+1}^{\infty} \alpha_j(s, z) - \frac{1-f(s)}{s} \right). \quad (40)$$

From (35) and (38) follows immediately

$$C(s, z) = [\Psi_1(s, z) - \Psi_2(s, z)]^{-1} [\chi_2(s, z) - \chi_1(s, z)]. \tag{41}$$

Now the representations (24), (26), (28) and (41) lead to the following main theorem:

Theorem 1 *The representation for the PGF of the LT of the conditional departure process in the M/G/1/K-type model with generally distributed server setup times is following:*

$$\begin{aligned} \tilde{h}_n(s, z) &= \sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st} \mathbf{P}\{h(t) = m | X(0) = n\} dt \\ &= [\Psi_1(s, z) - \Psi_2(s, z)]^{-1} [\chi_2(s, z) - \chi_1(s, z)] \left\{ R_{K-n+1}(s, z) \right. \\ &\quad \left. + \sum_{i=0}^{K-n} R_{K-n-i}(s, z) \left[\alpha_{i+1}(s, z) R_1(s, z) - \theta(s) (R_2(s, z) + \alpha_1(s, z) R_1^2(s, z)) \sum_{j=i+1}^{\infty} \alpha_j(s, z) \right] \right\} \\ &\quad + \sum_{i=0}^{K-n} R_{K-n-i}(s, z) \left(\theta(s) R_1(s, z) \frac{1-f(s)}{s} \sum_{j=i+1}^{\infty} \alpha_j(s, z) - \frac{1-f(s)}{s} \right), \end{aligned} \tag{42}$$

where the formulae for $\alpha_i(s, z)$, $R_i(s, z)$, $\theta(s)$, $\Psi_1(s, z)$, $\chi_1(s, z)$, $\Psi_2(s, z)$ and $\chi_2(s, z)$ are given in (22), (29), (33), (36), (37), (39) and (40), respectively.

Remark 1 Let us note that from (42) the formula for the conditional mean number $E_n h(t)$ of packets completely processed until the fixed time epoch t (where $0 \leq n \leq K$) can be found. Namely, we have

$$E_n h(t) = \frac{\partial}{\partial z} [\mathcal{L}^{-1}(\tilde{h}_n(s, z))]_{z=1}, \tag{43}$$

where the notation $\mathcal{L}^{-1}[\cdot]$ stands for the inverse Laplace transform.

5 Numerical Results

Let us consider the stream of packets of average sizes 100 B, arriving into the node of a wireless sensor network according to a Poisson process. Consider three different arrival intensities 300, 400 and 500 Kb/s, which give $\lambda = 375$, $\lambda = 500$ and $\lambda = 625$ packets per second, respectively. Moreover, let us assume that a radio transmitter/receiver of the node is switched off during an idle period and needs an exponentially distributed setup time with mean 4 ms to become ready for processing. Besides, let packets are being transmitted with speed 500 Kb/s according

to 2-Erlang service distribution, that gives the mean processing time 1.6 ms (hence the parameter of the 2-Erlang service distribution is $\mu = 1250$). Under the assumptions about arrival and serving rates, the utilization factor ρ of the system equals to 0.6, 0.8 and 1.0, respectively. Mean number $E_n h(t)$ of packets completely processed until the fixed time epoch t can be found from (43). Moreover, we can estimate the transient loss ratio function as $LR_n(t) \approx 1 - \frac{E_n h(t)}{\lambda t}$, where λt is mean number of packets in the arrival stream up to the time t . Transient evolutions of the mean number $E_0 h(t)$ of completely processed packets and the estimations of the loss ratio $LR_0(t)$ for given values of system parameters are presented in Figs. 1 and 2, respectively.

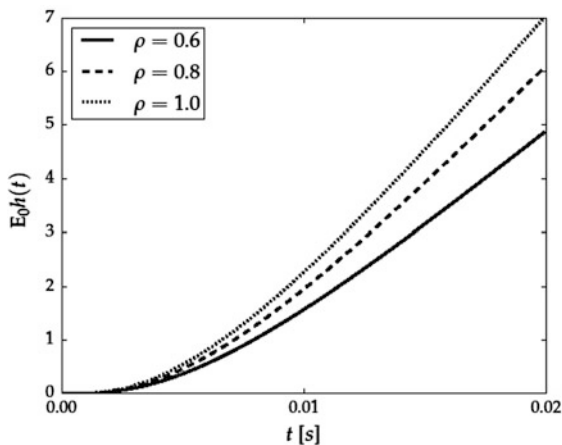


Fig. 1 Transient mean number $E_0 h(t)$ of completely processed packets for $\rho = 0.6, 0.8$ and 1.0

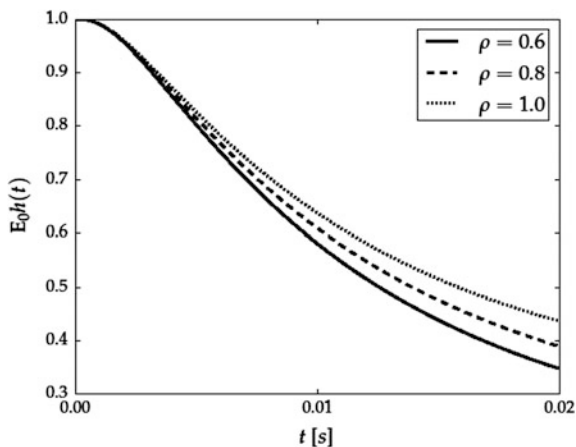


Fig. 2 Estimation of transient loss ratio $LR_0(t)$ for $\rho = 0.6, 0.8$ and 1.0

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