

Study on Transient Queueing Delay in a Single-Channel Queueing Model with Setup and Closedown Times

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Abstract. A single-channel queueing model with finite buffer capacity, Poisson arrival stream and generally distributed processing times is considered. After each busy period the service station is being switched off but this operation requires a randomly distributed closedown time. Similarly, after the idle time, the first service in a new busy period is preceded by a random setup time, during which the processing is still suspended and the server achieves full readiness for the service process. A system of integral equations for transient conditional queueing delay distribution is derived, by using the idea of embedded Markov chain and the formula of total probability. The solution of the corresponding system written for Laplace transforms is obtained via the linear algebraic technique. Numerical examples are attached as well.

Keywords: Closedown time · Finite buffer · Queueing delay · Setup time · Transient state

1 Introduction and Preliminaries

As it is commonly known, different-type real problems occurring in telecommunication and computer networks, in manufacturing processes, in transport and logistics, tracking systems and similar can be efficiently modelled by using appropriate queueing models (e.g. see [18, 19]). Systems with one or more mechanisms limiting the access to the service station seem to be of special importance. One of such mechanisms are server setup and closedown times, occurring at the beginning and at the end of each busy period of the system, respectively. Indeed, due to the energy saving strategy, the server is being switched off when there are no customers present, and is being switched on when a job arrives into the empty system. Switching off requires a time, usually random, as, similarly, switching on during which the service station achieves full

readiness for processing. In the paper we deal with a single-channel queueing model with finite buffer capacity and the service process organized according to the FIFO service discipline, in which generally distributed closedown and setup times are implemented. The considered model has potential practical applications, e.g. in SVC (switched virtual connection) modelling, where the setup time corresponds to the time for building a new SVC by using the signalling procedure, and the closedown time stands for the deactivation timer during which the SVC resource (as, e.g. routing information or the bandwidth frequency) is reserved anticipating more packets from the same flow (see [12]).

One of the crucial operating characteristics in each queueing model is queueing delay, i.e. the time the potentially entering customer needs to wait until the start its processing. In the paper, using the technique based on the idea of embedded Markov chain, the total probability law and linear algebra, we obtain a compact-form representation for the LT (=Laplace transform) of the queueing delay conditional CDF (=cumulative distribution function).

As it is easy to note, in the literature most results for stochastic features of different-kind queueing systems are obtained for the stationary case, when the system is stable. However, transient (time-dependent) analysis is often desired or even necessary, e.g. due to high volatility of the arrival process (as it can be observed in the TCP/IP traffic) or in the case of large traffic in distributed systems [21, 22] or server breakdowns, when the stochastic stabilization of the system is longer or more difficult.

In [3] the M/G/1-type vacation queueing system with server breakdowns, setup and closedown times, in which the length of the vacation period is controlled either by the number of arrivals during the vacation period, or by a timer is investigated. The study on a batch-arrival queue with multiple vacation policy and server setup and closedown times in the stationary case can be found in [1]. Results for models with group arrivals are also presented in [4, 10, 20]. In [11] a discrete-time model is analyzed, where the mixed-type mechanism of a multiple vacation policy combined with setup/closedown periods is implemented. The case of a multi-server queue is investigated in [2]. Transient results for infinite-buffer systems with server setup times can be found in [5, 6]. Analytical solution for time-dependent queue-size distribution in the queueing model with finite buffer capacity and machine setup and closedown times is derived in [7] (similar technique is also applied in [8]). Evolutionary algorithms are used in [13, 14] for the analysis of a finite-buffer queue with temporary unavailable server. In various data management systems (see [15]) we can find applications of queueing systems to enable faster requests management and therefore improved data mining (see e.g. [16, 17]).

The remaining part of the article is organized as follows. In the next Sect. 2 we give a precise mathematical description of the considered queueing model. In Sect. 3 we derive a system of integral equations for conditional CDF of transient queueing delay. In Sect. 4 we obtain the corresponding system for LTs and write it in a specific form. Section 5 contains main result: the closed-form representation for the LT of conditional CDF of queueing delay and the last Sect. 7 gives a short conclusion.

2 System Description

In the paper we deal with a single-server queueing model with a reliable service station and finite capacity of the buffer for jobs waiting for service. Assume that the arrival process of jobs (packets, customers, calls, etc.) is described by a Poisson process with intensity λ , while the processing time of each one is generally distributed random variable with a CDF $F(\cdot)$. A number of jobs simultaneously present in the system is bounded by a non-random value N , i.e. we have a buffer with $N - 1$ places and one place in service station. Initially, at time $t = 0$, a buffer may contain a number of jobs waiting for service. Every time when the system becomes empty (a service of a job finishes and there are no waiting jobs in the buffer), the service station initializes a randomly distributed closedown time with a CDF $C(\cdot)$. If at the completion epoch of a closedown time a buffer contains at least one job waiting for service, the processing begins immediately, otherwise the server waits in the standby mode for the first arrival. The first processing after the idle period is always preceded by a setup time, duration of which is a random variable with a CDF $S(\cdot)$. Moreover, if a job enters the system during the closedown time, after its completion epoch the server immediately begins the setup time and next the processing. As it was mentioned in the previous section, the closedown time is needed for a server to switch off according to energy saving strategy. Similarly, the setup time is needed for a server to achieve full operational readiness after the idle period. Successive jobs are being processed according to the FIFO service discipline. Moreover, if the arriving job find the buffer being saturated, it is lost. We assume that the delay of the customer being lost equals 0.

3 Integral Equations for Conditional CDF of Queueing Delay

Let $v(t)$ be the (virtual) queueing delay at time t , i.e. the waiting time of a job entering the system exactly at time t . Introduce the conditional CDF of $v(t)$ in the following way:

$$V_n(t, x) \stackrel{\text{def}}{=} P\{v(t) < x \mid X(0) = n\}, 0 \leq n \leq N, t > 0, x > 0, \tag{1}$$

where $X(0)$ denotes the number of jobs present in the system at the opening.

Assume, firstly, that the system is empty at the start epoch $t = 0$. In such a case we treat this epoch as a time at which the last processing completes in a busy period, so, in consequence, a closedown time begins. In fact, one can distinguish eight different mutually excluding possibilities:

1. the first jobs arrives during the closedown time, and both the closedown and setup times finishes before time t (denote this random event by E_1);
2. the first job enters during the closedown time but that period completes after t (E_2);
3. the first job occurs during the closedown time and the time t is “inside” the following setup time (E_3);
4. the first job arrives after the closedown time but before t , and the setup time finishes before t (E_4);

5. the first job enters after the closedown time but before t , and the setup time ends after t (E_5);
6. the first jobs arrives after t , but the closedown time “closes” before t , so at the first arrival epoch the setup time begins (E_6);
7. the first job arrives after t but still “inside” the closedown time (E_7);
8. the first jobs joins the system after t and after the closedown time that “contains” the time t (E_8).

Introducing the following notation:

$$V_{0,i}(t, x) \stackrel{\text{def}}{=} P\{(v(t) < x)\} \cap E_i \mid X(0) = 0\}, 1 \leq i \leq 8, \quad (2)$$

we can write the following representations:

$$\begin{aligned} V_{0,1}(t, x) = & \int_{u=0}^t dC(u) \int_{v=0}^{t-u} dS(v) \int_{y=0}^u \left[\sum_{k=0}^{N-2} \frac{[\lambda(u+v-y)]^k}{k!} e^{-\lambda(u+v-y)} V_{k+1}(t-u-v, x) \right. \\ & \left. + V_N(t-u-v, x) \sum_{k=N-1}^{\infty} \frac{[\lambda(u+v-y)]^k}{k!} e^{-\lambda(u+v-y)} \right] \lambda e^{-\lambda y} dy; \end{aligned} \quad (3)$$

$$V_{0,2}(t, x) = \int_{u=t}^{\infty} dC(u) \int_{y=0}^t \lambda e^{-\lambda y} \sum_{k=0}^{N-2} \frac{[\lambda(t-y)]^k}{k!} e^{-\lambda(t-y)} dy \int_{v=0}^{\infty} F^{(k+1)*}(x-v-u+t) dS(v); \quad (4)$$

$$V_{0,3}(t, x) = \int_{u=0}^t dC(u) \int_{y=0}^u \lambda e^{-\lambda y} \sum_{k=0}^{N-2} \frac{[\lambda(t-y)]^k}{k!} e^{-\lambda(t-y)} dy \int_{v=t-u}^{\infty} F^{(k+1)*}(x-v-u+t) dS(v); \quad (5)$$

$$\begin{aligned} V_{0,4}(t, x) = & \int_{u=0}^t dC(u) \int_{y=u}^t \lambda e^{-\lambda y} dy \int_{v=0}^{t-y} \left[\sum_{k=0}^{N-2} \frac{(\lambda v)^k}{k!} e^{-\lambda v} V_{k+1}(t-y-v, x) \right. \\ & \left. + V_N(t-y-v, x) \sum_{k=N-1}^{\infty} \frac{(\lambda v)^k}{k!} e^{-\lambda v} \right] dS(v); \end{aligned} \quad (6)$$

$$V_{0,5}(t, x) = \int_{u=0}^t dC(u) \int_{y=u}^t \lambda e^{-\lambda y} \sum_{k=0}^{N-2} \frac{[\lambda(t-y)]^k}{k!} e^{-\lambda(t-y)} dy \int_{v=t-y}^{\infty} F^{(k+1)*}(x-y-v+t) dS(v); \quad (7)$$

$$V_{0,6}(t, x) = C(t) \int_t^{\infty} \lambda e^{-\lambda y} S(x-y+t) dy; \quad (8)$$

$$V_{0,7}(t, x) = \int_t^\infty S(x - u + t)(e^{-\lambda t} - e^{-\lambda u})dC(u); \tag{9}$$

and

$$V_{0,8}(t, x) = \int_{u=t}^\infty dC(u) \int_{y=u}^\infty \lambda e^{-\lambda y} S(x - y + t)dy. \tag{10}$$

In the formulae above $F^{j*}(\cdot)$ stands for the j -fold Stieltjes convolution of the CDF $F(\cdot)$ with itself.

Let us comment briefly some of the formulae (3)–(10). The first summands on the right sides of (3) and (6) refer to the situation in which there is at least one free place in the buffer at the completion epoch of the setup time, while the second one to the case in which the buffer becomes saturated during the setup time. In (4) and (7) the sum is taken only for $k = N - 2$. Indeed, in the case t is “inside” the period during which the service is suspended (the closedown or the setup time) and the buffer becomes saturated before t , the “virtual” jobs occurring exactly at time t is lost.

Now, let us analyze the case where at $t = 0$ the buffer contains at least one job, i.e. the level of buffer saturation equals n , where $1 \leq n \leq N$. Since successive service completion epochs are Markov moments in the evolution of the system, then, applying the continuous version of the total probability law with respect to the first service completion epoch after the start of the system, the following system of integral equations can be written:

$$\begin{aligned} V_n(t, x) = & \sum_{k=0}^{N-n-1} \int_0^t V_{n+k-1}(t - y, x) \frac{(\lambda y)^k}{k!} e^{-\lambda y} dF(y) \\ & + \sum_{k=N-n}^\infty \int_0^t V_{N-1}(t - y, x) \frac{(\lambda y)^k}{k!} e^{-\lambda y} dF(y) \\ & + \sum_{k=0}^{N-n-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \int_t^\infty F^{(n+k-1)*}(x - y + t) dF(y), \end{aligned} \tag{11}$$

where $1 \leq n \leq N$. Indeed, the first term on the right side of (11) relates to the situation in which the buffer is not full before the first processing completion epoch, while the second one describes the case in which the buffer becomes saturated before the first departure moment. The last summand refers to the situation in which the first service completes after time t .

4 System of Equations for LTs of Delay Conditional CDFs

In this section we derive the system of equations for LTs of conditional CDFs of the queueing delay in the considered model and write it in a specific form. Thus, let us introduce the following notation:

$$\tilde{v}_n(s, x) \stackrel{\text{def}}{=} \int_0^\infty e^{-st} V_n(t, x) dt = \int_0^\infty e^{-st} P\{v(t) < x | X(0) = n\} dt, \quad \Re(s) > 0, \quad (12)$$

$x > 0, 0 \leq n \leq N.$

Moreover, due to the fact that $\tilde{v}_0(s, x) = \sum_{i=1}^8 \int_0^\infty e^{-st} V_{0,i}(t, x) dt$, then just by virtue of the Eqs. (3)–(10), we obtain

$$\tilde{v}_0(s, x) = \sum_{k=0}^{N-2} \tilde{v}_{k+1}(s, x) \alpha_k(s) + \tilde{v}_N(s, x) \sum_{k=N-1}^\infty \alpha_k(s) + \beta(s, x), \quad (13)$$

where

$$\alpha_k(s) \stackrel{\text{def}}{=} \frac{\lambda^{k+1}}{(k+1)!} \int_{u=0}^\infty e^{-(\lambda+s)u} dC(u) \int_{v=0}^\infty e^{-(\lambda+s)v} \left[(v+u)^{k+1} - v^{k+1} \right] dS(v) + \frac{\lambda}{\lambda+s} \tilde{c}(\lambda+s) \int_0^\infty e^{-(\lambda+s)v} \frac{(\lambda v)^k}{k!} dS(v); \quad (14)$$

$$\tilde{c}(s) \stackrel{\text{def}}{=} \int_0^\infty e^{-st} dC(t); \quad (15)$$

$$\begin{aligned} \beta(s, x) \stackrel{\text{def}}{=} & \int_{t=0}^\infty e^{-(\lambda+s)t} \sum_{k=0}^{N-2} \frac{\lambda^{k+1}}{(k+1)!} \left\{ \left[t^{k+1} - (t-u)^{k+1} \right] \int_{u=0}^t dC(u) \int_{v=t-u}^\infty F^{(k+1)*}(x-v-u+t) dS(v) \right. \\ & \left. + t^{k+1} \int_{u=t}^\infty dC(u) \int_{v=0}^\infty F^{(k+1)*}(x-v-u+t) dS(v) \right\} dt \\ & + \int_{t=0}^\infty e^{-st} dt \int_{u=0}^t dC(u) \int_{y=u}^t \lambda e^{-\lambda y} \sum_{k=0}^{N-2} \frac{[\lambda(t-y)]^k}{k!} e^{-\lambda(t-y)} dy \int_{v=t-y}^\infty F^{(k+1)*}(x-y-v+t) dS(v) \\ & + \int_{t=0}^\infty e^{-st} C(t) dt \int_{y=t}^\infty \lambda e^{-\lambda y} S(x-y+t) dy \\ & + \int_{t=0}^\infty e^{-st} dt \int_{u=t}^\infty S(x-u+t) (e^{-\lambda t} - e^{-\lambda u}) dC(u) \\ & + \int_{t=0}^\infty e^{-st} dt \int_{y=t}^\infty \lambda e^{-\lambda y} S(x-y+t) [C(y) - C(t)] dy. \end{aligned} \quad (16)$$

In the similar manner, by defining the following functional sequences:

$$a_k(s) \stackrel{\text{def}}{=} \int_0^\infty e^{-(\lambda+s)t} \frac{(\lambda t)^k}{k!} dF(t), \quad (17)$$

$$\phi_n(s, x) \stackrel{\text{def}}{=} \int_{t=0}^{\infty} e^{-(\lambda+s)t} \sum_{k=0}^{N-n-1} \frac{(\lambda t)^k}{k!} dt \int_{y=t}^{\infty} F^{(n+k-1)*}(x-y+t) dF(y), \tag{18}$$

we obtain from (11) the following system of equations:

$$\tilde{v}_n(s, x) = \sum_{k=0}^{N-n-1} a_k(s) \tilde{v}_{n+k-1}(s, x) + \tilde{v}_{N-1}(s, x) \sum_{k=N-n}^{\infty} a_k(s) + \phi_n(s, x), \tag{19}$$

where $1 \leq n \leq N$. Let us apply to (13) and (19) the following substitution:

$$\tilde{\omega}_n(s, x) \stackrel{\text{def}}{=} \tilde{v}_{N-n}(s, x) \tag{20}$$

Now, the representations (13) and (19) can be rewritten as follows:

$$\tilde{\omega}_N(s, x) = \sum_{k=1}^{N-1} \tilde{\omega}_k(s, x) \alpha_{N-k-1}(s) + \tilde{\omega}_0(s, x) \sum_{k=N-1}^{\infty} \alpha_k(s) + \beta(s, x), \tag{21}$$

and

$$\sum_{k=-1}^n a_{k+1}(s) \tilde{\omega}_{n-k}(s, x) - \tilde{\omega}_n(s, x) = \Delta_n(s, x), 0 \leq n \leq N-1, \tag{22}$$

where

$$\Delta_n(s, x) \stackrel{\text{def}}{=} a_{n+1}(s) \tilde{\omega}_0(s, x) - \tilde{\omega}_1(s, x) \sum_{k=n+1}^{\infty} a_k(s) - \phi_{N-n}(s, x). \tag{23}$$

5 Transient Solution for LT of CDF of Queueing Delay

The system of equations of type (22) but with infinite number of equations, namely written for $n \geq 0$, was investigated in [9]. It was proved there that each its solution can be written in the form (we give here the representation for functional sequences, however, in [9], the equations with coefficients given by ordinary number sequences are considered):

$$\tilde{\omega}_n(s, x) = R_{n+1}(s)K(s, x) + \sum_{k=0}^n R_{n-k}(s)\Delta_k(s, x), \quad n \geq 0, \tag{24}$$

where $K(s, x)$ is independent on n , and successive terms of the functional sequence $R_k(s), k \geq 0$, can be found recursively from coefficients $a_k(s), k \geq 0$, of the system as follows:

$$R_0(s) = 0, R_1(s) = a_0^{-1}(s), R_{k+1}(s) = R_1(s) \left(R_k(s) - \sum_{i=0}^k a_{i+1}(s) R_{k-i}(s) \right), \quad k \geq 1. \tag{25}$$

Obviously, if we find closed-form representations for $\tilde{\omega}_0(s, x)$ and $\tilde{\omega}_1(s, x)$, which are present in the formula (23) for $\Delta_n(s, x)$, and the formula for $K(s, x)$, it will be possible to state a representation for $\tilde{\omega}_n(s, m)$ for arbitrary n , basing on (24).

Taking $n = 0$ in (24), we obtain

$$K(s, x) = a_0(s) \tilde{\omega}_0(s, x). \tag{26}$$

Similarly, substituting $n = 0$ into (22) and referring to (23), we have

$$\tilde{\omega}_1(s, x) = \frac{1}{\sum_{k=0}^{\infty} a_k(s)} [\tilde{\omega}_0(s, x) - \phi_N(s, x)] = \frac{\tilde{\omega}_0(s, x) - \phi_N(s, x)}{f(s)}, \tag{27}$$

where

$$f(s) \stackrel{\text{def}}{=} \int_0^{\infty} e^{-st} dF(t), \Re(s) > 0. \tag{28}$$

In order to find the formula for $\tilde{\omega}_0(s, x)$ we must compare the right sides of (21), being in fact a kind of a boundary condition, and (24) taken for $n = N$. Executing necessary calculations, we obtain

$$\tilde{\omega}_0(s, x) = \tilde{v}_N(s, x) = \frac{\beta(s, m) + \gamma(s, x)}{H_N(s) - \sum_{k=1}^{N-1} \alpha_{N-k-1}(s) H_k(s) - \sum_{k=N-1}^{\infty} \alpha_k(s)}, \tag{29}$$

where

$$\begin{aligned} \gamma(s, x) \stackrel{\text{def}}{=} & \sum_{k=0}^N R_{N-k}(s) \left[(f(s))^{-1} \phi_N(s, x) \sum_{i=k+1}^{\infty} a_i(s) - \phi_{N-k}(s, x) \right] \\ & + \sum_{k=1}^{N-1} \alpha_{N-k-1}(s) \sum_{i=0}^k R_{k-i}(s) \left[(f(s))^{-1} \phi_N(s, x) \sum_{j=i+1}^{\infty} a_j(s) - \phi_{N-i}(s, x) \right] \end{aligned} \tag{30}$$

and

$$H_k(s) \stackrel{\text{def}}{=} R_{k+1}(s) a_0(s) + \sum_{i=0}^k R_{k-i}(s) \left[a_{i+1}(s) - (f(s))^{-1} \sum_{j=i+1}^{\infty} a_j(s) \right]. \tag{31}$$

After collecting the formulae (23)–(31), we can state the following main theorem:

Theorem 1. *The representations for the LT $\tilde{v}_n(s, x)$ of conditional CDF of the queueing delay in the M/G/1/N-type finite-buffer model with generally distributed setup and closedown times are following:*

$$\tilde{v}_n(s, x) = R_{N-n+1}(s)a_0(s)q_N(s) + \sum_{k=0}^{N-n} R_{N-n-k}(s) \left[a_{k+1}(s)\tilde{v}_N(s, x) - (f(s))^{-1}(\tilde{v}_N(s, x) - \phi_N(s, x)) \sum_{i=k+1}^{\infty} a_i(s) - \phi_{N-n}(s, x) \right], \tag{32}$$

where $0 \leq n \leq N - 1$, and

$$\tilde{v}_N(s, x) = \frac{\beta(s, m) + \gamma(s, x)}{H_N(s) - \sum_{k=1}^{N-1} \alpha_{N-k-1}(s)H_k(s) - \sum_{k=N-1}^{\infty} \alpha_k(s)}, \tag{33}$$

where the formulae for $\beta(s, x)$, $\alpha_k(s)$, $\phi_k(s, x)$, $R_k(s)$, $f(s)$ and $\gamma(s, x)$ are given in (16), (17), (18), (25), (28) and (30), respectively.

6 Computational Examples

Let us consider the stream of packets of sizes 400 [B] arriving to the node of a network with buffer capacity 50 packets. The arrival stream is governed by a Poisson process with intensity 600 [kb/s], while the throughput of the output link is 800 [kb/s], so the traffic load equals 0.75. The mechanism of the node setup/closedown times is implemented. Three different situations of exponentially distributed setup and closedown times are investigated (see Table 1).

Table 1. Three different variants for the relationship between setup and closedown times

Variant	Mean setup time	Mean closedown time
SET < CLO	0.050 [s]	0.100 [s]
SET > CLO	0.100 [s]	0.050 [s]
SET = CLO	0.067 [s]	0.067 [s]

In Fig. 1 the behavior of the waiting time distribution in the stable system is shown for all considered cases. It is easy to note that for SET < CLO the probability of long waiting times for the arriving packets is relatively smallest one. In Fig. 2 the case of the arrival intensity 768 kb/s is considered (now the traffic load equals 0.96, so the system is close to the critical loading). Let us observe that in such a case the probability of waiting time less than e.g. 0.1 [s] is relatively large (equals approximately 0.4–0.5), while for the system less loaded equals even about 0.8.

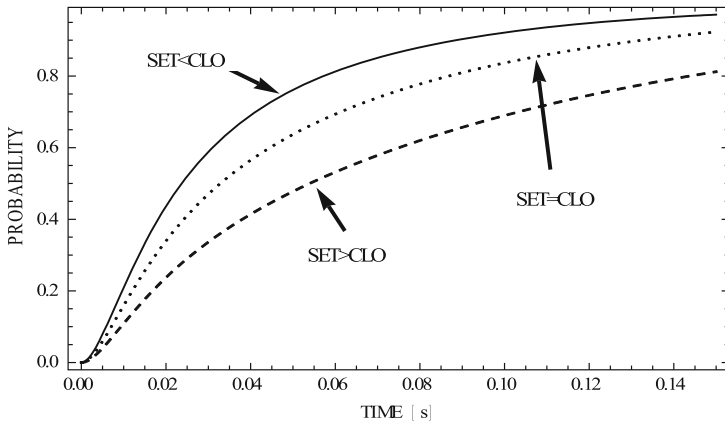


Fig. 1. Waiting time distribution for the stable system with traffic load 0.75

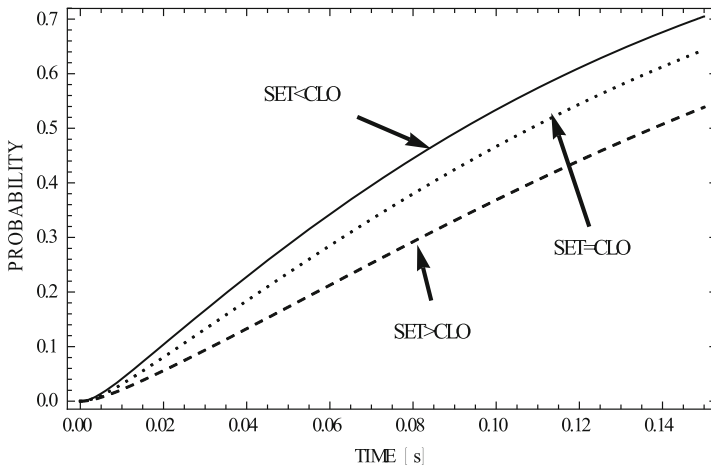


Fig. 2. Waiting time distribution for the stable system with traffic load 0.96

7 Conclusion

In the paper a single-channel queuing model with finite buffer capacity and server setup-closedown times is considered. The arrival process is governed by a single Poisson stream while the processing times are generally distributed random variables. By using the analytical approach based on the concept of embedded Markov chain, total probability law and linear algebra a closed-form representation for the LT of conditional CDF of the queuing delay is obtained. The final formulae are written in terms of “input” system characteristics and certain functional sequence, defined recursively, connected with them. Numerical examples, illustrating theoretical results, are attached.

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