# **Suitable Aggregation Operator for a Realistic Supplier Selection Model Based on Risk Preference of Decision Maker**

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**Abstract.** In this paper, we propose (a) the realistic models for Supplier Selection and Order Allocation (SSOA) problem under fuzzy demand and volume/quantity discount constraints, and (b) how to select the suitable aggregation operator based on risk preference of the decision makers (DMs). The aggregation operators under consideration are additive, maximin, and augmented operators while the risk preferences are classified as risk-averse, risk-taking, and risk-neutral ones. The fitness of aggregation operators and risk preferences of DMs is determined by statistical analysis. The analysis shows that the additive, maximin, and augmented aggregation operators are consistently suitable for risk-taking, risk-averse, and risk-neutral DMs, respectively.

### **1 Introduction**

Selecting appropriate suppliers is one of the critical business decisions faced by purchasing managers, and it has a long term impact on a whole supply chain. For most firms, raw material costs account for up to 70 % of product cost as observed in Ghodspour and O'Brien (2001). Thus, a supplier selection process is an important issue in strategic procurement to enhance the competitiveness of the firm [\[1\]](#page-13-0). Effective selection of appropriate suppliers involves not only scanning the price list, but also requirements of organization which are increasingly important due to a high competition in a business market. Typically, Dickson (1996) indicated that major requirements are meeting customer demand, reducing cost, increasing product quality and on time delivery performance [\[2](#page-13-1)]. Hence, supplier selection is a Multi-Criteria-Decision-Making (MCDM) problem which includes both qualitative and quantitative data, and some of which may be conflicting. In a case of conflicting criteria, DMs need to compromise among criteria. To do so, decision criteria are transformed to objective functions or constraints. The relative importance (weight) of each criterion may be also applied to the model.

Essentially, to prevent a monopolistic supply base as well as to meet all the requirements of firms, most firms have multiple sources which lead to the problem of how many units of each product should be allocated to each of suppliers. Thus, it becomes a Supplier Selection and Order Allocation (SSOA) problem.

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Interestingly, to attract large order quantities, suppliers frequently offer trade discounts. Commonly, volume and quantity discounts are popular trade-discount strategies. The quantity-discount policy aims to reduce a unit cost, while the volume-discount encourages firms to reduce the total purchasing cost. Both discounts are triggered at a certain purchasing level. For example, buyers purchase at \$20 per unit from \$25 per unit when they purchase more than 100 units or receive a 10 % discount when the total purchase cost of all products is greater than \$1000. It is interesting here to observe that the trade discount complicates the allocation of order quantities placed to suppliers. Thus, determining the joint consideration of different pricing conditions is a crucial task of DMs to make the most beneficial buying decision.

Practically, firms try to place an order at the level of predicted demand to avoid excess inventory. However, when trade discounts are offered, firms usually purchase more than predicted demand to receive a lower price. Hence, to flexibly optimize the benefit, fuzzy demand is incorporated in models. Note that the satisfaction of demand criteria decreases whenever the order quantity is greater or less than predicted demand. Regarding the issue of uncertainty (fuzziness), fuzzy set theory (FST) developed by Zadeh (1965) has been extensively used to deal with uncertain data, like in this case [\[3](#page-13-2)].

During the last decades, we have witnessed many decision techniques for handling MCDM problem. Among several techniques suggested in Ho et al. (2010) [\[4](#page-13-3)], linear weighting programming model proposed by Wind and Robinson (1968) [\[5](#page-13-4)], is widely applied to assess the performances of suppliers. The model is relatively easy to understand and implement. Later, with the use of pairwise comparisons, an analytical hierarchy process (AHP) allows more accurate scoring method [\[6\]](#page-13-5). Generally, this technique decomposes the complex problem into multiple levels of hierarchical structure. Similarly, Analytic Network Process (ANP), Goal Programming(GP), Neural Network (NN), etc., are also introduced to deal with the MCDM problem.

Although several advanced techniques have been proposed to deal with the MCDM problem, little attention has been addressed to determine which aggregation operator is suitable for a specific risk preference of DMs. Basically, the risk preference of DMs can be distinguished into three types, namely, risk-taking, risk-averse, and risk-neutral. Another concerning issue is that previous research works related to the SSOA problem have been conducted based on either volume or quantity discount, not both of them at the same time.

Based on these motivations, this paper proposes realistic models with important practical constraints, especially volume and quantity discount constraints under fuzzy demand. Interestingly, three types of aggregation operators are applied to the models to determine which operator is suitable for risk-taking, risk-averse, and risk-neutral DMs. The aggregation operators are (1) additive, (2) maximin, and (3) augmented operators. The models are developed from Amid et al. [\[7](#page-13-6)], Amid et al. [\[8](#page-13-7)], and Feyzan [\[9\]](#page-13-8), accordingly. In addition, to test the sensitivity of the models as well as the effect of aggregation operators, statistical analysis is conducted based on two performance indicators, namely, the average and the lowest satisfaction levels.

The rest of this paper is organized as follows. In Sect. [2,](#page-2-0) related works are mentioned. Then, six developed models are presented in Sect. [3.](#page-2-1) In Sect. [4,](#page-9-0) statistical experiments are designed to analyze the performances of the aggregation operators using MINITAB software. Results are discussed in Sect. [5](#page-10-0) and some concluding remarks are presented in Sect. [6.](#page-12-0)

## <span id="page-2-0"></span>**2 Related Work**

To aggregate multiple criteria, many advanced aggregation operators have been proposed in decades. However, in this paper, three basic types of operators are investigated with relative importance of criteria.

**Additive Aggregation Operator.** The weighted additive technique is probably the best known and widely used method for calculating the total score when multiple criteria are considered. In [\[7](#page-13-6)], the objective function is

$$
\text{Max} \quad \Sigma_{i=1}^I w_i \lambda_i
$$

where  $w_i$  is the relative importance of criteria i and  $\lambda_i$  is the satisfaction level (SL) of criteria i. Note that to deal with multiple criteria, dimensions of criteria are transformed to SLs which are dimensionless.

**Maximin Aggregation Operator.** In [\[8\]](#page-13-7), this operator is looking for SL that meets the need of all criteria. Therefore, s is the smallest SL of all criteria.

Max s

**Augmented Aggregation Operator.** In [\[9\]](#page-13-8), the author propose this operator in order to keep both advantages of additive and maximin operators. The objective function is developed as follows.

$$
\text{Max} \quad s + \varSigma_{i=1}^I w_i \lambda_i
$$

## <span id="page-2-1"></span>**3 Model Development**

There are six proposed models for SSOA problem under fuzzy demand and volume/quantity discount constraints. These models are based on risk preference of DMs which are risk-taking, risk-averse, and risk-neutral. Models under consideration are shown in Fig. [1.](#page-3-0)

### **3.1 Problem Description**

In this study, DMs must properly allocate the order quantities to each supplier so that the maximum satisfaction is achieved. They have four criteria in mind: (1) the total cost, (2) the quality of product, (3) the on time delivery performance, and (4) the preciseness of demand, where relative importances of criteria

(weights) are given. We reduce dominant effects among criteria by transforming them into satisfaction levels (SLs) in a range from 0.0 to 1.0. Demand of each product is allowed to be fuzzy. As multiple products are considered, the overall demand SL is the least SL of all products. In addition, the price-discount models were developed from Xia and Wu (2007) [\[10\]](#page-13-9), Wang and Yang (2009) [\[11](#page-13-10)], and Suprasongsin et al. (2014) [\[12](#page-13-11)].



<span id="page-3-0"></span>**Fig. 1.** A combined model diagram

<span id="page-3-1"></span>**Fig. 2.** Experimental's factor of each data set

#### **3.2 Notations**

Let us assume that there are five products and five suppliers under consideration. Supplier  $k$   $(k = 1, ..., K)$  offers either volume discount or quantity discount when product  $j$   $(j = 1, ..., J)$  is purchased at a discount level  $c$   $(c = 1, ..., C)$ . It is also assumed that supplier 3 offers a volume discount policy, while other suppliers offer a quantity discount policy.

*Indices*



#### *Input parameters*

 $dc_j$  constant (crisp) demand of product *j* (unit)

- $h_{ik}$  capacity for product *j* from supplier *k* (unit)
- $u_j$  maximum number of supplier that can supply product *j* (supplier)
- $l_j$  minimum number of supplier that can supply product *j* (supplier)
- $o_{ik}$  minimum order quantity of product *j* from supplier *k* (unit)
- $sr_{ik}$  1 if supplier *k* supplies product *j*; 0 otherwise (unitless)
- $r_{ik}$  minimum fraction of total demand of product *j* purchased from supplier *k* (unitless)
- $p_{cik}$  price of product *j* offered from supplier *k* at discount level  $c$  (\$)
- $z1_{ik}$  unit price of product *j* from supplier  $k$  (\$)
- $z^2_{ik}$  quality score of product *j* from supplier *k* (scores)
- $z3_{ik}$  delivery lateness of product *j* from supplier *k* (days)
- $e_{cik}$  break point of quantity discount at level *c* of product *j* from supplier *k* (unit)
- g*ck* discount fraction of volume discount from supplier *k* at discount level *c* (unitless)
- $b_{ck}$  break point of volume discount at level *c* from supplier  $k$  (\$)
- $f_k$  1 if supplier *k* offers quantity discount; 0 otherwise (unitless)
- $w_i$  weight of criteria *i* (unitless)
- $\sigma$  weight of fuzzy demand (unitless)
- $mn_i$  minimum value of criteria *i* (\$, scores, days)
- $md<sub>i</sub>$  moderate value of criteria  $i$  (\$, scores, days)
- $mx_i$  maximum value of criteria *i* (\$, scores, days)
- $b_{0m}$  boundary of demand level *m* of product *j* (unit)

*Decision variables*

- $x_{cikn}$  purchased quantity at discount level *c* of product *j* from supplier *k* atdemandlevel *n* (unit)
- $v_{cjk}$  purchased quantity at discount level *c* of product *j* from supplier *k* (unit) at constant demand
- $\pi_{ik}$  1 if supplier *k* supplies product *j*; 0 otherwise (unitless)
- $t_{cjk}$  total purchasing cost *j* from supplier *k* atlevel *c* for quantity discount (\$)
- $a_{ck}$  total purchasing cost *j* from supplier *k* at level *c* for volume discount  $(\$)$
- $\alpha_{ck}$  1 if quantity discount level *c* is selected for supplier *k*; 0 otherwise (unitless)
- $\beta_{ck}$  1 if volume discount level *c* is selected for supplier *k*; 0 otherwise (unitless)
- $\lambda_i$  satisfaction level of criteria *i*; cost, quality and delivery lateness (unitless)
- s overall satisfaction level formulated by weighted maximin model (unitless)
- $sl$  the minimum of satisfaction levels of all criteria (unitless)
- $\gamma$  achievement level of fuzzy demand from all products (unitless)
- $z_{in}$  1 if demand level *n* is selected for product *j*; 0 otherwise (unitless)
- $sld<sub>j</sub>$  satisfaction level of fuzzy demand of each product *j* (unitless)
- $d_{in}$  total demand of product *j* at level *n* (unit)

### **3.3 Mathematical Formulation**

In this section, six models are presented as the following.

*Additive Model.* In this model, we assume that all criteria are equally important. The model aims to maximize the average SLs of all criteria including the achievement level of fuzzy demand as shown in [\(1\)](#page-4-0).

<span id="page-4-0"></span>Maximize

$$
(\Sigma_i \lambda_i + \gamma)/(I+1) \tag{1}
$$

<span id="page-4-1"></span>**Price Discount.** In quantity discount constraints  $(2-4)$  $(2-4)$ , the purchasing quantity  $x_{cikn}$  must be corresponding to a suitable discount level. Similarly, in volume discount constraints  $(5-7)$  $(5-7)$ , the business volume  $a_{ck}$  from supplier k should be in a suitable discount level c.

$$
\Sigma_c t_{cjk} \cdot f_k = \Sigma_c \Sigma_n p_{cjk} \cdot x_{cjkn} \cdot f_k \qquad \forall j, k \tag{2}
$$

<span id="page-5-0"></span>
$$
e_{c-1,jk} \cdot \alpha_{ck} \cdot f_k \le \Sigma_j x_{cjkn} \cdot f_k < e_{cjk} \cdot \alpha_{ck} \cdot f_k \qquad \forall c, k, n \tag{3}
$$

$$
\Sigma_c \alpha_{ck} \cdot f_k \le 1 \qquad \forall k \tag{4}
$$

$$
\Sigma_c a_{ck} \cdot (1 - f_k) = \Sigma_c \Sigma_j \Sigma_n z_{1jk} \cdot x_{cjkn} \cdot (1 - f_k) \qquad \forall k \tag{5}
$$

<span id="page-5-2"></span><span id="page-5-1"></span>
$$
b_{c-1,k} \cdot \beta_{ck} \cdot (1 - f_k) \le a_{ck} \cdot (1 - f_k) < b_{ck} \cdot \beta_{ck} \cdot (1 - f_k) \qquad \forall c, j, k \tag{6}
$$

$$
\Sigma_c \beta_{ck} \cdot (1 - f_k) \le 1 \qquad \forall k \tag{7}
$$

**Available Supplier.** A supplier may supply only some products but not all of the products.

$$
\pi_{jk} \le sr_{jk} \quad \forall j, k \tag{8}
$$

**Capacity.** The total purchasing quantity  $x_{cikn}$  must be less than the supply capacity  $h_{jk}$  and it is active only if the assigned  $\pi_{jk}$  is equal to 1.

$$
\Sigma_c \Sigma_n x_{cjkn} \le h_{jk} \cdot \pi_{jk} \quad \forall j, k \tag{9}
$$

**Limited Number of Supplier.** The number of suppliers cannot exceed the available suppliers.

$$
l_j \le \Sigma_k \pi_{jk} < u_j \quad \forall j \tag{10}
$$

**Minimum Order Quantity.** The total purchasing quantity  $x_{cjkn}$  must be greater than the required minimum order quantity of product  $j$  from supplier  $k$ 

$$
o_{jk} \cdot \pi_{jk} \le \Sigma_c \Sigma_n x_{cjkn} \quad \forall j, k \tag{11}
$$

**Relationship.** The agreement with a supplier k that a firm will purchase the product j at least some percentage of the total demand from this supplier  $k$ .

$$
r_{jk} \cdot \Sigma_n d_{jn} \le \Sigma_c \Sigma_n x_{cjkn} \quad \forall j, k \tag{12}
$$

**Fuzzy Demand.** Total purchasing quantity x*cjkn* must be in a range of minimum  $bo_{m,j}$  and maximum  $bo_{m+1,j}$  demand levels and only one demand level  $z_{jn}$ must be selected.

<span id="page-5-4"></span>
$$
bo_{mj} \cdot z_{jn} \le d_{jn} < bo_{m+1,j} \cdot z_{jn} \qquad \forall j, m, n \tag{13}
$$

$$
\sum_{c} \sum_{k} x_{cjkn} = d_{jn} \qquad \forall j, n \tag{14}
$$

$$
\Sigma_n z_{jn} = 1 \qquad \forall j \tag{15}
$$

<span id="page-5-3"></span>**Satisfaction Level.** Constraints [\(16–](#page-5-3)[18\)](#page-6-0) describe the SLs of cost, quality, and delivery lateness criteria. Constraints  $(19-21)$  $(19-21)$  calculate the SL (called achievement level) of the fuzzy demand.

$$
\lambda_1 \le \frac{mx_1 - \Sigma_c \Sigma_j \Sigma_k t_{cjk} \cdot f_k + \Sigma_c \Sigma_k a_{ck} \cdot (1 - g_{ck}) \cdot (1 - f_k)}{mx_1 - md_1} \tag{16}
$$

$$
\lambda_2 \le \frac{\sum_c \sum_j \sum_k \sum_n z_{jk} \cdot x_{cjkn} - mn_2}{md_2 - mn_2} \tag{17}
$$

<span id="page-6-1"></span><span id="page-6-0"></span>
$$
\lambda_3 \le \frac{mx_3 - \Sigma_c \Sigma_j \Sigma_k \Sigma_n z_{jk} \cdot x_{cjkn}}{mx_3 - md_3} \tag{18}
$$

$$
sld_j \le \frac{bo_{3j} - \Sigma_n d_{jn}}{bo_{3j} - bo_{2j}} \qquad \forall j \tag{19}
$$

<span id="page-6-2"></span>
$$
sld_j \le \frac{\sum_n d_{jn} - bo_{1j}}{bo_{2j} - bo_{1j}} \qquad \forall j \tag{20}
$$

$$
\gamma \le sld_j \qquad \forall j \tag{21}
$$

<span id="page-6-6"></span><span id="page-6-3"></span>**Non-negativity Conditions and the Range of Values.** Constraints [\(22](#page-6-3)[–24\)](#page-6-4) are non-negativity conditions and the range of values.

$$
0 \le \lambda_i < 1 \qquad \forall i \tag{22}
$$

<span id="page-6-4"></span>
$$
0 \le sld_j < 1 \qquad \forall j \tag{23}
$$

$$
0 \le \gamma < 1\tag{24}
$$

*Weighted Additive Model.* A basic concept of this model is to use a single utility function representing the overall preference of DMs corresponding to the relative importance of each criterion.

Maximize

$$
(\Sigma_i w_i \cdot \lambda_i) + (\sigma \cdot \gamma) \tag{25}
$$

All constraints are the same as those of the additive model  $(2-24)$  $(2-24)$ .

*Maximin Model.* Different from the additive model, the maximin model attempts to maximize the minimum SLs of all criteria, rather than maximize the average value of all SLs. In this model, all criteria are equally important.

Maximize

$$
sl \tag{26}
$$

Constraints  $(2-24)$  $(2-24)$  are used and three non-negativity constraints are added.

$$
sl \le \gamma \tag{27}
$$

<span id="page-6-8"></span><span id="page-6-7"></span>
$$
sl \le \lambda_i \qquad \forall i \tag{28}
$$

$$
0 \le sl < 1\tag{29}
$$

*Weighted Maximin Model.* It is similar to the maximin model but weights are considered. Constraints [\(31–](#page-6-5)[36\)](#page-7-0) are adapted from constraints [\(16](#page-5-3)[–21\)](#page-6-2).

Maximize

$$
s \tag{30}
$$

<span id="page-6-5"></span>The constraints are subjected to  $(2-15)$  $(2-15)$ ,  $(23)$  and the following constraints.

$$
w_1 \cdot s \le \frac{mx_1 - \sum_c \sum_j \sum_k t_{cjk} \cdot f_k + \sum_c \sum_k a_{ck} \cdot (1 - g_{ck}) \cdot (1 - f_k)}{mx_1 - md_1} \tag{31}
$$

$$
w_2 \cdot s \le \frac{\sum_{c} \sum_{j} \sum_{k} \sum_{n} z_{jk} \cdot x_{cjkn} - mn_2}{md_2 - mn_2} \tag{32}
$$

$$
w_3 \cdot s \le \frac{mx_3 - \sum_c \sum_j \sum_k \sum_n z3_{jk} \cdot x_{cjkn}}{mx_3 - md_3} \tag{33}
$$

$$
\sigma \cdot sld_j \le \frac{bo_{3j} - \Sigma_n d_{jn}}{bo_{3j} - bo_{2j}} \tag{34}
$$

<span id="page-7-0"></span>
$$
\sigma \cdot s l d_j \le \frac{\sum_n d_{jn} - b o_{1j}}{b o_{2j} - b o_{1j}} \qquad \forall j \tag{35}
$$

$$
s \le sld_j \qquad \forall j \tag{36}
$$

$$
0 \le s < 1\tag{37}
$$

*Augmented Model.* Technically, to maximize the average SLs and the minimum SLs of all criteria simultaneously, the objective function is changed to [\(38\)](#page-7-1).

<span id="page-7-1"></span>Maximize

$$
(sl + (\Sigma_i \lambda_i + \gamma))/(I + 1)
$$
\n(38)

All constraints are drawn from the maximin model  $(2-24)$  $(2-24)$  and  $(27-29)$  $(27-29)$ .

*Weighted Augmented Model.* Weighted augmented model is developed from augmented model by taking weights into account. All constraints are the same as augmented model (Tables [1,](#page-7-2) [2,](#page-7-3) [3,](#page-7-4) [5,](#page-8-0) [6,](#page-8-1) [7,](#page-8-2) [8,](#page-8-3) [9](#page-9-1) and [10\)](#page-9-2).

Maximize

$$
sl + (\Sigma_i w_i \cdot \lambda_i + \sigma \cdot \gamma) \tag{39}
$$

<span id="page-7-2"></span>

**Table 1.** Weight sets  $(w_i, \sigma)$ 

<span id="page-7-3"></span>**Table 2.** Crisp demand of each product  $(dc_j)$ 

	Product Predicted demand
$\mathbf{1}$	500
$\mathfrak{D}$	30
3	100
	700
5	2500

**Table 3.** Narrow(N) and wide(W) demand range  $(bo_{mi})$ 

<span id="page-7-4"></span>

<span id="page-8-4"></span>**Table 4.** Unit (LIST) price, quality score and delivery lateness for Incomplete tradeoff(I) and Complete trade-off(C);  $(z1_{jk})$ , $(z2_{jk})$  and  $(z3_{jk})$ 

Data	P/S	S <sub>1</sub>		S <sub>2</sub>		S <sub>3</sub>		S4		S <sub>5</sub>		
		T	$\mathcal{C}$	I	$\mathcal{C}$	I	$\mathcal C$	I	$\mathcal{C}$	I	$\mathcal{C}$	
Unit (list) price	P <sub>1</sub>	50	50	40	40	55	55	50	50	45	45	
	P <sub>2</sub>	$\overline{0}$	$\overline{0}$	200	200	$\overline{0}$	$\overline{0}$	230	230	$\overline{0}$	$\overline{0}$	
	P3	70	70	75	75	72	69	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	
	P <sub>4</sub>	$\overline{0}$	$\overline{0}$	$\theta$	$\overline{0}$	8	8	10	10	5	5	
	P5	$\overline{0}$	$\Omega$	$\theta$	$\overline{0}$	$\theta$	$\overline{0}$	20	20	20	20	
Quality score	P <sub>1</sub>	3	3	5	8	6	6	$\overline{2}$	$\overline{2}$	4	$\overline{4}$	
	P <sub>2</sub>	$\overline{0}$	$\theta$	6	6	$\theta$	$\overline{0}$	7	$\overline{7}$	$\overline{0}$	$\boldsymbol{0}$	
	P3	5	5	$\overline{7}$	7	6	8	$\overline{0}$	$\theta$	$\overline{0}$	$\theta$	
	P4	$\overline{0}$	$\Omega$	$\overline{0}$	$\overline{0}$	8	8	10	10	5	5	
	P5	$\Omega$	$\theta$	$\theta$	$\overline{0}$	$\theta$	$\overline{0}$	8	8	9	9	
Delivery lateness	P <sub>1</sub>	3	3	1	1	$\overline{2}$	$\overline{2}$	$\overline{4}$	$\overline{4}$	3	3	
	P <sub>2</sub>	$\overline{0}$	$\Omega$	4	4	$\theta$	$\overline{0}$	3	3	$\overline{0}$	$\overline{0}$	
	P <sub>3</sub>	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	
	P <sub>4</sub>	$\overline{0}$	$\theta$	$\theta$	$\overline{0}$	3	3	5	5	4	$\overline{4}$	
	P5	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	5	5	3	3	

<span id="page-8-0"></span>**Table 5.** Limited number of supplier  $(u_j, l_j)$ 

<span id="page-8-1"></span>**Table 6.** Break point of volume discount  $(b_{ck})$  and volume discount percentage  $(g_{ck})$ 





<span id="page-8-2"></span>Table 7. Available<br>supplier for each supplier for product (*srjk*)

 $4 \t |0 \t |0 \t |1 \t |1$  $5 \t 0 \t 0 \t 0 \t 1 \t 1$ 

<span id="page-8-3"></span>**Table 8.** Price of each product for quantity discount levels  $(p_{cjk})$ 

			P/S S1 S2 S3 S4 S5	Level/sup. S1			S <sub>2</sub>			S3				S5						
							$P1 P3 P2.4.5 P1 P2 P3 P4-5 P1 P2 P3 P4 P5 P1 P2-3 P4 P5$													
$\overline{2}$	$\Omega$		$\overline{0}$	Level 1		50 70 0			40 200 75 0				50 230 0				32 20 45 0		29 20	
3			$\overline{0}$	Level 2		45 68 0			39 180 74 0				48 220 0				30 18 43 0		28 17	
4				Level 3		43 65 0			38 170 73 0				46 210 0				28 16 42 0		25 14	

Level $ S1 $		S <sub>2</sub>		S <sub>4</sub>	IS5	
		supplier $P1-5$ $P1,3,4,5$ $P2$ $P1,3,4,5$ $P2$ $P1-5$				
Level 1	$\Omega$	0	0	0	0	
Level $2 100$		100		50 100	20	100
Level $3 500$		500		60 500	30	500

<span id="page-9-1"></span>**Table 9.** Break point of quantity discount at level (e*cjk*)

<span id="page-9-2"></span>**Table 10.** Boundaries of each criterion  $(mn_i, md_i, mx_i)$ 



### <span id="page-9-0"></span>**4 Design of Experiment to Statistically Analyze Effects of Each Aggregation Operator**

To statistically analyze the sensitivities of optimal solutions and the advantages of aggregation operators, we generate five data sets by varying randomly the capacity, the number of supplier, the minimum order quantity, and the relationships to suppliers. In designing the experiment, independent and dependent variables are required. Models investigate how independent variables have significant effects on dependent variables. The experimental results are analyzed by MINITAB software (Table [11\)](#page-9-3).

**Independent Variable.** Four independent variables are considered in this study: (1) two sets of weights, (2) two types of demand ranges(wide and narrow demand ranges), (3) six models, and (4) two types of trade-offs (Incomplete and Complete trade-off). Incomplete trade-off means that there are some dominant

<span id="page-9-3"></span>**Table 11.** Capacity (h*jk*), Minimum order quantity  $(MOQ)$   $(o_{ik})$  and Min % of demand to be purchased  $(\%$ Demand) $(r_{ik})$ 



<span id="page-9-4"></span>**Table 12.** Optimal purchasing quantity of weighted additive technique: weight set1, complete tradeoff, narrow demand range



suppliers. For example, supplier 1 is considered as a dominant supplier if supplier 1 provides the lowest cost, the highest quality and the lowest delivery lateness. Each data set consists of 48 combinations as illustrated in Fig. [2.](#page-3-1)

**Dependent Variable.** Dependent variables are the performance indicators and are used as responses in MINITAB software. The average SL and the lowest SL are two responses in this study.

## <span id="page-10-0"></span>**5 Results and Discussion**

Results are evaluated in four aspects, namely, verification of reasonable results, average SL, lowest SL, dominated solution, and how to select the aggregation operator to match the risk preferences of DMs.

## **5.1 Reasonable Result Verification**

From Table [12,](#page-9-4) it can be seen that the model yields reasonable results as follows. For Product 4  $(P4)$ , it is supplied by 3 suppliers. Unquestionably, if there is only cost criterion, all units must be ordered from S5 due to the lowest price offered. As multiple criteria are concerned, the model is required to make tradeoffs among criteria with respect to assigned weights from DMs. As we have seen from Table [4,](#page-8-4) the quality score of S4 is greater than S5 (10:5) and the delivery lateness of S5 is less than S4 (4:5). Thus, to achieve the highest satisfaction of DMs, DMs purchase P4 at a bit higher price and gain a much better quality and a bit worse delivery lateness. In addition, as the fuzzy demand has the highest weight (32 %), DMs prefer to purchase at the amount closed to the predicted demand. Hence, the total demand of P4 in this model is exactly 700 units.

## **5.2 Level of Average Satisfaction**

By means of statistical analysis, a two-level full factorial design of experiment is applied and each insignificant factor is gradually deleted each time beginning with the highest p-value of interaction factors, until only significant factors are left. The results show that the method and demand range have significant interaction effects. Using Tukey test presented in Fig. [3](#page-11-0) and interaction plot in Fig. [4,](#page-11-1) techniques with the additive operators (Tech.1 and 4) have significantly higher average SL than those of augmented operators (Tech.3 and 6) and maximin operators (Tech.2 and 5) in both environments. Although, in Fig. [4,](#page-11-1) the demand range and method have significant interaction effect, conclusion can be concluded in the same way.



<span id="page-11-1"></span>

<span id="page-11-0"></span>**Fig. 3.** Grouping for the average SL **Fig. 4.** Interaction plot of method and demand range for the average SL

#### **5.3 Level of the Lowest Satisfaction**

The results show that an interaction between method and demand range is statistically significant. It is because the model has more ability to search for a better solution when demand range is wider. In Figs. [5](#page-11-2) and [6,](#page-11-3) the maximin aggregation operator (Tech.2) has significantly higher lowest SL than techniques based on the additive operators (Tech.1 and 4). The benefit of the maximin operator is to avoid very bad performance in any aspect. Paradoxically, although the weighted maximin technique is developed using maximin operator, it provides the lowest SL (Lowest  $SL = 0.1$ ), instead of the highest  $SL$  (Highest  $SL = 0.4$ ) as presented in Fig. [6.](#page-11-3)

```
Grouping Information Using Tukey
Method and 95.0% Confidence
Method
             \overline{N}Mean
                             Grouping
                     0.4\overline{\mathcal{L}}\epsilon\overline{A}3
              8
                     0.4\overline{A}6
                     0.4\mathbf{a}\overline{A}\mathbf{1}8
                     0.2_{\rm B}\overline{a}\mathbf{a}0.2\mathsf{C}5
              8
                     0.1\overline{D}Means that do not share a letter
are significantly different.
```
<span id="page-11-3"></span>

<span id="page-11-2"></span>**Fig. 5.** Grouping for the lowest SL **Fig. 6.** Interaction plot of method and demand range for the lowest SL

#### **5.4 Dominated Solution**

A solution is considered as a dominated solution whenever the SLs of all criteria are worse than or the same as those of other solutions. The results show that all techniques, except the weighted maximin technique, do not provide any dominated solution as shown in Table [13.](#page-12-1) We can see that every SLs of weighted maximin technique is lower than the weighted additive technique. This is because if SLs of all criteria are equal to their assigned weights, the weighted maximin technique will get the optimal solution (the sum of all  $SLs = 1.0$ ) and it has no effort to strive for a better solution. Thus, there is high chance that it will be dominated by the others since the sum of their SLs can be greater than one.

Method/criteria	$\cos t$	Quality	Delivery lateness	Demand
Weight set 2	0.38	0.28	0.11	0.23
Additive	0.99	0.6	0.18	0.57
Maximin	0.99	0.46	0.34	0.34
Augmented	0.99	0.46	0.34	0.34
Weighted additive	1	0.63	0.11	0.6
Weighted maximin	0.99	0.33	0.11	0.36
Weighted augmented	0.99	0.46	0.34	0.34

<span id="page-12-1"></span>**Table 13.** Dominated solution (weight set 2, complete trade-off, narrow demand range)

### **5.5 How to Select the Aggregation Operator to Match the Risk Preferences of DMs**

The risk-taking DM normally prefers the solution with relatively high value of average SLs of all criteria even some criteria may have very low or zero SL. The risk-taking DM will feel that scarifying a criterion for a betterment of many other criteria is worth to take a risk. In opposite, the risk-averse DM is very unhappy if a criterion has a very low or zero degree of satisfaction although many other criteria will have very high degree of satisfaction. The risk-neutral DM has a moderate opinion about risk which is somewhere between the risk-taking and risk-averse ones. This type of risk preference DM feels that the average SLs of all criteria is important but the lowest degree of satisfaction should not be too low. Based on the above mentioned characteristics of risk preference, most risk-taking DMs should prefer the additive aggregation operator while most risk-averse DMs should prefer the maximin operator. Similarly, most risk-neutral DMs will find that the augmented operator provides the most preferable solution for them.

### <span id="page-12-0"></span>**6 Concluding Remarks**

In this paper, we have proposed the realistic FMOLP models that involve volume and quantity discounts under fuzzy demand and how to select a proper aggregation operator based on risk preference of DMs. The effects of aggregation operator are statistically analyzed. The results reveal that solutions are reasonable with different sets of input parameters. The statistical results also show that the additive aggregation operator matches the preference of the risk-taking DMs since it offers relatively high average SL but a criterion may have very low SL. In opposite, the maximin aggregation operator is acceptable for the risk-averse DMs since it yields a solution with not too low degree of the lowest satisfaction. The augmented aggregation operator, which tries to combine the additive and maximin aggregation operators, provides the solution that is acceptable for the risk-neutral DMs. In addition, it also reveals that the weighted maximin technique should be applied with caution since it may generate a dominated solution.

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