Fundamentals of Risk Measurement and Aggregation for Insurance Applications

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Abstract. The fundamentals of insurance are introduced and alternatives to risk measurement are presented, illustrating how the size and likelihood of future losses may be quantified. Real data indicate that insurance companies handle many small losses, while large or extreme claims occur only very rarely. The skewness of the profit and loss probability distribution function is especially troublesome for risk quantification, but its strong asymmetry is successfully addressed with generalizations of kernel estimation. Closely connected to this approach, distortion risk measures study the expected losses of a transformation of the original data. GlueVaR risk measures are presented. The notions of subadditivity and tail-subadditivity are discussed and an overview of risk aggregation is given with some additional applications to insurance.

Keywords: Risk analysis · Extremes · Quantiles · Distortion measures

1 Introduction and Motivation

The insurance market is made up of customers that buy insurance policies and shareholders that own insurance companies. The latter are typically concerned about adverse situations and seek to maximize their profits, while the former search for the best market price, although they also need reassurance that they have opted for a solvent company.

Every insurance contract has an associated risk. Here, we analyse the caveats of measuring risk individually when we consider more than one contract and more than one customer, i.e., the aggregate risk in insurance.

Risk quantification serves as the basis for identifying the appropriate price for an insurance contract and, thus, guaranteeing the stability and financial strength of the insurance company. The aim of this article is to provide some fundamentals on how best to undertake this analysis. Once the individual risk associated with each contract has been calculated, the sum of the risk of all contracts provides an estimate of the overall risk. In this way, we also provide an overview of risk aggregation.

V. Torra et al. (Eds.): MDAI 2016, LNAI 9880, pp. 15–25, 2016.
DOI: 10.1007/978-3-319-45656-0_2

1.1 Basic Risk Quantification in Insurance

Let us consider a client who buys a car insurance policy that covers the risk of losses caused by accidents involving that vehicle for a period of one year. The insurance company needs to cover its expenses attributable to administration costs, regulatory mandates, advertising and IT systems. In other words, the company needs to fix a minimum price to cover the general expenses derived from its ordinary operations. The contract price is known as the premium.

The premiums collected can then be invested in the financial market, producing returns for the company before its financial resources are required for paying out compensation to its customers. A company that sells car insurance may sell thousands of one-year contracts but only those clients that suffer an accident, and who are covered, are compensated.

Each insurance contract has an associated profit or loss outcome, which can only be observed at the end of the contract. Two problems emerge when measuring this outcome. First, from an economic point of view, the production process of an insurance contract follows what is known as an inverted cycle, i.e., the price has to be fixed before the cost of the product is fully known. In a normal factory production process, the first step is to create and manufacture the product and, then, according to existing demand and the expenses incurred, a minimum price is fixed for the product. In the insurance sector, however, information on costs is only partial at the beginning of the contract, since accidents have yet to occur. Moreover, uncertainty exists. The eventual outcome of an insurance contract depends, first, on whether or not the policyholder suffers an accident and, second, on its severity. If an accident occurs, then the company has to compensate the insured party and this amount may be much greater than the premium initially received. Thus, the cost of any one given contract is difficult to predict and the eventual outcome may be negative for the insurer.

Despite the large financial component involved in the management of an insurance firm, insurance underwriting is based primarily on the analysis of historical statistical data and the law of large numbers. Here, recent advances in the field of data mining allow massive amounts of information to be scrutinized and, thus, they have changed the way insurance companies address the problem of fixing the correct price for an insurance contract. This price, moreover, has to be fair for each customer and, therefore, premium calculation requires a sophisticated analysis of risk. In addition, the sum of all prices has to be sufficient to cover the pool of insureds.

Insurance companies around the world are highly regulated institutions. An insurance company cannot sell its products unless they have been authorized by the corresponding supervisor. In Spain, supervision is carried out by the *Direccion General de Seguros y Fondos de Pensiones*, an official bureau that depends on the Ministry of Economics and which has adhered to European guidelines since January 2016. Under the European directive known as *Solvency II*, no company is allowed to operate in European territory unless it complies with strict legal requirements. This directive is motivated by the need to provide an overall assessment of the companys capacity to face its aggregate risk, even in the worst case scenario.

The choice of loss models and risk measures is crucial, as we shall illustrate in the sections that follow. We start by providing definitions and notations and include a simple example that illustrates the definition of losses and the risk measure. We present distortion risk measures and report key findings about their behaviour when aggregating losses. We then present a special family of distortion risk measures. The non-parametric approach to the estimation of distribution functions is discussed. An example using data from car insurance accidents is analysed and we conclude with a discussion of some possible lines of future research.

1.2 Notation

Consider a probability space and the set of all random variables defined on this space. A risk measure ρ is a mapping from the set of random variables to the real line [26].

Definition 1. ***Subadditivity.*** *A risk measure is subadditive when the aggregated risk, which is the risk of the sum of individual losses, is less than or equal to the sum of individual risks.*

Subadditvity is an appealing property when aggregating risks in order to preserve the benefits of diversification.

Value-at-Risk (VaR) has been adopted as a standard tool to assess risk and to calculate legal requirements in the insurance industry. Throughout this discussion, we assume without loss of generality that all data on costs are non-negative, so we will only consider non-negative random variables.

Definition 2. ***Value-at-Risk.*** *Value-at-Risk at level α is the α-quantile of a random variable X (which refers to a cost, a loss or the severity of an accident in our context), so*

$$\mathrm{VaR}_\alpha(X) = \inf\{x \mid F_X(x) \geq \alpha\} = F_X^{-1}(\alpha),$$

where F_X is the cumulative distribution function (cdf) of X and α is the confidence or the tolerance level $0 \leq \alpha \leq 1$.

VaR has many pitfalls in practice [23]. A major disadvantage when using VaR in the insurance context is that this risk measure does not always fulfill the subadditivity property [1,3]. So, the VaR of a sum of losses is not necessarily smaller than or equal to the sum of VaRs of individual losses. An example of such a case is presented in Sect. 5. VaR is subadditive for elliptically distributed losses [25].

Definition 3. ***Tail Value-at-Risk.*** *Tail Value-at-Risk at level α is defined as:*

$$\mathrm{TVaR}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_\lambda(X)\,d\lambda.$$

Roughly speaking, the TVaR is understood as the mathematical expectation beyond the VaR. The TVaR risk measure is subadditive and it is a coherent risk measure [18].

Since we are mainly concerned with extreme values, we consider the definition of *tail-subadditivity.* This means that we only examine the domain of the variables that lies beyond the VaR of the aggregate risk.

Definition 4. ***Tail-Subadditivity.*** *A risk measure is tail-subadditive when the aggregated risk (risk of the sum of losses) is less than or equal to the sum of individual risks, only in the domain defined by the VaR of the sum of losses.*

Additional information on the algorithm to rescale the risk measure in the tail is given below.

1.3 Exposure to Risk: A Paradox

An additional problem of measuring risk in insurance is that of exposure. The following simple example shows the importance of defining losses with respect to a certain level of exposure. For this purpose, we compare flying vs. driving.

There is typically much discussion as to whether flying is riskier than driving. In a recent paper published in *Risk Analysis* [24], a comparison of the risks of suffering a fatal accident in the air and on the highway illustrates that the construction and interpretation of risk measures is crucial when assessing risk. However, this example does not discuss the paradox that is described in [20], which argues that risk quantification also depends on how exposure is measured.

MacKenzie [24] calculates the probability of a fatal incident by dividing the total number of fatal incidents by the total number of miles travelled in the United States. He also approximates the distributions of the number of victims given a fatal incident occurs. The probabilities of a fatal incident per one million miles travelled compared to those calculated by Guillen [20] for 10,000 hours of travel (in parentheses) are 0.017 % (0.096 %) for air carriers, 22.919 % (45.838 %) for air taxis and commuters, and 1.205 % (0.843 %) for highway driving. The two approaches produce different outcomes in the probability of an accident with fatalities, because speed is not homogeneous across all transportation modes. However, regardless of whether miles travelled or hours of travel are considered, we always conclude that the safest means of transport is flying with a commercial air carrier if we look solely at the probability of an incident occurring.

However, if the expected number of fatalities per one million miles or per 10,000 hours of travel is compared, a contradiction emerges. The average number of victims per one million miles is 0.003 if we consider distance in terms of commercial aviation trips, whereas the average number of victims is 0.013 if we consider distance driven on highways. However, if we consider the time spent on the commercial aviation trip, the average is 0.017 victims compared to 0.009 when driving on highways. The conclusion we draw here is that highway trips are safer than commercial airline flights. This contradiction with respect to the previous discussion is caused by the use of the mathematical expectation of two different loss functions. This simple example shows the importance of knowing how to define the losses and the implications of the choice of the risk measure.

2 Distortion Risk Measures

Distortion risk measures were introduced by Wang [29,30] and are closely related to the distortion expectation theory [31]. A review of how risk measures can be interpreted from different perspectives is provided in [27], and a clarifying explanation of the relationship between distortion risk measures and distortion expectation theory is provided. Distortion risk measures are also studied in [4,17]. The definition of a distortion risk measure contains two key elements: first, the associated distortion function; and, second, the concept of the Choquet integral [15].

Definition 5. ***Distortion Function.*** *Let $g : [0,1] \to [0,1]$ be a function such that $g(0) = 0$, $g(1) = 1$ and g is injective and non-decreasing. Then g is called a distortion function.*

Definition 6. ***Choquet Integral.*** *The Choquet Integral with respect to a set function μ of a μ-measurable function $X : \Omega \to \overline{R}^+ \cup \{0\}$ is denoted as $\int X d\mu$ and is equal to*

$$\int X d\mu = \int_0^{+\infty} S_{\mu,X}(x)dx,$$

if $\mu(\Omega) < \infty$, where $S_{\mu,X}(x) = \mu(\{X > x\})$ denotes the survival function of X with respect to μ. See [16] for more details.

Definition 7. ***Distortion Risk Measure for Non-negative Random Variable.*** *Let g be a distortion function. Consider a non-negative random variable X and its survival function $S_X(x) = P(X > x)$. Function ρ_g defined by*

$$\rho_g(X) = \int_0^{+\infty} g(S_X(x))\,dx$$

is called a distortion risk measure.

3 GlueVaR Risk Measures

A new family of risk measures known as GlueVaR was introduced by Belles-Sampera et al. [5]. A GlueVaR risk measure is defined by a distortion function. Given confidence levels α and β, $\alpha \le \beta$, the distortion function for a GlueVaR is:

$$\kappa_{\beta,\alpha}^{h_1,h_2}(u) = \begin{cases} \dfrac{h_1}{1-\beta} \cdot u, & \text{if} \quad 0 \le u < 1-\beta \\ h_1 + \dfrac{h_2 - h_1}{\beta - \alpha} \cdot [u - (1-\beta)], & \text{if} \quad 1-\beta \le u < 1-\alpha \\ 1, & \text{if} \quad 1-\alpha \le u \le 1 \end{cases} \tag{1}$$

where $\alpha, \beta \in [0, 1]$ such that $\alpha \leq \beta$, $h_1 \in [0, 1]$ and $h_2 \in [h_1, 1]$. Parameter β is the additional confidence level besides α. The shape of the GlueVaR distortion function is determined by the distorted survival probabilities h_1 and h_2 at levels $1-\beta$ and $1-\alpha$, respectively. Parameters h_1 and h_2 are referred to as the heights of the distortion function.

The GlueVaR family has been studied by [5–7, 9], who showed that the associated distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}$ can be defined as being concave in $[0, 1]$. The concavity of the distortion risk measure is essential to guarantee tail-subadditivity.

Theorem 1. *Concave and continuous distortion risk measures are subadditive.*

Proof. A proof can be derived from [16].

Corollary 1. *If a distortion risk measure is subadditive, it is also tail-subadditive in the restricted domain.*

Theorem 2. *GlueVaR risk measures are tail-subadditive if they are concave in the interval* $[0, (1-\alpha))$.

Proof. For a GlueVaR risk measure, it suffices to check that its corresponding distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ is concave for $0 \leq u < (1-\alpha)$. Note that by definition the distortion function is also continuous in that interval. Then it suffices to restrict the domain so that the variable only takes values that are larger than the VaR of the sum of losses and apply the previous theorem. Note also that the VaR of the sum of losses is always larger or equal than the VaR of each individual loss, since we consider that all losses are non-negative. [5] also provide a proof.

Let us comment on the practical application of the above results. Given two random variables, X_1 and X_2. Let us denote by $m_\alpha = VaR_\alpha(X_1 + X_2)$. Then, we define the truncated variables $X_1|X_1 > m_\alpha$ and $X_2|X_2 > m_\alpha$. Likewise, we consider the truncated random variable $(X_1 + X_2)|(X_1 + X_2) > m_\alpha$, then tail-subadditivity holds whenever

$$\rho_g \left[(X_1 + X_2)|(X_1 + X_2) > m_\alpha\right] \leq \rho_g \left[X_1|X_1 > m_\alpha\right] + \rho_g \left[X_2|X_2 > m_\alpha\right]. \quad (2)$$

Put simply, expression (2) means that the risk of the sum of the losses of two contracts that exceed the value-at-risk of the sum is less than or equal to the sum of the risks of losses from each contract above the risk of the sum.

The algorithm to calculate the rescaled GlueVaR risk measure in the tail that we implement below in Sect. 5 is as follows. We have restricted our data set to all values greater than m_{α_0} for a given confidence level α_0. For these data, we subtract m_{α_0} from each data point and redefine the tolerance parameters, so that $\alpha = 0$ and $\beta = 1 - (1-\beta_0)/(1-\alpha_0)$, where α_0 and β_0 are the original levels of confidence. Once the GlueVaR has been calculated for this set of data and parameters, we add $\alpha_0 m_{\alpha_0}$ to return to the original scale.

4 Nonparametric Estimation of Standard Risk Measures

Let $T(\cdot)$ be a concave transformation where $Y = T(X)$ is the transformed random variable and $Y_i = T(X_i)$, $i = 1 \ldots n$ are the transformed observed losses and n is the total number of observed data. Then the kernel estimator of the transformed cumulative distribution function of variable X is:

$$\widehat{F}_Y(y) = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{y - Y_i}{b}\right) = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{T(x) - T(X_i)}{b}\right), \quad (3)$$

The transformed kernel estimation of $F_X(x)$ is:

$$\widehat{F}_X(x) = \widehat{F}_{T(X)}(T(x)).$$

where b and $K\left(\frac{x-X_i}{b}\right)$ are defined as the bandwidth and the integral of the kernel function $k(\cdot)$, respectively (see [10] for more details).

In order to obtain the transformed kernel estimate, we need to determine which transformation should be used. Several authors have analysed the transformed kernel estimation of the density function ([10, 14, 28]).

A double transformation kernel estimation method was proposed by Bolancé et al. [11]. This requires an initial transformation of the data $T(X_i) = Z_i$, where the transformed variable distribution is close to a Uniform $(0, 1)$ distribution. Afterwards, the data are transformed again using the inverse of the distribution function of a *Beta* distribution. The resulting variable, with corresponding data values $M^{-1}(Z_i) = Y_i$, after the double transformation is close to a *Beta* (see, [10, 12]) distribution, so it is quite symmetrical and the choice of the smoothing parameter can be optimized.

Following the double transformation of the original data, VaR_α is calculated with the Newton-Raphson method to solve the expression:

$$\widehat{F}_{T(X)}(T(x)) = \alpha$$

and once the result is obtained, the inverse of the transformations is applied in order to recover the original scale. The optimality properties and performance, even in small samples are studied by Alemany et al. [2].

When calculating the empirical $TVaR_\alpha$ a first moment of the data above VaR_α is used, but other numerical approximations based on the non-parametric estimate of the distribution function are also possible.

In general, a non-parametric estimation of distortion risk measures can be directly achieved in the transformed scale, which guarantees that the transformed variable is defined in a bounded domain. So, the non-parametric approach can simply be obtained by integrating the distorted estimate of the survival function of the transformed (or double transformed) variable $T(X)$, so:

$$\hat{\rho}_g(T(X)) = \sum_{i>1}^{n} g(1 - \widehat{F}_{T(X)}(T\left(X_{(i)}\right)))(T\left(X_{(i)}\right) - T\left(X_{(i-1)}\right)),$$

where subscript (i) indicates the ordered location. Once the result is obtained, the inverse of the transformations is applied in order to recover the original scale. The properties of this method have not yet been studied.

5 Example

Here we provide an example of the implementation of risk measurement and aggregation. The data have been provided by a Spanish Insurer and they contain information on two types of costs associated with the car accidents reported to the company. The first variable (X_1) is the cost of the medical expenses paid out to the insurance policy holder and the second variable (X_2) is the amount paid by the insurer corresponding to property damage. Medical expenses may contain medical costs related to a third person injured in the accident. More information on the data can be found in [13,21,22]. The sample size is 518 cases. The minimum, maximum and mean values of X_1 (in parentheses X_2) are 13 (1), 137936 (11855) and 1827.6 (283.9), respectively.

The empirical risk measures for different levels of tolerance are shown in Table 1. Risk in the tail region is shown in Table 2. The results in Table 1 confirm that VaR is not subadditive; nor is the GlueVaR example chosen here. However, tail-subadditivity holds in the tail, as shown in Table 2.

Table 1. Distortion risk measures (ρ) for car insurance cost data and subadditivity

ρ	α	$\rho(X_1)$	$\rho(X_2)$	$\rho(X_1+X_2)$	$\rho(X_1)+\rho(X_2)$	Subadditivity
VaR_α	95.0 %	6450.00	1060.00	7926.00	7510.00	No
	99.0 %	20235.00	4582.00	25409.00	24817.00	No
$TVaR_\alpha$	95.0 %	18711.78	3057.88	20886.81	21769.66	Yes
	99.0 %	48739.25	7237.02	53259.39	55976.27	Yes
$GlueVaR^*$	95.0 %	10253.39	1558.42	11996.87	11811.81	No
	99.0 %	24817.56	4988.43	29992.21	29805.99	No

*The GlueVaR parameters are $h_1 = 1/20$, $h_2 = 1/8$ and $\beta = 99.5\,\%$.

Nonparametric estimates of VaR are shown in Table 3. The results also indicate that subadditivity is not fulfilled for a level of $\alpha = 95\,\%$. Note also that compared to the empirical results, the non-parametric approximation produces higher values for larger tolerance levels because the shape of the distribution in the extremes is smoothed and extrapolated. So, in this case, subadditivity is found for $\alpha = 99\,\%$ and $\alpha = 99.5\,\%$.

Table 2. Distortion risk measures (ρ) for car insurance cost data and rescaled tail-measure

ρ	α	$\rho(X_1)$	$\rho(X_2)$	$\rho(X_1 + X_2)$	$\rho(X_1) + \rho(X_2)$	Tail-subadd.*
VaR_α	95.0 %	7603.70	18978.70	7529.7	26582,40	Yes
	99.0 %	36603.91	-	25409.00	-	-
$TVaR_\alpha$	95.0 %	20380.47	69517.70	20440.66	89898.17	Yes
	99.0 %	87142.91	-	49453.17	-	-
$GlueVaR$**	95.0 %	11740.70	27401.87	11588.64	39142.57	Yes
	99.0 %	45027.08	-	29398.90	-	-

*Only values above the corresponding $VaR_\alpha(X_1+X_2)$ are considered. For $\alpha = 99\,\%$, no values of X_2 are larger than this level.
**The GlueVaR parameters are $h_1 = 1/20$, $h_2 = 1/8$ and $\beta = 99.5\,\%$.

Table 3. Nonparametric estimates of Value-at-Risk (ρ) for car insurance cost data and subadditivity

α	$\rho(X_1)$	$\rho(X_2)$	$\rho(X_1 + X_2)$	$\rho(X_1) + \rho(X_2)$	Subadditivity
95.0 %	6357.58	1049.77	7415.80	7407.35	No
99.0 %	23316.56	4693.33	26606.16	28009.89	Yes
99.5 %	36967.12	7921.23	36968.11	44888.35	Yes

6 Conclusion

We highlight the importance of transformations in the analysis of insurance data that present many extreme values. Distortion risk measures transform the survival function to focus on extreme losses, while advanced non-parametric kernel methods benefit from the transformation of the original data to eliminate asymmetry.

Extreme value theory plays an important methodological role in risk management for the insurance, reinsurance, and finance sectors, but many challenges remain with regards how best to measure and aggregate risk in these cases. Tails of loss severity distributions are essential for pricing [19] and creating the high-excess loss layers in reinsurance.

Distortion risk measures constitute a tool for increasing the probability density in those regions where there is more information available on extreme cases. Yet, the selection of the distortion function is not subject to an optimization procedure. Regulators have imposed the use of some easy-to-calculate measures, for example, in Solvency II the central risk measure is the VaR, while in the Swiss Solvency Test, TVaR is the standard approach. Non-parametric methods for risk measurement are flexible and do not require any assumptions regarding the statistical distribution that needs to be implemented. As such, they certainly impose fewer assumptions than when using a given parametric statistical distribution. We believe that distortion risk measures could optimize an objective function that reflects attitude towards risk. The relationship between risk measures and risk attitude was initially studied by [8]. The analysis of the attitudinal position and the risk aversion shown by the risk quantifier have not been addressed here and remain matters for future study.

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