Applying ER-MCDA and BF-TOPSIS to Decide on Effectiveness of Torrent Protection

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Abstract. Experts take into account several criteria to assess the effectiveness of torrential flood protection systems. In practice, scoring each criterion is imperfect. Each system is assessed choosing a qualitative class of effectiveness among several such classes (high, medium, low, no). Evidential Reasoning for Multi-Criteria Decision-Analysis (ER-MCDA) approach can help formalize this Multi-Criteria Decision-Making (MCDM) problem but only provides a coarse ranking of all systems. The recent Belief Function-based Technique for Order Preference by Similarity to Ideal Solution (BF-TOPSIS) methods give a finer ranking but are limited to perfect scoring of criteria. Our objective is to provide a coarse and a finer ranking of systems according to their effectiveness given the imperfect scoring of criteria. Therefore we propose to couple the two methods using an intermediary decision and a quantification transformation step. Given an actual MCDM problem, we apply the ER-MCDA and its coupling with BF-TOPSIS, showing that the final fine ranking is consistent with a previous coarse ranking in this case.

Keywords: Belief functions · BF-TOPSIS · ER-MCDA · Torrent protection

1 Introduction

In mountainous areas, torrents put people and buildings at risk. Thousands of check dams, clustered in series, have been built to protect them. Risk managers must assess their effectiveness given several criteria such as their structural stability or their hydraulic dimensions. This is a Multi-Criteria Decision-Making (MCDM) problem. In practice, scoring each criterion is difficult and imperfect. Experts affect each check dam series to one of several qualitative evaluation

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classes of effectiveness (high, medium, low, no) [1]. Evidential Reasoning for Multi Criteria Decision Analysis (ER-MCDA) has been developed on the basis of fuzzy sets, possibility and belief function theories [2,3] to decide on such MCDM problems, taking into account imperfect assessment of criteria provided by several sources.

Given the final qualitative label for each check dam series, a coarse ranking of all of them can be provided, as shown in recent applications [1]. Nevertheless, risk managers need a finer ranking to choose the most effective one. To help it, the recent Belief Function-based Technique for Order Preference by Similarity to Ideal Solution (BF-TOPSIS) methods [4] are more robust to rank reversal problems than other classical decision-aid methods such as the Analytic Hierarchy Process (AHP) [5]. Nevertheless, the BF-TOPSIS methods are limited to MCDM problems with precise quantitative evaluation of criteria.

To help risk managers rank several check dam series according to their effectiveness, the BF-TOPSIS should take into account the initial imperfect assessment of criteria. Therefore, we propose to combine the ER-MCDA and BF-TOPSIS methods. We first detail the ER-MCDA process and apply it to an actual case with a final coarse ranking. We then combine ER-MCDA with BF-TOPSIS. Applying it to the same example, we finally show that the finer ranking result obtained is consistent with the previous coarse ranking result in this case.

2 Some Basics of Belief Function Theory

Shafer proposed belief function theory [6] to represent imperfect knowledge (imprecision, epistemic uncertainty, incompleteness, conflict) through a basic belief assignment (BBA), or belief mass $m(\cdot)$, given the frame of discernment (FoD) Θ. All elements $\theta_k, k = 1, \ldots, q$ of Θ are considered exhaustive and mutually exclusive. The powerset 2^Θ is the set of all subsets (focal elements) of Θ, the empty set included. Each body (or source) of evidence is characterized by a mapping $m(\cdot) : 2^\Theta \rightarrow [0, 1]$ with $m(\emptyset) = 0$, and $\sum_{X \subseteq \Theta} m(X) = 1, \forall X \neq \emptyset \in 2^\Theta$. For a categorical BBA denoted m_X, it holds that $m_X(X) = 1$ and $m_X(Y) = 0$ if $Y \subseteq \Theta \neq X$.

Given Θ, numerous more or less effective rules allow combining several BBAs. Before their combination, each BBA $m(\cdot)$ can be differently discounted by the source reliability or importance [7]. The comparison of the combination rules is not the main scope of this paper, and hereafter we use the 6th Proportional Conflict Redistribution (PCR6) fusion rule, developed within the framework of Dezert-Smarandache Theory (DSmT) [8] (vol. 3). The latter is a modification of belief function theory, designed to palliate the disadvantages of the classical Dempster fusion rule [9].

Given $m(\cdot)$, choosing a singleton $\hat{\theta} \in \Theta$ or subset $\hat{X} \subseteq \Theta$ is the decision issue. In general, it consists in choosing $\hat{\theta} = \theta_{k^\star}, k = 1, \ldots, q$ with $k^\star \triangleq \arg\max_k C(\theta_k)$, where $C(\theta_k)$ is a decision-making criterion. Among several $C(\theta_k)$, the most widely used one is the belief $\text{Bel}(\theta_k) \triangleq m(\theta_k)$ corresponding to a pessimistic attitude of the Decision-Maker (DM). On the contrary, the plausibility $Pl(\theta_k) \triangleq \sum_{X\cap\{\theta_k\}\neq\emptyset|X\in 2^\Theta} m(X)$ is used for an optimistic attitude. Between those two extreme attitudes, an attitude of compromise is represented by the decision based on the maximum probability. For this, the BBA $m(\cdot)$ is transformed into a subjective probability measure $P(\cdot)$ through a probabilistic transformation such as the pignistic one [10], the normalized plausibility transformation [11], etc.

In some cases, taking into account non-singletons $X \subseteq \Theta$ is needed to make a decision. As shown in [12], the minimum of any strict distance metric $d(m, m_X)$ between $m(\cdot)$ and the categorical BBA m_X can be used in Eq. (1). If only singletons of 2^Θ are accepted, the decision is defined by Eq. (2).

$$\hat{X} \triangleq \arg\min_{X\in 2^\Theta\setminus\{\emptyset\}} d(m, m_X) \tag{1}$$

$$\hat{\theta} \triangleq \theta_{k^\star} \triangleq \arg\min_{k=1,\ldots,q} d(m, m_{\{\theta_k\}}) \tag{2}$$

Among the few true distance metrics[1] between two BBAs $m_1(\cdot)$ and $m_2(\cdot)$, the Belief Interval-based Euclidean $d_{BI}(m_1, m_2) \in [0, 1]$ defined by Eq. (3) [13] provides reasonable results. It is based on the Wasserstein distance defined by Eq. (4) [14] with $[a_1, b_1] \triangleq BI_1(X) \triangleq [\mathrm{Bel}_1(X), \mathrm{Pl}_1(X)]$ and $[a_2, b_2] \triangleq BI_2(X) \triangleq [\mathrm{Bel}_2(X), \mathrm{Pl}_2(X)]$ for $X \subseteq \Theta$.

$$d_{BI}(m_1, m_2) \triangleq \sqrt{\frac{1}{2^{|\Theta|-1}} \cdot \sum_{X\in 2^\Theta} d_W^2(BI_1(X), BI_2(X))} \tag{3}$$

$$d_W\left([a_1, b_1], [a_2, b_2]\right) \triangleq \sqrt{\left[\frac{a_1+b_1}{2} - \frac{a_2+b_2}{2}\right]^2 + \frac{1}{3}\left[\frac{b_1-a_1}{2} - \frac{b_2-a_2}{2}\right]^2} \tag{4}$$

The quality indicator $q(\hat{X})$ defined by Eq. (5) evaluates how good the decision $\hat{X}$ is with respect to other focal elements: the higher $q(\hat{X})$ is, the more confident in its decision $\hat{X}$ the DM should be. If only singletons of 2^Θ are accepted, $q(\hat{X}) = q(\{\hat{\theta}\})$ is defined by Eq. (6).

$$q(\hat{X}) \triangleq 1 - \frac{d_{BI}(m, m_{\hat{X}})}{\sum_{X\in 2^\Theta\setminus\{\emptyset\}} d_{BI}(m, m_X)} \tag{5}$$

$$q(\{\hat{\theta}\}) \triangleq 1 - \frac{d_{BI}(m, m_{\{\hat{\theta}\}})}{\sum_{k=1}^{q} d_{BI}(m, m_{\{\theta_k\}})} \tag{6}$$

3 From ER-MCDA to Decision-Making

3.1 Multi-Criteria Decision-Making Problems

In a MCDM problem, the DM compares alternatives $A_i \in \mathcal{A} \triangleq \{A_1, A_2, \ldots, A_M\}$ through N criteria C_j, scored with different scales. Each C_j has an importance

[1] For any BBAs x, y, z defined on 2^Θ, a true distance metric $d(x, y)$ satisfies the properties of non-negativity ($d(x, y) \geq 0$), non-degeneracy ($d(x, y) = 0 \Leftrightarrow x = y$), symmetry ($d(x, y) = d(y, x)$), and triangle inequality ($d(x, y) + d(y, z) \geq d(x, z)$).

weight $w_j \in [0,1]$ assuming $\sum_{j=1}^{N} w_j = 1$. The N-vector $\mathbf{w} = [w_1, \ldots, w_N]$ represents the DM preferences between criteria. The AHP process helps extract it, comparing criteria pairwisely [5]. The DM gives an $M \times N$ score matrix $\mathbf{S} = [S_{ij}]$ in Eq. (7). S_{ij} is a score value of A_i according to the scoring scale of the criterion C_j. In practice, S_{ij} for each alternative A_i is given in hazardous situations, with no sensor and in a limited amount of time. The sources of information can therefore be imprecise, epistemically uncertain, incomplete and possibly conflicting.

Given the matrix $\mathbf{S}$,

$$\mathbf{S} \triangleq \begin{bmatrix} S_{11} & \ldots & S_{1j} & \ldots & S_{1N} \\ & & \vdots & & \\ S_{i1} & \ldots & S_{ij} & \ldots & S_{iN} \\ & & \vdots & & \\ S_{M1} & \ldots & S_{Mj} & \ldots & S_{MN} \end{bmatrix} \tag{7}$$

we consider two different decision-making assessments (DMA1 and DMA2). Given a final FoD $\Theta = \{\theta_1, \ldots, \theta_q\}$, DMA1 involves choosing a singleton $\hat{\theta}(A_i) \in \Theta$ for each alternative A_i, $i = 1, \ldots, M$. Given $\mathbf{S}$, DMA2 consists in totally ranking the M alternatives A_i and choosing the best one $A_{i^\star}$.

3.2 The ER-MCDA for the DMA1 Given Imperfect S_{ij}

- **Step1$_{\text{old}}$** ($\mathbf{M}^{\boldsymbol{\Theta}}$ construction): Given the FoD $\Theta = \{\theta_1, \ldots, \theta_q\}$ of qualitative labels, the set $\mathcal{A}$ of M alternatives, the N criteria C_j and w_j, the $M \times N$ BBA matrix $\mathbf{M}^{\boldsymbol{\Theta}} = [m_{ij}^{\Theta}(\cdot)]$ is provided in Eq. (8). For each criterion C_j, a possibility distribution π_{ij} [15] is provided by an expert through intervals F_{α_ι}, $\iota = 1, \ldots, \iota_{\max}$ with a confidence level. This represents the imprecise scoring of S_{ij} of each alternative A_i. The mapping [2] of each possibility distribution into q fuzzy sets θ_k, $k = 1, \ldots, q$ [16] provides each BBA $m_{ij}^{\Theta}(\cdot)$ on 2^{Θ} for each A_i, $i = 1, \ldots, M$ and C_j, $j = 1, \ldots, N$ in the BBA matrix M^{Θ}.

$$\mathbf{M}^{\boldsymbol{\Theta}} \triangleq \begin{bmatrix} m_{11}^{\Theta}(\cdot) & \ldots & m_{1j}^{\Theta}(\cdot) & \ldots & m_{1N}^{\Theta}(\cdot) \\ & & \vdots & & \\ m_{i1}^{\Theta}(\cdot) & \ldots & m_{ij}^{\Theta}(\cdot) & \ldots & m_{iN}^{\Theta}(\cdot) \\ & & \vdots & & \\ m_{M1}^{\Theta}(\cdot) & \ldots & m_{Mj}^{\Theta}(\cdot) & \ldots & m_{MN}^{\Theta}(\cdot) \end{bmatrix} \tag{8}$$

  The algorithm of the geometric mapping process is detailed in [2]. A BBA $m_{ij}^{X_j}(\cdot)$ is first extracted from each π_{ij}: the FoD is the scoring scale X_j of the criterion C_j; focal elements are the intervals F_{α_ι}, $\iota = 1, \ldots, \iota_{\max}$. Then each interval F_{α_ι} is mapped into each fuzzy set θ_k to obtain its geometric area $A_{\iota,k}$, with $A_\iota \triangleq \sum_{k=1}^{q} A_{\iota,k}$. A final BBA is then computed for the FoD Θ with $m_{ij}^{\Theta}(\theta_k) \triangleq \sum_{\iota=1}^{\iota_{\max}} m_{ij}^{X_j}(F_{\alpha_\iota}) \frac{A_{\iota,k}}{A_\iota}$.

- **Step2$_{\text{old}}$** (DMA1): We refer the reader to [3] for details. Each BBA $m_{ij}^{\Theta}(\cdot)$ is discounted by the importance weight w_j of each criterion C_j. For each $A_i \in \mathcal{A}$, the N BBAs $m_{ij}^{\Theta}(\cdot)$ are combined[2] with importance discounting [3] to obtain the BBA $m_i^{\Theta}(\cdot)$ for each i^{th}-row. Given that the FoD $\Theta = \{\theta_1, \ldots, \theta_k, \ldots, \theta_q\}$ and for each A_i, $\hat{\theta}(A_i) = \arg\min_{k=1,\ldots,q} d_{BI}(m_i^{\Theta}, m_{\{\theta_k\}})$ is chosen, where $m_{\{\theta_k\}}$ is the categorical BBA focused on the singleton $\{\theta_k\}$ only, based on the minimum of d_{BI} defined by Eq. (3).

Given a preference ranking of the q elements of Θ, comparing all the $\hat{\theta}(A_i)$ chosen for each A_i helps rank the A_i alternatives. Nevertheless, it is not necessarily a strict ranking since the label $\hat{\theta}(A_i)$ may be the same for several A_i.

3.3 BF-TOPSIS Methods for the DMA2 Given Precise S_{ij}

Four BF-TOPSIS methods were developed to decide on the corresponding $M \times N$ matrix $\mathbf{S} = [S_{ij}]$ (Eq. (7)), with the precise score value S_{ij}. Details are given in [4].

All BF-TOPSIS methods start with the same construction of the $M \times N$ matrix $\mathbf{M}^{\mathcal{A}} = [m_{ij}^{\mathcal{A}}(\cdot)]$ from $\mathbf{S}$ for the FoD $\mathcal{A} \triangleq \{A_1, A_2, \ldots, A_M\}$. In the sequel, $\bar{A}_i$ denotes the complement of A_i in the FoD $\mathcal{A}$. For each A_i and each C_j, the positive support $\text{Sup}_j(A_i) \triangleq \sum_{k\in\{1,\ldots,M\}|S_{kj}\leq S_{ij}} |S_{ij} - S_{kj}|$ measures how much A_i is better than other alternatives according to criterion C_j. The negative support $\text{Inf}_j(A_i) \triangleq -\sum_{k\in\{1,\ldots,M\}|S_{kj}\geq S_{ij}} |S_{ij} - S_{kj}|$ measures how much A_i is worse than other alternatives according to C_j. Given $A_{\max}^j \triangleq \max_i \text{Sup}_j(A_i)$ and $A_{\min}^j \triangleq \min_i \text{Inf}_j(A_i)$, each $m_{ij}^{\mathcal{A}}(\cdot)$ is consistently defined by the triplet $(m_{ij}^{\mathcal{A}}(A_i), m_{ij}^{\mathcal{A}}(\bar{A}_i), m_{ij}^{\mathcal{A}}(A_i \cup \bar{A}_i))$ presented on the FoD $\mathcal{A}$ by:

$$m_{ij}^{\mathcal{A}}(A_i) \triangleq \begin{cases} \frac{\text{Sup}_j(A_i)}{A_{\max}^j} & \text{if } A_{\max}^j \neq 0 \\ 0 & \text{if } A_{\max}^j = 0 \end{cases} \tag{9}$$

$$m_{ij}^{\mathcal{A}}(\bar{A}_i) \triangleq \begin{cases} \frac{\text{Inf}_j(A_i)}{A_{\min}^j} & \text{if } A_{\min}^j \neq 0 \\ 0 & \text{if } A_{\min}^j = 0 \end{cases} \tag{10}$$

$$m_{ij}^{\mathcal{A}}(A_i \cup \bar{A}_i) \triangleq m_{ij}^{\mathcal{A}}(\Theta) \triangleq 1 - (\text{Bel}_{ij}^{\mathcal{A}}(\bar{A}_i) + \text{Bel}_{ij}^{\mathcal{A}}(A_i)) \tag{11}$$

To help rank all alternatives $A_i \in \mathcal{A}$, the main idea of BF-TOPSIS methods is to compare each A_i with the best and worst ideal solutions. It is directly inspired by the technique for order preference by similarity to the ideal solution (TOPSIS) developed in [17]. The four BF-TOPSIS methods differ from each other in how they process the $M \times N$ matrix $\mathbf{M}^{\mathcal{A}}$ with an increasing complexity and robustness to rank reversal problems. In this paper, we focus on BF-TOPSIS3 (the 3rd BF-TOPSIS method using the PCR6 fusion rule) [4].

[2] with the PCR6 rule in this paper [8] (Vol. 3).

1. For each A_i, the N BBAs $m_{ij}^{\mathcal{A}}(\cdot)$ are combined to give $m_i^{\mathcal{A}}(\cdot)$ on $2^{\mathcal{A}}$, taking into account the importance factor w_j of each criterion C_j [7].
2. For each $A_i \in \mathcal{A}$, the best ideal BBA defined by $m_i^{\mathcal{A},\text{best}}(A_i) \triangleq 1$ and the worst ideal BBA defined by $m_i^{\mathcal{A},\text{worst}}(\bar{A}_i) \triangleq 1$ means that A_i is better, and worse, respectively, than all other alternatives in $\mathcal{A}$. Using Eq. (3), one computes the Belief Interval distance $d^{\text{best}}(A_i) = d_{BI}(m_i^{\mathcal{A}}, m_i^{\mathcal{A},\text{best}})$ between the computed BBA $m_i^{\mathcal{A}}(\cdot)$ and the ideal best BBA $m_i^{\mathcal{A},\text{best}}(\cdot)$. Similarly, one computes the distance $d^{\text{worst}}(A_i) = d_{BI}(m_i^{\mathcal{A}}, m_i^{\mathcal{A},\text{worst}})$ between $m_i^{\mathcal{A}}(\cdot)$ and the ideal worst BBA $m_i^{\mathcal{A},\text{worst}}(\cdot)$.
3. The relative closeness of each alternative A_i with respect to an unreal ideal best solution defined by A^{best} is given by $C(A_i, A^{\text{best}}) \triangleq \frac{d^{\text{worst}}(A_i)}{d^{\text{worst}}(A_i)+d^{\text{best}}(A_i)}$. Since $d^{\text{worst}}(A_i) \geq 0$ and $d^{\text{best}}(A_i) \geq 0$, then $C(A_i, A^{\text{best}}) \in [0, 1]$.
   If $d^{\text{best}}(A_i) = 0$, then $C(A_i, A^{\text{best}}) = 1$, meaning that alternative A_i coincides with A^{best}. On the contrary, if $d^{\text{worst}}(A_i) = 0$, then $C(A_i, A^{\text{best}}) = 0$, meaning that alternative A_i coincides with the ideal worst solution A^{worst}.
   Thus, the preference ranking of all alternatives $A_i \in \mathcal{A}$ is made according to the descending order of $C(A_i, A^{\text{best}})$.

3.4 BF-TOPSIS Coupled with ER-MCDA to Deal with Imperfect S_{ij}

To deal with the DMA2 and imperfect information, we propose to couple (mix) BF-TOPSIS with ER-MCDA according to the following steps:

- **Step1$_{\text{new}}$** = **Step1$_{\text{old}}$** ($\mathbf{M}^{\boldsymbol{\Theta}}$ construction): We use the same step 1 from ER-MCDA to obtain the matrix $\mathbf{M}^{\boldsymbol{\Theta}} = [m_{ij}^{\Theta}(\cdot)]$ defined by Eq. (8) for the FoD $\Theta = \{\theta_1, \ldots, \theta_k, \ldots, \theta_q\}$.
- **Step2$_{\text{new}}$** ($\mathbf{M}^{\mathcal{A}}$ construction): ER-MCDA is coupled with BF-TOPSIS in this step. We obtain the BBA matrix $\mathbf{M}^{\mathcal{A}} = [m_{ij}^{\mathcal{A}}(\cdot)]$ related to the FoD $\mathcal{A}$ from the BBA matrix $\mathbf{M}^{\boldsymbol{\Theta}}$ as follows:
  1. For each $m_{ij}^{\Theta}(\cdot), i = 1, \ldots, M, j = 1, \ldots, N$, restricting the decision to singletons, one chooses $\hat{\theta}(A_i, C_j)$ applying Eq. (2) with $m = m_{ij}^{\Theta}$. This gives the $M \times N$ matrix $\mathbf{S}^{\boldsymbol{\Theta}} = [\hat{\theta}(A_i, C_j)]$ with qualitative scores $\hat{\theta}(A_i, C_j)$. The corresponding quality indicator is computed by $q(\hat{\theta}(A_i, C_j))$ applying Eq. (5) with $m = m_{ij}^{\Theta}$.
  2. A quantitative transformation of each element θ_k in Θ is made to obtain the $M \times N$ matrix $\mathbf{S} = [S_{ij}]$, S_{ij} being the quantitative transformation of $\hat{\theta}(A_i, C_j)$. Several transformations are possible. We are aware that the choice of one can impact the final results. We introduce it as a general step and propose to analyze the results given different transformations in forthcoming publications.
  3. From the score matrix $\mathbf{S} = [S_{ij}]$, we use the formulas (9)-(11) to obtain the BBA matrix $\mathbf{M}^{\mathcal{A}} = [m_{ij}^{\mathcal{A}}(\cdot)]$ for $\mathcal{A} = \{A_1, A_2, \ldots, A_M\}$.

- **Step3$_{new}$** (ranking alternatives): We use $q(\hat{\theta}(A_i, C_j))$ as the reliability factor to discount each BBA $m_{ij}^{\mathcal{A}}(\cdot)$ using the Shafer discounting method [6]. For each A_i, we combine them with the PCR6 rule to obtain the BBA $m_i^{\mathcal{A}}(\cdot)$, taking into account the importance factor w_j of each criterion C_j [7]. As explained in points 2 and 3 of subsect. 3.4, the relative closeness factors $C(A_i, A^{best})$ are calculated, from which the preference ranking of all A_i is deduced.

4 Effectiveness of Torrential Check Dam Series

To reduce potential damage on at-risk housing, each torrential check dam series stabilizes the torrent's longitudinal profile to curtail sediment release from the headwaters. Their effectiveness in achieving this function depends on $N = 7$ technical criteria C_j with their importance weights w_j, as shown in Fig. 1. An expert assesses $M = 4$ check dam series A_i according to their effectiveness given an imperfect evaluation of each C_j and using ER-MCDA step 1. After this common step, ER-MCDA step 2 is used to assess (DMA1) the effectiveness of each A_i expressed by four qualitative labels (levels) in $\Theta = \{\text{high}, \text{medium}, \text{low}, \text{no}\}$ [1]. Then steps 2 and 3 of the method based on BF-TOPSIS3 developed in Sect. 3.4 are used to rank all A_i and to choose the most effective one, $A_{i^\star}$ (DMA2).

- **Step1$_{new}$** = **Step1$_{old}$** ($\mathbf{M}^{\boldsymbol{\Theta}}$ construction): The expert evaluates each criterion C_j for each A_i through possibility distributions. $N = 7$ fuzzy scales are specified, each one gathering the $q = 4$ fuzzy sets θ_k, $k = 1, \ldots, q$. The BBA matrix $\mathbf{M}^{\boldsymbol{\Theta}} = [m_{ij}^{\Theta}(\cdot)]$ obtained for $\Theta = \{\text{high}, \text{medium}, \text{low}, \text{no}\}$ is given in Table 1.
- **DMA1** (based on Step 2$_{old}$ described in Sect. 3.2): given $\mathbf{M}^{\boldsymbol{\Theta}}$ in Table 1, the column d_{BI}^{min} in Table 2 lists the minimal value obtained for $d_{BI}(m_i^{\Theta}, m_{\{\theta_k\}})$ defined by Eq. (3), for each $A_i \in \mathcal{A}$. The best label $\hat{\theta}(A_i)$ is chosen for each A_i. Three check dam series A_1, A_2, and A_4 are declared as medium, and A_3 is declared as low. The DM coarsely has $A_1 \succ A_3$, $A_2 \succ A_3$ and $A_4 \succ A_3$.
- **DMA2** (based on Step 2$_{new}$ and Step 3$_{new}$ described in Sect. 3.2): given $\mathbf{M}^{\boldsymbol{\Theta}}$ in Table 1, for each A_i and C_j, one computes $\arg\min_{k=1,\ldots,q} d_{BI}(m_{ij}^{\Theta}, m_{\{\theta_k\}})$ between each $m_{ij}^{\Theta}(\cdot)$ and the categorical BBA $m_{\{\theta_k\}}(\cdot)$, with $\Theta = \{\theta_1 = \text{high}, \theta_2 = \text{medium}, \theta_3 = \text{low}, \theta_4 = \text{no}\}$. The linear quantitative transformation: $\theta_1 = 4, \theta_2 = 3, \theta_3 = 2, \theta_4 = 1$ is assumed to establish the matrix $\mathbf{S} = [S_{ij}]$ in Table 3. For each A_i and C_j, the quality factor $q(\hat{\theta}(A_i, C_j))$ is also computed in Table 3 applying Eq. (5) with $m = m_{ij}^{\Theta}$.

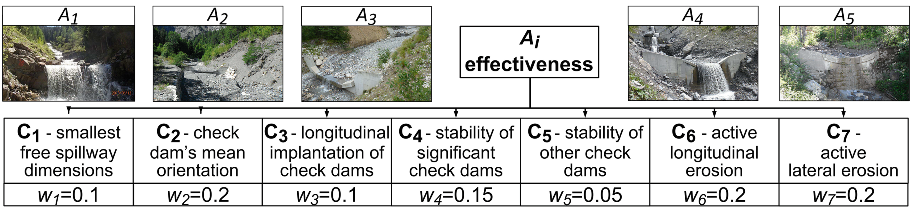

Fig. 1. Formalization of the actual MCDM problem.

Table 1. $\mathbf{M}^{\Theta}$ provided by Step 1_{new} = Step 1_{old}.

	A_i	Focal element	$m_{ij}^{\Theta}(\cdot)$						
			C_1	C_2	C_3	C_4	C_5	C_6	C_7
$\mathbf{M}^{\Theta}$	A_1	θ_1	0.2963	0.1755	0.0161	0.0000	0.0000	0.0000	0.1378
		θ_2	0.6270	0.7556	0.9107	0.0000	0.0391	0.1748	0.8083
		θ_3	0.0467	0.0389	0.0432	0.0009	0.4099	0.7786	0.0239
		θ_4	0.0000	0.0000	0.0000	0.9691	0.5210	0.0166	0.0000
		Θ	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
	A_2	θ_1	0.8446	0.0052	0.0310	0.9281	0.0693	0.6434	0.0073
		θ_2	0.1254	0.2677	0.9232	0.0419	0.3469	0.3266	0.9250
		θ_3	0.0000	0.6050	0.0158	0.0000	0.2670	0.0000	0.0377
		θ_4	0.0000	0.0921	0.0000	0.0000	0.2868	0.0000	0.0000
		Θ	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
	A_3	θ_1	0.7159	0.0019	0.6463	0.0000	0.0000	0.7154	0.0000
		θ_2	0.2541	0.1464	0.3237	0.0451	0.0338	0.2546	0.3769
		θ_3	0.0000	0.6655	0.0000	0.3786	0.2188	0.0000	0.5578
		θ_4	0.0000	0.1562	0.0000	0.5463	0.7174	0.0000	0.0353
		Θ	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
	A_4	θ_1	0.3372	0.3950	0.3849	0.0000	0.0576	0.0022	0.0000
		θ_2	0.4731	0.5676	0.2460	0.1562	0.3390	0.7030	0.5075
		θ_3	0.1597	0.0074	0.3391	0.7831	0.5147	0.2643	0.4371
		θ_4	0.0000	0.0000	0.0000	0.0307	0.0587	0.0005	0.0254
		Θ	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300

Table 2. Final results for **DMA1** based on ER-MCDA step 2_{old} from $\mathbf{M}^{\Theta}$.

A_i	$d_{BI}^{\min}$	$\hat{\theta}(A_i)$	Final class	Ranking
A_1	0.3769	θ_2	Medium	1-3
A_2	0.4837	θ_2	Medium	1-3
A_3	0.5096	θ_3	Low	4
A_4	0.3911	θ_2	Medium	1-3

Table 3. S_{ij} and $q(\hat{\theta}(A_i, C_j))$ $(= q(i,j))$ provided by Step 2_{new} from $\mathbf{M}^{\Theta}$.

C_j, w_j	$C_1, 0.1$		$C_2, 0.2$		$C_3, 0.1$		$C_4, 0.15$		$C_5, 0.05$		$C_6, 0.2$		$C_7, 0.2$	
$A_i \downarrow$	S_{i1}	$q(i,1)$	S_{i2}	$q(i,2)$	S_{i3}	$q(i,3)$	S_{i4}	$q(i,4)$	S_{i5}	$q(i,5)$	S_{i6}	$q(i,6)$	S_{i7}	$q(i,7)$
A_1	3	0.8747	3	0.9226	3	0.9754	1	0.9921	1	0.8332	2	0.9287	4	0.9404
A_2	4	0.9505	2	0.8708	3	0.9794	4	0.9797	3	0.8737	4	0.8754	3	0.9794
A_3	4	0.9029	2	0.8953	4	0.8765	1	0.8435	1	0.9078	4	0.9027	2	0.8469
A_4	3	0.8747	3	0.9226	4	0.9754	2	0.9921	2	0.8332	3	0.9287	3	0.9404

Table 4. Final results for **DMA2** based on step 3_{new} from Table 3.

A_i	$d^{best}(A_i)$	$d^{worst}(A_i)$	$C(A_i, A^{best})$	Ranking
A_1	0.5965	0.3061	0.3391	3
A_2	0.4930	0.4069	0.4521	1
A_3	0.6431	0.2683	0.2944	4
A_4	0.5033	0.4090	0.4483	2

After the reliability discounting of BBAs from Table 1 by the factors $q(\hat{\theta}(A_i, C_j))$ from Table 3, one obtains $\mathbf{M}^{\mathcal{A}} = [m_{ij}^{\mathcal{A}}(\cdot)]$ for $\mathcal{A} = \{A_1, A_2, \ldots, A_M\}$. After applying BF-TOPSIS3, we obtain the relative closeness $C(A_i, A^{best})$ values in Table 4. The ranking of all A_i according to their effectiveness is consistent with the DMA1 results: $A_2 \succ A_4 \succ A_1 \succ A_3$. The most effective check dam series is A_2.

5 Conclusion

The ER-MCDA helps provide a coarse ranking of torrential check dam series according to their effectiveness, taking into account several imperfectly scored criteria. Given the same imperfect MCDM problem, risk managers may need a finer ranking. For this purpose, we suggested coupling the ER-MCDA and BF-TOPSIS methods. We have shown the consistency of coarse and finer ranking results for only one example. Further studies are needed to determine whether such consistency holds in general or for certain classes of examples. Moreover, an intermediary decision step and a quantitative transformation are needed to meet this goal. The sensitivity of results to their definition is under evaluation and will be reported in forthcoming publications.

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