On Internal Conflict as an External Conflict of a Decomposition of Evidence

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Abstract. Conflictness is an important a priori characteristic of combining rules in the belief functions theory. A new approach to the estimation of internal conflict offered in this article. This approach is based on the idea of decomposition of the initial body of evidence on the set of bodies of evidence by means of some combining rule. Then the (external) conflict of this set of beliefs is estimated. The dependence of change of internal conflict from the choice of the combining rules is analyzed in this study.

Keywords: Internal conflict · Belief functions theory · Combining rules · Imprecision index

1 Introduction

Conflictness is an important a priori characteristic of combining rules in the belief functions theory [5,17]. Usually the conflict of two or more pieces of evidence is evaluated by a functional (measure), taking values in [0,1]. The conflict of pieces of evidence characterizes the information inconsistency given by corresponding bodies of evidence. Historically, the functional associated with Dempster's combining rule is the first conflict measure [5]. Recently the study of a conflict measure in the framework of the belief functions theory was allocated as a separate problem. So, the axiomatic of a conflict measure defined on pairs of bodies of evidence was discussed in [6,15]. An axiomatic of a conflict measure defined on arbitrary subsets of a finite set of bodies of evidence was considered in [3]. There are several approaches to the estimation of conflict of evidence. The metric approach is one of the most popular approaches [9,10,14]. A structural approach was considered in [15]. The degree of inclusion of focal elements of one evidence in the focal elements of other evidence took into account in this approach. The algebraic approach to the estimation of a conflict was discussed in [12]. In this case, the conflict measure was defined as a bilinear form satisfying a certain conditions.

Also, conflictness of single evidence is considered together with the conflict between the bodies of evidence. In the first case we talk about the external conflict, in the second case we talk about the internal conflict. For example, we have the following evidence in which a large internal conflict is observed:

J. Vejnarová and V. Kratochvíl (Eds.): BELIEF 2016, LNAI 9861, pp. 25–34, 2016.
DOI: 10.1007/978-3-319-45559-4_3

the value of the company shares will be tomorrow in the interval [0,10] or [30,35]. The internal conflict considered beginning in the early 1980s. This conflict estimated with the help of different measures: dissonance, confusion, discord, strife etc. [11]. Also the axiomatic of an internal conflict was considered in [1]. In [4] internal conflict was determined in the case of a finite set of alternatives as minimum of the belief function, which is taken over all subsets of alternatives that complement the singletons to the entire set. In [16] internal conflict was defined as a conflict among the so-called generalized simple support functions on which the original evidence decomposes uniquely.

In this paper we will consider and study another approach (but also used the idea of decomposition, as in [16]) to the definition of internal conflict. The following assumption is the basis of this approach. Evidence with a great internal conflict has been obtained as a result of aggregating information from several different sources with the help of some combining rule. Then the (external) conflict of the decomposed set of evidence can be regarded as an internal conflict of the original evidence. It is understood that the decomposition result (and hence the value of the internal conflict) is ambiguous. Therefore we can talk only about the upper and lower estimates of the internal conflict in this case. In addition, it is necessary to introduce some additional restrictions on the set of combinable evidence in order to the result is not trivial or degenerate. These restrictions are related with the character of combining rules, as will be shown below. Thus the optimization problem formulates in this paper to estimation of the internal conflict of evidence. The solution of this problem is studies for Dempster's rule and Dubois and Prade's disjunctive consensus rule. The dependence of change of internal conflict from the choice of the combining rules is analyzed in this study. The decomposition method described above discussed in detail for the case of two alternatives set of evidence.

2 Basic Concepts of the Belief Functions Theory and a Conflict Measure

Let X be a finite set and 2^X be a powerset of X. The mass function is a set function $m : 2^X \to [0,1]$ that satisfies the conditions $m(\emptyset) = 0$, $\sum_{A \subseteq X} m(A) = 1$. The value $m(A)$ characterizes the relative part of evidence that the actual alternative from X belongs to set $A \in 2^X$.

The subset $A \in 2^X$ is called a focal element, if $m(A) > 0$. Let $\mathcal{A} = A$ be a set of all focal elements of evidence. The pair $F = (\mathcal{A}, m)$ is called a body of evidence. Let $F_A = (A, 1)$ (i.e. $\mathcal{A} = A$ and $m(A) = 1$), $A \in 2^X$ and $\mathcal{F}(X)$ be a set of all bodies of evidence on X.

If we know the body of evidence $F = (\mathcal{A}, m)$ then we can estimate the degree of confidence that the true alternative of X belongs to set B with the help of belief function [17] $g : 2^X \to [0,1]$, $g(B) = \sum_{A \subseteq B} m(A)$.

The belief function corresponding to body of evidence $F_A = (A, 1)$ is called a categorical belief function and it is denoted as η_A. In particular η_X is called a vacuous belief function because the body of evidence $F_X = (X, 1)$ is totally uninformative.

Let us have two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$. For example, these bodies of evidence can be obtained from two information sources. We have a question about a conflict between these bodies of evidence. Historically, the conflict measure $K_0(F_1, F_2)$ associated with Dempster's rule [5,17] is the first among conflict measures:

$$K_0 = K_0(F_1, F_2) = \sum_{\substack{B \cap C = \emptyset, \\ B \in \mathcal{A}_1, C \in \mathcal{A}_2}} m_1(B) m_2(C). \quad (1)$$

The value $K_0(F_1, F_2)$ characterizes the amount of conflict between two sources of information described by the bodies of evidence F_1 and F_2. If $K_0 \neq 1$, then we have the following Dempster's rule for combining of two evidence:

$$m_D(A) = \frac{1}{1 - K_0} \sum_{B \cap C = A} m_1(B) m_2(C), \quad A \neq \emptyset, \quad m_D(\emptyset) = 0.$$

Below in this paper we will consider only the conflict measure (1).

Dubois and Prade's disjunctive consensus rule is a dual rule to Dempster's rule in some sense. This rule is defined by a formula [8]:

$$m_{DP}(A) = \sum_{B \cup C = A} m_1(B) m_2(C), \quad A \in 2^X. \quad (2)$$

3 Decomposition of Evidence

In general case we can assume that some evidence describing with the help of body of evidence $F = (\mathcal{A}, m)$ has a great internal conflict, if its information source is a heterogeneous. For example, information about the prognostic value of shares was obtained with the help of several different techniques. In this case we can consider that the body of evidence $F = (\mathcal{A}, m)$ is a result of combining of several bodies of evidence $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1, ..., l$ with the help of some combining rule R: $F = R(F_1, ..., F_l)$. Therefore we can estimate the internal conflict by the formula

$$K_{in}^R(F) = K(F_1, ..., F_l)$$

assuming that

$$F = R(F_1, ..., F_l),$$

where K is some fixed (external) conflict measure, R is a fixed combining rule. Since the equation $F = R(F_1, ..., F_l)$ has many solutions then we can consider the optimization problem of finding the largest $\overline{K}_{in}^R(F)$ and smallest $\underline{K}_{in}^R(F)$ conflicts:

$$\overline{K}_{in}^R(F) = \operatorname*{arg\,max}_{F = R(F_1, ..., F_l)} K(F_1, ..., F_l), \quad \underline{K}_{in}^R(F) = \operatorname*{arg\,min}_{F = R(F_1, ..., F_l)} K(F_1, ..., F_l). \quad (3)$$

Let $S_n = \{(s_i)_{i=1}^n : s_i \geq 0, i = 1, ..., n, \sum_{i=1}^n s_i = 1\}$ be a n-dimensional simplex. Let us consider some special cases of this problem.

Decomposition of Evidence with the Help of Dempster's Rule. Let $R = D$ be Dempster's Rule. Then optimization problems (3) for $l = 2$ have the following formulation. We have to find the bodies of evidence $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1, 2$, that satisfy the condition

$$K_0(F_1, F_2) = \sum_{\substack{B \cap C = \emptyset, \\ B \in \mathcal{A}_1, C \in \mathcal{A}_2}} m_1(B) m_2(C) \to \max \qquad (\min) \tag{4}$$

with constraints

$$(m_1(B))_{B \in \mathcal{A}_1} \in S_{|\mathcal{A}_1|}, \qquad (m_2(C))_{C \in \mathcal{A}_2} \in S_{|\mathcal{A}_2|}, \tag{5}$$

$$(1 - K_0(F_1, F_2))\, m(A) = \sum_{\substack{B \cap C = A, \\ B \in \mathcal{A}_1, C \in \mathcal{A}_2}} m_1(B) m_2(C), \qquad A \in \mathcal{A}. \tag{6}$$

This is a problem of quadratic programming with linear (5) and quadratic (6) restrictions. Note, that in the case of the general formulation (4)–(6) $\underline{K}_{in}^D(F) = 0$ and this value is achieved on the pair $F_1 = F$, $F_2 = F_X$. In the same time we have $\overline{K}_{in}^D(F) = 1$ and this value achieved for such $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1, 2$, that $B \cap C = \emptyset \quad \forall B \in \mathcal{A}_1, \ \forall C \in \mathcal{A}_2$. The latter being bodies of evidence are not related with the initial body of evidence F. Therefore, in general formulation the problem (4)-(6) to finding $\overline{K}_{in}^D(F)$ and $\underline{K}_{in}^D(F)$ is not meaningful.

At the same time, Dempster's rule is an optimistic rule in the following sense. If one evidence argues that the true alternative belongs to the set A, and the other – to the set B, then after combination of evidence in accordance with Dempster's rule we get that the true alternative belong to the set $A \cap B$ (see [13]). Therefore, we can require from unknown bodies of evidence $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1, 2$ that their imprecision would not be less than imprecision of initial evidence F:

$$f(F) \leq f(F_i), \qquad i = 1, 2, \tag{7}$$

where $f : \mathcal{F}(X) \to [0, 1]$ is a some imprecision index [2], for example, the generalized Hartley measure [7]:

$$f(F) = \frac{1}{\ln |X|} \sum_{A \in \mathcal{A}} m(A) \ln |A|.$$

It is known that the estimation (7) is always true for any linear imprecision index f and non-conflicting set of evidence (see [13]). Note that the conditions (7) are performed for the bodies of evidence $F_1 = F$ and $F_2 = F_X$ since $f(F_X) = 1$. Therefore we have always $\underline{K}_{in}^D(F) = 0$. Then the problem can be put to find bodies of evidence with the largest conflict (4) and satisfying the conditions (5)–(7).

In addition, the form of initial body of evidence $F = (\mathcal{A}, m) \in \mathcal{F}(X)$ and the combining rule defines a class of evidence in which we should seek solutions.

Example 1. It is necessary to estimate the internal conflict of evidence given by a belief function

$$g = m_0\eta_X + \sum_{i=1}^{n} m_i\eta_{\{x_i\}}, \quad (m_i)_{i=0}^{n} \in S_{n+1}.$$

In other words, we have the following set of focal elements $\mathcal{A} = \{\{x_1\}, \ldots, \{x_n\}, X\}$ and $m(\{x_i\}) = m_i$ for $i = 1, ..., n$, $m(X) = m_0$. Let us assume that Dempster's rule is used to combine of belief functions. In this case combinable belief functions g_1 and g_2 should have a form similar to function g:

$$g_1 = \alpha_0\eta_X + \sum_{i=1}^{n} \alpha_i\eta_{\{x_i\}}, \quad g_2 = \beta_0\eta_X + \sum_{i=1}^{n} \beta_i\eta_{\{x_i\}}.$$

Then

$$K_0(g_1, g_2) = \sum_{\substack{B\cap C=\emptyset, \\ B\in\mathcal{A}_1, C\in\mathcal{A}_2}} m_1(B)m_2(C) = \sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} \alpha_i\beta_j =$$

$$(1-\alpha_0)(1-\beta_0) - \sum_{i=1}^{n} \alpha_i\beta_i. \tag{8}$$

The conditions (5)–(6) have the following form

$$(\alpha_i)_{i=0}^{n} \in S_{n+1}, \quad (\beta_i)_{i=0}^{n} \in S_{n+1}, \tag{9}$$

$$\left(1 - (1-\alpha_0)(1-\beta_0) + \sum_{i=1}^{n} \alpha_i\beta_i\right) m_i = \alpha_i\beta_i + \alpha_i\beta_0 + \alpha_0\beta_i, \quad i = 1, ..., n, \tag{10}$$

$$\left(1 - (1-\alpha_0)(1-\beta_0) + \sum_{i=1}^{n} \alpha_i\beta_i\right) m_0 = \alpha_0\beta_0.$$

The last equation follows from (9) and (10). The condition (7) for the generalized Hatrley measure (and for any linear imprecision index [2]) has the form

$$m_0 \leq \alpha_0, \quad m_0 \leq \beta_0. \tag{11}$$

Thus, the problem of finding the largest internal conflict $\overline{K}_{in}^{D}$ has a form: it is necessary to find the largest value of the function (8) with constraints (9)–(11).

Decomposition of Evidence with the Help of Dubois and Prade's Disjunctive Consensus Rule. Let $R = DP$ be a Dubois and Prade's disjunctive consensus rule (2). Then the conditions (2) will be used instead of the conditions (6) in the problem of finding the internal conflict. In addition (see [13]), the following estimation holds for Dubois and Prade's disjunctive consensus rule and any linear imprecision index f [2]:

$$f(F) \geq f(F_i), \qquad i = 1, 2, \tag{12}$$

i.e. imprecision of evidence is not reduced after the application of this combining rule. The inequalities (12) reflect the pessimism of Dubois and Prade's disjunctive consensus rule. If the one evidence states that true alternative belongs to the set A and another evidence states that the true alternative belongs to the set B then true alternative should be belong to the set $A \cup B$ after combining of these evidence with the help of Dubois and Prade's disjunctive consensus rule.

Thus, we have a problem of finding of bodies of evidence having the largest (smallest) conflict (4) and satisfying constraints (2), (5), (12).

Note that it is convenient to consider that the empty set can also be a focal element of evidence in the case of using Dubois and Prade's disjunctive consensus rule. This can be interpreted as $x \notin X$ and a value $m(\emptyset)$ characterizes the degree of belief to the fact $x \notin X$. Then the largest value of conflict measure (4) satisfying conditions (2), (5), (12) will be equal $\overline{K}_{in}^{DP}(F) = 1$. This value is achieved for the following decomposition body of evidence F: $F_1 = F$, $F_2 = F_\emptyset$ (in this case we assume by definition that $f(F_\emptyset) = 0$ for any imprecision index f).

4 Estimates of the Internal Conflict in the Case $|X| = 2$

Decomposition with the Help of Dempster's Rule. We solve the problem of finding of measuring internal conflict for body of evidence F with the help of its decomposition by using Dempster's rule, if $X = \{x_1, x_2\}$. In this case the information is described by a belief function $g = m_0\eta_X + m_1\eta_{\{x_1\}} + m_2\eta_{\{x_2\}}$ with $\mathbf{m} = (m_i)_{i=0}^2 \in S_3$. Since $\underline{K}_{in}^D(F) = 0$, then we will find the maximum of the function (8) with constraints (9)–(11) for computing of $\overline{K}_{in}^D(F)$. We have

$$K_0(g_1, g_2) = \alpha_1\beta_2 + \alpha_2\beta_1$$

after the exclusion of variables α_0, β_0 and conditions (9)–(11) can be rewritten as

$$(1 - \alpha_i)(1 - \beta_i) = (1 - \alpha_1\beta_2 - \alpha_2\beta_1)(1 - m_i), \qquad i = 1, 2, \tag{13}$$

$$\alpha_1 + \alpha_2 \leq m_1 + m_2, \quad \beta_1 + \beta_2 \leq m_1 + m_2, \quad \alpha_i \geq 0, \ \beta_i \geq 0, \quad i = 1, 2. \tag{14}$$

Let $\Omega = \{(\alpha_1, \alpha_2) \in [0, 1]^2 : \ \alpha_1 + \alpha_2 \leq m_1 + m_2\}$. We solve the system (13) with respect to β_1, β_2. The determinant $\Delta(\alpha_1, \alpha_2)$ of this system is equal

$$\Delta(\alpha_1, \alpha_2) = (1 - \alpha_1)(1 - \alpha_2) - (1 - m_2)\alpha_1(1 - \alpha_1) - (1 - m_1)\alpha_2(1 - \alpha_2)$$

and $\Delta(\alpha_1, \alpha_2) \geq 0$ in Ω. We have $\Delta(\alpha_1, \alpha_2) > 0$, if $m_0 = 1 - m_1 - m_2 > 0$. We consider precisely this case ($m_0 > 0$). Then

$$\beta_i(\alpha_1, \alpha_2) = \tfrac{1}{\Delta(\alpha_1, \alpha_2)}(m_i - \alpha_i + \alpha_i m_{3-i} - \alpha_{3-i} m_i), \quad i = 1, 2.$$

Conditions (14) define the set

$$\Omega_0 = \{(\alpha_1, \alpha_2) \in [0,1]^2 : \ (1 - m_{3-i})\alpha_i + m_i \alpha_{3-i} \leq m_i, \ i = 1,2\} \subseteq \Omega.$$

Thus, finding the largest internal conflict $\overline{K}_{in}^D$ reduces to the solution of the problem

$$K_0 = \frac{\alpha_1 \beta_2(\alpha_1, \alpha_2) + \alpha_2 \beta_1(\alpha_1, \alpha_2)}{\Delta(\alpha_1, \alpha_2)} \to \max, \quad (\alpha_1, \alpha_2) \in \Omega_0.$$

The unique stationary point $\alpha_i^0 = 1 - \frac{\sqrt{1-m_i}}{\sqrt{1-m_1}+\sqrt{1-m_2}-\sqrt{1-m_1-m_2}}$, $i = 1, 2$, of this function is a saddle point. The solution of problem is achieved on the border $\partial\Omega_0$ and

$$\overline{K}_{in}^D = K_0(0, \tfrac{m_2}{1-m_1}) = K_0(\tfrac{m_1}{1-m_2}, 0) = \tfrac{m_1 m_2}{(1-m_1)(1-m_2)} = \tfrac{m_1 m_2}{(m_0+m_1)(m_0+m_2)}.$$

The set Ω_0 and level lines of K_0 for $m_1 = 0.4$, $m_2 = 0.3$ are shown on Fig. 1.

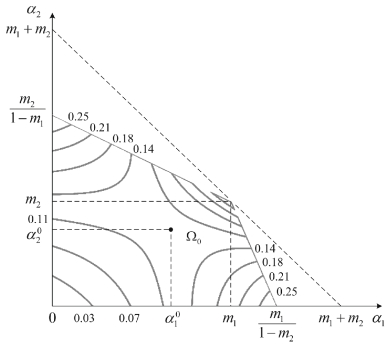

Fig. 1. The set Ω_0 and level lines of K_0 for $m_1 = 0.4$, $m_2 = 0.3$.

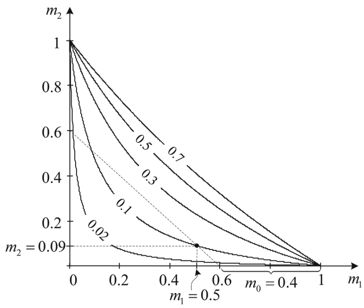

Fig. 2. Level lines of $\overline{K}_{in}^D$.

We have $\overline{K}_{in}^D \approx 1$ if $m_0 \ll \min\{m_1, m_2\}$ (see Fig. 2). In particular, the last condition is fulfilled when $m_0 \approx 0$ and $\min\{m_1, m_2\} \gg 0$, i.e. the belief function is close to probability measure but not a Dirac measure. Since $\underline{K}_{in}^D(F) = 0$, then the uncertainty of internal conflict will be maximum in this case. At that the value $\overline{K}_{in}^D$ is more when the distance $|m_1 - m_2|$ is less for one and the same value of m_0.

Conversely, we have $\overline{K}_{in}^D \approx 0$ (and hence $K_{in}^D \approx 0$), if the belief function is either close to the Dirac measure $m_1 \approx 1 \vee m_2 \approx 1$, or it is closer to the vacuous belief function η_X $(m_0 \approx 1)$.

Decomposition with the Help of Dubois and Prade's Disjunctive Consensus Rule. Now we will estimate the internal conflict in the case of $X = \{x_1, x_2\}$ in suggestion that Dubois and Prade's disjunctive consensus rule is used for decomposition of evidence and the external conflict is computed in the formula (1). The conditions (2), (9), (12) can be rewritten as

$$m_1 = \alpha_1\beta_1, \quad m_2 = \alpha_2\beta_2, \tag{15}$$

$$\alpha_1 + \alpha_2 \geq m_1 + m_2, \quad \beta_1 + \beta_2 \geq m_1 + m_2, \tag{16}$$

$$(\alpha_i)_{i=0}^2 \in S_3, \quad (\beta_i)_{i=0}^2 \in S_3 \tag{17}$$

correspondingly. We should find the minimum (maximum) of $K_0(g_1, g_2) = \alpha_1\beta_2 + \alpha_2\beta_1$ with constraints (15)–(17) for calculation of conflict measure's borders $\underline{K}_{in}^{DP}$ and $\overline{K}_{in}^{DP}$. We solve this problem assuming that $m_1 \neq 0$, $m_2 \neq 0$. Then our problem is reduced to finding minimum (maximum) of the function

$$K_0 = \frac{\alpha_1}{\alpha_2}m_2 + \frac{\alpha_2}{\alpha_1}m_1$$

in the set

$$\Omega_1(m_1, m_2) = \left\{(\alpha_1, \alpha_2) \in (0, 1]^2 : \alpha_1 + \alpha_2 \leq 1, \frac{m_1}{\alpha_1} + \frac{m_2}{\alpha_2} \leq 1\right\}.$$

The set $\Omega_1(m_1, m_2) \neq \emptyset \;\Leftrightarrow\; m_0 = 1 - m_1 - m_2 \geq 2\sqrt{m_1 m_2}$. We have

$$\underline{K}_{in}^{DP}(F) = (K_0)_{\min} = 2\sqrt{m_1 m_2}, \quad \overline{K}_{in}^{DP} = (K_0)_{\max} = m_0 = 1 - m_1 - m_2.$$

Let

$$M = \left\{(m_1, m_2) \in \mathring{S}_2 : \; \Omega_1(m_1, m_2) \neq \emptyset\right\} = \left\{(m_1, m_2) \in \mathring{S}_2 \colon \sqrt{m_1} + \sqrt{m_2} \leq 1\right\}.$$

The level lines are shown in Fig. 3 for $\underline{K} = \underline{K}_{in}^{DP}$ and $\overline{K} = \overline{K}_{in}^{DP}$ on the set M, which indicated by grey color. In particular, we have $\underline{K}_{in}^{DP}(F) \approx 0$ and $\overline{K}_{in}^{DP} \approx 1$, if $m_0 \approx 1$ ($m_1 \approx 0 \wedge m_2 \approx 0$). In this case the uncertainty of estimating conflict is maximal.

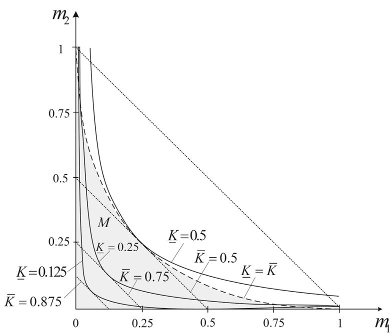

Fig. 3. Level lines of $\underline{K} = \underline{K}_{in}^{DP}$, $\overline{K} = \overline{K}_{in}^{DP}$.

If $m_0 \approx 0$, then the belief function is close to a Dirac measure and $\overline{K}_{in}^{DP} \approx 0$ in this case (and consequently, $K_{in}^{DP} \approx 0$).

The estimation of internal conflict results in a unique $\underline{K}_{in}^{DP} = \overline{K}_{in}^{DP} = 2\sqrt{m_1}\left(1 - \sqrt{m_1}\right)$, $0 < m_1 < 1$, on the curve $\sqrt{m_1} + \sqrt{m_2} = 1$, which noted by dashed line in Fig. 3. In particular, this unique value is maximal and it is equal to 0.5 for belief function $g = \frac{1}{2}\eta_X + \frac{1}{4}\eta_{\{x_1\}} + \frac{1}{4}\eta_{\{x_2\}}$.

We can make the following conclusions comparing decompositions with the help of Dempster's rule and Dubois and Prade's disjunctive consensus rule. The obtained estimations of an internal conflict are different but do not contradict each other. In addition, it is easy to show also, that $\overline{K}_{in}^{D}(m_1, m_2) < \underline{K}_{in}^{DP}(m_1, m_2)$ for all $(m_1, m_2) \in \Omega_1$. This means that the estimation of an internal conflict obtained with the help of optimistic Dempster's rule is always less than the estimation of an internal conflict obtained with the help of pessimistic Dubois and Prade's disjunctive consensus rule.

5 Conclusions

The approach to the estimation of internal conflict of evidence based on the decomposition of the body of evidence on the set of bodies of evidence with the help of some combining rule and later computing of external conflict measure of decomposed set of evidence is considered in this article. This approach is discussed in more detail for decomposition with the help of Dempster's rule and Dubois and Prade's disjunctive consensus rule. The decomposition method discussed in detail for the case of a set of evidence with two alternatives. In particular, it is shown that:

- interval estimations of internal conflict obtained with the help of decomposition by Dempster's rule and Dubois and Prade's disjunctive consensus rule do not intersect;
- in the case of decomposition by Dempster's rule, the greatest uncertainty ($0 \le K_{in}^{D} \le 1$) is achieved for the belief function close to a probability measure but not close to a Dirac measure; the value $K_{in}^{D} \approx 0$ is achieved for belief function close to a Dirac measure either it is close to the vacuous belief function;
- in the case of decomposition by Dubois and Prade's disjunctive consensus rule, the greatest uncertainty ($0 \le K_{in}^{DP} \le 1$) is achieved for a vacuous belief function $F = \eta_X$; the value $K_{in}^{DP} \approx 0$ is achieved for a Dirac measure.

Acknowledgments. The financial support from the Government of the Russian Federation within the framework of the implementation of the 5-100 Programme Roadmap of the National Research University Higher School of Economics is acknowledged. This work was supported by the grant 14-07-00189 of RFBR (Russian Foundation for Basic Research).

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