Chapter 3
2-Cycle Systems Including Some Digit Tally Systems

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Abstract During Lean's (1992) analysis of counting systems in Papua New Guinea (PNG) using Salzmann's (1950) notions of frame pattern, cycle, and operative pattern (see Chapter 1, pp. 20–21 for more details), he found that the most prominent cycle to be 2. Further analysis showed that there were a number of modified 2-cycle systems that he denoted by 2′ and 2″ as they had a non-compound word for 3 or 4 respectively. Furthermore, many of these systems had additional cycles especially 5- and 5- and 20- cycle systems. He also noted some that had 4- and 8- and 10- cycles. His map of the spatial occurrences of these types of systems is given in Figure 3.1. Further data (Mimica, 1988; C. Muke, 2000) and that obtained by Owens indicate that these systems were developed from cultural beliefs and practices to incorporate large numbers.

Keywords 1-2 counting • 2-cycle systems • Neo-2 cycle systems • (2, 5) cycle systems • (2, 5, 20) cycle systems • Digit-tally systems • Iqwaye counting systems • Large number counting in highlands of Papua New Guinea • Wahgi counting systems

The Existence and Nature of 2-Cycle Systems

Schmidt (1929), in an article on *Numeral Systems* in the 1929 edition of the *Encyclopaedia Britannica*, said:

> there is no language without some numerals; the notion of unity and plurality is expressed at least in the formation of "one" and "two", though "two" is often equal to "much", thus concluding a numeration that has only just started. (p. 614)

The type of numeration in which we have two, or perhaps three or four, numerals which are not compounded to form larger numbers and of which the largest is the limit of counting, Schmidt termed "systemless", and noted that it is doubtful that it really exists "as it is mostly reported of peoples that are vaguely known" (W. Schmidt, 1929, p. 614). Lean adopted the view, consistent with Schmidt's, that the basic criterion that must be met in order for a set of distinct numerals to qualify as a "counting system" is that there must exist some method for forming composite number words which extend the limit of counting beyond the numeral of greatest magnitude. The simplest such system is that for

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which we have the numerals "one" and "two" and all larger numerals are compounds of these so that, for example, 3 is a compound of "two" and "one", 4 is a compound of "two" and "two", and 5 is a compound of "two", "two", and "one", and so on. This type of system is known variously in the literature as the "pure 2-system" (Seidenberg, 1960, p. 219), the "pair system", (W. Schmidt, 1929, p. 614) and the "pure 2-count"(Van der Waerden & Flegg, 1975, p. 15). In Salzmann's (1950) terminology, the system has a frame pattern (1, 2), a cyclic pattern (2), and an operative pattern which may be summarised as (3=2+1, 4=2+2, 5=2+2+1, …). As we shall see subsequently, there are other systems which have a primary cycle of 2 but which have a secondary cycle as well. However in our search for the existence of "pure" 2-cycle systems we will require that the system has at least: 3=2+1, 4=2+2, and 5=2+2+1. This requirement does not of course preclude the possibility of a secondary cycle of magnitude greater than 5, e.g. 10, and we should bear this in mind if the data are insufficient to determine whether or not this is the case.

At this point it should be noted that Pica, Lemer, Izard, and Dehaene (2004) and Davis (2009), based on their studies in South America, noted that approximations were more appropriate than exact counting practices in some Indigenous cultures. So it is possible that some languages that seemed to have only 1 and 2 as numerals actually had a different approach to quantifying.

Seidenberg (1960, p. 281; 1976) listed four geographical locations where "pure 2-counting" is found: South America, Australia, South Africa and New Guinea (West Papua and Papua New Guinea). For South America, Schmidt noted that "it is found among the ethnologically oldest tribes - the Fuegian tribes" (W. Schmidt, 1929, p. 614), which are located near Tierra del Fuego at the southernmost tip of the American continent. Conant, in his book *The Number Concept* (Conant, 1896, p. 111), listed seven examples of 2-cycle systems ("binary scales") from South America, different from those given by Schmidt. Similarly, Seidenberg (1960, p. 281) provided nine examples from South America, all of which derive from Kluge (1939), and three of which are identical with those given by Conant. Between the various sources cited, a total of about 18 language groups are listed as possessing 2-cycle systems. By contrast, for the entire African continent, the same sources indicate that only the counting system of the Bushmen, located in southern Africa, possesses a 2-cycle system. Schmidl (1915, p. 195) gave an example of a system from one of the Bushmen dialects: this appears to have three numerals 1, 2, and 3, the last numeral being compounds of the first two so that 4=2+2, 5=2+(2+1), 6=2+2+2, and so on until 10 is reached.

Australian 2-Cycle Systems

In Australia, there is evidence of widespread use of the 2-cycle system. Schmidt said that it is found "among the tribes ethnologically the oldest - the Kurlin-Kurnai of the south-east, the Narrinyeri of the south; several of them count up to "ten" in this manner" (W. Schmidt, 1929, p. 614). Conant gave 35 examples of "Australian and Tasmanian number systems", each of which shows evidence of a 2-cycle (Conant, 1896, pp. 106-110). Only six of the examples, however, give data beyond the numeral 4 and thus it is impossible to establish whether the remaining 29 systems given have secondary 5-cycles or whether they are "pure" 2-cycle systems. Kluge (1938) had an extensive collection of numeral lists of Australian languages taken mainly from sources published in the nineteenth century. Of the several hundred lists only a few give data beyond the first four numerals. About 25 numeral lists clearly indicate examples of 2-cycle systems with 3=2+1, 4=2+2, and 5=2+2+1. Those lists which give only the first four numerals include about 150 which clearly indicate the existence of a 2-cycle in which we have either 3=2+1 or 4=2+2, or both. It is quite clear from the data presented in Kluge's manuscript that other types of counting systems exist in Australia; some of these will be discussed in the next chapter. It would nevertheless appear to be the case that variants of the 2-cycle system predominate in Australian language groups and that these take one of the three forms: (a) (1, 2, 2+1, 2+2), or (b) (1, 2, 3, 2+2), or (c) (1, 2, 2+1, 4). Only a small proportion of these types combine the numerals 2 and 1 to form numerals larger than 5. Both Dawson (1881, pp. xcvii-xcviii) and Kluge (1938, p. 52) gave examples of type (b) where a secondary 5-cycle operates (so that 6=5+1, 7=5+2, etc.), and Kluge also had many examples where type (a) also has a secondary 5-cycle. In summary, there are examples of language groups that possess a "pure" 2-cycle system but there is a larger number of groups that possess variants as given above, and these include systems with a (2, 5) cyclic pattern. These early references provide background for sections of the argument in Chapter 10 on the Prehistory of Number in Papua New Guinea and Oceania.

2-Cycle Systems in New Guinea

Various 2-cycle systems are found in mainland New Guinea (PNG and West Papua), the Torres Strait Islands, and the island of New Britain to the east of the mainland. Nowhere are they found in the rest of Island Melanesia, Polynesia or Micronesia. As was found in the case of the Australian languages, it is possible to distinguish several variants of the 2-cycle system. Similar variants are also possessed by New Guinea language groups and each of these will be discussed separately below. Altogether Lean distinguished five types:

(a) The "pure" 2-cycle system which has two basic numerals, 1 and 2, all other larger numerals being compounded from these two so that no secondary cycle exists,
(b) The (2, 5) or (2, 5, 20) type which is such that the first four numerals follow the 2-cycle pattern: 1, 2, 2+1, 2+2, but subsequently a secondary 5-cycle comes into play so that, for example, 6=5+1, 7=5+2, and 8=5+2+1, and so on. In some cases this sequence is continued until a tertiary 20-cycle comes into operation.

Lean characterised two variants, (c) and (d), of the 2-cycle system as "modified" or "quasi" 2-cycle systems:

(c) This type is such that we have at least three distinct numerals 1, 2, and 4; the numeral 3 is compounded as (2+1). This system usually has a secondary 5-cycle and occasionally a tertiary 20-cycle. To distinguish this variant the notation 2′-cycle is used and the cyclic patterns of the system are expressed as (2′, 5) or (2′, 5, 20).
(d) The other "quasi" 2-cycle system is such that we have at least three distinct numerals 1, 2, and 3; the numeral 4 is compounded as (2+2). This system, too, usually possesses a secondary 5-cycle and occasionally a tertiary 20-cycle as well. The type (d) variant is distinguished by the notation 2″ cycle and the cyclic patterns of the system are expressed as (2″, 5) or (2″, 5, 20).
(e) It was noted that the type (c) system, defined above, usually has a secondary 5-cycle. There is a single variant of the 2′-cycle system, however, which instead possesses a secondary 4-cycle so that the system may be represented as: 1, 2, 2+1, 4, 4+1, 4+2, 4+2+1, and so on. This variant, type (e), has a tertiary 8-cycle so that the sequence continues: 8, 8+1, 8+2, 8+2+1, 8+4, 8+4+1, and so on. The cyclic pattern of this type is expressed as (2′, 4) or (2′, 4, 8).

In establishing these categories, he is merging and crossing over Smith's (1988) classifications but it assists to present the arguments of Chapter 9 on the diffusion theory critique and Chapter 10 on the prehistory of number.

Type (a): The "Pure" 2-Cycle

It is recalled that the total number of Papuan (NAN) languages in Lean's counting systems database is 430; the total number of Austronesian (AN) languages in the database is 453, of which 226 are found in the West Papua and Papua New Guinea region. Of these totals, 42 (or less than ten percent)

Non-Austronesian (NAN) language groups possess "pure" 2-cycle systems and 2 Austronesian (AN) language groups also have them, that is less than one percent. In none of these cases is there evidence of a secondary 5-cycle; also, in most cases, it is not possible to determine whether there is a larger secondary cycle, 10 for example, as the data are usually not sufficient. Of the 42 NAN systems, 18 of these are associated with body-part tally systems which are discussed in later sections of this chapter and in Chapter 4. Several examples of this type of system, as it occurs on the New Guinea mainland, are given in Table 3.1.

Table 3.1
"Pure" 2-Cycle Systems in Three NAN Languages of New Guinea

	Southern Kiwai, Western Prov.	Giri, Madang Prov.	Gende, Madang Prov.
1	*na'u*	*ibabira*	*mapro*
2	*netowa*	*ppunini*	*oroi*
3	*netowa na'u bi*	*ppuni kagine*	*oro gu mago*
4	*netowa netowa*	*ppunini ppuninin*	*oroi oroi*
5	*netowa netowa na'u*	*ppunini ppunini kagine*	*oroi oroi mago*

Note. Southern Kiwai data are from Smith (1978), Giri from Stanhope (1972) and Gende from Kluge (1941, p. 280) and Counting System Questionnaire data.

It may be observed that none of the systems shown in Table 3.1 is precisely "regular" in that "one" in combination is not identical to the numeral "one". The pattern displayed here is quite common among those systems classified as Type (a). In some cases the numeral 3 may be translated as "two and another"; in other cases copula may be used to link the compounding numerals so that 4 may be translated as "two and two" rather than just "two two". Some systems have compound numerals which incorporate grammatical or syntactic elements. Nomad, in the Western Province of PNG (Lean's data based on SIL and IMP/SIL data in the 1970s) had:

2 *benau*
4 *benau-ili benau-ili*
6 *benau-ili benau-ili benau-ili*

where *benau* is the numeral "two" and *-ili* is the dual form of the third person plural, that is "they-2", which appears only in compound numerals.

However, 2-cycle systems have been developed innovatively into more complex systems. Sometimes speakers will refer to 6 in several ways first beginning with 2 + 2 + 1 for 5 + 1 or 2 + 1 + 2 + 1 possibly building on a three cycle. Donahue (2008) remarked that despite the restricted numerals,

> this does not mean that people are not capable of keeping careful track of precisely how much is owed to which parties in any transaction, with quantities reckoned routinely extending up to and beyond 50, indicating that the absence of verbal representation for numerals does not indicate their psychological absence (consistent with the discussion in Gelman & Gallistel, 2004). (p. 424)

It is useful to comment at this point on the fact that two AN language groups are found to possess "pure" 2-cycle systems. While it is not known whether specific counting system types were associated with the languages ancestral to the NAN languages now found in the New Guinea region, this is not the case for the Oceanic AN languages. There is good evidence that the reconstructed Proto Oceanic (POC) numerals form a 10-cycle system and indeed the putative form of these numerals derives from the many 10-cycle systems belonging to contemporary daughter languages of POC.

However the data for the AN languages Sissano, Sandaun Province, and Middle Watut, Morobe Province indicate that instead of there being at least ten distinct numerals we have in fact only two

(Table 3.2). It is apparent that, contrary to expectation, language groups which have a counting system with a given primary cycle may be influenced to change their system in the direction of a *reduction* in the magnitude of the primary cycle, in this case from 10 to 2. As we shall see, these are not two isolated cases and that throughout the New Guinea and Melanesian Islands region a large number of AN languages have counting systems which have a primary cycle of less than 10 (several examples are given in this chapter and Chapter 5).

Table 3.2
Examples of Two Austronesian Languages With 2-cycle Numeral Systems

	Sissano, Sandaun Prov.	Middle Watut, Morobe Prov.
1	*puntanen*	*morots*
2	*eltin*	*serok*
3	*eltin puntanen*	*serok a morots*
4	*eltin eltin*	*serok a serok*
5	*eltin eltin puntanen*	*serok a serok a morots*

Note. Sissano data come from early data given by M. Schmidt (1900, p. 360), Ray (1919, p. 330), Kluge (1938, p. 177) and included the village of Arop. Middle Watut data are based on Holzknecht (1989).

Type (b): With a (2, 5) or (2, 5, 20) Cyclic Pattern

This is the most common type of counting system found among the NAN language groups. Altogether 109 of these languages have systems which possess a primary 2-cycle and which have a secondary 5-cycle; for those languages for which sufficient data exist there is almost always a tertiary 20-cycle in operation as well. Included in this group are variants, similar to those mentioned in the previous section. One of these is such that the numeral 3, normally expressed as a simple combination of the numerals “two” and “one” (i.e., $2+1$ and never, incidentally, $1+2$), is expressed as “two and another” and in which the numeral “one” does not appear explicitly. Another variant which occurs in only one language group, the Fore people of the Eastern Highlands of PNG has $4=2+2$ but 3 can be interpreted as having the construction “$1+1+1$”. Examples of the Type (b) counting system from NAN languages – Sulka, East New Britain, Tairora and Fore from Eastern Highlands Province – are given in Table 3.3.

Table 3.3
Systems Which Have (2, 5, 20) Cyclic Patterns: Examples From Three NAN Languages

	Sulka	Tairora	Fore
1	*a tiang*	*vohaiqa*	*ka:*
2	*a lomin (lo)*	*taaraqanta*	*tara*
3	*kor lo tige*	*taaraqanta vohaiqa*	*kakaga*
4	*kor lo lo*	*taaraqanta taaraqanta*	*tara-wa tara-wa-ki*
5	*a ktiek*	*kauqu-ru*	*naya:ka-mu*
10	*a lo ktiek*	*kauqa-tanta*	*naya:tara-mu*
20	*a mhelum*	*vohaiqa vaiinta*	*ka:kina*

Note. Sulka sources are fairly similar e.g., Müller (1915/1916), pp. 82-3), Kluge (1941, p. 28, p. 196). Taitora and Fore data are from G. Scott’s (1978) compilation for the IMP/SIL project.

Each of these systems has a secondary 5-cycle and a tertiary 20-cycle. This type of system is, in fact, a 2-cycle system augmented by a digit-tally. The number words for 5 and 10 each contain "hand" morphemes and those for 20 contain a "man" (or "being") morpheme. This type of system often contains grammatical or syntactic elements when compound numerals are formed; the compounds, too, may contain words which are tally prescriptions or directions. The Asaro dialect of Gahuku-Asaro illustrates both of these aspects (Lean's database on Gahuku-Asaro, from IMP/SIL questionnaire) as shown in Table 3.4.

Table 3.4
Translation of Tallying Terms of the Asaro Dialect of the Gahuku-Asaro Language

	Gahuku-Asaro	Translation
1	*hamo've*	one it is
2	*sita've*	two it is
3	*sito-hamo've*	two-one it is
4	*sita've sita've*	two it is two it is
5	*ade hela osu'live*	hand-our this at finished it is
6	*ade hela osu'livo hamo've*	hand-our this at finished being one it is
10	*ade hela hela osu'live*	hand-our this at this at finished it is
20	*evene'hamo'gizene ana osu'live*	person one foot-his hand-his finished it is

Source. Data from SIL/IMP questionnaire.

It is pertinent here to provide other Gahuku-Asaro or Alekano systems provided in 2004 and 2006. When visiting different villages, there were often different versions given for this counting system indicating people would "play" with the words when counting. The data in Table 3.5 indicate some interesting results in terms of school, language, Tok Pisin, and humour.

Table 3.5
Alternative Words for Gahuku-Asaro Counting from Gavehumuto (Asaro area) and Kaveve (Gahuku area) villages.

	Gavehumuto village (2004)
1	*hamo*
2	*losi*
3	*losive makole*
4	*losive losive*
5	*ligizani luga*
6	*ligizani luga hamo*
7	*ligizani luga losi*
8	*ligizani luga losive makole*
9	*ligizani luga losive losive*
10	*ligizani luga luga or asasi hamo*
11	*asasigi hamoki*
12	*asasigi losigi*
13	*asasigi losi hamo*
14	*asasigi losive losive*
15	*asasigi ligizani luga*
16	*asasigi ligizani luga hamo*
17	*asasigi ligizani luga losi*
18	*asasigi ligizani luga losive makole*
19	*asasigi ligizani luga losive losive*
20	*asasi losi*
21	*asasi losi hamo*
100	*asasi ligizani luga luga* (stick)
200	*go'hamo* (bilum)
1000	*mulisi* (hip=heap)

(continued)

Table 3.5 (continued)

	Kaveve village (2006)		
	From Stan Aize (Kaveve)	From Zuzai Hizole (Kaveve)	From Elder
1	*hamako*	*hamo*	*amoko*
2	*logosi*	*logosita*	*lohosive*
3	*luguha (logosigi moka)*	*logidigi hamoki*	*luguhave*
4	*logosigi logosi* or *logosi*^{2} *(logosi logosi* or 2 + 2*)*	*logosivi logosive*	*lohosi lohosi*
5	*logosigi luguhagi*	*logosigi logosi hamo* or *nigizani hamo asu igo* (one hand finished)	*lohosigi lohosigi makoki*
6	*luguha logosi*	*luguha luguha*	
7	*luguha logosigi makoki (segininaga)*	*luguha luguha hamoki*	
8	*logosi*^{4} *(*means *logosi logosi logosi logosi,* really *2 + 2 + 2 + 2)*	*nigizani hamo asu o'oko makotoka logsive hamo ol o malago*	
9	*luguhagi luguhagi luguhagi*	*nigizani hamo a su o'ko logosive logosive oli'o malago*	
10	*golaha*	*nigizani logosi asu igo (nagahuni hamo)*	
11	*golohaki hamakoki*	*nigizani logosi asu oko ligisaloka hamo oli'o malago (*2 hands finished and one on the leg*)*	
12	*golohaki logosigi*	*nigizani logosi asu oko ligisa loka logosi oli'omalago (*2hands finished and two on the leg*)* or *nagahuni makoki logosita*	
13	*golohaki luguhagi*	*nagahuni makoki logosigi makoki*	
14	*golohaki logosigi logosigi*	*nagahuni makoki logosi logosi*	
15	*golohaki luguhagi logosigi*	*nagahuni makoki logosi logosi hamo or nigizani logosi asu 'olo ligisa hamo asuigo (*2 hands finished-one leg finished*)*	
16	*golohaki luguhagi logosigi*	*nagahuni hamo luguha luguha*	
17	*golohaki segini nagaki*	*nagahuni hamo luguha luguha hamoko*	
18		*nagahuni hamo, luguha luguhagi logosi* or *nigizani logosi asu'oko ligisahamo asuiko mako toka luguha oli'o'mallago (*2 hands finished, lone leg finished and 3 on the other side*)*	
19		*nagahuni logosi, hamo hakene igo* (20-1) or *nigizani logosi asu okake nigisa mako asu oko makotoka logosigi logosi oli'o'malago*	
20		*nagahuni logosi* or *nigizani logosi aso'oko nigisa logosi asu igo (*2 hands finish, 2 legs finish*)*	

Note. See comment in text on the use of the power of 2. These data were obtained during field trips by Owens in 2004 and 2006.

The symbol for power of 2 is linked to reduplication which is common in many languages of PNG as well as the lingua franca Tok Pisin. Here it refers to addition rather than multiplication which is its use in school mathematics and perhaps indicates diminishing knowledges due to interference of schooling and the local languages. It also indicates the willingness and ability to "play with numbers" with reordering them and also a sense of ownership of what they are doing with mathematics. Gavehumuto data were from a teacher fluent in her home language unlike some of the other elementary school teachers. She did not hesitate to group count in threes even though the language had cycles of 2 and 5 suggesting that this is not a difficult task in any counting system. In Kaveve, both men were educated and hard working in different careers and positions. It is also likely that the neighbouring Bena Bena language influenced various people who tried to count in their home language in this village. In general, across Eastern Highlands the rapid increase of Tok Pisin among all age groups is influencing thought, knowledge of language, and counting systems so that a group of 30 teachers from this area in 2014 were not really sure of their language's counting system and the more prominent speaker, if not the most fluent in language, provided a simple 1, 2 system repeating the words without further meaning, e.g. 5 was *losive losive hamo* and 6 was *losive losive losive*.

There are 18 Austronesian (AN) language groups that possess this type of system and 13 of these belong to the Markham Family (Holzknecht, 1989) from the Markham Valley of the Morobe Province of PNG. The others are Sera (Sandaun), Roinji (Morobe), Dawawa (Milne Bay), Igora (Milne Bay) and Tomoip (East New Britain). (Tomoip details are also given in Appendix C). However, Roinji at least is based on pairing e.g. 2+2+2+1 for 7; 10=2x5. This provides further evidence of AN languages having had a change in the cyclic nature of their counting systems from, originally, a (putative) 10-cycle to systems with (2, 5) or (2, 5, 20) cyclic patterns. Three examples – Tomoip, East New Britain, Wampar and Duwet, Markham Valley, Morobe – are given in Table 3.6.

Table 3.6
Three Examples of AN Languages With Counting Systems Having (2, 5) Cyclic Patterns

	Tomoip	Wampar	Duwet
1	*denan*	*orots*	*taginei, taine, ta*
2	*ro huru*	*serok*	*seik*
3	*horum detu*	*serok orots*	*seik mba ta*
4	*horumu horum*	*serok a serok*	*seik mba seik*
5	*ko liem*	*bangi-d ongan*	*lima-ngg alinan*
10	*liem*	*bangi-d serok*	*lima-ngg a seik,*
20	*tamdil*	*ngaeng orots*	*lima-ngg a seik a mbei-ngg seik*

Note. The Tomoip data are from Parkinson (1907, p. 780), and Kluge (1941, p. 197) while Wampar and Duwet data are from Holzknecht (1989).

As was found in the case of the NAN systems, the 2-cycle system is augmented by a (5, 20) digit-tally system. The number words for 5 and 10 in each of the examples above contain a "hand" morpheme. For Tomoip and Wampar, the word for 20 contains a "man" morpheme while that for Duwet contains both a "hand" and a "foot" morpheme.

Type (c): With a (2′, 5) or (2′, 5, 20) Cyclic Pattern

This type of system, which has a compound numeral "three" such that 3=2+1 but has a distinct numeral "four", is relatively uncommon. No AN language group has this type of system while 27 NAN groups have it. Six of those also have body-part tally systems as well (Awin, South Kati, North Kati, Yongom, and Ninggirum, situated near the border in either Western Province, PNG or West Papua) while a further 14 have systems with (2′, 5, 20) cyclic patterns. Three examples of the Type (c) system – Kol-Sui in East New Britain, Biangai in Morobe, and Au in Sandaun – are given in Table 3.7. (See Lean's appendices for other data, also in Lean (1992)).

Table 3.7
Examples of the Type (c) 2′-Cycle Numeral System

	Kol-Sui	Biangai	Au
1	*'pusup*	*wame-nak*	*kiutip*
2	*te'tepe*	*na-yau*	*wiketeres*
3	*tetepe kosup*	*nayau keya nak*	*wikak*
4	*ke'a so*	*mango-bek-tau ono*	*tekyait*
5	*'a:meleng*	*mele-na-zik*	*his pinak*
10	*melem'be:ga*	*mele yau*	*his wien*

Note. Kol-Sui data are from SIL word list, 1981; Biangai is from IMP/SIL project (1978) and SIL word list (1975); Au from IMP/SIL project (1978).

Members of the (2′, 5, 20) group are largely located in the Oro Province and the southern part of the Morobe Province. It is possible that one language group, Biangai, may have a numeral 4 which has the meaning “one less than five” and of which we find several examples in the next type of system.

Type (d): With a (2″, 5, 20) Cyclic Pattern

This system, with a distinct numeral “three” but with the numeral “four” compounded such that 4=2+2, is not uncommon. Some 12 examples are found among the AN languages of PNG, five of these (Vehes, Mapos Buang, Manga Buang, Mumeng, and Piu) are members of the Buang Family, located in the Morobe Province (details are given in Appendix C), and a further 4, perhaps 5 (Anuki, Paiwa, Boianaki, Wedau, and, possibly, Taupota), are located in the Milne Bay Province. Among the NAN languages there are 40 examples of this system; three of these may not have a secondary 5-cycle and are associated with body-part tallying methods (Bine, Gidra in Western Province and Oksapmin in the south of Sandaun Province have 2″-cycle system). One AN and two NAN examples are given in Table 3.8.

Table 3.8
Examples of the Type (d) 2″-Cycle Numeral System

	Mumeng (AN)	Kwanga (NAN)	Usarufa (NAN)
1	*ti*	*findara*	*morama*
2	*yuu*	*frisi*	*kaayaqa*
3	*yon*	*lamor*	*kaomomo*
4	*yuu di yuu*	*frisi frisi*	*kaayate kaayate(qa)*
5	*vige vilu*	*tabanangki*	*mora tiyaapaqa*

Note. Mumeng (Morobe Province) data are from IMP/SIL project (1978); Kwanga (East Sepik Province) is from SIL word list (1960); Usarufa (Eastern Highlands Province) data are from SIL word list (1968). It should be noted that some data (IMP/SIL) from Mumeng suggest a 20-cycle but the older data available are used here.

Type (e): With a (2′, 4, 8) or (2′, 4, 8, 10) Cyclic Pattern

There is a rare example of this type of system and this occurs in the Melpa dialect of the Hagen (NAN) language. The structure of the system is: 1, 2, 2+1, 4, 4+1, 4+2, 4+2+1, 8, 8+1, (10), … There is an argument for including this system under the 4-cycle classification as the putative 2+1 construction for the numeral may not in fact be valid and there may be a distinct, uncompounded numeral 3. There are two representations for the numeral 10: one has the meaning "hands one", that is "the hands of one man"; the other representation is as the compound 8 + 2 as given in Table 3.9.

Table 3.9
Numerals of the Melpa Dialect of the Hagen Language

	Hagen (Melpa dialect)
1	*tenta*
2	*ralg*
3	*raltika*
4	*timbakaka*
5	*timbakaka pamb ti*
6	*timbakaka pamb ralg*
7	*timbakakagul raltika*
8	*engaka*
9	*engaka pamb ti*
10	*engaka pamb ralg pip,* **or**, *ki tenta,* **or**, *ki ti*

Note. Data are from IMP/SIL project in 1978 but are not unlike the Vicedom and Tischner (1948) data.

Lean provided other alternatives and gave numbers well beyond 10 indicating the 8 cycle system but this will be discussed in Chapter 7 and Appendix B.

Summary of 2-Cycle Data

If all the 2-cycle variants are combined we have a total of 218 NAN languages which have counting systems belonging to this category, just over half the total sample (430) of NAN languages. The 218 languages are distributed mainly among the major phyla, the Sepik-Ramu, the Torricelli, and the Trans New Guinea, as given in Table 3.10. Thus more than half (58%) of the total sample of the Trans New Guinea Phylum languages possess a 2-cycle system variant, as do 92% of the Torricelli Phylum sample and 36% of the Sepik-Ramu Phylum sample.

Table 3.10
The Distribution of 2-Cycle Variants Among Three of the Major Phyla of the NAN Languages

Type	Sepik-Ramu	Torricelli	Trans New Guinea
a	3	0	39
b	5	16	86
c	5	3	17
d	3	5	31
Totals	16	24	173
Total Sample	44	26	298

Altogether there are 32 AN languages which have a 2-cycle variant. All of these are Oceanic and are found in PNG. Twenty-four of these 32 languages belong to Ross's North New Guinea Cluster, 7 belong to the Papuan Tip Cluster, and 1 belongs to the Meso-Melanesian Cluster (see Appendix E, based on Ross, Pawley, and Osmond (2003)). We should note that only two of the AN systems appear to be "pure" 2-cycle systems while the remaining 30 all have a secondary 5-cycle, and in many cases a tertiary 20-cycle as well.

Figure 3.1 shows the distribution of the 2-cycle variants (a) to (d) in the New Guinea region for both the NAN and the AN languages. The distribution is widespread throughout the region and is found in both the coastal and inland highland areas. Type (e) is discussed in Chapter 7.

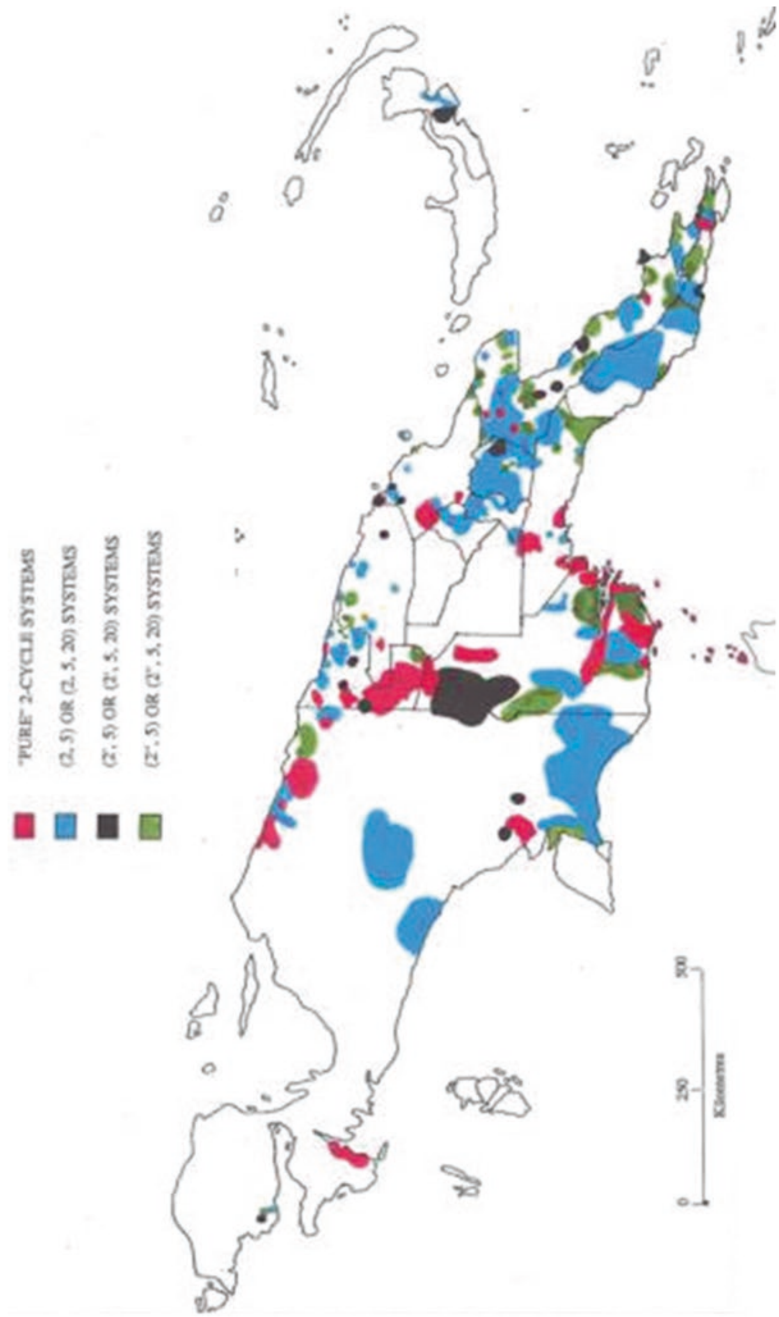

Figure 3.1. Distribution of 2-cycle variants in New Guinea.
Source. Lean's (1992) thesis.

Digit Tally Systems – Embedded in Culture and Extending Mathematical Thinking

However, there is more to say about digit tally systems as pointed out by Mimica (1988) for a Yagwoia language, Iqwaye. Yagwoia, a member of the Angan Stock-Level Family, is spoken in two separated mountainous areas in the extreme south-east of the Eastern Highlands Province and in the south-west of the Morobe Province in the Menyamya area. Wurm and Hattori (1981, Map 8) gave the total resident population of Yagwoia-speakers as a little over 6 000, based on statistics compiled in the early 1970s but Mimica gave the total of 9 000 in 1988 with the Iqwaye numbering about 2 000. Smith (1984, p. 264) had some Yagwoia number data in his study of the counting systems of the Morobe Province. Mimica (1988) had data on the Iqwaye (Ikwaye) counting system. The Yagwoia system is a digit-tally one with a (2, 5, 20) cyclic pattern. Mimica's description of the Iqwaye system (Lean, 1992, citing an earlier unpublished paper from Mimica) was that:

> Counting on digits is commonly done in such a way that a person starts with the thumb of either open hand, uttering the numerals simultaneously as he (she) does so and proceeds systematically to all the fingers of one hand (1-5), then through all the fingers of the other (6-10). Following this, the person then carries on by counting the toes of firstly one foot (11-15), then of the next (16-20). (Mimica, 1984, p. 9, cited in Lean, 1992)

The tally of 20 having been reached, the tally may continue to higher numbers by counting on the fingers and toes of another person. Mimica emphasised that:

> it does not follow that for a person, in order to be able to count or to express in numbers above 20, there should necessarily be the corresponding number of digits and persons … in principle no more than those of one person's, and for practical reasons, no more than those of two are required (Mimica, 1984, p. 10, cited in Lean, 1992).

Mimica indicated that the number word for 5 means "hand" (an arm morpheme but closely linked to gestures of counting involving fingers and the hand) and that for 10 means "two hands". The number word for 15 contains a "leg" morpheme *hyule* and has the gloss "half of the legs, all (digits)". The number word for 20 may be expressed either as "two hands two legs" or as "one person". Data from Lean's study are given in Table 3.11.

Table 3.11
(2, 5, 20) Cycle System of Iqwaye on the Eastern Highlands and Morobe Border

	Iqwaye Counting Words	Yagwoia
1	*ungwonangi*	*'kwananoi*
2	*huwlaqu*	*u'laako*
3	*huwlaqungwa* or *huwlaqanga*	*u'laangwa*
4	*hyaqu-hyaqu*	*'yaako 'yaako*
5	*hwolyempu*	*walyampu*
6	*hwolye indeumoni ungwonangi*	
7	*hwolye indeumoni huwlaqu*	
8	*hwolye indeumoni huwlaqungwa*	
9	*hwolye indeumoni hyaqu-hyaqu*	
10	*hwolye kaplaqu*	*'walya 'mplaako*
11	*hyule yengwonye ungwonangi*	
15	*hyule umance hyelaq*	*hyula walyampa*
20	*hwolye kaplaqu hyule kaplaqu* or *amnye ungwonangi*	*apni*
30	*anmye ungwonangi amnye ungwoli amnye ineumoni hwolye kaplaqu*	
40	*[amnye hyule hwolye hyepu], amnye huwlaqu*	
100	*amnye hwolyempu kokoleoule hwolye hyelaqapu*	

Note. Iqwaye data were provided by Mimica (1988, pp. 32-33), while the Yagwoia were supplied by Smith (1984, p. 264)

The deliberate use of digits when counting indicates that the fingers and toes are important representations of number that links counting to their whole bodily existence and the relationship with others when using the fingers of another. Even the numbers can incorporate this deictic of "this" and "that" or "mine" and "yours" or "another". Nevertheless, this close link with the physical body parts does not prevent the Iqwaye of thinking of numbers more abstractly. While the first two and a second twenty are useful for counting purposes, the Iqwaye are also able to use the set of fingers and toes to denote not only ones but each group of 20 as they are being counted and even to a third level, thus making it possible for the Iqwaye to think of large numbers (powers of 20) appropriately.

While the Iqwaye generally do not count large numbers except of cowrie shells, they are able to do it and explain their system. They also make use of cowrie sticks of 5 shells *ungye hilyce* so when counting these, for example, 3 or 7 *ungye hilyce,* they are referring to 15 or 35 individual shells. A rope of shells may also be used for comparison. This is important in bride price exchanges which are generally not accepted on the first offer. One-to-one matching of ropes and shells also occurs for other occasions such as provision of warriors, the rope representing a sufficient number of warriors for combat. This sense of wholeness of a rope and a sense of the body, namely the digits, as embodiments of the counting scheme also provide relationships between numbers that are significant for different purposes. Quantifying is not necessarily the end point but the decision required for which counting or its equivalent objects might assist. The objects and digit assist with sense of number size and comparison. Thus "the background of number use [is evident in] … the Iqwaye perception of quantities in their appearances as deemed equivalent and different, substitutable, exclusive or commensurate" (Mimica, 1988, p. 18). Thus there is a flow of objects—money, food and other things—and the actual object (shell or crisp banknote) is valued. This qualitative structuring of substitutes and commensurability is evident too in the classifications of marriage arrangements (Mimica, 1988).

The body digits are important from birth and are involved in rhymes with each digit of the infant being associated with a line of verse and an action on an edible animal. The numerical series for which the body digits can be used provides for both abstraction of number and for large numbers. Thus the one digit may represent one, 20, 20×20, or $20 \times 20 \times 20$ depending on the size of numbers under consideration. The Iqwaye thus express 400 without a numeral as

Aa' 'mnye, aa'mnye, toqwotni tepu hyelaqa kokoloule hyule hwolye hyelaqapu

Person person this-me this that-all their leg hand that-all

'[as] this many persons [as] me this [one] person [speaker] all their legs and hands'

… 1000 …. 'two persons [as] me this [one] person all their legs and hands and to another person's two hands (= ten persons) all their legs and hands' (Mimica, 1988, pp. 35-36)

The lingual form of numbers and the system is deictic. The lexemes *indeumoni* (to the next) is a horizontal shift on hands and feet; *ineumoni* (to the side of) is a shift to the next person; and *yengwonyemoni* (down-to) for a vertical shift from the hands to feet occur in the appropriate numbers. The word for two contains the morpheme *–aqu* (two) which occurs in other words such as the third person dual reference while *hyaqu-hyaqu* (four) also contains the demonstrative *hye* (that) (Mimica, 1988, p. 46). Furthermore, the pattern of numbers to 20 is divided into four segments of five related to the natural body segments but the numbers are in a specific series of one, two, two-one, two-two, hand and five is conceived not as the addition of 5 ones but as 2 twos and 1 giving 5 as an indivisible whole but also a marker of a pattern (Mimica, 1988, pp. 48-52). According to Mimica (1988), this system based on one and two, the links (e.g. in gestures of the hands) to joining together of two for procreation, the myth of the bow and arrow relationship, and the generating of 20 parts for a whole give the numeric system a mythopoeic perspective.

However, the body crouched with fingers and toes touching may be a macro-view of the world and culture, a part of which is the person with four sets of five digits that generates self-similarity of 20s. With the digits then being able to represent 20 persons, then 20 x 20 then 20 x 20 x 20 for an infinite appreciation of abstract number. It seems that out of the binary system of 1 and 2, the sense of self-generation over time resulted in the relationships that could be represented by powers of 20. It seems that a cosmic view linked to numbers can be associated with generating large numbers. It is not recorded for related Angan languages of the same area and is not necessarily present in other (2, 5, 20) digit tally systems as far as current data can provide (Mimica, 1988).

While Mimica recorded the general size equivalents often given for large numbers, he also noted the importance to the Iqwaye of precise money exchanges. The importance of this comparison is in placing attitudes to numbers and their values within a social context where numbers are valued for their relationships to culture rather than of themselves. This view of number in a cultural activity and worldview then provides a new perspective on number.

This perspective of generating numbers is also known in the Western Province where the act of doing many times or by many people creates the sense of "many" or large in a numeric sense. This is in contrast to "few" for small numbers of objects or times (personal communication with SIL linguist, 2013). The next section provides another example of representing large numbers in a (2, 5) cycle system.

A (2, 5, 20) Cycle System With Large Numbers

Muke's Study of Mid-Wahgi (Yu Wooi) Counting in a Cultural Context

Charly Muke (2000) used an ethnographic research design to explore how easily patterns of mathematical practices are recognised in a culture. He mainly used a two-page questionnaire gathering quantitative data to support the claims of recognition as well as a conversational interview over several nights with his father, an Elder and keeper of knowledge. He used data from 30 teachers from the Yu Wooi language and neighbouring groups (Kumai on the Simbu border, Yu Nimbang variant of mid-Wahgi and North Wahgi speakers). Seven different tribes were represented in the Yu Wooi group and four different tribes were included in the North Wahgi language group. There were 40 school students from eight Yu Wooi tribal groups and two from Yu Nimbang dialect. Two non-teaching members of the cultural group also completed the questionnaire. He observed people in their own territory and interacted with them in their own language and on their terms in order to study the knowledge of counting subsumed within a cultural context. Yu Wooi means real talk and constitutes accurate talk compared to parables, folded talk, jokes, exaggerated talk, swear talk, war talk, magical utterances, stories and legends, oratory talk, and lies as identified by J Muke (1993).

His father gave words of the ancestors: *endi, tak, takendeka, taksi taksi and angek yemdo* (1, 2, 3, 4, 5 respectively). The majority of teachers gave the same words (95%, 100%, 75%, 61%, 50% respectively) with *angek yemto* also given for 5 in a further 36% of cases. After 5, there was mainly 38% agreement except for 7 which dropped to 29% while 41% agreed 10 was *angek yem yem*. As the numbers increased, the number of variations tended to increase between 6 and 9. This variation reflects much of the work of Lean especially from his counting system questionnaire. Some variation is just in pronunciation or the use of additional or different phonemes or morphemes. The students all provided the same words for 1, 2 and 4 (*endi, tak*, and *taksi taksi)* with half of them giving 3 as *takendek* while nearly all the rest gave *takendika.*

Table 3.12 provides the most commonly used words. At least one variation used the names of fingers for adding onto the hand almost as a vestige of a body-part tally system. Furthermore, when it came to counting and having the opportunity to use hands, people often counted in twos. They would

Table 3.12
Yu Wooi (Mid-Wahgi) Number Words

Numeral	Yu Wooi	Comment
¼	*kekep*	quarter of an animal for food
½	*arhka*	half an animal for food
½	*yem*	half of other things
1	*endi*	
2	*tak*	
3	*takendeka*	2 and 1
4	*taksi taksi*	2 and 2
5	*angek yem*	hands half
6	*angek yemsi endi*	hands half one
7	*angek yemsi tak*	hands half two
8	*angek yemsi takendeka*	hands half three
9	*angek yemsi taksi taksi*	hands half two two (four)
10	*angek yem yem*	hands half half
11	*angek yem yem endi*	hands half half one
15	*angek yem yem simb yem*	hands half half leg half
20	*simb angek yem yem*	legs hands half half
27	*angek yem yemsi simb yem yemsi, angek yemsi, tak*	both hands + both legs + half hand + 2
40	*hi ende simb angek*	2^{nd} man's legs and hands
80	*ala hi ende simb angek*	4^{th} man's legs and hands
100	*ala hi ende simb angek*	5^{th} man's legs and hands
250	*simb angek mongum pel angek yem yem kunum taksi angek yem yem*	20 x 10 + 2 x 20 + 10 hands and legs (20)

Note. The most commonly supplied words are used except for 250 for which the non-English was chosen. The data are from C. Muke (2000, pp. 199, 118-119)

fold down two fingers at a time saying *eraksi* meaning "take 2". This was accompanied by 2 then 2 on one hand followed by 2 and 2 on the other and bringing the four folded fingers of each hand together being *mam erak* followed by the 2 thumbs with the words *angek yem yem* being said (some also folded 2, 2, then thumb *eraksi eraksi el* and then 2 and 2 and thumb repeated before bringing together and saying *angek yem yem*). This use of 4 and 4 and then 2 to make 10 is reminiscent of the Hagen language as described in Chapter 7 in more detail and mentioned above. The counters in pairs would also continue with the toes.

Variations were analysed by Muke (2000) to fall into a number of categories:

(a) the use of the suffix –*si* for numbers from 6-9,
(b) use of the suffix –*to*,
(c) pronunciation especially those influenced by North-Wahgi dialect,
(d) shortening of the word for three especially for the number eight,
(e) some additional words used in phrases such as "and" and "you will do it" signaling that two numbers were used to form compound numbers,
(f) variations on 10 being "hand half half" because of suffixes, pronunciation, repeat use of the word "hand" before the second "half", or additional words like "both of them".
(g) However, in some cases, the word for "fingers" *mongum* was used in line with Lean's (1992) records.

Large Numbers

Like other counting systems as illustrated for the Iqwaye, Muke showed the idea of whole person tally. Some people chose to count items in hundreds by grouping them in tens and then tally each ten using their fingers and toes by twos. Thus by using fingers and toes they reached 200 and would then use another person's fingers and toes. One participant said "for six hundred pigs, they would say that they will kill pigs equal to the hands and legs of three man" (C. Muke, 2000, p. 134). Table 3.13 indicates how tallying occurs.

Table 3.13
Tallying of Quantities in Yu Wooi

Numeral	*Yu Wooi*	Explanation
2	*eraksi*	1st two (left hand first)
4	*eraksi*	2nd two
6	*eraksi*	3rd two
8	*eraksi*	4th two
10	*eraksi* or *angek yem yemsi elsi*	5th two or 1st ten (fingers are folded)
20	*elsi*	this one, 2nd ten
30	*elsi*	3rd ten
…	*elsi* for each ten	
100	*elsi* or *angek yem yemsi peng ngond*	10th ten or head (ordered from head to leg)
100	*peng*	head – 1st 100
200	*komuk*	ear – 2nd 100
300	*gnumb*	nose – 3rd 100
400	*gupe*	mouth – 4th 100
500	*angek woiro*	right hand – 5th 100
600	*angek daro*	left hand – 6th 100
700	*buk*	back – 7th 100
800	*kumbuk*	belly – 8th 100
900	*simb woiro*	right leg – 9th 100
1000	*simb daro* or *hi ende simb angek poro bekenj*	left leg – 10th 100 (whole body parts of one person)
2000	*hi tak*	two persons
3000	*hi takendeka*	three persons

Note. Data are from C. Muke (2000, p. 199).

For the larger thousands, people used the fingers and toes for groups of ten and when they had ten groups of ten, they gave each on a specific body part, starting from the head towards the legs. Muke's father said "the body parts were used as representation that helped us remember how many pigs we were asked to pay or spread the news of the event accurately" (C. Muke, 2000, p. 135). This is illustrated in Figure 3.2. When they had tallied 1000 items in this way they would say *hi end simb angek begenj* (I have tallied all body parts of a person) and so for two or three thousand they would use two or three persons and so on.

When deciding the number of pigs to be given by each person in a compensation claim, the leader asked people to take the number they would give from a bunch of small banana fruit. When everyone had offered as they wished, then the banana fruit were put together and tallied in groups of 10, each group matched with a digit tally parts starting with the fingers. In another ceremony a parcel of sweet potato would equal a quarter of a pig to be received in the big pig kill. In yet another ceremony, pigs were lined up one-to-one matching of pigs from each clan in two parallel rows.

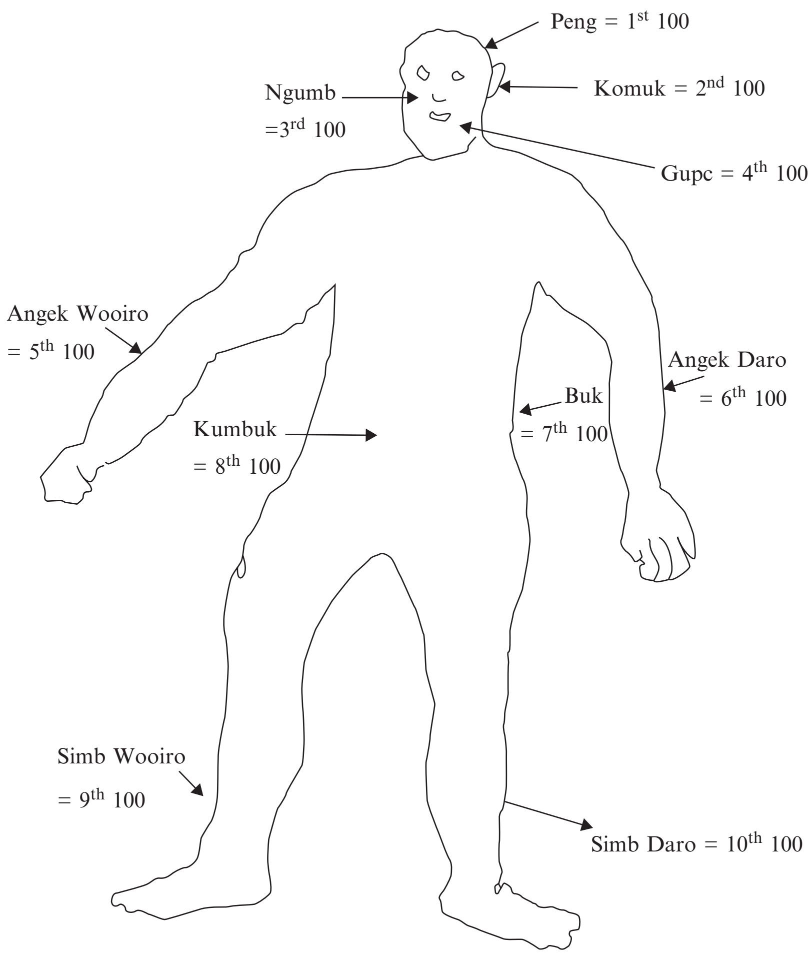

Figure 3.2. *Yu Wooi* (*Mid-Wahgi*) tallying of thousands using body parts.
Data are from C. Muke (2000, p. 135).

Nevertheless, there was one idea introduced after colonisation when brideprices began to escalate. It was the use of tying knots. Knots might represent pigs then a knot was used to separate kinds of items such as feathers or shells. The rope could also be carried to the bride's parent to indicate the amounts being negotiated. The girl's family returned a rope indicating what they would give and these two ropes could be compared as a form of subtraction. Another newer method was that of having stakes for the pigs and poles for money, for example a pole might represent 100 or 1000 *kina*.

Cultural Beliefs Associated With Number

Yu Wooi speakers avoid using numbers, especially totals, when presenting goods to others. This is particularly the case when giving to maternal uncles. For this reason, they use alternative methods of tallying to indicate amounts and display these tallies. It was also important to show wealth and status. The desire to tally also encouraged large numbers as shown above. In each case, the tally point was a one-to-many matching as we found with the Iqwaye speakers.

Analysis of the language itself indicates that *si* means "take" and *eraksi* appears to mean "take these two" (*rak* sounding like *tak* and *e* meaning "these") but to use the morpheme *ka* would appear to be related to "good" or "base/root". It is however clear that like most digit-tally systems numbers from 6 to 9 and in this case 10 are the word for $5+n$ where $n=1, 2, 3, 4$, or 5. However, it is in the use of words greater than 20, that the ideas of tens and twenties as a composite unit became apparent and was conceptually understood allowing respondents to provide a meaningful expression for numbers such as 27, 250 and 1376 and that there were not too many variations suggesting a reasonable acceptance of these methods by the community. In de Abreu's (2002) terminology, this valued practice can be accepted as a valorisation of the community.

Another aspect of culture was the effect of the neighbouring languages. While the Ek Nii influenced the Yu Nimbang dialect with whom they shared a land border, the result was more about individual words rather than the system since the Ek Nii had a (5, 10) system while the Yu Wooi has a (2, 5) system. Within Yu Wooi, the dialect for one of the tribes borrowed the Ek Nii word for 3 giving the set of frame words as 1, 2, 3, 5, rather than 1, 2, 5. In Lean's categorisation, Ek Nii probably has a neo-2 cycle system with a special word for 3 but it also has a special word for 10 (containing the morpheme for one). In all other respects it was similar to Yu Wooi but the 10 cycle required a different classification in Lean's terms. The North-Wahgi language was also a (2, 5) system except that it used different words for 1, 2 and 5.

One other aspect of note is the use of different words for "half". This is associated with classification of objects and ideas, a relatively common occurrence in different parts of PNG in terms of counting numbers. In this case, the community use *kekep* and *arhka* for ¼ and ½ respectively for dividing up animals with four or two legs and these words come from the words "to cut". The animal name is used eg. *kong arhka* for half a pig. However, *yem* is used for half of other things, e.g., pair of hands as noted for five as *angek yem.* C. Muke (2000) considered whether classifiers may have been used for counting words but there is no evidence of this in this language group or neighbouring groups.

Finally, comparing was an important aspect of culture whether it was achieved through counting or displays of tallying.

In summary, the Yu Wooi language involved two forms of quantifying: they used number words for small group counting and they used a matching and tallying approach for larger numbers. Counting in pairs and counting first the fingers of both hands before adding the thumbs is reminiscent of Hagen counting (not far away along the Highlands Highway) in which the people also count in fours then eights but superimpose the decimal system marking the two thumbs being added. This is appropriate too in ceremonies for counting where a second person tallies the 10s.

Conclusion

This chapter presents evidence that 2-cycle systems may provide sufficient numeral representation to generate large numbers and ways of thinking systematically in terms of the number sense of quantity. This generation of systematic thinking about quantity is embedded in cultural ways of thinking. Thus it is important not to separate mathematical thinking from the context of counting if we are to appreciate the systematic ways of thinking. Number then is represented not only in words but also in gestures and begins a better appreciation of digit-tally systems that may not be limited to just (5, 20) cycle systems. The chapter certainly disputes suggestions that 2-cycle systems are "less sophisticated" systems than the base 10 system. In place of place value notation in non-written languages, the body provides an important recording tool visible for communication with others together with the associated language words. The chapter also indicates that innovation is highly likely in the Papua New Guinea and Oceania region rather than counting systems being diffused from far away.

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