Peculiarities in Multidimensional Regional Poverty

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1 Introduction and Background Data

Some basic tools of partial order theory are employed to analyze poverty patterns in the European Union (EU) regions. Poverty is intrinsically a multidimensional concept and its measure must take into account a wide variety of aspects that do not always display identical patterns.

The starting point is a multi-indicator system (MIS) comprising three indicators measuring regional poverty in 88 different regions and countries of the European Union (Annoni and Weziak-Bialowolska 2016): Absolute Poverty Index (*API*), Relative Poverty Index (*RPI*), and Earnings and Incomes Index (*EII*), respectively (Annoni et al. 2015). These indexes evaluate poverty in absolute and in relative terms, taking into account monetary and non-monetary aspects. A total of 13 raw indicators are used to compute the three poverty indexes (Fig. 1).

API measures the individual capacity of affording basic needs, as classically defined in the literature (Törmälehto and Sauli 2010). It is based on seven non-monetary indicators of material deprivation including material deprivation rate and intensity, capacity of making ends meet, quality of the housing, and affordability of health and dental care.

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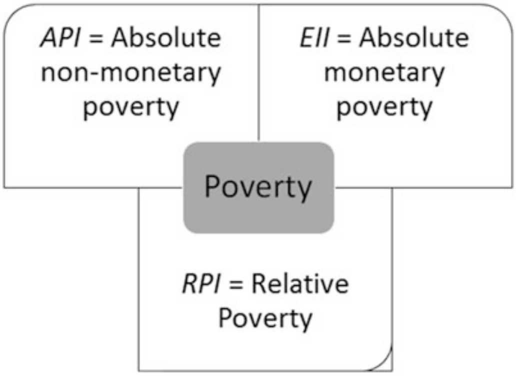
M. Fattore, R. Bruggemann (eds.), *Partial Order Concepts in Applied Sciences*,
DOI 10.1007/978-3-319-45421-4_8

Fig. 1 The three indexes included in the poverty MIS



RPI is computed by aggregating three well-known poverty statistics: poverty incidence P_0, depth P_1, and severity P_2 (Foster et al. 1984). These statistics are based on the national poverty line defined as 60 % of the national country income median. Relative poverty statistics capture the condition of living of an individual compared to the people surrounding him. The general formulation of a relative poverty statistics is defined for each integer, non-negative number α as:

$$P_\alpha(\boldsymbol{y}, z) = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z - y_i}{z}\right)^\alpha \tag{1}$$

where $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$ is a vector of properly defined income in increasing order, $z > 0$ is a predefined poverty line, $(z - y_i)$ is the income gap of individual i, within a group of n individuals, and q is the number of individuals having income not greater than the poverty line z. When $\alpha = 0$, $P_\alpha = P_0$ is the share of poor people where poverty is defined with respect to the poverty line and is known as at-risk-of-poverty rate. When $\alpha = 1$, $P_\alpha = P_1$ is the normalized income gap measure and indicates the average relative gap between the incomes of poor individuals and the poverty line. Finally, when $\alpha = 2$ $P_\alpha = P_2$ is called severity indicator because it measures the degree of inequality in the distribution of income among poor people.

EII describes different types of income and includes: (1) the median regional income, computed from the individual income distribution within each region, and two other measures derived from the Eurostat database (ec.europa.eu/eurostat/data/database); (2) compensation of employees; and (3) net adjustable household income.

Each poverty index, *API*, *RPI*, and *EII*, is computed as a generalized mean of order 0.5 of the selected indicators, after standardization (Annoni and Weziak-Bialowolska 2016; Decancq and Lugo 2013).

To better clarify the orientation of poverty indexes used here, we remind that:

- Poverty, as conventionally understood, has a negative orientation, so if we say “high poverty” it means that people are generally poor;

- However, for technical reasons linked to the aggregating functional form chosen in the original analysis, the three poverty indexes are all oriented in order to have the higher the region score, the lower the level of poverty.

Region scores on the three poverty indexes are shown in the Table in Appendix, where high scores mean high levels of quality of life, i.e., low levels of poverty. The distribution of the poverty indexes across the regions shows the presence of severe pockets of poverty, as *API* and *RPI* distributions are significantly negatively skewed. On the other hand, the *EII* distribution is characterized by few but very high values showing that there are certain regions with extremely high income levels.

We follow a multi-criteria approach to shed more light into the poverty pattern of the EU regions.

The chapter is organized as follows. Section 2 provides a brief overview of partial order concepts applied in the analyses. Section 3 presents and discusses main results while Sect. 4 concludes the chapter.

2 Partial Order Approach, General Remarks

The concepts of partial order theory allow for an alternative view on MIS because the need to derive a one-dimensional scalar as a ranking index is avoided. Instead, a simultaneous view on the role of the three poverty indexes is provided, which is not easily possible when aggregation methods are applied. The analysis of multidimensional poverty and deprivation can also be done applying a conceptual evaluation framework as the one recently discussed in Fattore (2015).

2.1 The Hasse Diagram Technique

The central point in partial order theory is the introduction of a binary relation between any two objects, which fulfills the axioms of partial order (Birkhoff 1984; Neggers and Kim 1998; Trotter 1992).

Let $x, y \in X$, where X is the set of objects of interest, and $q_i \in IB$, IB being the set of indicators observed on the objects in X. The governing relation between the objects derived from IB is given by:

$$x \leq y : q_i(x) \leq q_i(y) \;\text{ for all } q_i \in IB \tag{2}$$

Relation (2) is the basic of the special variant of partial order theory, which is known as Hasse diagram technique (HDT), see for example Bruggemann and Patil (2011) for an overview of the technique. In partial order terminology, objects which follow (2) are said to be comparable, i.e., they can be ordered. Objects which do not follow (2) are said to be incomparable (notation: $x \| y$). Given object x, the number of objects incomparable to x is called $|U(x)|$.

Usually, a partial order based on (2) is visualized by a Hasse Diagram, HD. Comparable objects are connected by a sequence of lines. A HD provides insight on subsets of objects for which a complete ranking can be found, called chains (see the formal definition below), without the need of aggregation or additional information such as weights. Also, it allows for identifying subsets of incomparable objects whose scores will be influenced by the particular type of aggregating function used.

As a downside, a HD can be very complicated as can be seen from the HD corresponding to the here studied poverty MIS (Fig. 2). There exist software packages, such as PyHasse used here (Bruggemann et al. 2014), which provide support in navigating across a HD.

Some further relevant definitions are:

Chain: A subset $X' \subseteq X$, where all objects fulfill (2) is called a chain. A chain has a length, which is $|X'| - 1$. For objects within a chain, say from the bottom to the top of the chain, all indicator values are simultaneously non-decreasing.

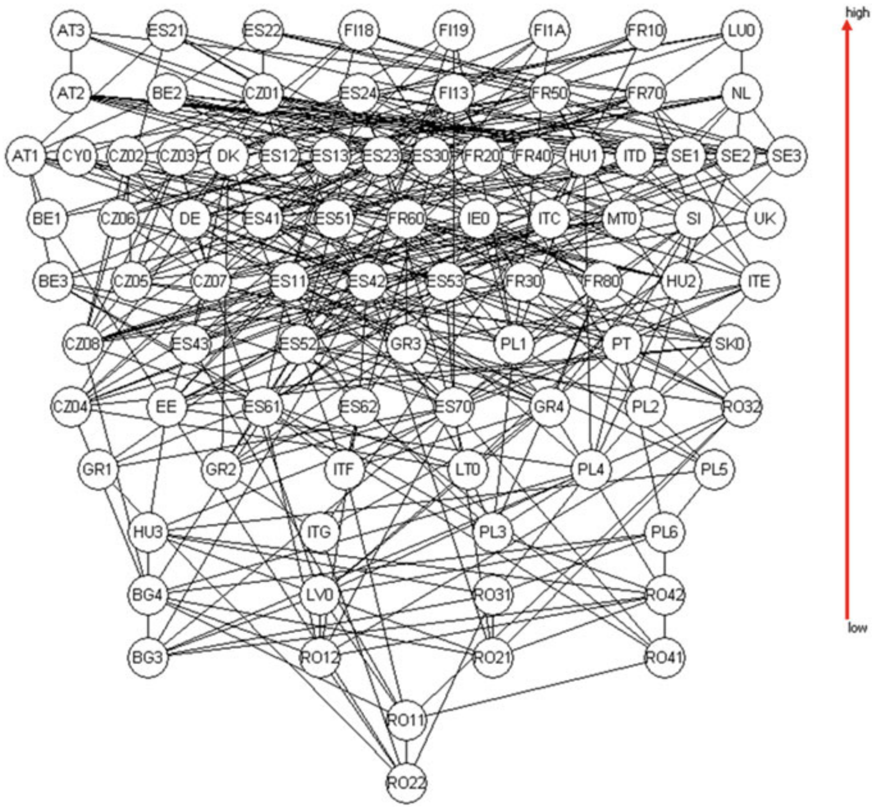


Fig. 2 Hasse Diagram of the 88 regions based on the three poverty measures API, RPI, and EII (PyHasse software)

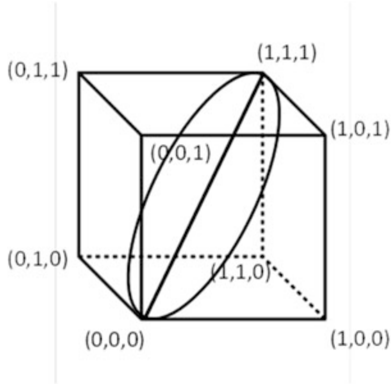


Fig. 3 The extreme points of the cube $h(3)$

Antichain: A subset $X' \subseteq X$, where no object fulfill (2), i.e., all objects in X' are mutually incomparable, is called an antichain. Thus, for any two objects within an antichain there is a conflict in indicator values.

Local HD: An object is selected and all the objects comparable upwards and downwards are visualized. One says: object x is "generating" its local HD.

2.2 The Peculiarity Rule

The Hasse diagram, shown in Fig. 2, reveals by its set of connecting lines only the set of all comparisons among the regions. It is possible to further classify the regions in order to identify peculiar ones. The first step is then to define the peculiarity concept.

In the general case with m indicators, a simple rule to define peculiarity is through normalizing the indicator values in the interval [0,1] and considering the m-dimensional hypercube $h(m)$. In our case $m = 3$; hence, we have to analyze the $h(3)$ (Fig. 3) as a classifier (Bruggemann and Carlsen 2014; Carlsen et al. 2015). The three dimensions of the cube represent the three poverty indexes, *API*, *RPI*, and *EII*. The vertexes of the cube $h(3)$ can be then interpreted in terms of extreme poverty levels: for example, the points (0,0,0) represents the worst and (1,1,1) the best condition, respectively, while, e.g., (1,0,0) is the point where *API* is the best possible, but *RPI* and *EII* are the worst. In general, there are 2^m extreme points (vertexes). Often an increase in one indicator will be accompanied by an increase in the other indicators. Thus, regions which are centered around the line between (0,0,0) and (1,1,1) can be denoted as "main stream" regions. Whereas regions close to one of the other vertices, e.g., (1,0,0) or (0,1,1) can be classified as peculiar regions, i.e., regions deviating from the "main stream" (Fig. 3).

A "near-enough-factor"-$f \in [0, 1]$-can be introduced to define the level of proximity of the points to the vertices of $h(3)$ (Bruggemann and Carlsen 2014). In the three-dimensional case, the maximum squared Euclidian distance between (0,0,0) and (1,1,1) in $h(3)$ is $D_{\max} = 3$. This means that in the three-dimensional case the (squared) distance between two given points (regions) is always in the interval [0, 3]. Regions are considered as peculiar if their distance, d, to one of the corners in $h(3)$ is $d < f \cdot 3$. The closer d to 0, the closer the region, characterized by its (normalized) values with respect to *API*, *RPI*, and *EII*, will be to one of the vertices of $h(3)$. As (0,0,0) and (1,1,1) are the extreme cases of the "mainstream," the interest is focused on the set of corners $h(3)' := h(3) - \{(0,0,0), (1,1,1)\}$. Regions apart from (0,0,0) and (1,1,1) are denoted peculiar regions due to their imbalanced pattern relative to the regions belonging to the main steam. If f is set to 0.05 only regions with highly imbalanced poverty measure profiles will be classified as peculiar. The value $f = 0.05$ can be interpreted as 5 % of the maximal distance, i.e., $d \leq 0.15$ to one of the peculiar vertices.

By looking at the actual vertex of proximity, the particular poverty index that causes the peculiarity can be spotted out. Thus, for example a region close to the (1,0,0) vertex has a markedly higher API than regions located in the "main stream," meaning lower levels of absolute poverty.

As an increase of one poverty index does not necessarily imply an increase of another poverty index, the cloud of points will encompass points which are considered as mutually incomparable.

3 Results

3.1 Hasse Diagram

Figure 2 shows the Hasse Diagram—HD—associated to the poverty MIS. As expected, given the relatively high number of objects, not much information can be directly extracted from this HD. However, it can be seen that:

- Poor regions, as measured by MIS are found at the bottom of the diagram; for example, BG3, RO21, and RO22;
- Rich regions are at the top of the diagram; for example, AT3, ES21, and LU0;
- The number of incomparable regions is very high (1754 total number of incomparabilities);
- There is a high number of comparable regions; for example, those in the chain AT3 > AT2 > SE1 > DE or NL > SE1 > UK > ES70 > GR1 > RO22. Two thousand seventy-four comparabilities are found in total.

The last point means that the HD is characterized by many chains of different lengths. As already said, the presence of chains is interesting as each chain is a realization of a unique ranking for a subset of objects. As an example, the chain:

AT3 > AT2 > ES12 > MTO > CZO5 > SKO > CZO4 > LTO > PL3 > RO42 > BG3

is informative of the fact that all the three poverty measures decrease (weakly) monotonously from AT3 with profile (5.8, 6.4, 5.8) to BG3 with profile

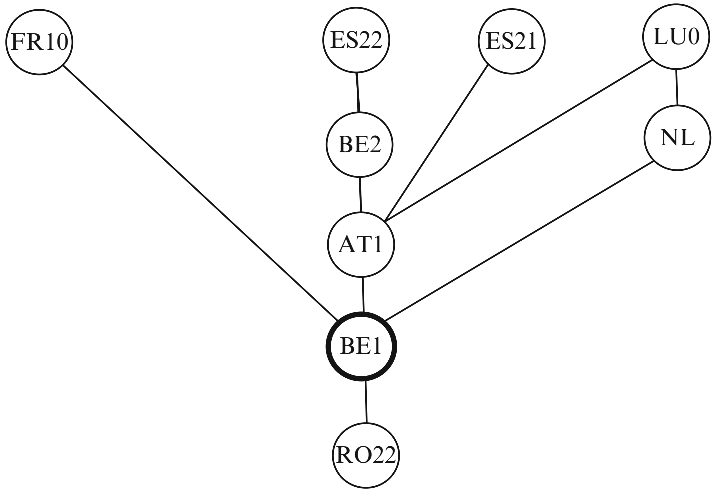


Fig. 4 Local partial order for the region BE1 with profile (4.6, 2.1, 5.9) visualized by the HD

(1.6, 3.5, 2.8) (see Table in Appendix). Many other chains connecting AT3 with BG3 are present (in total 66 chains including 11 regions).

3.2 Local HD

The analysis of a complex HD as the one showed in Fig. 2 needs proper navigation tools.

The concept of local HD tool is here applied. Assume to select the object $x = \text{BE1}$, then a "local HD" can be constructed as visualized in Fig. 4. The local HD is the visualization of the partial order of all objects y fulfilling either $y < \text{BE1}$ or $y > \text{BE1}$, so that only regions which are comparable with BE1 are shown. In this case, there are seven regions higher then BE1, showing a lower level of multidimensional poverty; however, only one region is lower than BE1, having worse poverty levels, namely RO22. Many of these regions with a lower degree of poverty than BE1 are mutually incomparable, indicating that the reasons for having a lower degree of poverty are different. For example, it is found $\text{FR10} > \text{BE1}$ as well as $\text{ES21} > \text{BE1}$, but FR10 || ES21. Specifically, $\text{FR10} < \text{ES21}$ with respect to the poverty measure *API*; but $\text{FR10} > \text{ES21}$ with respect to *EII*.

Region BE1 is comparable to 8 other regions only, while the number of incomparable regions with BE1 is rather high (79). Hence, 91 % of the 87 regions are incomparable with BE1. The profile of BE1 (API = 4.6, RPI = 2.1, EII = 5.9) interestingly shows that BE1 has a very low level of *RPI*. Only one region has even lower values, namely RO22 (RPI = 1.84). Thus, almost all regions perform better than BE1 with respect to *RPI*. As a result of this, the data profile of BE1 has a potential to crisscross (a high/low value of one poverty measure implying a low/high value in all the others) with all those regions whose data profile display higher values in *RPI* and lower values for *API* and *EII*, respectively. In particular, BE1 is incomparable with all those regions with a more balanced data profile, i.e., with scores located closely around the center of the distribution as for example ES11 (5.3, 5.3, 4.9).

3.3 Peculiar Regions

To identify some extremely conflicting regions, we apply the peculiarity rule (Sect. 2.2) with $f = 0.05$. Four regions are defined as peculiar (Table 1): the Belgian region hosting the capital Brussels (BE1) (consistent with the finding in the former section), Cyprus (CY0), the western part of Hungary (HU2), and the United Kingdom.

Table 1 The four regions identified as peculiar by the peculiar method with $f = 0.05$

Region/ country	Profile	(0–1) normalized data	Distance from vertex	Interpretation
BE1	[0, 0, 1]	[0.354, 0.011, 0.920]	0.126	This Belgian region, which includes Brussels, features deep pockets of poverty with respect to the rest of the population in the country (very low values of RPI). It is also characterized by very high income levels (EII > 0.9). This is a clear signal of the presence of important levels of inequality
CY0	[0, 1, 1]	[0.322, 0.897, 0.828]	0.144	The region, which coincides with the country Cyprus, is relatively well off in terms of income and relative poverty even if absolute poverty levels are higher that in the other EU regions/countries. This signals an equal society with few very rich but also few very poor people
HU2	[0, 1, 0]	[0.276, 0.851, 0.195]	0.137	The region, which is the western Hungarian macro-region, is relatively worse-off than CY0 because of low income levels. The region's profile indicates an equal society but toward low level of wealth (low *API* and *EII*)
UK	[1, 0, 1]	[0.782, 0.184, 0.770]	0.134	This country[a] stands at the opposite side than HU2, featuring high levels of incomes and low levels of poverty in absolute terms (high *API* and *EII*). The relatively low value of RPI indicates instead the presence of groups of people significantly poorer than the rest of the population (deep pockets of poverty)

[a]Due to data availability issues, the United Kingdom has been described at the national level only

Let us examine two peculiar regions, the Belgian BE1 near to the corner (0,0,1) and the Hungarian HU2, which is found to be in the neighborhood of the corner (0,1,0). Their normalized profiles respectively are [$API = 0.354$, $RPI = 0.011$, $EII = 0.920$] and [$API = 0.276$, $RPI = 0.851$, $EII = 0.195$], so that *RPI* is the most contrasting poverty index. This does not come as a surprise given that relative poverty is by construction a relative concept, which basically captures the level of inequality across the population. The three indicators from which *RPI* is constructed all refer to the poverty line that is the median national income. High values of *RPI* do not always imply that the population is rich; it indicates a low level of heterogeneity of poverty across the population. On the contrary, the earnings and incomes measure is a purely monetary measure of poverty. Relative poverty in BE1 is very high while income levels are within the highest in the European Union. HU2 shows the opposite pattern, quite low values of relative poverty but, at the same time, low wealth. This is a clear indication of the Belgian region being overall rich but highly unequal, while of the Hungarian region being poorer but less unequal (i.e., more homogenous).

The remaining two peculiar regions are briefly discussed in Table 1.

The analysis shows that several regions need attention with respect to compensation effects (Munda 2008). By further aggregating the three poverty indexes, there is the risk to miss relevant information. A population in a region is certainly well-off when low values of relative poverty are associated with low values of monetary and non-monetary absolute poverty. But low values of relative poverty with high values of absolute poverty, either monetary or non-monetary, are a signal of a widespread poverty where, however, there are low levels on inequality. Instead high values of relative poverty and low values of absolute poverty indicate the presence of deep and severe, but not widespread, pockets of poverty.

4 Conclusions

The chapter aim is to show the potential of partial order tools to analyze multi-indicator systems, MIS. When the indicators included in the MIS are carrying different messages, aggregation should be avoided. Partial order tools can be employed to assess how much information is lost due to compensation effects, always hidden in any aggregation process even if to different degrees according to the actually used aggregation technique.

On the flip side, by definition the partial order approach does not solve the incomparabilities between objects due to contrasting indicators. However, the analysis of these incomparabilities can be highly informative. To illustrate the value that a posetic analysis can add, we take as case study the MIS on the multidimensional measure of poverty in the EU regions. The MIS consists of three poverty indexes, Absolute Poverty, Relative Poverty, and Earnings and Incomes, estimated for 88 regions/countries across the European Union.

The analysis of incomparabilities in the poverty case sets a flag on particular regions where the aggregation can hide important contrasting patterns across the poverty measures. In our case, the relative poverty index is mostly contrasting with the remaining two. Relative poverty is in fact a “relative” concept that, by definition, captures the level of deprivation of people with respect to those living in the same area. Low values of relative poverty do not necessarily imply that people are well-off; it shows a low level of heterogeneity of poverty across the population. On the contrary, the earnings and incomes measure is a purely monetary one.

Some regions are detected as peculiar if they deviate from the main pattern of poverty profiles, as defined by the peculiarity rule. Four regions belong to this group with interesting profiles featuring contrasting levels of the poverty indexes.

Appendix

Starting multi-indicator system matrix (scores are oriented in order to have: the higher the score, the lower the level of poverty)

Country	Region code	Region name	Absolute poverty index	Relative poverty index	Earnings and income index
AT	AT1	Ostösterreich	5.4	5.7	6.0
AT	AT2	Südösterreich	5.8	6.1	5.6
AT	AT3	Westösterreich	5.8	6.4	5.8
BE	BE1	Région de Bruxelles-Capitale/Brussels Hoofdstedelijk Gewest	4.6	2.1	5.9
BE	BE2	Vlaams Gewest	5.8	6.1	6.0
BE	BE3	Région Wallonne	5.1	4.7	5.4
BG	BG3	Severna i Iztochna Bulgaria	1.6	3.5	2.8
BG	BG4	Yugozapadna i Yuzhna Centralna Bulgaria	2.7	5.0	3.2
CY	CY0	Κύπρος/Kypros	4.5	6.1	5.6
CZ	CZ01	Praha	5.5	6.4	5.7
CZ	CZ02	Střední Čechy	5.5	6.2	4.2
CZ	CZ03	Jihozápad	5.5	6.3	4.1
CZ	CZ04	Severozápad	5.0	5.0	3.8
CZ	CZ05	Severovýchod	5.2	5.9	4.0
CZ	CZ06	Jihovýchod	5.5	5.8	4.1
CZ	CZ08	Moravskoslezsko	4.9	5.0	4.0
DE	DE	Deutschland	5.5	4.4	5.4
DK	DK	Danmark	5.9	5.1	4.6

(continued)

Country	Region code	Region name	Absolute poverty index	Relative poverty index	Earnings and income index
CZ	CZ07	Střední Morava	5.3	5.5	3.9
EE	EE	Eesti	4.9	5.2	3.6
ES	ES11	Galicia	5.3	5.3	4.9
ES	ES12	Principado de Asturias	5.5	5.9	5.4
ES	ES13	Cantabria	5.7	5.9	5.3
ES	ES21	País Vasco	5.7	5.9	6.1
ES	ES22	Comunidad Foral de Navarra	5.8	6.5	6.0
ES	ES23	La Rioja	5.7	5.0	5.3
ES	ES24	Aragón	5.9	5.6	5.5
ES	ES30	Comunidad de Madrid	5.5	5.0	5.8
ES	ES41	Castilla y León	5.5	4.5	5.1
ES	ES42	Castilla-La Mancha	5.4	3.8	4.7
ES	ES43	Extremadura	5.3	3.2	4.5
ES	ES51	Cataluña	5.5	4.9	5.5
ES	ES52	Comunidad Valenciana	5.3	4.8	5.0
ES	ES53	Illes Balears	5.3	4.5	5.3
ES	ES61	Andalucía	5.0	3.3	4.6
ES	ES62	Región de Murcia	5.2	3.3	4.6
ES	ES70	Canarias	4.6	3.6	4.7
FI	FI13	Itä-Suomi	6.0	5.4	4.6
FI	FI18	Etelä-Suomi	5.9	5.9	5.2
FI	FI19	Länsi-Suomi	6.0	5.6	4.8
FI	FI1A	Pohjois-Suomi	6.2	5.7	4.7
FR	FR10	Île de France	5.2	5.9	6.8
FR	FR20	Bassin Parisien	5.5	6.1	5.3
FR	FR30	Nord - Pas-de-Calais	5.3	5.4	5.1
FR	FR40	Est	5.39	6.16	5.36
FR	FR50	Ouest	5.56	6.43	5.25
FR	FR60	Sud-Ouest	5.35	5.59	5.35
FR	FR70	Centre-Est	5.62	6.14	5.54
FR	FR80	Méditerranée	4.99	4.79	5.30
GR	GR1	Voreia Ellada	4.46	2.67	4.64
GR	GR2	Kentriki Ellada	4.42	2.99	4.54
GR	GR3	Attiki	4.51	4.51	5.35
GR	GR4	Nisia Aigaiou, Kriti	4.11	4.38	4.82
HU	HU1	Közép-Magyarország	3.97	6.22	4.90
HU	HU2	Dunántúl	4.34	6.08	3.68
HU	HU3	Alföld És Észak	3.69	5.15	3.40
IE	IE0	Ireland	5.40	5.76	5.32

(continued)

Country	Region code	Region name	Absolute poverty index	Relative poverty index	Earnings and income index
IT	ITC	Nord-Ovest	5.04	5.68	5.72
IT	ITD	Nord-Est	4.95	6.07	5.61
IT	ITE	Centro (I)	4.82	5.52	5.48
IT	ITF	Sud	4.04	3.15	4.49
IT	ITG	Isole	3.73	2.97	4.54
LT	LT0	Lietuva	4.20	4.86	3.76
LU	LU0	Luxembourg (Grand-Duché)	5.93	5.86	8.15
LV	LV0	Latvija	2.99	3.93	3.34
MT	MT0	Malta	5.41	5.94	5.18
NL	NL	Nederland	5.87	5.48	6.02
PL	PL1	Region Centralny	3.90	5.51	4.00
PL	PL2	Region Południowy	3.99	5.34	3.59
PL	PL3	Region Wschodni	3.88	4.55	3.06
PL	PL4	Region Północno-Zachodni	3.91	5.06	3.48
PL	PL5	Region Południowo-Zachodni	3.70	5.23	3.53
PL	PL6	Region Północny	3.68	5.17	3.40
PT	PT	Portugal	4.32	5.19	4.25
RO	RO11	Nord-Vest	2.47	3.03	2.81
RO	RO12	Centru	2.67	3.31	2.79
RO	RO21	Nord-Est	2.18	3.37	2.57
RO	RO22	Sud-Est	2.45	1.84	2.74
RO	RO31	Sud - Muntenia	3.35	5.05	2.78
RO	RO32	Bucureşti - Ilfov	2.94	4.77	4.19
RO	RO41	Sud-Vest Oltenia	3.28	3.23	2.86
RO	RO42	Vest	3.33	4.36	3.01
SE	SE1	Östra Sverige	5.67	5.31	5.62
SE	SE2	Södra Sverige	5.75	5.13	5.18
SE	SE3	Norra Sverige	5.79	4.91	4.99
SI	SI	Slovenija	4.92	6.11	5.23
SK	SK0	Slovenská Republika	5.16	5.70	3.87
UK	UK	United Kingdom	5.56	3.83	5.47

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