Design and Experimental Characterization of a Flexure Hinge-Based Parallel Four-Bar Mechanism for Precision Guides

P. Gräser, S. Linß, L. Zentner and R. Theska

Abstract This paper presents the investigation of the influence of the flexure hinge contour in compliant linkage mechanisms for precision engineering applications. Especially the influence on the precision of the path of motion and the stroke of the compliant mechanism is reflected. Based on previous results on optimized single polynomial flexure hinges, the validity of proposed guidelines is analyzed for a combination of several flexure hinges in one compliant mechanism. A parallel crank mechanism is used as an example for a compliant rectilinear guiding mechanism with constant link orientation. The parameters of the approximated linear motion are investigated for the rigid-body model, a compliant analytic model and a FEM model. Finally these results are compared with measurement results taken at manufactured prototypes.

Keywords Compliant mechanism · Parallel crank mechanism · Linear motion · Flexure hinge · Contour optimization · Precision guide

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1 Introduction

In precision engineering linkage mechanisms with optimized flexure hinges gain in importance. They are distinguished from common mechanisms with conventional hinges by numerous positive properties. Since they are based on elastic bending there is no external friction, no wear and no need of maintenance or lubrication. Therefore compliant linkage mechanisms are suitable for the use under vacuum and clean room conditions. They are preferred in many applications in the semiconductor industry, astronautics or precision measurement.

Compliant mechanisms can be subdivided based on different attributes (Zentner 2014). One option is the distribution of the compliance in the hinge. A distinction is made between distributed and lumped compliance based on the ratio of the length and the smallest thickness of the hinge (Zentner 2014). This article will only address fully compliant mechanisms with lumped compliances.

In precision engineering, decisive for the use of compliant mechanisms are the reproducibility, the precision of the path of motion, and the maximum stroke. These parameters are affected by the characteristics of flexure hinges. Based on the nature of flexure hinges there is no stationary rotation axis, causing a deviation of the path of motion compared to the rigid-body mechanism. The maximum stroke of the compliant mechanism is limited by the admissible strain in the flexure hinges. First investigations of the authors in generalized guidelines for the synthesis of compliant mechanisms for the optimization via geometric parameters of the flexure hinges have shown positive results. Therefore optimized flexure hinges with polynomial contours have been investigated. The validity of the derived guidelines will be demonstrated on a planar compliant linkage mechanism. The example used in the presented investigations is a compliant parallel crank with the task of motion of approximate linear planer guidance.

2 Motion Task

The starting point is a motion task which should be realized by a compliant mechanism via synthesis. Based on this approach, firstly guidelines for the combination of several flexure hinges need to be formulated. Secondly existing design rules for polynomial contours need to be validated. Therefore a procedure is recommended which leads from a given motion task to a suitable compliant mechanism. The procedure includes the selection of an appropriate rigid-body mechanism, the constructional realization of the compliant mechanism and the goal-oriented design of the geometric parameters of the flexure hinges. These synthesis steps are discussed in detail in Chap. 3.

A frequent task in precision engineering is to guide each point of a plane along a straight line. The deviation of this line should be minimal and highly reproducible. Furthermore, the smallest possible rotation of the moved link should occur during the motion. Such a task is necessary for adjusting and positioning of elements in precision manufacturing or in metrology. Therefore, a parallel crank mechanism is chosen due to the practical relevance, demonstrated in a large number of publications e.g. in (Liaw and Shirinzadeh 2008; Luo et al. 2015; Yang et al. 2010). In these cases, the trajectory of each coupler point of the parallel crank mechanism is equivalent to a circular translation. However, the motion can be approximated as a straight line with deviations within the set boundaries.

After the determination of the motion task and selection of a rigid-body model the requirements for the subsequent implementation of the mechanism are specified. The following three requirements are defined which are related directly to the output motion: The range of motion along the guiding line v_x, the maximum error in the orthogonal direction v_y and the maximum rotation angle of the guided coupler link δ (cf. Sect. 3.1). Another requirement arises due to the flexure hinges. According to the state of the art and preliminary investigations the rotation angle φ^* for each hinge is limited to 10 degrees because otherwise the maximum admissible strain ε_{zul} is exceeded.

3 Synthesis of the Compliant Linkage Mechanism

The used synthesis approach is the rigid-body replacement method (Howell et al. 2013). Therefore, the rigid-body hinges are replaced by prismatic flexure hinges in the compliant mechanism. The approach includes the following synthesis steps (Linß 2015):

- Determination of a suitable rigid-body mechanisms for the required path of motion,
- Design and implementation of the compliant mechanism,
- Geometric design and optimization of the flexure hinges,
- Verification of results, and of the need for further adjustment and optimization.

The selection of the rigid-body mechanism includes the choice of an appropriate model and the dimensioning of associated geometrical parameters. This is done with respect to required path of motion. Based on the specified concrete model the path of motion can be determined.

A further possibility of the analytical calculation of the motion parameters is the use of mathematical models for the compliant mechanism. In this way, the deformations can be determined by equations. An example for this is a model of the parallel spring guide derived by Nönnig (1980) with and without stiffened crank arms. Higher accuracy in the calculation of the motion path can be achieved with FEM simulations based on the CAD models.

After the choice of the kinematic parameters and the constructional realization of the compliant mechanism the geometric design and optimization of the four flexure hinges is a crucial step in the mechanism synthesis. For a simplified determination of suitable flexure hinge contours in comparison to computerized optimization

FEM-based guidelines and design graphs can be used (Linß 2015). Therefore at least the relative rotation angle in the rigid-body model φ^* for each hinge must be analyzed. It has been shown that hinge contours based on polynomial functions are particularly suitable in this case (Gräser et al. 2015).

3.1 Rigid-Body Mechanism

The rigid-body mechanism describes an ideal path of motion which is the reference in the following qualitative studies in addition to the exact straight line. The path of motion corresponds to an analytically predictable function, which results from the geometric parameters of the mechanism. The function can be characterized by the displacement in x and y direction of one particular point on the coupler of the mechanism. In the chosen example of the parallel crank, these parameters are the coupler length c and the crank length a. The point which is analyzed during motion is the point K on the coupler. This point is called coupler point, see Fig. 1.

For dimensioning of a compliant parallel crank mechanism the following requirements for the guiding of a plane along a straight line are given:

- Stroke of motion $\left|v_{\mathrm{x}}^*\right| \geq 10\,\mathrm{mm}$,
- Guiding deviation in y direction $\left|v_{\mathrm{y}}^*\right| \leq 1\,\mathrm{mm}$,
- Rotation angle of the coupler $\delta \leq 90''$.

The requirements are defined based on the addressed application in precision engineering. Additionally the angle of rotation should not exceed the permissible angle of the used plane mirror interferometer for the length measurement, see Sect. 5. With the given requirements and in consideration of the maximum deflection angle of the hinges φ^* suitable values for the parameter a of the parallel crank can be chosen with the help of the created design graph in Fig. 2.

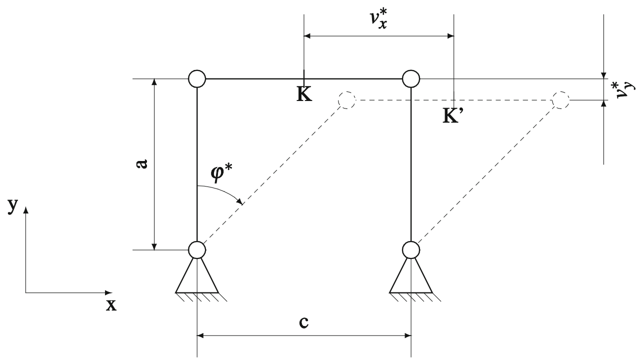

Fig. 1 Rigid-body model of a parallel crank mechanism with the coupler point K respectively K', the geometrical parameters (a, c) and the parameters of motion ($v_{\mathrm{x}}^*, v_{\mathrm{y}}^*$)

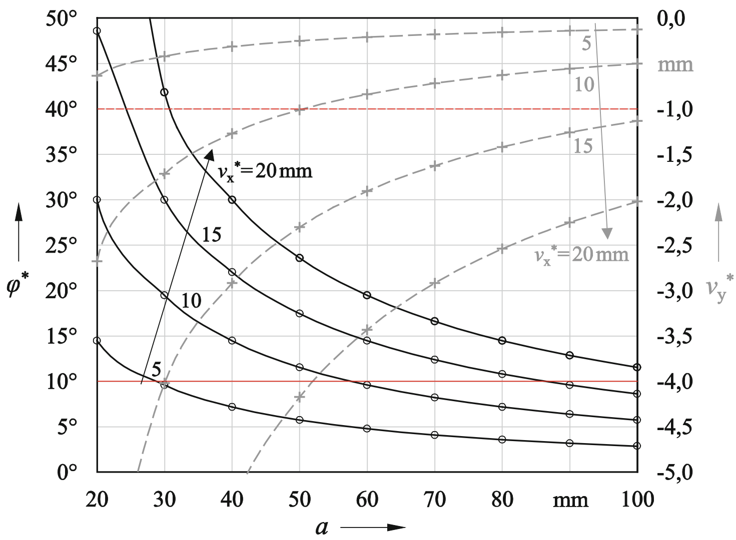

Fig. 2 Design graph for the determination of parameters for the rigid-body mechanism

In the design graph, the functions of the angle of the hinges φ^* and the straight line deviation v_y^* are given in dependence of the crank length a for different maximum strokes v_x^*. With the condition v_x = 10 mm and the selected crank length of a = 80 mm a maximum hinge angle of φ^* = 7.1° and a maximum straight line deviation of v_y^* = −0.6 mm result. The basis for this design graph is the following equation for the rigid-body mechanism:

$$v_y^* = \frac{v_x^*}{\tan(\varphi^*)} - a, \text{ with } \varphi^* = \arcsin\left(\frac{v_x^*}{a}\right). \tag{1}$$

3.2 *Compliant Mechanism*

After the dimensioning of the rigid-body mechanism the implementation of the compliant mechanism is discussed in this section. First the idealized rigid-body joints are replaced by flexure hinges. These are connected by links which have a significant higher stiffness than the hinges. Thus, the deformation of these connecting link segments can be neglected. The path of motion of the coupler point can be calculated in various ways. In this work, the compliant parallel crank mechanism is calculated by the model of a concentrated parallel spring guide according to Nönnig (1980), see Fig. 3 and Eq. 2 and 3.

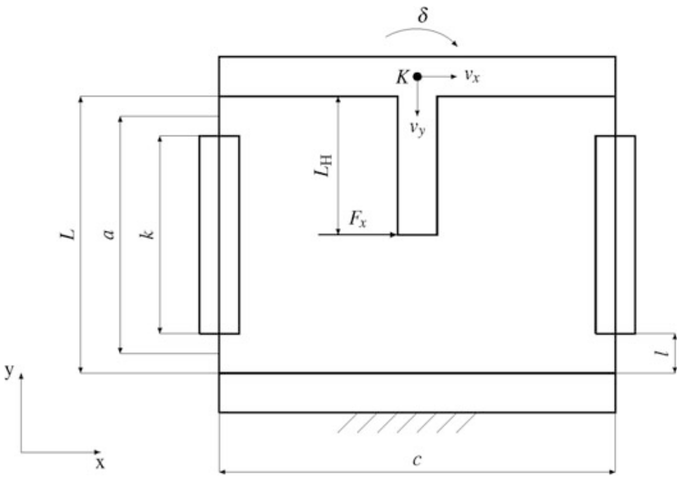

Fig. 3 Analytical model for the calculation of the path of motion of the parallel crank mechanism with concentrated compliance

$$v_y = \frac{3}{5}\frac{v_x^2}{L}\eta, \quad \eta = \frac{1 - \frac{5}{2}m^3 + \frac{3}{2}m^5}{\left(1 - m^3\right)^2}, \quad m = \frac{k}{L} \tag{2}$$

$$\delta = \left(16 \cdot 10^{-5}\frac{L^3\,\frac{L-2L_H}{2c}}{H_0 c}F_x + \frac{L^7\,\frac{L-2L_H}{2c}}{201600 H_0^3 c}F_x^3\right)\psi \tag{3}$$

The parameter η is a path factor in the case of stiffened model and m is the stiffening ratio. The parameter ψ is a correction factor which depends on the stiffening ratio m. With v_x and v_y the motion of the coupler point K of the compliant parallel crank mechanism can be described. In addition to the rigid-body model without coupler rotation, in the case of the compliant counterpart the rotation angle of the coupler δ must be regarded, because a minimal rotation occurs.

With a given maximum rotation δ and the guiding deviation v_y suitable values for the coupler length c can be determined as shown in Fig. 4. For the example of the investigated parallel crank mechanism a value of c = 100 mm results.

3.3 Deployment of Design Guidelines for Flexure Hinge Contour Optimization

The polynomial function derived by (Linß et al. 2015) allows the description of contours from elementary to very complex shape. Since all flexure hinges in the compliant parallel crank mechanism are equally shaped they all can be described by

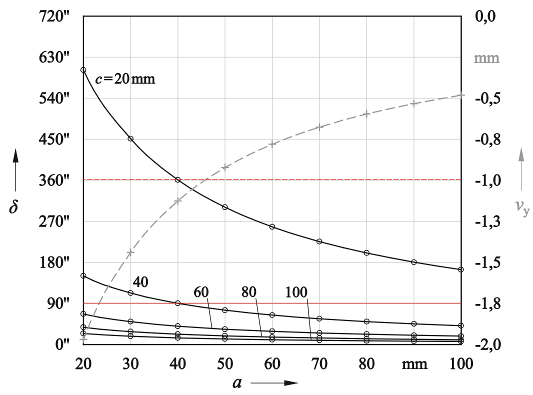

Fig. 4 Design graph for the determination of the coupler length c of the compliant parallel crank mechanism for the listed requirements ($v_x = 10\,\text{mm}, v_y < 1\,\text{mm}, \delta < 90''$)

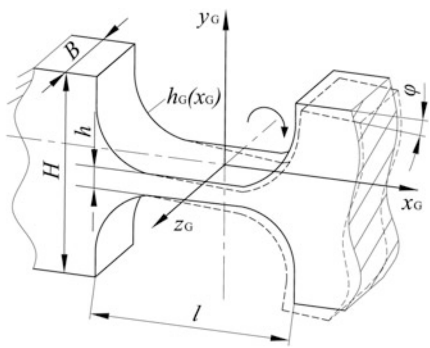

Fig. 5 Geometric parameters of a flexure hinge for deflection with the angle φ

the variable hinge height $h_G(x_G)$ based on each hinge coordinate system, see Fig. 5. To determine the geometric design it needs the definition of the basic shape of the hinge contour and the geometric parameters.

Compared to common circular or corner-filleted contours, special polynomial contours offer a great potential for optimization deploying a comparably simple contour modeling, see Fig. 6. For the investigations symmetrical flexure hinges with the minimum hinge height h in the joint center height are considered.

With regard to the precision and the possible stroke of the compliant mechanism the following geometric parameters have to be considered:

- the hinge length ratio l/H,
- the hinge height ratio h/H and
- the used order n of the polynomial function.

To determine appropriate parameter values for the flexure hinges with polynomial contours derived guidelines can be used. For the simultaneous decrease of the two objectives path deviation and maximum strain (which limits the motion range

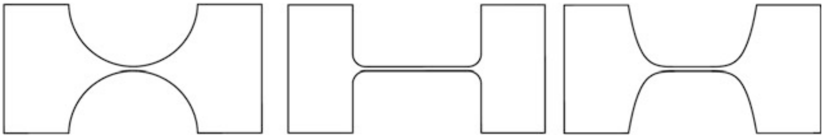

Fig. 6 Investigated flexure hinge contours for the compliant parallel crank mechanism: semi-circular, corner-filleted and 6th-order polynomial contours

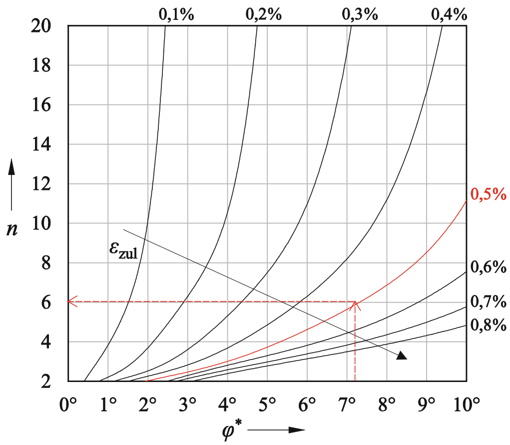

Fig. 7 Design graph for the determination of the order n of the polynomial function for the contour of the flexure hinge in the mechanism with given admissible strain ε_{adm} and rotation angle $\varphi^* \left(\frac{l}{H} = 1, \frac{h}{H} = 0.03\right)$

respectively possible stroke) the use of the hinge length ratio $l/H = 1$ and the hinge height ratio $h/H = 0.03$ is suggested (Linß 2015). The mechanism width B can be chosen arbitrarily, because it affects only the necessary force to deflect the mechanism. In the presented example the width is chosen as $B = 6\,\text{mm}$.

After defining the basic hinge dimensions $l = H = 10\,\text{mm}$ and $h = 0.03\,\text{mm}$, a suitable polynomial order n can be determined by means of the created design graph in Fig. 7.

For instance the admissible maximum strain of the utilized aluminum alloy EN AW 7075 is $\varepsilon_{adm} = 0.5\,\%$. Together with the maximum relative rotation angle of the hinges φ^* of a polynomial order of at least $n = 6$ is needed.

4 Simulative Investigations of the Parallel Mechanism

4.1 Rigid-Body Simulation

To investigate the path of motion of the rigid-body model the regarded mechanism is simulated with the software SAM. The parameters for the link lengths and the position of the hinges are the input for the program. The result is a model of the rigid-body mechanism, see Fig. 8. The motion in the simulation of the parallel

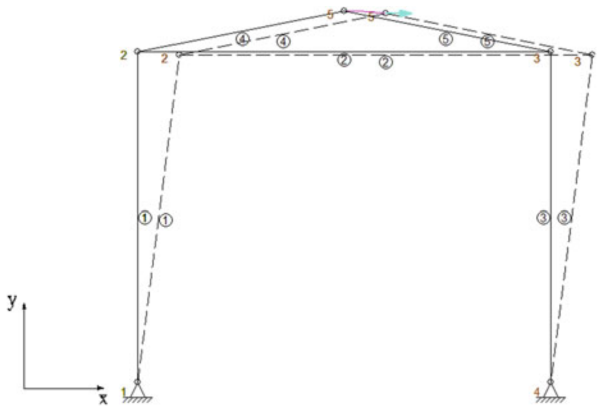

Fig. 8 Simulation of the motion path of the rigid-body mechanisms with the software SAM (coupler point K and K′ in point 5)

crank is specified by a deflection of a point of the coupler in positive x direction $v_x^* = 10\,\text{mm}$. An additional parameter for the evaluation of the path of motion is the displacement of the point K in y direction v_y^*. A rotation of the coupler δ^* does not occur in this case.

4.2 FEM Simulation

To proof the potential of the synthesis method based on suitable polynomial contours, the influence of the variation of the hinge contour on the mechanism properties is investigated by means of a static-mechanical FEM analysis. For this purpose, any flexure hinge is modeled with the same polynomial order $n = 6$, while the centers of all flexure hinges correspond to the coordinates of each revolute joint in the rigid-body mechanism.

An additional parameter that results due to the constructional implementation is the orientation of the hinges. This is initially chosen for the flexure hinges so that the longitudinal axes of the flexure hinges are aligned parallel to the crank. As a reference two compliant mechanisms, one with usual semicircular and one with corner-filleted hinge contours, are considered.

The simulation model was created using the software ANSYS Workbench 16.2 in consideration of large deformations. The deflection of the mechanism with a linear actor is given by a force with constant x direction at the coupler. The coupler displacement in y direction and the rotation around the z axis are free. The compliant parallel crank mechanism with the used polynomial contours of 6th-order is shown in Fig. 9 for the non-deformed and deformed state at full input $|v_x| = 10\,\text{mm}$. To obtain accurate simulation results comparable to the metrological investigation for the selected non upright position the following three aspects are considered in the FEM model too: The influence of gravity on the deformation in the z direction, a boundary condition to simulate the weight compensation

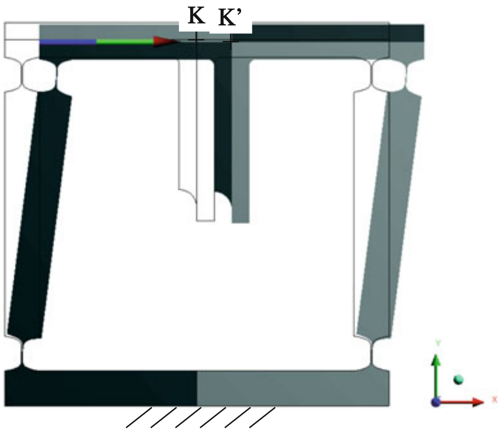

Fig. 9 Simulation of path of motion of the compliant mechanism with ANSYS Workbench

implemented in the measurement setup by means of a guided ball and the weight of the measuring mirrors used for interferometric length measurement.

After specifying the input displacement $v_x = v_x^*$ the guiding properties of the coupler link can be determined. Thus, the position of the regarded coupler point K in the compliant mechanism results in comparison to the ideal straight line (straight line deviation v_y) and in comparison to the rigid-body model (lateral path deviation Δv_y). In addition, the angle of rotation δ of the coupler is calculated based on the coordinates of two points. The analysis of the maximum strain according to von Mises ε_V in the mechanism allows the calculation of the realizable range of motion of the manufactured mechanisms according to the admissible strain for the chosen material. The mechanisms with corner-filleted and polynomial contours can be examined for full deflection of $|v_x| = 10\,\mathrm{mm}$, while the mechanism with semi-circular contours can only be deflected with an input displacement in x direction of $|v_x| = 4\,\mathrm{mm}$.

5 Measurements of Parallel Mechanism Prototypes

To verify the presented model prototypes which were fabricated by means of wire-cut EDM are measured. Three parallel crank mechanisms with different flexure hinge contour have been investigated: semi-circular, corner-filleted and polynomial function of 6th-order. The shapes of the contours are the semi-circular, the corner-filleted and the polynomial function of 6th-order. The hard aluminum alloy EN AW 7075 is used, as well as in the simulations.

The metrological investigation of the path of motion of the prototypes is made with a specially constructed test bench. The investigated parameters are corresponding to those of the simulation, the displacement v_x, the displacement v_y and

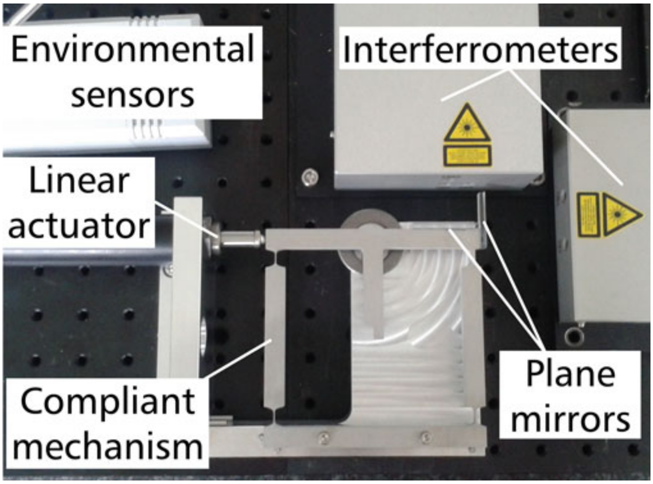

Fig. 10 Setup for the high precision measurement of the path of motion

the coupler rotation δ. The two displacements can be determined directly be means of length measurements. To measure the rotation, it is necessary to use two parallel linear measurements carried out at a known distance and from their difference to determine the angle. The length measurements are made by plane mirror interferometers, which have a resolution of 0.1 nm. For realizing the described three length measurements, a single-beam interferometer and orthogonal a two-beam interferometer are used, see Fig. 10.

The initial driving motion is directly applied by a precision linear actuator at the coupler of the parallel crank, for all mechanisms see Fig. 10. The resolution of the linear actuator is 0.1 µm. To compensate the dead load of the mechanism and the mirrors a rolling ball is used additionally, which is placed below the coupler and is moved along during deflection.

To ensure a statistically firm analysis, multiple measurements for each prototype have been made. The mean values are presented as functions in the following diagrams, while the indication of the confidence intervals is not shown due to a better clarity.

5.1 Linear Deviation and Path Deviation of the Coupler

The evaluation of the measurement results and the verification of the different models are made under consideration of two different motion parameters. They differ by the desired trajectory of the compliant mechanism. A first option is to refer directly to an ideal straight line. The resulting error is called here straight line deviation v_y (cf. Fig. 11 left).

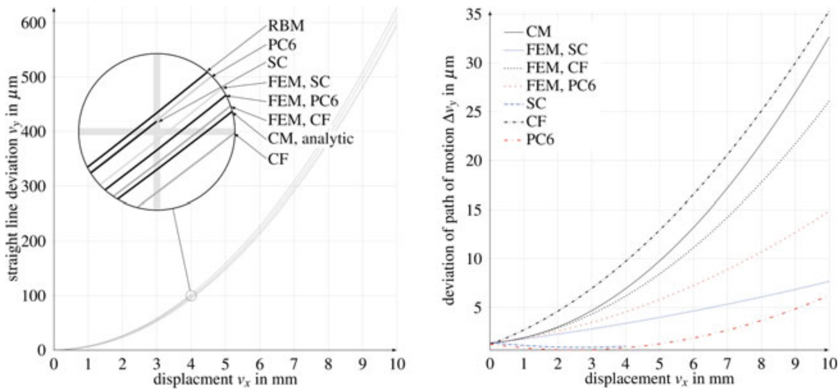

Fig. 11 Measurement results for the different models and prototypes for the parallel crank mechanism: rigid-body mechanism (RBM); analytically calculation of compliant mechanism (CM); FEM simulation with different flexure hinge contours: semi-circular (FEM, SC), corner-filleted (FEM, CF), 6th-order polynomial (FEM, PC6); manufactured prototypes with different flexure hinge contours: semi-circular (SC), corner-filleted (CF), 6th-order polynomial (PC6)

Secondly, the motion path is compared to the result derived with the rigid-body model. This comparison shows how accurately the constructed compliant mechanism realizes the motion path of the rigid body mechanism. For the parallel crank mechanism this path deviation is called Δv_y (see Fig. 11 right).

In the diagram of the straight line deviation (Fig. 11 left) the trajectories of the various models and prototypes are shown. The individual motion patterns have the same characteristic, but differ in their distances from the straight line. Differences between the parallel crank mechanisms with different notch contours occur. The arrangement in the diagram is the same for the simulation and the measurements. Differences between simulation and measurement can be attributed to several reasons. Either, the simulation model inaccuracies as well as tolerances of the manufactured mechanisms compared with the CAD models, lead to deviations. The trajectory of the prototype with semicircular contour ends already at $v_x = 4$ mm, because from this value the maximum admissible strain is exceeded in the flexure hinges.

Compared with the trajectories of the rigid-body model (Fig. 11 right) the semi-circular and also the polynomial contours lead to better values than the corner-filleted contours. But the advantage of the polynomial contour in comparison with the semi-circular contours is the larger range of motion. The mechanism with 6th-order polynomial flexure hinges can be deflected without exceeding the strain limit at $v_x = 10$ mm. Thus, the polynomial flexure hinges represent a good compromise between semi-circular and corner-filleted contours regarding the precision and the stroke.

5.2 Rotation of the Coupler

A rotation of the coupler δ occurs due to the replacement of the rigid-body mechanism with the compliant mechanism. The rotation δ results from transversal forces and moments generated by the flexure hinges. For the analytical model of the compliant mechanism and the FEM simulation the maximum rotation of the coupler for the predetermined range of motion results.

For the manufactured prototypes the rotation angle δ of the coupler is examined according to the resolution of the proposed measurement system. The resolution is mainly limited by the surface quality of the mirror and it is $\pm 4''$. Thus, it can be concluded that the rotation of the manufactured prototypes is below $\delta = 10''$. Compared with the state of the art this minimal angle of rotation is very low for a straight line guiding of a link and sufficient for a variety of applications in precision engineering.

6 Conclusion and Outlook

The investigations in this paper show that the design of the notch geometry of flexure hinges in compliant linkage mechanisms has an influence on both, the accuracy of the path of motion as well as the maximum range of motion. A flexure hinge contour with polynomial functions offer an advantage compared to conventional contours like semi-circular contours, as these allow an increased stroke for the same precision. For the selection of suitable polynomial orders the presented design graphs can be used. These provide recommendations regarding the geometric parameters of the compliant mechanism and its flexure hinge design.

The metrological investigation confirms that analytical calculations allow only an approximation of the motion. For the detailed design of compliant mechanisms a numerical solution for example by means of FEM simulation is necessary. The increase in accuracy results from the consideration of the exact nonlinear deflection of the flexure hinges, which is neglected in the analytic calculations. Thus, a simplified design with respect to precision and range of motion for compliant linkage mechanisms with optimized flexure hinge contours is possible by means of special design graphs and guidelines.

Another parameter for validating the FEM model is the characteristic stiffness of the mechanism. This can be evaluated by monitoring the driving force of the actuator as a function of the displacement. Simulations show the dependence on the hinge contour. This will be demonstrated by existing prototypes in further investigations. Furthermore, there is an initial investigation of the hinge orientation in the compliant mechanism, i.e. the spatial orientation of each hinge. The consideration of the different deflection angles of each hinge leads to the conclusion to use different notch geometries for flexure hinges deflected with different rotation angle. Through a combination of hinges with different orders of the polynomial function it is possible to increase the range of motion of a mechanism without a loss in precision of the path of motion. It is even possible to improve it. More aspects to be examined are manufactured-based effects and the use of different materials.

Further investigations should validate existing synthesis methods and provide new guidelines for the mechanisms synthesis with a focus on high precision and reproducibility of the path of motion.

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References

Gräser P, Linß S, Zentner L, Theska R (2015) Ultraprecise linear motion generated by means of compliant mechanisms. In: Proceedings of euspen's 15th International Conference & Exhibition, Leuven, Belgium, 01–05 June 2015, pp 241

Howell LL, Magleby SP, Olsen BM (2013) Handbook of compliant mechanisms. Wiley, New York

Liaw HC, Shirinzadeh B (2008) Robust generalised impedance control of piezo-actuated flexure-based fourbar mechanisms for micro-nano manipulation. Sens Actuators, A 148(2):443

Linß S (2015) Ein Beitrag zur geometrischen Gestaltung und Optimierung prismatischer Festkörpergelenke in nachgiebigen Koppelmechanismen. Dissertation, TU Ilmenau

Linß S, Milojevic A, Pavlovic ND et al (2015) Synthesis of compliant mechanisms based on goal-oriented design guidelines for prismatic flexure hinges with polynomial contours. In: Proceedings of the 14th World Congress in Mechanism and Machine Science, Taipei, 25–30 Oct 2015, pp 6

Luo Y, Liu W, Wu L (2015) Analysis of the displacement of lumped compliant parallel-guiding mechanism considering parasitic rotation and deflection on the guiding plate and rigid beams. Mech Mach Theory 91:50–68

Nönnig R (1980) Untersuchungen an Federgelenkführungen unter besonderer Berücksichtigung des räumlichen Verhaltens. Dissertation, Ilmenau: Technische Hochschule Ilmenau

Yang X, Li W, Wang Y, Ye G (2010) Output displacement analysis for compliant single parallel four-bar mechanism. In: Proceedings of the International Conference on Mechatronics and Automation (ICMA) 2010. International Conference on Mechatronics and Automation (ICMA). Xian, China. 04–07 Aug 2010. pp 1354

Zentner L (2014) Nachgiebige Mechanismen. De Gruyter Oldenbourg, München