

# Modeling Internalizing and Externalizing Behaviour in Autism Spectrum Disorders

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**Abstract.** This paper presents a neurologically inspired computational model for Autism Spectrum Disorders addressing internalizing and externalizing behaviour. The model has been verified by mathematical analysis and it is shown how by parameter tuning the model can identify the characteristics of a person based on empirical data.

## 1 Introduction

Over the years, much research has been performed in the area of Autism Spectrum Disorders (ASD); e.g., [9, 19]. Most people think of people with autism as shy or socially disabled, but ASD can occur in different forms. It is sometimes difficult to find a suitable way of counseling somebody with ASD. Persons with ASD often need extra counseling, for example during high school. For such counseling to be effective it is important to have insight in the specific variant the person has, and which specific mental processes and behaviours occur. A computational model can be a basis for someone to get more understanding of such mental processes and behaviours.

One distinction that can be made within ASD is between persons who are internalizing versus those who are externalizing; e.g., [14]. The former type of persons may show anxiety whereas the latter type may show aggression. The computational model presented here addresses these two types of mental processes and behaviours, and enables to model both internalizing persons and externalizing persons with ASD, depending on the settings of certain parameters representing characteristics of the person. Besides this, the model also covers other characteristics that can occur in persons with ASD, such as enhanced sensory processing sensitivity (e.g., [1]), reduced mirror neuron activation (e.g., [12]), imperfect self-other distinction (e.g., [5]), and reduced emotion integration (e.g., [11], pp. 73–74). To cover the latter aspects as well, and to obtain an integrative model, elements of an earlier model from [20, 21] were incorporated.

This new model extends earlier models [20, 21] with different behavior types and contributes to the understanding externalizing and internalizing behavior of persons

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Authorships are based on comparable contribution.

with ASD. This can be used as a basis for human-aware or socially aware computing applications within the field of ASD. In the Sect. 2 neurological background information is discussed. After that, in Sect. 3 the model is presented. Section 4 discusses some simulation experiments. Section 5 contributes verification of the model by mathematical analysis. In Sect. 6 it is shown how the model can be used to automatically identify the characteristics of a person, based on empirical behavioural data and a parameter tuning method. Finally, Sect. 7 is a discussion.

## 2 Neurological Background

The proposed computational model was designed on the basis of findings and theories from Cognitive and Social Neuroscience and Developmental Psychology. In this section these are briefly discussed. Persons with ASD often have an enhanced sensitivity of their sensory processing. Incoming stimuli easily result in a level of stress that has to be handled in some way. Being an internalizing or externalizing human being, is one of the differences between persons with ASD [14]. Internalizing feelings means that you do not show them, but you do feel them. In [14], internalizing persons are described as being withdrawn-depressed, anxious-depressed and having somatic complaints. One of the findings in [23] is that anxiety is often comorbid with ASD, as it is related to enhanced sensitivity and problems with emotion regulation.

Anxiety can be seen as an internalizing behaviour, as the behaviours are not clearly shown to the outside world. However, other persons with ASD are showing more externalizing behaviour; this means that those persons do express their feelings, mostly negative feelings like anger. [14] summarizes externalizing behaviour as aggressive and rule-breaking behaviour. Anxiety is not the only behaviour that can result from bad emotion regulation; [13] describes that aggressive behaviour of persons with ASD can be caused by poor emotion regulation as well. Since aggressive behaviour is an externalizing behaviour type this behaviour is linked to externalizing children. [18] showed that aggressive behaviour in children with ASD could be caused by a combination of poor emotion regulation and impaired understanding of emotions of others.

Anxiety can be seen as a defensive reaction to a potential threat [6], in this case the avoiding of the gaze of the other person is a defensive reaction on the threat of a stimulus for which the person is highly sensitive. She or he does not communicate with the other person, which can be interpreted as a flight response. The externalizing person shows more a fight response and expresses aggressiveness, looks the other person in the eye and communicates with the other person. Such a person does not express anxiety.

The model introduced in Sect. 3 takes into account the two opposite behaviour types, internalizing and externalizing behaviour, as discussed above. The model addresses how these behaviour types relate to the dynamics of processes involving a number of internal states and the expressions of the body, an avoiding gaze and communication.

### 3 The Computational Model

The computational model has been designed using the temporal-causal network modelling approach described in [22]. According to this general dynamic modeling approach a model is designed at a conceptual level, for example, in the form of a graphical conceptual representation or a conceptual matrix representation. A graphical conceptual representation displays nodes for *states* and arrows for *connections* indicating causal impacts from one state to another (e.g., as shown in Fig. 1 below), and includes some additional information in the form of a *connection weight* for each connection (for the strength of the impact), and for each state a *speed factor* (for the timing of the effect of the impact), and the type of *combination function* used (to aggregate multiple impacts on the state). In Table 1 the states used in the model are briefly explained. These states are depicted as nodes in Fig. 1. Sensory representation states  $srs_s$ ,  $srs_{self}$ ,  $srs_B$  and  $srs_b$  are used for stimulus  $s$ , the agent *self*, other agent  $B$ , and body states  $b$  that embody and label emotional states; for body state  $b$  two instances are considered: *anx* for an anxious and *agg* for aggressive. For some of these ( $s$  and  $B$ ), which refer to the external world also sensor states  $ss_s$  and  $ss_B$  are used to incorporate sensing from the world states. Two types of preparation states are considered:  $ps_b$  for body states  $b$  and  $ps_B$  for communication to the other agent  $B$ . The preparation states

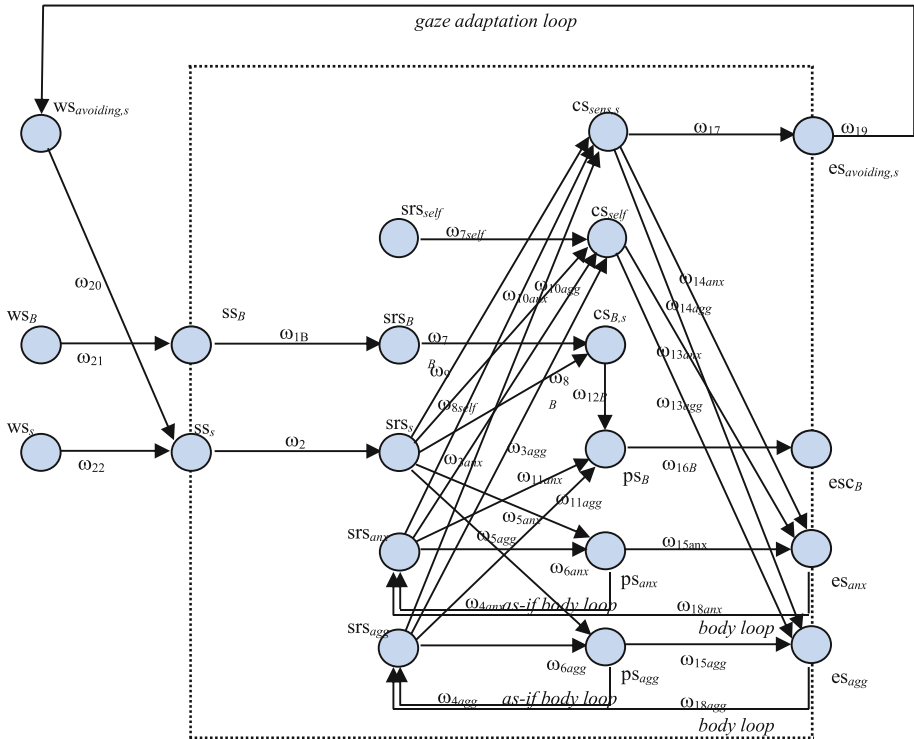


Fig. 1. Conceptual representation of the computational model

**Table 1.** States used in the model

State	Explanation
$ss_s$	sensor state for stimulus $s$
$ss_B$	sensor state for $B$
$srs_s$	sensory representation state of stimulus $s$
$srs_{self}$	sensory representation state of agent <i>self</i>
$srs_B$	sensory representation state of other agent $B$
$srs_b$	sensory representation state of body state $b$
$ps_b$	preparation state for body state $b$
$ps_B$	preparation state for communication to other agent $B$
$cs_{B,s}$	control state for self-other distinction concerning agent $B$
$cs_{sens,s}$	control state for enhanced sensory sensitivity for $s$
$cs_{self}$	control state for the agent itself
$es_b$	execution state for body state $b$
$es_B$	execution state for communication to $B$
$es_{avoiding,s}$	execution state for avoidance of $s$
$ws_s$	world state for stimulus $s$
$ws_B$	world state for other agent $B$
$ws_{avoiding,s}$	world state for gaze avoiding stimulus $s$

$ps_{anx}$  and  $ps_{agg}$  for each of the body states  $anx$  and  $agg$  are affected by the representation states  $srs_{anx}$  and  $srs_{agg}$  for these body states, and in turn affect in a cyclic manner these representation states  $srs_{anx}$  and  $srs_{agg}$ , both by an as-if body loop and a body loop, following [7, 8].

Execution (or expression) states  $es_{anx}$  and  $es_{agg}$  are included for these two types of preparations for body states, plus an execution state  $es_{avoiding,s}$  for a stimulus  $s$  avoiding gaze. The actual execution or expression of preparations for body states and gaze is controlled by control states  $cs_{self}$  for *self* and  $cs_{sens,s}$  for enhanced sensory processing sensitivity for  $s$  (emotion regulation; e.g., [10, 15]). The preparation for communication to the other agent gets control from the control state  $cs_{B,s}$  for self-other distinction (e.g., [12], pp. 201–202, [5]). By such control states specific internal monitoring and control functions are modeled that usually are attributed to specific areas within the prefrontal cortex.

In Table 2 for each state it is indicated which impacts from other states it gets, via which connections and with which weights. In Fig. 1 these weights are depicted as labels for the arrows. Note that as the nodes represent states, the processes happen between these states, as indicated by the arrows representing causal impact; the terms used in the fourth column in Table 2 refer to the types of processes.

The conceptual representation of the model as shown in Fig. 1 and the tables can be transformed in a systematic or even automated manner into a numerical representation of the model as follows [22]:

- At each time point  $t$  each state  $Y$  in the model has a real number value in the interval  $[0, 1]$ , denoted by  $Y(t)$

**Table 2.** Connections and their weights

From states	To state	Weight	Connection
$WS_s$	$SS_s$	$\omega_{22}$	sensing stimulus $s$
$WS_{avoiding,s}$		$\omega_{20}$	suppressing sensing of $s$
$WS_B$	$SS_B$	$\omega_{21}$	sensing agent $B$
$SS_s$	$SFS_s$	$\omega_2$	representing $s$
$SS_B$	$SFS_B$	$\omega_{1B}$	representing $B$
$ps_b$	$SFS_b$	$\omega_{4b}$	predicting $b$
$es_b$		$\omega_{18b}$	effectuating $b$
$SFS_s$	$ps_b$	$\omega_{5b}$	responding $b$
$SFS_b$		$\omega_{6b}$	amplifying $b$
$SFS_{anx}$	$ps_B$	$\omega_{11anx}$	responding communication
$SFS_{agg}$		$\omega_{11agg}$	to anxiety and aggression
$CS_{B,s}$		$\omega_{12B}$	controlling communication
$SFS_B$	$CS_{B,s}$	$\omega_{7B}$	monitoring $B$ for self-other
$SFS_s$		$\omega_{8B}$	monitoring $s$ for self-other
$SFS_s$	$CS_{sens,s}$	$\omega_9$	monitoring $s$ for sensitivity
$SFS_{anx}$		$\omega_{10anx}$	monitoring anxiety
$SFS_{agg}$		$\omega_{10agg}$	monitoring aggression
$SFS_{self}$	$CS_{self}$	$\omega_{7self}$	monitoring $self$
$SFS_s$		$\omega_{8self}$	monitoring $s$
$SFS_{anx}$		$\omega_{3anx}$	monitoring $anx$
$SFS_{agg}$		$\omega_{3agg}$	monitoring $agg$
$CS_{self}$	$es_b$	$\omega_{13b}$	controlling response $b$
$CS_{sens,s}$		$\omega_{14b}$	suppressing response $b$
$ps_b$		$\omega_{15b}$	executing response $b$
$ps_B$	$esc_B$	$\omega_{16}$	executing communication
$CS_{sens,s}$	$es_{avoiding,s}$	$\omega_{17}$	executing avoidance of $s$
$es_{avoiding,s}$	$WS_{avoiding,s}$	$\omega_{19}$	effectuating avoidance of $s$

- At each time point  $t$  each state  $X$  connected to state  $Y$  has an *impact* on  $Y$  defined as  $\mathbf{impact}_{X,Y}(t) = \omega_{X,Y} X(t)$  where  $\omega_{X,Y}$  is the weight of the connection from  $X$  to  $Y$
- The *aggregated impact* of multiple states  $X_i$  on  $Y$  at  $t$  is determined using a *combination function*  $\mathbf{c}_Y(\cdot)$ :

$$\mathbf{aggimpact}_Y(t) = \mathbf{c}_Y(\mathbf{impact}_{X_1,Y}(t), \dots, \mathbf{impact}_{X_k,Y}(t)) = \mathbf{c}_Y(\omega_{X_1,Y}X_1(t), \dots, \omega_{X_k,Y}X_k(t))$$

where  $X_i$  are the states with connections to state  $Y$

- The effect of  $\mathbf{aggimpact}_Y(t)$  on  $Y$  is exerted over time gradually, depending on *speed factor*  $\eta_Y$ :

$$Y(t + \Delta t) = Y(t) + \eta_Y [\mathbf{aggimpact}_Y(t) - Y(t)] \Delta t$$

or  $\mathbf{d}Y(t)/\mathbf{d}t = \eta_Y [\mathbf{aggimpact}_Y(t) - Y(t)]$

- Thus the following *difference* and *differential equation* for  $Y$  are obtained:

$$Y(t + \Delta t) = Y(t) + \eta_Y [\mathbf{c}_Y(\omega_{X_1,Y} X_1(t), \dots, \omega_{X_k,Y} X_k(t)) - Y(t)] \Delta t$$

$$\mathbf{d}Y(t)/\mathbf{d}t = \eta_Y [\mathbf{c}_Y(\omega_{X_1,Y} X_1(t), \dots, \omega_{X_k,Y} X_k(t)) - Y(t)]$$

As an example, according to the pattern described above the difference and differential equation for  $ps_{anx}$  are as follows:

$$ps_{anx}(t + \Delta t) = ps_{anx}(t) + \eta_{ps_{anx}} [\mathbf{c}_{ps_{anx}}(\omega_{5anx} srs_s(t), \omega_{6anx} srs_{anx}(t)) - ps_{anx}(t)] \Delta t$$

$$\mathbf{d}ps_{anx}/\mathbf{d}t = \eta_{ps_{anx}} [\mathbf{c}_{ps_{anx}}(\omega_{5anx} srs_s(t), \omega_{6anx} srs_{anx}(t)) - ps_{anx}(t)]$$

So, for any set of values for the connection weights, speed factors and any choice for combination functions, each state of the model gets a difference or differential equation assigned. For the model considered here this makes a set of 17 coupled difference or differential equations, that together, in mutual interaction describe the model's behaviour. Note that the speed factors enable to obtain a realistic timing of the different states in the model, for example, to tune the model to the timing of processes in the real world.

For all states except the sensor state  $ss_s$ , for the combination function either the *identity function*  $\mathbf{id}(\dots)$  or the *advanced logistic sum combination function*  $\mathbf{alogistic}_{\sigma,\tau}(\dots)$  is used [22]:

$$\mathbf{c}_Y(V) = \mathbf{id}(V) = V$$

$$\mathbf{c}_Y(V_1, \dots, V_k) = \mathbf{alogistic}_{\sigma,\tau}(V_1, \dots, V_k) = \left( \frac{1}{1 + e^{-\sigma(V_1 + \dots + V_k - \tau)}} - \frac{1}{1 + e^{\sigma\tau}} \right) (1 + e^{-\sigma\tau})$$

Here  $\sigma$  is a *steepness* parameter and  $\tau$  a *threshold* parameter. The advanced logistic sum combination function has the property that activation levels 0 are mapped to 0 and keeps values below 1. The identity function  $\mathbf{id}(\dots)$  is used for the 6 states with a single impact:  $ss_B$ ,  $srs_B$ ,  $srs_s$ ,  $esc_B$ ,  $es_{avoiding,s}$ ,  $ws_{avoiding,s}$ . For example, the difference and differential equation for  $srs_s$  are as follows:

$$srs_s(t + \Delta t) = srs_s(t) + \eta_{srs_s} [\omega_{2ss_s}(t) - srs_s(t)] \Delta t$$

$$\mathbf{d}srs_s(t)/\mathbf{d}t = \eta_{srs_s} [\omega_{2ss_s}(t) - srs_s(t)]$$

The function  $\mathbf{alogistic}_{\sigma,\tau}(\dots)$  is used as combination function for the 10 states with multiple impacts, except the sensor state  $ss_s$  for stimulus  $s$ :  $srs_{anx}$ ,  $srs_{agg}$ ,  $cs_{sens,s}$ ,  $cs_{self}$ ,  $cs_{B,s}$ ,  $ps_B$ ,  $ps_{anx}$ ,  $ps_{agg}$ ,  $es_{anx}$ ,  $es_{agg}$ . For example, the difference and differential equation for  $ps_{anx}$  are as follows:

$$ps_{anx}(t + \Delta t) = ps_{anx}(t) + \eta_{ps_{anx}} [\mathbf{alogistic}_{\sigma,\tau}(\omega_{5anx} srs_s(t), \omega_{6anx} srs_{anx}(t)) - ps_{anx}(t)] \Delta t$$

$$\mathbf{d}ps_{anx}/\mathbf{d}t = \eta_{ps_{anx}} [\mathbf{alogistic}_{\sigma,\tau}(\omega_{5anx} srs_s(t), \omega_{6anx} srs_{anx}(t)) - ps_{anx}(t)]$$

For the sensor state  $ss_s$ , the effect of the avoiding gaze is modelled by the following combination function  $\mathbf{c}_{ss_s}(V_1, V_2)$ , where  $V_1$  refers to the impact  $\omega_{22} ws_s(t)$  from  $ws_s$  on  $ss_s$  and  $V_2$  to the impact  $\omega_{20} ws_{avoiding,s}(t)$  from  $ws_{avoiding,s}$  on  $ss_s$ :

$$\mathbf{c}_{ss_s}(V_1, V_2) = V_1(1 - V_2)$$

This function makes the sensing of stimulus  $s$  inverse proportional to the extent of avoidance; e.g., sensing  $s$  becomes 0 when avoidance is 1, and  $V_1$  when avoidance is 0. According to this combination function the difference and differential equation for  $ss_s$  are as follows:

$$\begin{aligned} ss_s(t + Dt) &= ss_s(t) + \eta_{ss_s} [\omega_{22} ws_s(t)(1 - \omega_{20} ws_{avoiding,s}(t)) - ss_s(t)] \Delta t \\ \mathbf{d}ss_s/\mathbf{d}t &= \eta_{ss_s} [\omega_{22} ws_s(t)(1 - \omega_{20} ws_{avoiding,s}(t)) - ss_s(t)] \end{aligned}$$

## 4 Simulation Experiments

The numerical representation of the model discussed above (in the form of the 17 difference equations for the 17 states) has been implemented in Python. In this section simulations are discussed that show the two different types of behaviours. In order to let the model show the behavior of an externalizing or internalizing person the parameters are constrained. Using the constraints shown in Table 3 the behavior of an externalizing or internalizing case will be shown in a realistic manner, according to the neurological background of Sect. 2. Internalizing persons suppress the aggressive response (high  $\omega_{14agg}$ ) more than the anxious response (low  $\omega_{14anx}$ ); for externalizing people this is opposite. Externalizing persons have a strong control over their communication (high  $\omega_{12}$ ), while internalizing persons don't (low  $\omega_{12}$ ). Externalizing persons have a low avoidance of stimuli (low  $\omega_{17}$ ), while internalizing people have a stronger tendency to avoid stimuli (high  $\omega_{17}$ ). The model also functions with parameters disregarding the constraints. However, in these cases the behavior cannot be classified as either internalizing or externalizing.

For the simulations discussed here the step size  $\Delta t$  was 0.5, all speed factors were 1, and all connection weights except the four in Table 3 were always 1. Moreover, for the states that use an advanced logistic sum combination function the threshold and steepness values were as shown in Table 4.

The initial values of all states were 0, except for the world states  $ws_s$  and  $ws_B$  for stimulus  $s$  and the other agent  $B$ , which as a form of input for the agent had constant

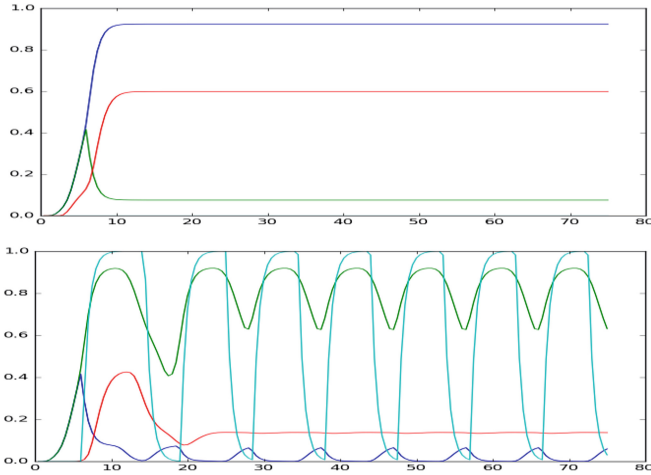
**Table 3.** Intervals for parameters for externalizing and internalizing

Weight	Externalizing	Internalizing
$\omega_{14agg}$	[-0.3, 0]	[-1, -0.7]
$\omega_{14anx}$	[-1, -0.7]	[-0.3, 0]
$\omega_{12}$	[0.7, 1]	[0, 0.3]
$\omega_{17}$	[0, 0.3]	[0.7, 1]

**Table 4.** Parameter values used for steepness  $\sigma$  and threshold  $\tau$ 

state	$\tau$	$\sigma$	state	$\tau$	$\sigma$	state	$\tau$	$\sigma$
srs <sub>anx</sub>	0.8	8	cs <sub>B,s</sub>	1	40	es <sub>anx</sub>	1.5	5
srs <sub>agg</sub>	0.8	8	cs <sub>self</sub>	1	40	es <sub>agg</sub>	1.5	5
ps <sub>anx</sub>	1	8	cs <sub>sens,s</sub>	1.2	40	esc <sub>B</sub>	0.5	2
ps <sub>agg</sub>	1	8	ps <sub>B</sub>	1.5	2	es <sub>avoiding,s</sub>	0.5	40

value 1 for the whole time. The upper graph in Fig. 2 shows simulation results for an externalizing, aggressive person ( $\omega_{12} = 1$ ,  $\omega_{14agg} = 0$ ,  $\omega_{14anx} = -1$  and  $\omega_{17} = 0$ ), i.e., there is no suppression of the aggressive response, but there is suppression of the anxious response, there is no stimulus avoiding gaze, and there is communication. In the first few time steps all values go up: in the beginning, there is an input for the sensor states in the model and it takes some time to reach all other states, and in particular the control states.



**Fig. 2.** Simulations of an externalizing person (upper graph) and an internalizing person (lower graph). Horizontal axis: time. Vertical axis: activation value. Light blue:  $es_{avoiding,s}$  (gaze avoiding  $s$ ). Green:  $es_{anx}$  (expressing anxiety). Red:  $esc_B$  (communication). Dark blue:  $es_{agg}$  (expressing aggressiveness). (Color figure online)

In time step 6, the anxious expression declines fast. This is due to the suppressive effect of  $\omega_{14anx}$  (from  $cs_{sens,s}$  to  $es_{anx}$ ). The expressed aggression goes up and stays activated; the person is aggressive as long as the stimulus is present. Because in this simulation the stimulus never fades away, the aggressiveness stays too. When the person becomes aggressive, he/she faces the stimulus (no avoidance) and starts to



communicate to the other agent (e.g., yelling at somebody). In the simulation the aggressiveness level never becomes 1; there is always a little bit of anxiousness present.

The lower graph in Fig. 4 shows a simulation of an internalizing, anxious person ( $\omega_{12} = 0$ ,  $\omega_{14agg} = -1$ ,  $\omega_{14anx} = 0$ , and  $\omega_{17} = 1$ ); i.e., there is suppression of the aggressive response, but there is no suppression of the anxious response, there is an avoiding gaze, and there is no communication. In the beginning, the values of all states go up again, but in time step 6, it can be seen that the aggressiveness declines rapidly. As part of this drop, body representation state  $srs_{agg}$  drops, which has influence on the preparation state  $ps_B$  for communication. Also body representation state  $srs_{anx}$  (which is high) has an influence on  $ps_B$ . This causes that  $ps_B$  is not declining immediately. Because the control state  $cs_{sens,s}$  is high, the execution of the avoiding gaze  $es_{avoiding,s}$  becomes also high which causes the person to look away, this causes that the stimulus fades away for that person and so is the expression of anxiousness. Because the anxiousness drops, and body representation state  $srs_{agg}$  is still low, the communication preparation state  $ps_B$  becomes lower which causes the communication to stop. When the person is less anxious, the control state  $cs_{sens,s}$  becomes lower which causes less suppression of aggressive expression  $es_{agg}$  and therefore the aggressiveness is shown a little. Because the stimulus is fading away for the person, there is no reason not to look at  $s$  anymore and the person looks at it again. The process goes on like this, ending up in a repeating (limit cycle) pattern. The communication is not coming back, this makes sense because if a person is internalizing, he or she withdraws him/herself socially and does not communicate anymore.

## 5 Verification by Mathematical Analysis

In this section, it is discussed how a mathematical analysis was performed of the equilibria of the model, in order to enable verification of (the implementation of) the model. A state  $Y$  has a *stationary point* at  $t$  if  $dY(t)/dt = 0$ . The model is in *equilibrium* at  $t$  if every state  $Y$  of the model has a stationary point at  $t$ . From the specific format of the differential or difference equations it follows that state  $Y$  has a stationary point at  $t$  if and only if

$$Y(t) = \mathbf{c}_Y(\omega_{X_1,Y} X_1(t), \dots, \omega_{X_k,Y} X_k(t))$$

where  $X_i$  are the states with connections to state  $Y$ , and  $\mathbf{c}_{X_i}(\dots)$  is the combination function for  $Y$ . If the values of the states for an equilibrium are indicated by  $\underline{\mathbf{X}}_i$  then being in an equilibrium state is equivalent to a set of 17 equilibrium equations for the 17 states  $X_i$  of the model:

$$\underline{\mathbf{X}}_i = \mathbf{c}_{X_i}(\omega_{X_1,X_i} \underline{\mathbf{X}}_1, \dots, \omega_{X_k,X_i} \underline{\mathbf{X}}_k)$$

For example, for state  $srs_s$  the identity function is used as a combination function; then the above equilibrium equation is

$$\underline{srs}_s = \omega_2 \underline{ss}_s$$

The 5 equilibrium equations for  $ss_B$ ,  $srs_B$ ,  $esc_B$ ,  $es_{avoiding,s}$ ,  $ws_{avoiding,s}$  are similar to this one. As another example, for state  $ps_{anx}$  the combination function is the advanced logistic function; then the equilibrium equation is

$$\underline{ps}_{anx} = \mathbf{alogistic}_{\sigma,\tau}(\omega_{5anx} \underline{srs}_s, \omega_{6anx} \underline{srs}_{anx})$$

The 9 equilibrium equations for  $srs_{anx}$ ,  $srs_{agg}$ ,  $cs_{sens,s}$ ,  $cs_{self}$ ,  $cs_{B,s}$ ,  $ps_B$ ,  $ps_{agg}$ ,  $es_{anx}$ ,  $es_{agg}$  are similar to this more complicated one. Finally, the equilibrium equation for  $ss_s$  is:

$$\underline{ss}_s = \omega_{22} \underline{ws}_s (1 - \omega_{20} \underline{ws}_{avoiding,s})$$

These 17 equilibrium equations cannot be solved analytically in an explicit manner, due to the 10 equations among them involving a logistic function. However, they still can be used for verification of the model. This can be done by substituting the values found in a simulation at the end time in these equations, and then check whether the equations hold. This indeed has been done for a number of arbitrary cases (for different parameter values for  $\omega_{12}$ ,  $\omega_{14agg}$ ,  $\omega_{14anx}$ , and  $\omega_{17}$ ) for the externalizing type of person, and the equations turned out to always hold (with an accuracy  $10^{-15}$  or lower).

For the internalizing type the equilibrium equations never hold, as then the pattern becomes a limit cycle with state values changing all the time. However, in a limit cycle each state fluctuates between a minimum and a maximum value. At the time points for these minima and maxima the state has a stationary point, which means that the equation

$$Y(t) = \mathbf{c}_Y(\omega_{X_1,Y} X_1(t), \dots, \omega_{X_k,Y} X_k(t))$$

should be fulfilled. This can be verified as well. This indeed has been done for the minima and maxima within the limit cycle of the internalizing type with  $\omega_{12} = 0$ ,  $\omega_{14agg} = -1$ ,  $\omega_{14anx} = 0$ , and  $\omega_{17} = 1$  (using step size 0.05). It turned out that the stationary point equations were indeed fulfilled for all states of the model, with an average accuracy of 0.0041 over the minima and maxima of all states (the maximal deviation among the minima and maxima of the different states was 0.027, which is still reasonable, as in that case a very high steepness  $\sigma = 40$  was applied, which can lead to sharp turning points).

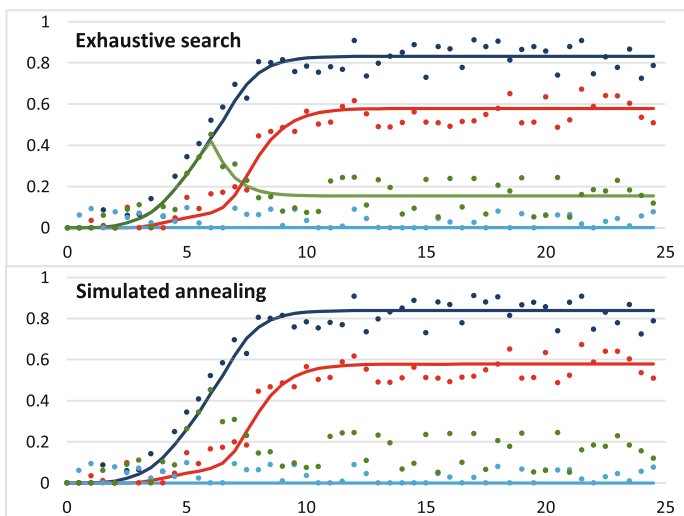
The period of the limit cycles was also analyzed. The period was analyzed after the first drop, as only from there the graph becomes stable. Analyzing this, for the most clear internalizing type it was found that the period is constant for all states, with value 9.5. This is coherent with the theory behind the model that the same process is repeated all the time. The length of the period depends on the exact parameter values for  $\omega_{12}$ ,  $\omega_{14agg}$ ,  $\omega_{14anx}$ , and  $\omega_{17}$ . For other values within the constraints it can reach 17. Unfortunately, there are no methods known to analyse this period mathematically.

The verification outcomes provide evidence that the model (as implemented in Python) does what is expected.

## 6 Tuning Characteristics in the Model

The model can be used to model different types of persons, with different characteristics. These personal characteristics are represented by the values of parameters in the model. For practical use of the model, for a given person these values have to be found. These values could be based on questionnaires filled by the person, but it would be more convenient if the characteristics can be determined automatically based on observed behaviour of the person; this can be used as a form of automated diagnosis. This section shows how this indeed can be done. In order to test this, some observable empirical data are needed.

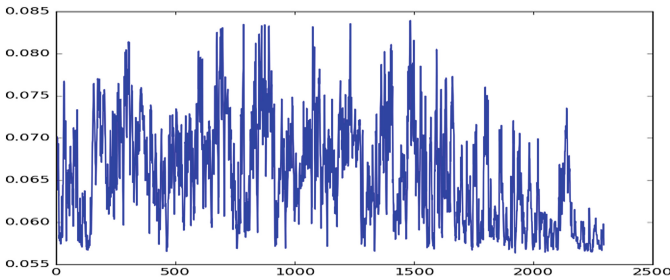
Because real empirical data were not available yet, the (pseudo-empirical) data used to explain and test the approach were generated (by a third person) based on the model with random parameter values for  $\omega_{12}$ ,  $\omega_{14agg}$ ,  $\omega_{14anx}$ , and  $\omega_{17}$  that satisfy the constraints described in Table 3, after which noise was added to make the data more like realistic. These parameters have been addressed as these parameters determine the type of characteristics of a person as addressed in this paper. The observable states used are the body and communication execution (or expression) states  $es_{agg}$ ,  $es_{anx}$ ,  $es_{avoidance,s}$ , and  $esc_B$ . The dots in Fig. 3 show these expression states according to the pseudo-empirical data. The dots show high values for  $es_{agg}$  and  $esc_B$ , and low values for  $es_{anx}$  and  $es_{avoidance,s}$ . This already indicates that the person can be an externalizing person. To find the parameter values characterizing the person represented by these data, two parameter estimation methods were used: exhaustive search and simulated annealing. For both cases an error function based on the sum of squares of the deviations was used.



**Fig. 3.** The model (lines) using parameter values found by exhaustive search (upper graph) and simulated annealing (lower graph) in comparison to the data (dots). Horizontal axis: time. Vertical axis: activation value. Light blue:  $es_{avoiding,s}$  (gaze avoiding  $s$ ). Green:  $es_{anx}$  (expressing anxiety). Red:  $esc_B$  (communication). Dark blue:  $es_{agg}$  (expressing aggressiveness). (Color figure online)

The constraints for the intervals of the possible values of the parameters limit the set of possible parameter values. Therefore *exhaustive search* can be feasible. This method has the advantage that a set with all possibilities is created, which gives the certainty that the correct set (global optimum) is among them. Other methods may only come up with a local optimum. Using exhaustive search with grain size 0.01, the following weights were found:  $\omega_{12} = 0.71$ ,  $\omega_{14agg} = -0.18$ ,  $\omega_{14anx} = -0.84$ , and  $\omega_{17} = 0.3$ . The accuracy for these weights was found to be 0.0563. Such values indeed are expected to represent somebody who does externalizing. This shows that it is indeed possible to identify the characteristics of a person expressed in terms of the found parameter values, using exhaustive search applied to behavioural data. In Fig. 3 (upper graph) it is shown how for these parameter values the model fits to the data.

As an alternative parameter tuning method, also a *simulated annealing* approach was applied. The lower graph in Fig. 3 shows how for these parameter values the model fits to the data. It can be seen that over time (and decreasing temperature) the changes become smaller. The graph in Fig. 4 shows the plot of the error during this process.



**Fig. 4.** Progression of the error during the simulated annealing. Horizontal axis: number of iterations. Vertical axis: error value

The best weights that were found are  $\omega_{12} = 0.721$ ,  $\omega_{14agg} = -0.195$ ,  $\omega_{14anx} = -0.884$ ,  $\omega_{17} = 0.117$ , with an accuracy of 0.0573. Note that this accuracy for simulated annealing is just a bit worse than what was found by exhaustive search. Moreover, note that apparently the value of  $\omega_{17}$  does not matter much, as it can be 0.117 or 0.3 without much difference in accuracy. This indeed can be explained from the model, as the threshold of  $es_{avoiding,s}$  is 0.5 and steepness 40 (see Table 4). This means that all values of  $\omega_{17}$  from 0 to 0.3 will lead to (practically) no activation of  $es_{avoiding,s}$  (which also can be seen in Fig. 3: the flat light blue line).

## 7 Discussion

This paper presented a computational model for persons with ASD enabling to distinguish two different behaviour types that are prevalent in ASD: internalizing and externalizing behaviour [14]. The model was inspired by findings and theories from Cognitive and Social Neuroscience and designed as a network of mental states

according to the temporal-causal network modeling approach presented in [22]. By simulation experiments and mathematical analysis the model was verified.

The presented model specifically addresses findings and theories concerning internalizing and externalizing behaviour types within ASD. However, as elements from the model described in [20, 21] were adopted as well, the model also integrates some other aspects of ASD, addressed by theories: reduced mirror function or poor self-other distinction; e.g., [5, 12].

The model represents specific personal characteristics by specific values of parameters included in the model, such as connection weights. By using proper choices for these connection weights, the model can either simulate an internalizing person or an externalizing person. Moreover, it was shown how based on a given data set concerning a person's behaviour, by parameter estimation methods the behaviour type of this person can be identified automatically.

The computational model proposed here can be used as an ingredient to develop human-aware or socially aware computing applications (e.g. [16, 17, 19]) that can provide help in getting more understanding of the different behaviour types and their influence on the behaviour of a person with ASD. More specifically, in [2, 19] it is shown how such applications can be designed with knowledge of human and/or social processes as a main ingredient represented by a computational model of these processes which is embedded within the application. Such computational models can have the form, for example, of qualitative causal models, or of dynamical numerical models. As an example, in [3, 4] this design approach is illustrated to obtain a human-aware software agent supporting professionals in attention-demanding tasks, based on an embedded dynamical numerical model for attention. The computational model for ASD proposed here can be used in a similar manner to design a human-aware or socially aware software agent to support persons with ASD. This might be helpful in particular for those who are counseling or supervising persons with ASD. The method shown to identify the behaviour type of a person based on empirical behavioural data can be useful, for example, in choosing a counseling approach.

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