Chapter 5
A Deep Understanding of Fractions Supports Student Success in Algebra

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Abstract Algebra is frequently referred to as the "gateway" course for high school mathematics in much the same way as calculus can "open" or "close" doors for students interested in pursuing degrees in science, technology, engineering, and mathematics (STEM) areas. This chapter presents the idea that students' challenges with algebra begin well before their first course in algebra and that these challenges are embedded in a complex set of issues. Weak or incomplete mathematical understanding of rational number concepts has a profound impact on students' success in algebra and subsequently, courses that follow where students are expected to confidently, competently, and efficiently address situations in which "and the rest is just algebra" is invoked. Recognizing that developing students' deep understanding of rational number concepts requires years of nurturing and care by capable, well-prepared teachers, both in terms of content and pedagogical knowledge, and a discussion of issues related to teacher preparation and teacher shortages and how these impact students' preparedness for algebra and their success in mathematics is presented.

Keywords Fractions • Algebra • Teacher education • Rational numbers • Proportional reasoning • Teacher shortages

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5.1 Introduction

Over the last two decades, numerous reports have been written that focus on the need for improved mathematics and science teaching and learning in the United States. The pressure for global competitiveness and ever-changing demands of the workforce in the areas of science, technology, engineering, and mathematics (STEM) have propelled the conversation forward with intensity regarding learning outcomes in the STEM areas. In light of increased attention on STEM learning outcomes, the need for individuals prepared to enter the STEM fields, and, in general, the "need for more powerful learning focused on the demands of life, work, and citizenship in the twenty-first century" (Darling-Hammond, 2010), more students are taking algebra courses. The link to increased educational and economic opportunities has also been linked to the increase in the number of students taking algebra courses (Gamoran & Hannigan, 2000; Moses & Cobb, 2001; Nord et al., 2011; Rampey, Dion, & Donahue, 2009). Further, over the past several decades, and particularly since 2002 when the reauthorization of the Elementary and Secondary Education Act (ESEA) of 1965 commonly known as the "No Child Left Behind" Act attached passing exams based on algebra courses to graduation, more states require the passing of an algebra course for all students for graduation.

Algebra is frequently referred to as the "gateway" course for high school mathematics in much the same way as calculus can "open" or "close" doors for students interested in pursuing degrees in STEM areas. Stein, Kaufman, Sherman, and Hillen (2011) state that "[h]istorically, algebra has served a gatekeeper to advanced mathematics and science course taking and entry into high-paying, technical careers. Increased recognition of this phenomenon has led to a growing trend, . . . , for more students taking algebra in eighth grade" (p. 483). Their study examines algebra enrollment trends using data from the Early Childhood Longitudinal Study, Kindergarten class of 1988–99 (ECLS-K), the High School Transcript Study, National Assessment of Educational Progress (NAEP), NAEP Long-Term Trends, and Trends in International Mathematics and Science Study (TIMMS) and reveals a significant increase in algebra enrollment in eighth grade over the past two decades. Analysis of these sources provides empirical data that from the late 1980s to the early 1990s, enrollment in algebra for the nation's eighth graders had increased from 15 to 20 % to around 30 % in 2009. Additionally, their study reveals consistent lower enrolment in "eighth and ninth grade algebra among minorities and low-income students" (p. 460). This finding, along with an examination of policies related to who takes algebra and when students take algebra, called into question the preparedness of the students taking algebra. If the policies in place are universal, then it is likely many students taking algebra may not be prepared for the rigor and abstraction required for algebra. However, if the policies related to who takes algebra and when they take algebra allow for selection, evidence suggests that some prepared students from traditionally marginalized groups may be excluded from taking algebra prior to high school.

5.2 The Challenges of Algebra Preparedness

Research from various fields including mathematics education, mathematics teacher education, and mathematics reveals there is a confluence of issues that impact students' preparedness for algebra (e.g., Ball, 1993; Booth & Newton, 2012; Booth & Siegler, 2006, 2008; Harvey, 2012; Lamon, 2012; Ma, 1999; Newton, 2008; NMAP, 2008; Wu, 2001). Students' mathematical background and abilities, misconceptions and limitations related to their mathematical understanding, student self-confidence related to mathematics, policies related to the mathematics required in school prior to the taking of algebra, and teacher preparation for teaching mathematics at the elementary and middle school levels are among the chief contributors to this problem. In keeping with the title of this volume, "and the rest is just algebra," this chapter will present the argument that students' challenges with algebra begin well before their first course in algebra and that these challenges are embedded in a complex set of issues. While recognizing the complexity of this problem, this chapter will specifically explore the impact of weak or incomplete mathematical understanding of rational number concepts on students' success in algebra and subsequently, courses that follow. Also, included will be a discussion of issues related to teacher preparation and teacher shortages and how these impact students' preparedness for algebra and their success in mathematics.

5.3 Fraction Understanding Supports Algebra

The National Mathematics Advisory Panel (NMAP, 2008) suggests that a central goal of student's mathematical development is the conceptual understanding of fractions and procedural fluency with rational numbers and further implies that these competencies provide the critical foundation for algebra learning. Research corroborates the suggestions made by NMAP regarding the impact of weak or limited mathematical understanding at the elementary and middle school level and the significant impact it has on the future mathematical success of students and their educational possibilities (e.g., Booth & Newton, 2012; Wu, 2001). Brown and Quinn (2007) state that "vague fraction concepts and misunderstood fraction algorithms will ultimately be generalised into vague algebraic concepts and procedures. The lack of precise definitions and reliance upon shortcuts that are thoughtlessly given to students are likely to hinder performance in algebra" (p. 29).

Research has shown that much of the basis for algebraic understanding and algebraic thinking is contingent on a clear understanding of rational number concepts (Driscoll, 1982; Kieren, 1980; Lamon, 1999; Wu, 2001) and the ability to manipulate common fractions. For example, Booth and Newton (2012) found that "knowledge of fraction magnitudes—more so than whole number magnitude . . . is related to students' skill in early algebra" (p. 251). Beyond simply using fractions and their related operations with fractions to solve algebraic problems involving fractions, students depend on their understanding of rates and ratios, often represented as fractions, to make sense of the key concepts of rate and variability in algebra. Wu (2001) claims that since operations with fractions can be generalized, fractions provide an opportunity to introduce students to the use of variables. Further, fractions are found throughout algebra. From coefficients to the slope of linear equations, from constants to solutions, from linear equations to completing the square, from solving systems of linear equations to solving rational equations, and from simple probabilities to the binomial theorem, algebra is brimming with examples that are directly and indirectly related to fractions. Wu (2001) suggests that "[w]ith proper infusion of precise definitions, clear explanations, and symbolic computations, the teaching of fractions can eventually hope to contribute to mathematics learning in general and the learning of algebra in particular" (p. 17).

Unfortunately, rational number concepts and fractions are challenging for many students, and students' understanding of rational numbers, or fractions, and misconceptions students might develop about fractions have a profound impact on their ability to learn algebra. According to Lamon (2012):

> Understanding fractions marks only the beginning of the journey toward rational number understanding. By the end of the middle school years, as a result of maturation, experience, and fraction instruction, it is assumed that students are capable of a formal thought process called proportional reasoning. This form of reasoning opens the door to high-school mathematics and science, and eventually, to careers in the mathematical sciences. The losses that occur because of the gaps in conceptual understanding about fractions, ratios, and related topics are incalculable. The consequences of doing, rather than understanding, directly or indirectly affect a person's attitudes towards mathematics, enjoyment and motivation in learning, course selection in mathematics and science, achievement, career flexibility, and even the ability to fully appreciate some of the simplest phenomena in everyday life (p. xi).

Algebra is replete with fractions and understanding many of the concepts found within algebra is dependent on student understanding of the multiple interpretations of fractions.

5.4 Deep Understanding of Fractions

Helping students develop a deep understanding and rich number sense about fractions and rational numbers including conceptual understanding and procedural fluency is not an easy task. It requires deep content knowledge specific to rational numbers on behalf of the teachers and requires several years to develop in students. Kieren (1988) reported that students in the United States rely heavily on rote memory of rules to solve fraction problems. The 2004 National Assessment of Educational Progress (NAEP), often referred to as the Nations Report Card, reported that 50 % of eighth grade students could not order three fractions from least to greatest and that fewer than 30 % of 17-year-olds correctly translated 0.029 as 29/1000 (Kloosterman, 2010). Further, Rittle-Johnson, Siegler, and Alibali (2001) conducted one-on-one controlled experiments and found that when asked which of two decimals 0.274 and 0.83 is greater, most fifth and sixth graders choose 0.274. Siegler et al. (2010) suggest that the lack of student conceptual understanding includes students not viewing fractions as numbers, viewing fractions as meaningless symbols that need to be manipulated in a variety of ways to produce answers that satisfy a teacher, focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number, and confusing properties of fractions with those of whole numbers. They go on to state that "A high percentage of U.S. students lack conceptual understanding of fractions, even after studying fractions for several years; this, in turn, limits students' ability to solve problems with fractions and to learn and apply computational procedures involving fractions" (pp. 6–7).

The challenges are significant in the United States with regard to fraction and rational number teaching and student understanding. In light of these and other concerning findings, understanding this challenge and working to improve student learning related to fractions and rational numbers have been a focus of the mathematics education community for several decades. In the late 1980s, the publication of the National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989) and several other NCTM publications in the decade that followed helped drive the charge for change in fraction instruction. Since that time, there have been continual calls for fraction instruction to move from a procedural focus to one aimed at developing deep conceptual understanding (Lamon, 2012; Van de Walle, 2007). Understanding fractions concepts with depth is a complex endeavor and requires that teachers understand the work on fraction meanings and constructs. Kieren's work in the 1970s revealed the complexity of fraction understanding suggesting that the concept of fractions consists of several sub-constructs or meanings (1976).

In his work, Kieren suggested that one must understand each sub-construct independently and jointly in order to have a general understanding of fractions. Initially, Kieren identified four meanings for fractions: measure, ratio, quotient, and operator. Originally, the notion of the part-whole relationship served as a basis for the development of the other sub-constructs and as such was not included in the list

Interpretations of $\frac{4}{5}$	Meaning
Part-Whole Comparisons with Unitizing "4 parts out of 5 equal parts"	$\frac{4}{5}$ means four parts out of five equal parts of the unit, with equivalent fractions found by thinking of the parts in terms of larger or smaller chunks.
Measure "4 ($\frac{1}{5}$ – units)"	$\frac{4}{5}$ means a distance of 4 ($\frac{1}{5}$ – units) from 0 on the number line or 4 ($\frac{1}{5}$ – units) of a given area.
Operator "$\frac{4}{5}$ of something"	$\frac{4}{5}$ gives a rule that tells how to operate on a unit (or on the result of a previous operation); multiply 4 and divide your result by 5 or divide by 5 and multiply the result by 4. This results in multiple meanings for $\frac{4}{5}$; 4 ($\frac{1}{5}$ – units), 1 ($\frac{4}{5}$ – units), and $\frac{1}{5}$ (4 – units).
Quotient "4 divided by 5"	$\frac{4}{5}$ is the amount each person receives when 5 people share a 4 – unit of something.
Ratios "4 to 5"	4:5 is a relationship in which there are 4 A's compared, in a multiplicative rather than an additive sense, to 5 B's.

Fig. 5.1 Fraction interpretations and meanings (adapted from Lamon, 2012)

as a separate construct. Kieren's ideas were later expanded by Behr, Lesh, Post, and Silver (1983) who recommended that the part-whole relationship as seen by Kieren to be embedded in the four other meanings be considered a distinct sub-construct of fractions (see Fig. 5.1). Their work connected the part-whole meaning of fractions with the notion of portioning and establishing it as a distinct sub-construct of fractions. Behr et al.'s (1983) work revealed that the process of partitioning and the part-whole sub-construct of rational numbers are fundamental for developing a deep understanding of the four other constructs of fractions. Since that time, others (Lamon, 1999, 2012; Mack, 2001; Simon, 1993; Tobias, 2012) have suggested that conceptualizing the whole is important for understanding many significant mathematical concepts including contextualizing situations, understanding procedures, and interpreting solutions.

The notion of part-whole as a construct for fractions and rational numbers occupies a significant place in curricular materials for elementary children throughout the world. This is based on the assumption that conceptualizing the whole and understanding part-whole relationships is fundamental to many important mathematical concepts including the four constructs of fractions identified by Kieren (1976) and that operations with fractions are connected to the part-whole relationship (Behr et al., 1983). Lamon (2012) discusses the idea that more emphasis should not be placed on one sub-construct, or interpretation, of fractions and that rather, teachers should understand that no single interpretation is a panacea. Cramer and Whitney (2010), however, suggested that the part-whole sub-construct is a good place for children to begin to develop an understanding of fractions.

Many researchers agree but also believe that while the part-whole meaning of fractions is the most commonly relied upon interpretation in curricular materials, placing more emphasis on other interpretations would help students gain a better understanding of fractions (e.g., Clarke, Roche, & Mitchell, 2008; Siebert & Gaskin, 2006). This suggests that an emphasis on the part-whole construct or interpretation of fractions, while perhaps serving as a basis for understanding fractions, is not sufficient by itself for deep understanding and flexibility with fractions. Lamon (2007) believes that we have a tremendous problem related to fraction teaching and learning due to the fact that most teachers only understand and teach fractions from a part-whole understanding. The findings of a study conducted by Reeder and Utley (under review) focused on prospective elementary teachers corroborates this claim. The prospective elementary teachers in their study relied almost exclusively on part-whole understanding of fractions as part of a whole to answer basic questions about fractions, and when asked how they would explain the concept of fractions to their students, the majority of the participants provided a part-whole explanation.

5.5 The Importance of Proportional Reasoning for Algebra

While there are functional differences between each of the five sub-constructs of fractions, they are interrelated. In addition, it is believed that, fractions should be taught in such a way that students develop a holistic understanding of fractions that includes the multiple perspectives of each of the sub-constructs. In this way, students may be able to work more flexibly within varied contexts, with more representations, and develop the higher-order thinking needed for proportional reasoning (Lesh, Post, & Behr, 1988). However, the sub-construct of ratio and rates is most related to proportional reasoning which makes it of paramount importance for student success in algebra. Proportional reasoning has been referred to as the cornerstone of higher levels of mathematics success (Kilpatrick, Swafford, & Findell, 2001; Lamon, 1999; Lesh et al., 1988). Wright (2005) states that proportional reasoning involves “making multiplicative comparisons between quantities” (p. 363), and Lesh et al. (1988) add that it is “the ability to mentally store and process several pieces of information” (p. 93). According to Lamon (1999), “proportional reasoning is one of the best indicators that a student has attained understanding of rational numbers” (p. 3).

The ability to reason proportionally involves a student’s ability to understand variation and covariation and make multiple comparisons. It involves students’ abilities to differentiate between relative and absolute meanings of “more” and determine which of these is a proportional relationship, compare ratios without using common denominator algorithms, differentiate between additive and multiplicative processes and their effects on scale and proportionality, and interpret graphs that represent proportional relationships or direct and indirect variation. These abilities are directly related to the kind of thinking and reasoning needed for algebraic reasoning and developing an understanding of functions. For example, Lobato and Thanheiser (2002) discuss the need for students to understand slope "as a ratio that measures some attribute in a situation" (p. 174). They go on to argue that helping students understand "the modeling and proportional reasoning aspects of ratio-as-measure tasks, can in turn help students develop an understanding of slope that is more general and applicable" (p. 174) and important for success in algebra.

5.6 Challenges Regarding Preparation of Teachers of Mathematics

Research in mathematics education has also well documented the challenges of teaching rational number concepts and the impact of teachers' limited content knowledge on their students' learning (e.g., Ball, 1993; Harvey, 2012; Lamon, 2012; Ma, 1999; Newton, 2008). The Conference Board of the Mathematical Sciences (CBMS, 2012) states that "a critical pillar of a strong PreK–12 education is a well-qualified teacher in every classroom" (p. 14). Unfortunately, that is not always the case with regard to teachers of mathematics at all grade levels. The paths to teacher certification in the United States are varied allowing for significant difference in what and how much mathematics is required for credentialing. Many states, due to a decade's long shortage of mathematics teachers, allow individuals prepared to teach elementary, many of whom have had little college level mathematics, to simply pass an exam to receive credentials to teach middle level mathematics—in some cases up through Algebra II. Sadly, with these extreme teacher shortages across the nation, some states are allowing significant numbers of individuals into mathematics classrooms with little or no background in mathematics.

In the case of teachers who have completed a teacher preparation program, the challenges and limitations related to their content knowledge for teaching have been a focus of the mathematics education community for decades and have been well documented in the mathematics education literature (Ball, 1993; CBMS, 2001, 2012; Ma, 1999; Shulman, 1986). For education practice, policy, and research, teachers' mathematical content knowledge continues to be a major focus (CBMS, 2012; Greenberg & Walsh, 2008; National Mathematics Advisory Panel, 2008). Despite this ongoing focus, a great number of teachers, particularly those teaching in elementary, intermediate, and middle level mathematics, continue to be under-prepared and uncomfortable with the mathematics content they are expected to teach (Greenberg & Walsh, 2008). This is often due to a variety of factors including, but not limited to, their own experiences with mathematics, their beliefs and ideas about mathematics teaching and learning, and their preparation as teachers related to mathematics content knowledge and pedagogical knowledge for teaching mathematics (Reeder, Utley, & Cassel, 2009; Utley & Reeder, 2012).

Prior to their teacher preparation coursework, most prospective teachers have spent many years learning mathematics from teachers whose pedagogical practices primarily reflect a traditional orientation focused on procedural understanding rather than a balanced approach that attends to both conceptual understanding and procedural fluency (National Research Council, 2001). When they arrive in their undergraduate degree, they are typically engaged with mathematics similar to their prior experiences through lecture style teaching methods and a show-and-repeat procedures approach. Further, many teacher preparation programs require prospective teachers to take mathematics coursework that is disconnected from the mathematics they will teach. Prospective mathematics teachers are required in many states to take a course in College Algebra which may extend their own mathematical understanding but does not do much to deepen their understanding of rational numbers, for example. This certainly shapes teachers' attitudes about mathematics and their ideas about what constitutes mathematics teaching and learning (Reeder et al., 2009). Likewise, most secondary mathematics education programs preparing teachers to teach grades 6–12 mathematics require, if not a degree in applied mathematics, the coursework equivalent. Prospective secondary mathematics teachers are typically required to take coursework well beyond what many consider as necessary the strong content knowledge needed for teaching but very well may not understand rational number concepts with depth. The CBMS recommends more mathematics coursework specifically developed to meet the needs of teachers and improve content knowledge specifically needed for teaching (2012).

Specific to this chapter, existing research demonstrates that prospective and in-service teachers' knowledge of fractions is limited (Ball, 1990; Becker & Lin, 2005; Chinnappan & Forrester, 2014; Cramer, Post, & del Mas, 2002; Harvey, 2012; Ma, 1999; Newton, 2008; Zhou, Peverly, & Xin, 2006). Additionally, research has documented that teaching and learning fraction concepts are a difficult and complex undertaking (Ball, 1993; Harvey, 2012; Lamon, 2012; Ma, 1999; Newton, 2008). Newstead and Murray (1998) purport that fractions are among the most complex mathematical concepts that elementary students encounter, and Charalambous and Pitta-Pantazi (2005) and Harvey (2012) assert that the teaching and learning of fractions have traditionally been problematic. Lamon (2007) believes that most teachers are not prepared to teach content other than the part-whole construct of fractions which leaves their students with an incomplete and shallow understanding of fractions and rational numbers.

5.7 The Growing Problem of Teacher Shortages

The United States is in the midst of a teacher shortage crisis. For decades there has been a chronic shortage in particular teaching content areas such as mathematics, science, special education, and bilingual education, but the current situation is widespread and involves almost every state in the nation. From California to

Oklahoma to New York, school districts are scrambling to hire teachers, and unfortunately, in many cases this is regardless of their credentialing. In October 2015, US World and News Report reported that school districts in the state of California were still trying to fill 21,500 vacant teaching positions. In this same report, Partelow stated that "while it may be too early to tell whether this year's reported shortages are a blip or part of a long-term systemic trend, we do know that fewer college students are enrolling in teacher training programs and surges of teachers are retiring" (2015, para. 4). If this trend continues, it will not only lead to greater numbers of unfilled teaching positions in the future but will also lead to classrooms likely filled with teachers who are not as well prepared as needed.

When the school year begins each fall and there are not enough teachers to fill the classrooms in each building, students do not sit in empty rooms. Rather, school districts begin filling classrooms, in some cases, with anyone they can find regardless of the person's credentials. This results in credentialed teachers teaching outside of their content area, long-term substitutes filling teaching positions, preservice teachers beginning teaching before they are fully prepared, and allowances for individuals to be "emergency certified" often without any teacher preparation. California, for example, has been particularly hard-hit following the loss of more than 80,000 teaching jobs between 2008 and 2012 (Rich, 2015). Now, with a recovering economy, there is a need for more teachers and they simply are not enough. Rich (2015) reported that "[b]efore taking over a classroom solo in California, a candidate typically must complete a post-baccalaureate credentialing program, including stints as a supervised student teacher. But in 2013–2014, the last year for which figures are available, nearly a quarter of all new teaching credentials issued in California were for internships that allowed candidates to work full time as teachers while simultaneously enrolling in training courses at night or on weekends" (para. 13). Additionally, from 2012 to 2013, the number of emergency permits issued in California to allow individuals who have no teaching credentials to fill teaching positions jumped by more than 36 %. This increase has been unfortunately paralleled in other states in the past few years. Partelow (2015), citing Oklahoma as an example, stated that "[u]nfortunately some states have instead responded [to the teacher shortage] by lowering the (arguably too low already) bar for entry into the profession. Oklahoma approved over 800 emergency certificates in July and August allowing non-credentialed teachers to teach in classrooms of their own" (para. 7). In October of the fall 2015 semester, over 1000 teacher vacancies remained unfilled in Oklahoma.

The teacher shortage will undoubtedly have an impact on students' mathematical preparedness. More classrooms will be filled with teachers who do not have the specialized content knowledge needed for teaching mathematics or the pedagogical content knowledge to teach mathematics effectively. Without deep content knowledge or sophisticated and well-developed pedagogical practices, teachers typically resort to teaching via rote methods and memorization—methods that do not account for a holistic approach to teaching fractions and rational number concepts with the five sub-constructs in mind.

5.8 Discussion

In a recent report published by the National Academy of Sciences, the author stated that the phrase "STEM education is shorthand for an enterprise that is as complicated as it is important" (Beatty, 2011, p. 1). She goes onto to say that:

> what students learn about the science disciplines, technology, engineering, and mathematics during their K-12 schooling shapes their intellectual development, opportunities for future study and work, and choices of career, as well as their capacity to make informed decisions about political and civic issues and about their own lives. A wide array of public and personal issues—from global warming to medical treatment to social networking to home mortgages—involve science, technology, engineering, and mathematics (STEM). Indeed, the solutions to some of the most daunting problems facing the national will require not only the expertise of top STEM professionals but also the wisdom and understanding of its citizens. (Beatty, 2011, p. 1)

Clearly, helping students succeed in STEM fields and to live and succeed in a global economy is important, and simply engaging students in the mastery of basic skills is not sufficient to meet this goal.

In his popular book, *The Checklist Manifesto*, Atul Gawande (2010) addresses the idea that despite our modern world and tremendous advances in health care, government, the law, and financial industry, challenges still plague us. He examines the nature of problems we frequently face and elaborates on the nature and complexity of said problems. Referencing the work of Glouberman and Zimmerman (2002), Gawande presents three different kinds of problems in the world: the simple, the complicated, and the complex. Simple problems, he notes, "are ones like baking a cake from a mix. There is a recipe and a few basic techniques to follow but once these are mastered, following the recipe brings a high likelihood of success" (p. 49). Complicated problems on the other hand, are ones like sending a rocket to the moon. "They can sometimes be broken down into a series of simple problems but there is no straightforward recipe. Success frequently requires multiple people, often multiple teams, and specialized expertise" (Gawande, 2010, p. 49), but once you learn to send a rocket to the moon, you can repeat the process with other rockets and perfect it—one rocket is typically like another rocket. "Complex problems, however, are like raising a child. Although raising one child may provide experience, it does not guarantee success with the next child" (Gawande, 2010, p. 49). Expertise is valuable but likely not sufficient because unlike rockets, every child is unique. Each child may require an entirely different approach from the previous one. Another feature of complex problems is that their outcomes remain highly uncertain. "Yet we all know that it is possible to raise a child well. It's complex, that's all" (Gawande, 2010, p. 49). Likewise, helping students be prepared for algebra is a complex endeavor.

Preparing students well for algebra involves many years of working with them to develop a deep understanding of fractions and proportional reasoning and ensuring that our teachers not only understand but are able to teach rational number concepts holistically. The challenge is multifaceted involving policy and practice, beliefs about mathematics teaching and learning, beliefs about what is mathematics and what it means to know mathematics deeply, teacher preparation, and ensuring every classroom of students has a well-qualified teacher. The complex challenge of helping students be prepared for algebra and the important mathematics beyond algebra involve many years of work and development. Equally important is the specialized content and the pedagogical knowledge of many skillful teachers who teach mathematics.

Darling-Hammond (2010) states that we can meet the challenges of our current education system by developing a new paradigm for national and state education policy that is guided "by twin commitments to *support meaningful learning* on the part of students, teachers, and schools and to *equalize access to educational opportunity*, making it possible for all students to profit from more productive schools" (p. 278). If, as an education community, we believe in the importance of preparing students to live happily and succeed in a global economy, then we need to insist that the mastery of basic skills that the emphasis on accountability has brought is not sufficient. Cortese and Ravitch (2008) noted that "[W]hat we need is an education system that focuses on deep knowledge, that values creativity and originality, and that values thinking skills" (p. 4). As an education community, we can advocate for policies and practices that support the teaching of deep knowledge and support teachers in helping students learn meaningfully. These challenges can be met and when they are, we can be confident that students in calculus courses and beyond will respond competently and efficiently when addressing "and the rest is just algebra."

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