Chapter 18
A Story of Teaching Using Inquiry

Christine von Renesse

Abstract This chapter tells my story of learning how to teach mathematics using inquiry and becoming a facilitator of professional development (PD) workshops on inquiry-based learning (IBL) for teachers of kindergarten to graduate school. I am an associate professor at Westfield State University in Massachusetts and an integral part of the project "Discovering the Art of Mathematics" (DAoM). In this chapter, I describe a salsa rueda activity used to teach mathematics via inquiry to liberal arts students who often are not interested in or even fear mathematics. I present a vignette of a PD workshop activity designed to teach participants in an inquiry-based way how to teach using inquiry. The chapter also summarizes results of students' beliefs and attitudes surveys as evidence of the effectiveness of IBL. I close with plans for future work and a reflection on the challenges I face as I step into a leadership role.

Keywords Inquiry-based learning • IBL • Discovering the art of mathematics • Salsa rueda • Inquiry-based learning workshop

18.1 Introduction

This chapter tells my story of learning how to teach mathematics using inquiry and becoming a facilitator of professional development (PD) workshops on inquiry-based learning (IBL). According to Laursen et al. (2014),

MSC Codes
97U30
97U20
97B50

C. von Renesse (✉)
Department of Mathematics, Westfield State University,
577 Western Avenue, Westfield, MA 01086, USA
e-mail: cvonrenesse@westfield.edu

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> IBL methods invite students to work out ill-structured but meaningful problems … Following a carefully designed sequence of tasks rather than a textbook, students construct, analyze, and critique mathematical arguments. Their ideas and explanations define and drive progress through the curriculum. In class, students present and discuss solutions alone at the board or via structured small-group work, while instructors guide and monitor this process (p. 407).

This description of IBL fits both my mathematics classrooms and my workshop environment.

Section 18.2 relates my educational background to how I came to teach inquiry-based mathematics classes at Westfield State University in Massachusetts. Section 18.3 describes the project "Discovering the Art of Mathematics" (DAoM) that promotes teaching mathematics for liberal arts courses using inquiry-based teaching and learning. Through DAoM, I started facilitating PD workshops for professors and teachers on IBL across the US. The mathematics for liberal arts course I teach, "Mathematical Exploration", is explained in detail in Sect. 18.4, including student learning goals, the inquiry-based activity "salsa rueda dancing" as a teaching sample, and student beliefs and attitudes evaluations. In Sect. 18.5 I reflect on collaborating with colleagues on teaching techniques. Section 18.6 describes the PD workshops and their goals, includes a workshop vignette, and outlines some future plans for workshop evaluations. The chapter concludes with a plan for future work and some personal reflections on project leadership.

18.2 Personal Background

18.2.1 Education in Germany

Besides being educated as an elementary school teacher, I minored in music at the HDK (university of arts) and received my Diplom (master) of mathematics at the Technical University Berlin. I did my practicum for the elementary licensure in a 4th grade class that was taught using methods of discovery. The children decided themselves what they would work on during the day and the teacher acted as a coach rather than a knowledge dispenser. This teaching method inspired me and set the base for my desire to teach using discovery or inquiry-based methods in my own classes. Parallel to my mathematics education I pursued music and dancing. I led an a capella choir, competed in ballroom dancing and composed my own songs. At this point in my career, the arts (music and dancing) and mathematics were parallel paths that seemed to have little connection with each other.

18.2.2 Graduate Education in the US

In 2003 I immigrated to the US to enter the PhD program in mathematics at the University of Massachusetts. I also danced several times a week and learned how to

teach salsa rueda, a Cuban dance form in which couples dance synchronously in a circle. After I received my PhD in algebraic geometry, I was finally ready to move on to full-time teaching. I was still interested in teaching at the K-12 level but also really craved the academic freedom of a college professor. In the German university system there are no tenured *teaching* positions and so I applied for jobs in the US.

18.2.3 Teaching Position at Westfield State University

Westfield State University (WSU) is a public university in Western Massachusetts with about 4500 students, mostly undergraduate. Many students are first generation university students and many enter college with fairly low SAT scores. Traditionally a teaching college, more than half of our mathematics majors are pursuing their secondary education teaching license. The mathematics department consists of 12 full time professors all of whom use progressive teaching methods. The teaching position at WSU was, and is, a perfect fit for me. Most of my new colleagues were open to collaboration, sharing materials, discussing student learning and new teaching ideas: I could finally develop into the teacher I wanted to be. I could not have become the teacher and facilitator I am today without their support and feedback.

At WSU I now teach mathematics for liberal arts (MLA) classes in which I use my music and dance background to motivate mathematical ideas (see Sect. 18.4). I was particularly drawn to the teaching methods of my colleagues Julian Fleron, Phil Hotchkiss, and Volker Ecke in their MLA classes. Together we started the project "Discovering the Art of Mathematics" (DAoM, www.artofmathematics.org) with the goal of bringing authentic mathematical inquiry into MLA classes. I describe DAoM in the next section.

My first years at Westfield I avoided the calculus sequence. It seemed more natural (and easy) to teach using inquiry in MLA, classes for prospective teachers, and in upper-level courses. Thanks to the availability of many resources for teaching calculus using inquiry (e.g., www.iblcalculus.com) I now feel very comfortable (von Renesse 2014). There are always aspects to improve, and treating all the topics on the syllabus is a struggle (Yoshinobu and Jones 2012). However, compared to the way I used to teach calculus, I notice that my students understand the material more deeply now. My broad experience in using inquiry methods in many different courses and levels enables me to coach other teachers and faculty in adopting inquiry.

I also teach courses for future K-12 teachers and spend a lot of time coaching and co-teaching in K-12 classrooms. Working with K-12 teachers and the workshops I lead for college faculty have a surprising overlap. It seems to me that, at the core, teaching mathematics using inquiry relies on the same principles, regardless of whether the students are children, young adults, college students, teachers or professors. The many seemingly disconnected areas (music, dance, and mathematics) that I have studied over the years have finally come together and I enjoy deeply that I can use "all of me" in my current position.

I focus the remainder of this chapter on my work with DAoM. It has had the biggest influence on my development as a facilitator of learning mathematics and it connects all the professional ideas that I feel passionate about.

18.3 Discovering the Art of Mathematics

DAoM was founded in 2009 by Julian Fleron (PI), Phil Hotchkiss, Volker Ecke and myself with this vision:

> *Mathematics for Liberal Arts students will be actively involved in authentic mathematical experiences that*
>
> - *are both challenging and intellectually stimulating,*
> - *provide meaningful cognitive and metacognitive gains, and,*
> - *nurture healthy and informed perceptions of mathematics, mathematical ways of thinking, and the ongoing mathematics not only on STEM fields but also on the liberal arts and humanities.*

DoAM's website (www.artofmathematics.org) provides a wealth of resources to help interested instructors realize this vision in their MLA courses: a library of 11 inquiry-based learning guides, extensive teacher resources, and many professional development opportunities. Thanks to the National Science Foundation (NSF) and Mr. Harry Lucas, we can offer our materials and workshops at no cost. DAoM was the perfect way to connect my interest in music and dancing to my passion for the teaching and learning of mathematics. Over time DoAM's goals have broadened and we now support faculty in including more inquiry-based techniques into *all* their mathematics classes, not just MLA.

18.4 Mathematics for Liberal Arts Course at WSU

The MLA course "Mathematical Explorations" at WSU is "an introductory course designed to provide the liberal arts major with an opportunity to develop a broader appreciation of mathematics by exploring ways in which the artistic, aesthetic, intellectual, and humanistic aspects of mathematics are as important as its utility" (official WSU course description). The professor can choose the specific content that he or she would like to use for this purpose. There is no prerequisite for this course and students tend to be weak in algebra. The course is part of WSU's common core curriculum and we offer about 6 sections of 30 students each semester. DAoM has developed specific student learning goals for this course:

1. Students will appreciate mathematics as a human endeavor, which is one of our most fundamental intellectual pursuits.
2. Students will understand that mathematics is a vital, rapidly growing field of inquiry with a dedicated cohort of practitioners.

3. Students will understand the continued impact of mathematics in shaping history, culture, logic, philosophy, and knowledge, as well as its role as a humanistic and aesthetic discipline.
4. Students will understand the ubiquitous role of mathematics in the world around them.
5. Students will strengthen their reasoning skills and become better problem solvers.
6. Students will strengthen their skills in reading, writing, argumentation and speaking.
7. Students will become more self-monitoring, reflective learners and take greater personal responsibility for their learning.
8. Students will approach mathematics more positively and gain a balanced perspective of mathematics.
9. Students will improve their mathematical confidence.
10. Students will develop awareness of the negative impact of broadly-held societal views of mathematics.
11. Students will be capable of and interested in considering mathematics outside of the confines of the classroom, understanding the value of life-long learning in mathematics.

Students in our course are being assessed through observing their abilities and progress during group work and presentations, larger student projects (e.g. posters, papers, art work, solving the Rubik's cube), journal writing, regular conceptual homework and sometimes a final exam (Fleron et al. 2014).

18.4.1 Dancing Salsa Rueda

Using just the existing DAoM materials, an instructor could teach different topics for 11 semesters. There are so many because we keep inventing activities around topics we feel passionate about (von Renesse 2012; von Renesse and Ecke 2011, 2016; Fleron and Ecke 2011; Livingston and Fleron 2012; Fleron 2012; von Renesse & Ecke 2015).

One of my favorite activities is dancing salsa rueda with my students. Movement can be very helpful in learning (Jensen 2000). In my class, the movement motivates the mathematical questions (see example below). It also helps to engage all students, bringing lightness and laughter into the classroom and building a strong classroom community. Moving together for the first time can be frightening for the students and the instructor. Many students have shared with me that they are as afraid of dancing as they are of mathematics! But the dancing we do is really about moving, with no attachment to "doing it perfectly". Doing the exact steps at the right time is not as important as the desired position in the circle. Similarly, performing arithmetic is not as important in my class as understanding a big idea. Students are often afraid of making mistakes instead of valuing them. Using dancing

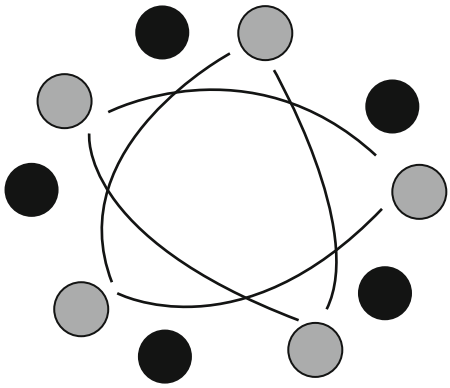

Fig. 18.1 The path of a rueda dance using only dame dos

I can model how to enjoy learning from mistakes and being persistent. In my experience this translates to valuing mistakes and being more persistent when we are learning mathematics: this is important for student learning (Kapur 2011).

In salsa rueda,[1] pairs of dancers stand in a circle and dance the same salsa moves simultaneously. There is a leader of the circle who calls out the moves and everyone is supposed to listen and react in time. Each pair has a leader and a follower, which students determine by preference and confidence, instead of by gender.

The mathematical content goals for this activity include understanding how greatest common factors and least common multiples are related, finding patterns in star polygons, and making conjectures and proving conjectures about star polygons.[2] During the dance move "dame" ("give me" in Spanish) each leader passes their follower to the next leader to the left and gets in turn a new follower from the right. This can be done skipping one or several leaders, leading to an interesting pattern. In fact, star polygons emerge from following the dancer's path. See Fig. 18.1 for a path of "dame dos" ("give me the second"–skipping one leader), with the black circles representing leaders and the gray circles followers. Notice that in each pair the follower stands on the leader's left.

In the language of the dance we are wondering:

1. Does each follower return eventually to his or her original leader? Does the answer depend on the number of dancers?
2. Does every follower get to dance with every leader in the circle? Does the answer depend on the number of dancers?
3. How many times does the follower move around the circle before returning to his or her original leader (if that happens).

[1] The video at https://artofmathematics.org/media/video-497 shows students dancing salsa rueda in my Mathematical Explorations class in Fall 2013.

[2] We create an (n,k) star polygon by taking n equidistant points on a circle and connecting each point to its kth neighbor (going to the right around the circle).

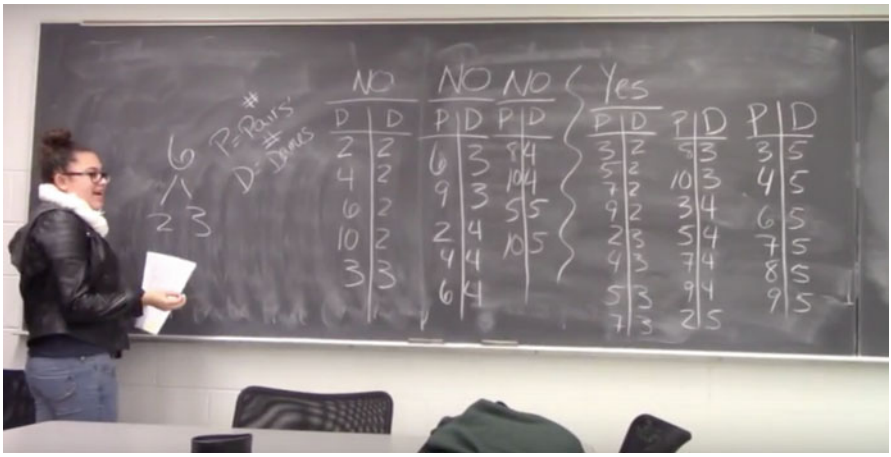

Fig. 18.2 Student presenting her work on question 2

Students spend several class periods looking for patterns, creating conjectures, and trying to figure out if and why their conjectures are true. At various points, whole class discussions allow the students to build on their classmates' ideas.[3] Figure 18.2 shows a table where a student recorded the number of partners (P) and the kind of dame (D). The "NO" table records all examples where followers don't dance with every leader, the "YES" table all others. She noticed how, for most examples, D divides P but that there are some "weird ones" like 6 and 4.

18.4.2 Creating Investigations

It is a difficult task to invent investigations that are "just right" for our students. If we guide the students too much, they don't do mathematics and lose sight of the larger picture. If we don't guide them enough, they get frustrated and give up. Mairead Greene and I (Greene and von Renesse 2016) described the processes we use to fine-tune our tasks and make sure that they align with our goals. The investigations lead students to make conjectures and eventually prove them. A proof doesn't have to be formal, but it has to establish an explanation that helps us make sense of *why* the conjecture is always true (Fleron et al. 2016). In my class, I encourage the students to alternate between thinking about the mathematics and returning to the movement to test their conjectures. Students need to consider greatest common factors and least common multiples to state their conjectures and

[3] In the video clip at https://artofmathematics.org/media/video-394, a student from my class presents her work on question (2). It shows how she explains her process and how open she is about what she doesn't know (yet).

their sense-making often includes models like the number line, factor trees and star polygons. Student work from my class is included in von Renesse (2016).

18.4.3 Student Evaluation: Does Inquiry-Based Learning Work?

Research indicates that active learning is effective (Freeman 2014; Kogan and Laursen 2014). But not many studies have been done on the MLA audience. In our project we have used pre- and post-surveys to measure changes in students' beliefs and attitudes changes since 2009. Fleron has collected data from over 1000 students since 1997. The accumulated changes on the survey all occurred in the desired direction and some of the differences were statistically significant. The survey questions we used and many of our results appeared in Ecke (2015). One of our results was that students are much more likely to perceive beauty in mathematics after the course: the percent of students agreeing that there is something in mathematics that they think is beautiful and that they could describe increased from 15 to 55 % (see Fig. 18.3). We are also interested in having but struggling to find a written test that measures students' skills (reasoning, creativity, persistence, flexibility, etc.) independently of the content treated during the semester.

In addition to data from the pre-and post-surveys such as that found in Fig. 18.3, we have collected student journals over the years. Journal entries like the following show that many students have achieved the specific learning goals DAoM developed for the course:

> From the very first moment we started the Stone Game, I was able to understand it and was actually excited to discover more ways in which a player could win. After the class ended, I remember I went back to my dorm room, tore up a piece of paper into little squares, and

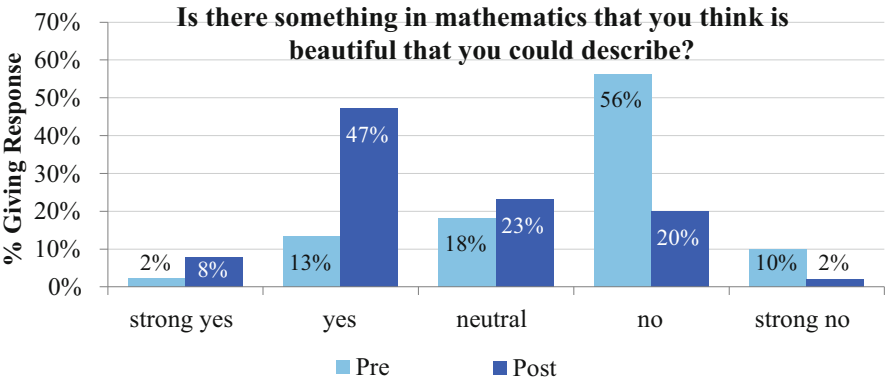

Fig. 18.3 Evaluation result

> began to search for more ways that a player could win. After I was finished working through some of my ideas, I looked around at all the little scraps of paper that were covering my desk, and I was amazed with myself. That had been the first time I had ever gotten so excited about working on a math problem outside of the classroom.
>
> DAoM Student, Fall 2015

> I agree with Lockhart[4] that students should explore math for themselves and discover equations so that they understand it better than being given a formula sheet and told to memorize them. In our class we did not use any equations or formulas and I feel that I learned more in this class than any other math class before.
>
> DAoM Student, Fall 2015

18.5 Collaborating on Teaching Techniques

Being part of the DAoM project has taught me much about teaching techniques. Visiting my colleagues' classrooms and collaborating weekly to create inquiry materials led to many questions, disagreements and discussions. This process helped me discern who *I* want to be in the classroom and why *I* prefer some teaching techniques to others. Even though all four of us clearly support inquiry, our teaching methods vary. I like to use a "large" investigation prompt to start off a new topic and then mix group work with whole class discussions to reach a conclusion (von Renesse and Ecke 2013). Fleron, on the other hand, prefers to give his students handouts with smaller prompts for the groups to work on (Fleron 2014), often without whole class discussions. The guiding principle is the same: we act as coaches and the students do the mathematics. The difference lies in how much guidance we give our students. Through many conversations we have seen the advantages and disadvantages of each approach and have found ourselves trying out and liking each other's methods of facilitating inquiry. Lately Fleron has been using my approach of the larger prompts in his classes and has really enjoyed the discoveries that his students made (Fleron 2015).

Our approaches to motivating the class also differ. I tend to pick topics that seem "easy" and engaging, (music, dance, games,…) or investigations that I know will lead students to disagree with each other, for example, wondering if $0.999\ldots=1$ or not. Then I tried out some of Fleron's materials and noticed that my students were not as excited about mathematics. I didn't know that he likes to choose topics that are rooted in pure mathematics (understanding the infinite, number theory,…) and generates motivation by regularly telling students about "cool" connections in mathematics (von Renesse 2015). Engaging a crowd by giving exciting minilectures

[4] The students read "A mathematician's lament" by Paul Lockhart (see https://www.maa.org/external_archive/devlin/LockhartsLament.pdf).

doesn't come naturally to me yet—but I am improving. The deep collaboration in our project over the last eight years is clearly visible in the peer-reviewed publications and blogs that we have written together.

18.6 Professional Development Workshops

18.6.1 History of Our Workshop Development

When our project DAoM made our learning guides freely available (www.artofmathematics.org/books) we assumed that now anyone could replicate what we do. But it is not easy for teachers and professors to change their teaching style. In addition to the curriculum materials, there are many teaching decisions that we make unconsciously or that we had not communicated in our learning guides. This realization led to the creation of our classroom web page and our traveling workshops.

While we developed these resources, I took on more and more the role of a leader in our group. As the most senior faculty member of our project, Fleron had been the natural leader of our efforts but his interests were more focused on curriculum development than leading workshops or creating pedagogical support. My background and interest in leading K-12 professional development workshops put me, the most junior faculty member and only woman of the project, suddenly in a leadership position.

18.6.2 Workshop Goals and Description

Our short traveling workshops usually include nine official contact hours. Additionally we like to plan at least one group meal to give us time to connect and network. We also offer to visit classes before the workshop to watch, model, or co-teach with the teaching participant. The classroom visits allow us to meet some of the participants and assess their needs before the workshops starts. The goals of our traveling workshops are to help faculty:

- Experience as a student what mathematical inquiry can feel like in a MLA class,
- Investigate particular content areas that might resonate with their students,
- Reflect on the interaction of teacher, student, mathematics, and inquiry materials in the classroom.

The 20–30 participants of a typical workshop are mostly professors from 2- to 4-year colleges and universities near the workshop site. We also ask the on-site workshop coordinator to invite some local K-12 teachers and graduate students since having a variety of participants promotes richer discussions and collaborations.

We believe that one learns best when using inquiry. Therefore the guiding principle for the workshop is to not lecture about how to teach using inquiry but to facilitate activities what will lead the participants to discover the teaching ideas themselves. As part of our activities we let participants experience what it feels like to be students in an inquiry-based learning environment. Participants report regularly how eye-opening this was for them and how much it motivated them to change their own teaching style. We also let participants practice their teacher moves[5] with the other workshop members, as in the workshop vignette in Sect.18.6.3.

My current research includes finding or inventing more workshop activities that help teachers and professors become successful in teaching using inquiry. The salsa rueda activity would be too long for a workshop activity, so we use shorter activities that are appropriate in difficulty level for the audience of teachers and professors. As long as the topic is approached via inquiry and the question is posed well (Weiss 2003) the specific content is not relevant for our purpose. The teacher moves that the participants engage with can be used with any mathematical topic.

In our workshops, we also use classroom videos containing other content from our MLA learning guides. For example, the video clip https://artofmathematics.org/media/video-395 shows how a week after the salsa rueda activity, I am recording students' conjectures about relating the greatest common factor and the least common multiple. This video clip is useful at workshops to demonstrate how I record ideas that are mathematically incorrect or irrelevant without losing my "poker face". The next section contains a vignette that presents a typical interaction that could have happened at any of our workshops.

18.6.3 Workshop Vignette

We just watched several classroom video clips to see how teachers use "talk moves"[6] to facilitate a good discussion. The 10 workshop participants (high school teachers and professors from 2- to 4-year colleges) described the teacher moves they observed, which include the talk moves from Susan Chapin's book Math Talk (Chapin et al. 2003):

- **Revoicing**: Let a student repeat what another student said
- **Rephrasing**: Facilitator rephrases what he/she heard the student say
- **Agree** or **Disagree**: Ask the class whether they agree or disagree with a statement
- **Adding On**: Invite students to add any observations, thoughts or questions

[5] A "teacher move" is any choice the teacher makes during class to enhance student learning. Examples are standing in a particular part of the room or clapping a rhythm to get students' attention.

[6] A "talk move" is a particular phrase the facilitator uses during a whole class discussion or a mathematical conversation with a student.

- **Wait Time**: Do nothing for at least 10 s, the hardest thing to do for most teachers
- **Sharing**: Decide who gets to share their ideas (and when)
- **Record**: Write/represent the essence of the discussion on the board

Some of the participants look a bit nervous since they know they will soon lead a discussion using talk moves in our group. I explain the goal of the role-playing: "This is your chance to practice facilitating a discussion in a space where it doesn't matter when we mess up. There are no students here; we are not trying to cover anything. Just see what it feels like." The first facilitator, Professor G, hands his problem to the groups of "students." They are supposed to decide if 0.999.,…=1 or not. While he walks between the groups I notice that he seems to "hover" over the groups with his arms crossed. I join him and suggest that he can pull up a chair to a group and tell them that he is "listening in." This little meta-conversation with Professor G doesn't distract the groups from playing with different proofs for different audiences. I observe that a few participants argue against the equality but are quickly shut down by their group members.

After 10 min of group work Professor G calls his "students" together as a whole "class". He decides to start the discussion with a group that uses the trick of multiplying $x = 0.999\ldots$ by 10 so that $10x - x = 9.999\ldots - 0.999\ldots = 9$, which makes $x = 1$. After they explain their idea he asks another participant to repeat their thinking. The next group he calls on uses a calculus approach with a geometric series. As they take over the board I wonder how the participants who don't believe in the equality are doing. Did Professor G. notice that they are there? Does he have a plan to bring them in? It seems to me that he will continue to let the groups share their thinking without connecting the ideas or bringing in disagreement. So I decide to interrupt him and address the group: "We have heard a lot of arguments in favor of equality. I wonder if there is anyone in this room who thinks 0.999… and 1 cannot be equal?" One participant immediately raises his hand to share that it doesn't feel right to him that these numbers could really be equal. "It just can never reach 1, something with the algebra must be wrong. Maybe we can't subtract the infinitely many places the way we did?" Now the energy rises in the room and a member of the first group starts to defend her algebraic thinking. Professor G. is busy recording their thinking and calls on a third group to repeat where the conflict lies.

When our allotted time for the discussion is over it is hard for me to get the participants to stop discussing the mathematics: "Take a deep breath, let go of the mathematics, and put your teacher hats back on." Pause "Thank you. Professor G., how did the discussion feel to you?" He shares that it was hard not to tell the "students" who was right and to let the conflict develop. His instinct was to call on the students who knew the correct answer and avoid the students who seemed to struggle. The question comes up when to step in and give feedback. Can we agree to disagree? Do we need to come to a conclusion before "class is over"? The discussion touches on many important aspects and challenges of an inquiry-based classroom and I try not to tell the participants how I would deal with this situation. After all, this is their moment to inquire about teaching, and not mine.

18.6.4 Workshop Evaluation

We are currently using pre- and post-workshop evaluations for our short workshops to measure the impact right after a workshop and a year later. Results are not compiled yet. Comments on our post-surveys suggest that our workshops are well received. Conversations with department chairs in the year after a traveling workshop indicate that participants use our ideas to change their class structure and work on their teaching moves. Written comments from workshop participants include:

> I believe the most effective aspect [of the workshop] was experiencing IBL as a student and unpacking what it is like as the facilitator. I think it gave me a real sense of what kinds of activities to utilize with my students.
>
> Workshop Participant, 2016

> [The most effective parts of the workshop were] explicitly looking at and practicing the teaching moves with a group of eager learners before I try it with my reluctant students.
>
> Workshop Participant, 2016

> This workshop exceeded my expectations. With a post-secondary focus, I expected a more typical 'quick fix' message. I felt the work we experienced here is much more fully and authentically aligned with current education research than any post-secondary PD I have ever experienced.
>
> Workshop Participant, 2015

I hope in the future to use a classroom observation protocol like O-TOP (Wainwright et al. 2004) or M-SCAN (Merritt et al. 2010) and videotape some classes that participants teach before and after having taken our workshop. Maybe this would allow us to measure in greater detail what they learned at our workshop.

18.7 Larger Audiences and Future Work

While the work of DAoM was intended to reach MLA instructors nationally we have seen huge interest in our materials and workshops from a much larger audience. Over the last 4 years, many K-12 schools have reached out to us for professional development. Mathematics departments from community colleges through universities are interested in our traveling workshops, international schools and universities have found our materials, and we have almost 900 followers of our monthly pedagogy blog. We are at the point where four people can't handle the work anymore, and we need more faculty leaders to help with implementing IBL workshops in mathematics.

The DAoM team just started collaborating with the Academy of Inquiry Based Learning (AIBL). The goal of the new project PRODUCT—PROfessional Development and Uptake through Collaborative Teams: Supporting Inquiry Based Learning in Undergraduate Mathematics—is to "train the trainers" so that more IBL workshops can be offered in the future. PRODUCT, under the leadership of Stan

Yoshinobu and Sandra Laursen, is supported by an NSF IUSE grant to increase the number of facilitators of IBL workshops.

DAoM is leading the short workshop development for PRODUCT, Professional Development and Uptake through Collaborative Teams. We will continue to offer short workshops at no cost until 2020 and develop materials that can support professors and teachers in offering short IBL workshops themselves. While DAoM and AIBL have been offering IBL workshops for the several years, our guiding principles differ from one another. Articulating our goals clearly, sharing our facilitation experience and deciding on joint strategies will be challenging and productive.

18.8 Personal Reflections

While I feel very comfortable teaching a class or leading a workshop, I still struggle with taking on a leadership role in a project among colleagues. How do we hold each other accountable for our respective work on the project? What do we do if we disagree on something? Where is the line between honesty and diplomacy? How does friendship intersect with work relationships? Who determines what is "fair" and how we get compensated for our respective contributions? There are more questions than answers at this point. I know that my strengths are planning ahead, working efficiently, getting my work done on time, and being an honest and clear communicator. But I am still learning how to be supportive of a colleague and give honest feedback (at the same time!), how to be a cheerleader instead of a taskmaster, and how to determine a reasonable workload instead of overcommitting.

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