
Modified Ghost Fluid Method for the Fluid Elastic-Perfectly Plastic Solid Interaction

S. Gao and T.G. Liu

Introduction

The purpose of this work is applying the MGFM [1], an effective multi-material treating technique, to decouple the fluid elastic-perfectly plastic solid interaction in the Eulerian coordinate system. The key element of the method is solving the multi-material Riemann problem defined at the interface. But the feature of solid strength, the resistance of the material to shear distortion, which makes it different from the fluid, will result in a leading elastic wave and a trailing plastic wave propagating simultaneously in the solid medium when it is under certain strong impact and complicate the fluid-solid interface wave analysis [2].

To deal with the multi-material interaction, approaches used are classified into two categories in general. Front capturing is rather apt to handle large deformation problems and relatively straightforward to extend to high dimensions [3–6]. However, numerical inaccuracies and nonphysical oscillations are inevitable. In contrast, front tracking can impose accurate boundary conditions at the interface, but entanglement of the Lagrangian meshes and quite complex extension to high dimensions come to its drawbacks [7, 8]. To combine the advantages of front capturing and front tracking, Fedkiw proposed a non-oscillatory Eulerian approach to treat the multi-material interface named the ghost fluid method (GFM) [9]. GFM considers the situations that pressure and velocity are continuous across the interface. However, when pressure and velocity suffer a sudden jump, for example, a strong shock impacting on the interface, the results will be unacceptable. To handle the interaction correctly in this case, Liu et al. modified the original GFM via solving a multi-material Riemann problem at the

interface instead of pressure and velocity assignment and entropy extrapolation used in GFM. MGFM works effectively and efficiently in various multi-material applications, even in very challenging problems. Compared with other prevalent techniques, the immersed boundary method (IBM) and its variation like immersed interface method (IIM) employed to decouple the fluid-structure interaction (FSI), which needs to add forcing terms to the governing equations at the moving immersed boundary, MGFM is more convenient and direct by solving the Riemann problem. Hybrid method of smoothed particle hydrodynamics (SPH) method and finite element method (FEM) also has taken attempt to high-velocity impact problem. But the shortages of low resolution for the SPH side and inability of large mesh distortion for the FEM side are inescapable, whereas finite difference (FD)-based MGFM can achieve high-resolution and large distortion instinctively.

Governing Equations

The governing equations for the one-dimensional compressible fluid are the Euler equations [10], neglecting the effects of body forces, viscous stresses, and heat conductivity:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0,$$

where

$$U = \begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}, F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (E + p)u \end{pmatrix}.$$

Here, the variables are the density ρ , the x -direction velocity u , the pressure p , and the total energy per unit volume E . For closure of the above systems, equations of state are required. The γ -law for the compressible gas is

S. Gao • T.G. Liu (✉)
LMIB and School of Mathematics and Systems Science, Beihang University, No. 37 Xueyuan Road, Beijing 100191, People's Republic of China
e-mail: liutg@buaa.edu.cn

$$p(\rho, e) = (\gamma_g - 1)\rho e,$$

where γ_g is the specific heat ratio for the particular gas and is set equal to 1.4 for the air. The Tait equation of state for the compressible water is

$$p(\rho, e) = (\gamma_w - 1)\rho e - \gamma_w(B - A),$$

where A , B , and the specific heat ratio for water γ_w are constants, $A = 10^5$ Pa, $B = 3.31 \times 10^8$ Pa, and $\gamma_w = 7.15$.

The one-dimensional governing equations to describe the elastic-perfectly plastic solid [11–13] are

$$\frac{\partial V}{\partial t} + \frac{\partial G(V)}{\partial x} = 0,$$

where

$$V = \begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}, \quad G(V) = \begin{pmatrix} \rho u \\ \rho u^2 - \sigma_x \\ (E - \sigma_x)u \end{pmatrix}.$$

Here, σ_x is the total stress in the x -direction. The total stress is decomposed into two parts, the hydrostatic component p and the deviatoric component s_x associated with the resistance of the material to shear distortion:

$$\sigma_x = -p + s_x.$$

And the hydrostatic pressure is assumed to equip with the “stiffened-gas” equation of state:

$$p(\rho, e) = c_0^2(\rho - \rho_0) + (\gamma_s - 1)\rho e,$$

where c_0 is the unshocked sound speed, ρ_0 is the reference density, and γ_s is the Grüneisen gamma. The von Mises yield condition is imposed to judge the material status. The solid is in the elastic region if the von Mises yield condition is satisfied:

$$s_x^2 \leq \left(\frac{2}{3}Y_0\right)^2.$$

Here, Y_0 is the yield strength of the material in simple tension. Hooke’s law is utilized to describe the elastic relationship:

$$\dot{p} = K \frac{\dot{\rho}}{\rho}, \quad \dot{s}_x = 2\mu \left(\dot{\epsilon}_x + \frac{1}{3} \frac{\dot{\rho}}{\rho} \right),$$

where K is the bulk modulus, μ is the shear modulus, and $\dot{\epsilon}_x$ is the strain rate in the x -direction. If the von Mises yield condition is violated, perfectly plastic flow occurs:

$$s_x = \pm \frac{2}{3}Y_0.$$

Note that the material yields in tension if $s_x > 0$ and in compression if $s_x < 0$.

Fluid-Solid Riemann Solver

Suppose that the fluid and the elastic-perfectly plastic solid are located at the left and right sides of the interface, respectively; thus, the one-dimensional fluid elastic-perfectly plastic solid Riemann problem in the vicinity of the interface can be given as, in the Euler-Euler coordinate system,

$$\begin{cases} \frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0, & \text{with } U|_{t=0} = U_L \text{ for } x < x_0, \\ \frac{\partial V}{\partial t} + \frac{\partial G(V)}{\partial x} = 0, & \text{with } V|_{t=0} = V_R \text{ for } x > x_0. \end{cases}$$

Here, x_0 is the interface location. U_L and V_R are the initial constant conservation vectors of the fluid and the solid media. For one-dimensional gas-gas or gas-water multi-material Riemann problem, study finds that the solution structure is similar to the pure gas Riemann problem [14]. In this case, the contact discontinuity is exactly the multi-material interface, and there will be one nonlinear wave, either shock wave or rarefaction wave, transmitting in each medium. However, because of the different behavior within the elastic region and the perfectly plastic region of the solid model, it could be two nonlinear waves, a leading elastic wave and a trailing plastic wave of the same type that propagate in the solid medium when the solid is undergoing elastic up to perfectly plastic deformation. To simulate the high-speed impact problem, we concentrate the case that both the nonlinear waves are shock waves when the solid is under compression. Thus, the solution structure of the fluid elastic-perfectly plastic Riemann problem consists of five constant states ($U_L, U^*_L, V^*_R, V_2, V_R$), where V_2 is the state at the elastic limit in the solid. Otherwise, the solution structure consists of four states (U_L, U^*_L, V^*_R, V_R) connected by three elementary waves when the solid is undergoing elastic deformation. In any event, the wave in the left fluid can be either a shock wave or a rarefaction wave and the waves in the right solid are always shock waves. The core part of the multi-material Riemann solver is the computational method of the pressure p^*_L in fluid and the total stress σ^*_R in solid.

Proposition 1

The solution of pressure p^*_L in fluid and total stress σ^*_R in solid for the fluid elastic-perfectly plastic solid Riemann problem is given by the root of the algebraic equation:

$$h(-\sigma_I) \equiv f_L(p_{*L}, W_L) + g_R(-\sigma_{*R}, W_R) + u_R - u_L = 0,$$

with $-\sigma_I \equiv p_{*L} = -\sigma_{*R}$. And $f_L(p_{*L}, W_L)$ is given by one of the following expressions:

$$f_L^S|_{g/w}(p_{*L}, W_L) = \sqrt{(p_{*L} - p_L) \left(\frac{1}{\rho_L} - \frac{1}{\rho_{*L}|_{g/w}} \right)}, \text{ if } p_{*L} > p_L,$$

$$f_L^R|_g(p_{*L}, W_L) = \frac{2a_{L|g}}{\gamma_g - 1} \left[\left(\frac{p_{*L}}{p_L} \right)^{\frac{\gamma_g - 1}{2\gamma_g}} - 1 \right], \text{ if } p_{*L} < p_L,$$

$$f_L^R|_w(p_{*L}, W_L) = \frac{2a_{L|w}}{\gamma_w - 1} \left[\left(\frac{p_{*L} + B - A}{p_L + B - A} \right)^{\frac{\gamma_w - 1}{2\gamma_w}} - 1 \right], \text{ if } p_{*L} < p_L.$$

Here superscripts ‘‘S’’ and ‘‘R’’ stand for the shock wave and the rarefaction wave, respectively. Subscripts ‘‘g’’ and ‘‘w’’ are short for the gas and the water. And $g_R(-\sigma_{*R}, W_R)$ is given by one of the following two expressions:

$$g_R^E|_{\text{sld}}(-\sigma_{*R}, W_R) = \sqrt{(\sigma_R - \sigma_{*R}) \left(\frac{1}{\rho_R} - \frac{1}{\rho_{*R}|_{\text{sld}}} \right)}, \text{ if } \sigma_2 < \sigma_{*R} < \sigma_R,$$

$$g_R^{\text{PE}}|_{\text{sld}}(-\sigma_{*R}, W_R) = \sqrt{(\sigma_R - \sigma_2) \left(\frac{1}{\rho_R} - \frac{1}{\rho_2|_{\text{sld}}} \right)} + \sqrt{(\sigma_2 - \sigma_{*R}) \left(\frac{1}{\rho_2} - \frac{1}{\rho_{*R}|_{\text{sld}}} \right)}.$$

$$\text{if } \sigma_{*R} < \sigma_2 < \sigma_R$$

Here subscript ‘‘E’’ denotes that the elastic shock wave is generated in the solid medium, while ‘‘PE’’ denotes that both the elastic and plastic shock waves are generated in the solid medium. Besides, subscripts ‘‘sld’’ are the abbreviation of solid.

Proposition 2

When the solid undergoes elastic up to perfectly plastic compression from the initial nonpressure and nonstress state, the hydrostatic pressure and deviatoric stress in V_2 region are given by

$$p_2 = \frac{KY_0}{2\mu}, \quad s_{x2} = -\frac{2}{3}Y_0.$$

Modified Ghost Fluid Method

To treat the fluid elastic-perfectly plastic solid interaction numerically, MGFm is applied to handle with the real-time multi-material interface at certain time t^n . Assuming that the computation of the whole domain has updated to n th time t^n and the left and right media are distinguished by the interface x_I^n , the goal is to obtain the results of the whole computational domain at the next time level t^{n+1} . Before standard numerical schemes are used for each medium, defining the ghost cells status is crucial to implement GFM-based approach. In another word, to solve the following left and right medium systems

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0, \text{ with } U|_{t=t^n} = \begin{cases} U_L^{\text{real}}|_{t=t^n}, & x < x_I^n, \\ U_L^{\text{ghost}}|_{t=t^n}, & x > x_I^n, \end{cases}$$

$$\frac{\partial V}{\partial t} + \frac{\partial G(V)}{\partial x} = 0, \text{ with } V|_{t=t^n} = \begin{cases} V_R^{\text{ghost}}|_{t=t^n}, & x < x_I^n, \\ V_R^{\text{real}}|_{t=t^n}, & x > x_I^n. \end{cases}$$

U_L^{ghost} and V_R^{ghost} have to be defined primarily and properly. U_L^{real} and V_R^{real} are exactly the conservative values at the particular real fluid and solid cells of the left and right media, respectively. U_L^{ghost} are the conservative vectors at additional right-end fluid cells of left medium that need to be defined. Similarly, V_R^{ghost} are the conservative vectors at additional left-end ghost solid cells of right medium that need to be defined.

As the MGFm is applied to treat the multi-material interfaces, the fluid elastic-perfectly plastic solid Riemann problem at time t^n is described as

$$\begin{cases} \frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0, \text{ with } U|_{t=t^n} = U_{\text{IL}} \text{ for } x < x_I^n, \\ \frac{\partial V}{\partial t} + \frac{\partial G(V)}{\partial x} = 0, \text{ with } V|_{t=t^n} = V_{\text{IR}} \text{ for } x > x_I^n. \end{cases}$$

where U_{IL} and V_{IR} are the values at the nodes next to the interface. Since the exact fluid-solid Riemann solver has been proposed, the ghost fluid and ghost solid statuses can be defined directly as

$$p_L^{\text{ghost}} = p_{*L}, \quad u_L^{\text{ghost}} = u_I, \quad \rho_L^{\text{ghost}} = \rho_{*L},$$

$$p_R^{\text{ghost}} = p_{*R}, \quad u_R^{\text{ghost}} = u_I, \quad \rho_R^{\text{ghost}} = \rho_{*R}, \quad s_{xR}^{\text{ghost}} = s_{xR}^*.$$

For two-dimensional MGFm implementation, level set method is included to track the boundary as the zero level of ϕ , and signed distance function ϕ is advected by solving the level set equation:

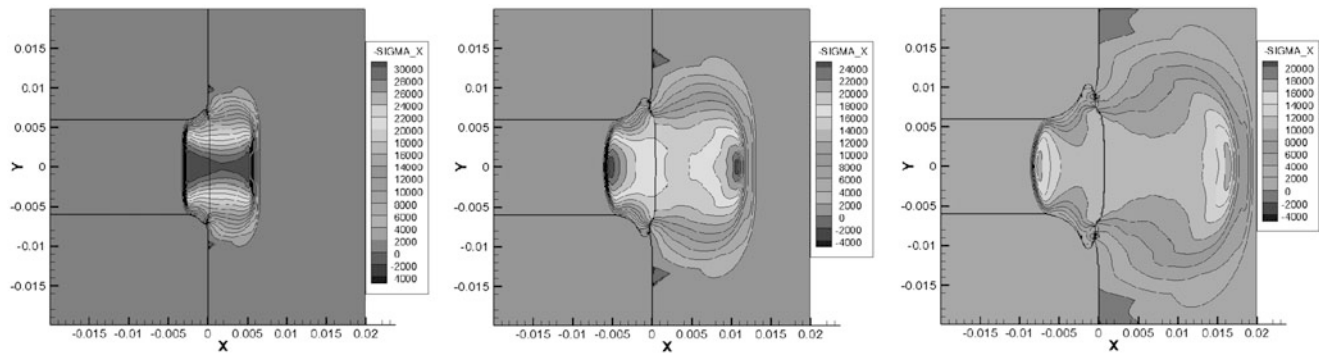


Fig. 1 A 1000 m/s semi-infinite long water-jet flow impacting on a semi-infinite aluminum target surrounded by static standard atmosphere in two dimensions depicted in contours of negative x -direction total stress $-\sigma_x$ at time $t = 0.00001, 0.00002, 0.00003$

$$\phi_t + u\phi_x + v\phi_y = 0,$$

for separate material region, together with the reinitialization equation:

$$\phi_t + S(\phi_0) \left(\sqrt{\phi_x^2 + \phi_y^2} - 1 \right) = 0.$$

After the interfaces are identified, it is straightforward to extend the one-dimensional MGFM algorithm to two dimensions along x - and y -directions approximately, and the tangential components at ghost fluid and solid nodes are copied from the inside real fluid nodes to avoid smearing out the jump in tangential components due to the numerical dissipation.

Numerical Results

MGFM is utilized to decouple the two-dimensional fluid elastic-perfectly plastic solid interaction in the following numerical test. A 1000 m/s semi-infinite long water-jet flow impacting on a semi-infinite aluminum target surrounded by static standard atmosphere in two dimensions. The entire computational domain is a square region $x \times y \in [-0.02, 0.02] \times [-0.02, 0.02]$, comprising a rectangle region $x \times y \in [-0.02, 0.00] \times [-0.006, 0.006]$ for the semi-infinite long jet flow and a rectangle region $x \times y \in [0.00, 0.02] \times [-0.02, 0.02]$ for the aluminum target at beginning time $t = 0$. A total of 400×400 uniform grid points are distributed in the whole computational domain. The nondimensional initial conditions for the left jet flow are $u_L = 100.0, v_L = 0.0, p_L = 1.0, \rho_L = 1.0$, while those for right solid target are $u_R = v_R = 0.0, p_R = 1.0, \rho_R = 2.7, s_{xR} = s_{yR} = s_{xyR} = 0.0$. The nondimensional parameters of the equation of state for the right solid target are $\rho_0 = 2.71, c_0 = 538.0, \gamma_s = 2.67, K = 740,000.0, \mu = 265,000.0, Y_0 = 3000.0$. The time step is set to be 0.0000001 and the terminal time is 0.00003. As shown in

Fig. 1, an elastic shock wave and a rather strong plastic wave form in the solid medium and a shock wave reflect back from the interface in the jet flow. The high-velocity impact results in a relative large deformation at the interfaces. The solid shock speeds are faster than that of the water, which can be observed apparently in the test.

Conclusions

In this paper, we propose the one-dimensional fluid elastic-perfectly plastic solid Riemann solver. It is rooted in the MGFM algorithm to decouple the fluid-solid interaction. Numerical experiments are carried out to test the performance of the simulation in two dimensions. The numerical method works effectively to simulate the high-speed impact problem with multi-material interfaces in the Eulerian coordinate system.

References

1. Liu, T.G., Khoo, B.C., Yeo, K.S.: Ghost fluid method for strong shock impacting on material interface. *J. Comput. Phys.* **190**, 651–681 (2003)
2. Liu, T.G., Xie, W.F., Khoo, B.C.: The modified ghost fluid method for coupling of fluid and structure constituted with hydro-elasto-plastic equation of state. *SIAM J. Sci. Comput.* **33**, 1105–1130 (2008)
3. Abgrall, R.: How to prevent pressure oscillations in multicomponent flow calculations: a quasi-conservative approach. *J. Comput. Phys.* **125**, 150–160 (1996)
4. Abgrall, R., Karni, S.: Computations of compressible multifluids. *J. Comput. Phys.* **169**, 594–623 (2001)
5. Saurel, R., Abgrall, R.: A multiphase Godunov method for compressible multifluid and multiphase flows. *J. Comput. Phys.* **150**, 425–467 (1999)
6. Shyue, K.M.: An efficient shock-capturing algorithm for compressible multicomponent problems. *J. Comput. Phys.* **142**, 208–242 (1998)
7. Glimm, J., Grove, J.W., Li, X.L., Shyue, K.M., Zeng, Y., Zhang, Q.: Three-dimensional front tracking. *SIAM J. Sci. Comput.* **19**, 703–727 (1998)

8. Glimm, J., Li, X.L., Liu, Y., Zhao, N.: Conservative front tracking and level set algorithm. *Proc. Natl. Acad. Sci. USA* **98**, 14198–14201 (2001)
9. Fedkiw, R.P., Aslam, T., Merriman, B., Osher, S.: A non-oscillatory Eulerian approach to interfaces in multimaterial flows (the ghost fluid method). *J. Comput. Phys.* **152**, 457–492 (1999)
10. Toro, E.F.: *Riemann solvers and numerical methods for fluid dynamics: a practical introduction*, 3rd edn. Springer, Berlin (2009)
11. Wilkins, M.L.: Calculation of elastic-plastic flow. *Mech. Comput. Phys.* **3**, 211–263 (1964)
12. Wilkins, M.L.: *Computer simulation of dynamic phenomena*. Springer, New York (1999)
13. Tyndall, M.B.: Numerical modelling of shocks in solids with elastic-plastic conditions. *Shock Waves* **3**, 55–66 (1993)
14. Smoller, J.: *Shock waves and reaction-diffusion equations*. Springer, New York (1983)