Balancing Rationality and Utility in Logic-Based Argumentation with Classical Logic Sentences and Belief Contraction

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Abstract. Compared to abstract argumentation theory which encapsulates the exact nature of arguments, logic-based argumentation is more specific and represents arguments in formal logic. One significant advantage of logic-based argumentation over abstract argumentation is that it can directly benefit from logical properties such as logical consistency, promoting adherence of an argumentation framework to rational principles. On the other hand, a logical argumentation framework based on classical logic has been also reported of its less-than-desirable utility. In this work we show a way of enhancing utility without sacrificing so much of rationality. We propose a rational argumentation framework with just classical logic sentences and a belief contraction operation. Despite its minimalistic appearance, this framework can characterise attack strengths, allowing us to facilitate coalition profitability and formability semantics we previously defined for abstract argumentation.

1 Introduction

Logic-based argumentation specialises Dung's abstract argumentation theory [8], representing arguments in formal logic. One significant advantage of logic-based argumentation over abstract argumentation is that it can directly benefit from logical properties such as logical consistency of the underlying formal logic. It promotes adherence of an argumentation framework to rational principles. There are studies (e.g. [2,5,10,15]) in this direction that identify logically desirable properties in argumentation frameworks.

It may appear trivial to attain such logical rigour among arguments and attacks at first sight. If we just assume that all arguments in an argumentation framework are classical logic sentences, any conflict between them could be just logical inconsistency. However, the use of logical inconsistency causes attack relation R to be necessarily symmetric, attacks meanwhile become necessarily cyclic, which was reported to restrict expressiveness of an argumentation framework [6]. A few approaches were proposed to bar the uniform symmetry. One could add a preference relation [12] to eliminate some of the members of R. One may also consider dividing an argument into its supports and its conclusion, facilitating differentiation of attacks. For instance, there is the rebutting

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where a conclusion of an argument attacks a conclusion of another argument, and there is the undercutting where a conclusion of an argument attacks a support of another argument. One may make use of both strict and defeasible rules, which is a popular approach in defeasible reasoning.

Either of them could reduce logical rigour, however. The preference relation, while powerful, does not elucidate the origin of the preference. The division or, to be more precise, the assumption that an argument is clear-cuttingly dividable into supports and a conclusion, too, is a source of logical incoherence. Consider a dialogue:

A(1): You finish your homework today.
B: No, dad. I still have one week. I will do it this weekend.
A(2): No arguing. You must listen to me.

Suppose $P_{1,2,3,4}$ are the following propositions: P_1: **B** finish homework today.; P_2: **B** still has one week.; P_3: **B** will do homework this weekend.; and P_4: **B** must listen to **A**. Suppose that (X, F) stands for an argument having its supports X, a set of sentences, and its conclusion F, a sentence. A possible encoding of the three arguments $\mathbf{A}(1), \mathbf{B}$ and $\mathbf{A}(2)$ is then: $(\emptyset, P_1)$, $(\{P_2, P_3\}, \neg P_1)$, and $(\{P_4\}, \neg\neg P_1)$. Assume that each conclusion follows logically from their supports. The problem is that it is not the unique encoding, since $(\emptyset, P_1)$, $(\{P_2\}, \neg P_1 \wedge P_3)$, $(\{\neg\neg P_1\}, P_4)$ is also plausible. Again, assume that each conclusion follows logically from their supports. It is usually not possible to totally demarcate supports and conclusions when arguments are obtained from natural expressions like shown above.

1.1 Contribution

In this work we show a way of enhancing utility without sacrificing so much of rationality by utilising a contraction operation from belief revision theories [1,7,13]. A contraction operation is informally an operation to rationally - as well as minimally - change a particular set of beliefs when existing beliefs are removed from it. A belief is often a formal sentence. We consider a rational argumentation framework $(\Gamma, \div)$ where Γ is a set of classical logic sentences, and $\div$, used for belief contraction, is a binary operation defined on a pair of the power set of the set of formal sentences. This framework as we will show has fair expressiveness despite its minimalistic appearance. The key observation is that $\div$ can be used to know attack strengths, in particular whether an attack is defeating or non-defeating. If we assume the concept of conflict-eliminability [3]: arguments can be grouped together so long as they do not defeat each other, then symmetric attacks are no longer such a big issue, for they do not imply symmetric defeats. We will show that our rational argumentation framework is expressive enough to represent certain features of coalition formation, specifically coalition profitability and formability semantics we described for abstract argumentation [3]. Though we pass the details to Sect. 2, this result - that coalition profitability and formability semantics can be just as well characterised in the rational framework as in abstract framework - is quite nice in light of gathering interests in "argument strength", to which this winter will incidentally dedicate a workshop.[1] Out of the key discussions expected in the venue, our work should offer insights into the following three questions.

1. Which factors influence the strength of an argument?
2. Can weaker arguments defeat and/or defend stronger arguments?
3. How do formal and informal approaches to argument strength relate?

In the rest, we will: go through preliminary materials (in Sect. 2); develop our rational argumentation frameworks (in Sect. 3); and characterise coalition profitability and formability semantics as well as detail relation to abstract argumentation (in Sect. 4), before drawing conclusions with related work.

2 Preliminaries

2.1 Abstract Argumentation Frameworks for Coalition Profitability and Formability

We recall abstract coalition profitability and formability we previously discussed [3]. Let $\mathbb{N}$ be the class of natural numbers including 0, let $\mathcal{A}$ be the set of abstract entities representing arguments, and let $\mathcal{S}$ be $\mathcal{A} \times \mathbb{N}$. We denote each element of $\mathbb{N}$ by i, j, k, m or n with or without a subscript, each element of $\mathcal{A}$ by a with or without a subscript, and each element of $\mathcal{S}$ by s with or without a subscript. We assume a projection operator π which is defined on ordered sets Σ and which is such that $\pi(n, \Sigma)$ is: $\{n$-th element of $\Sigma\}$ if $n \leq |\Sigma|$; and undefined, otherwise. For any $s \in \mathcal{S}$, we call $\pi(1, s)$ the argument identity of s and call $\pi(2, s)$ the argument capacity of s. We assume that an argumentation framework is a (S, R) for $S \subseteq \mathcal{S}$ and $R : 2^{\mathcal{S}} \times \mathcal{S} \rightharpoonup \mathbb{N}$ such that it satisfies all the following conditions. In the rest, a relation G (not the specific symbol G but any relation) being defined for something, say X (likewise, not the specific symbol X, but some entity on which G is defined), is synonymous to $G(X)$ being defined.

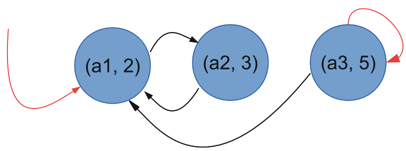
1. S is a finite set [Finite arguments].
2. For any $(a, n) \in S$, it holds that $n > 0$ [Positive argument capacity].
3. For any $(a, n) \in S$, there is no $m \neq n$ such that $(a, m) \in S$ [Unique argument identity].
4. R is undefined for $(\emptyset, s)$ for any $s \in \mathcal{S}$ [Attack coherence].
5. For any $S_1 \subseteq S$ and for any $s \in S$, if R is defined for (S_1, s), then R is defined for any (S_2, s) for $\emptyset \subset S_2 \subseteq S_1$. [Quasi-closure by subset relation].
6. For any $S_1, S_2 \subseteq S$ and for any $s \in S$, if R is defined both for (S_1, s) and for (S_2, s), then R is defined also for $(S_1 \cup S_2, s)$ [Closure by set union].
7. For any $S_1 \subseteq S$ and for any $s \in S$ such that R is defined for (S_1, s), it holds that $R(S_1, s) > 0$ [Attack with a positive strength].

[1] http://homepages.ruhr-uni-bochum.de/defeasible-reasoning/Argument-Strength-2016.html.

8. For any $(a, n), (a, m) \in \mathcal{S}$ such that $n \leq m$, if $R(S_1, s)$ - for some $s \in \mathcal{S}$ and for some $S_1 \subseteq_{\text{fin}} \mathcal{S}$ such that $(a, n) \in S_1$ - is defined, then $R(S_2, s)$ for $S_2 = (S_1 \backslash (a, n)) \cup (a, m)$ is defined, which is furthermore such that $R(S_1, s) \leq R(S_2, s)$ [Attack monotonicity 1 (source)].
9. For any $S_1, S_2 \subseteq S \subseteq_{\text{fin}} \mathcal{S}$ and for any $s \in S$, if R is defined for (S_1, s), (S_2, s) and $(S_1 \cap S_2, s)$, then $R(S_1 \cap S_2, s) \leq R(S_i, s)$ for both $i = 1$ and $i = 2$ [Attack monotonicity 2 (source)].
10. For any $(a, n), (a, m) \in \mathcal{S}$ such that $n \leq m$, it holds that if R is defined for $(S_1, (a, n))$ for some $S_1 \subseteq_{\text{fin}} \mathcal{S}$ such that $S_1 \cap \bigcup_{l \in \mathbb{N}} \{(a, l)\} = \emptyset$, then it is defined for $(S_1, (a, m))$, and, moreover, $R(S_1, (a, n)) \leq R(S_1, (a, m))$ [Attack monotonicity 3 (target)].
11. R is undefined for (S_1, s) if $S_1 \subseteq_{\text{fin}} \mathcal{S}$ and $s \in S_1$ [No self attacks].

Here and everywhere we may use and to emphasize truth-value comparisons. The expression '$S_1 \subseteq_{\text{fin}} \mathcal{S}$ and $s \in S_1$' is basically: 'it is the case that $S_1 \subseteq_{\text{fin}} \mathcal{S}$ and it is also the case that $s \in S_1$'.

The first three conditions are for $\pi(1, (S, R))$. The finiteness condition is assumed in many practical situations. The third condition enforces that each argument identified with an argument identity appears once in S. The second condition reflects an assumption that an argument capacity is proportional to meaningfulness of an argument, the greater the more meaningful, and 0 meaningless and not to be considered. Although the capacity is an abstract entity, it can be simplistically the number of sub-arguments of the argument : in the earlier example with **A(1)**, **B** and **A(2)**, we could give 1 to **A(1)**, 2 to **B**, and 2 to **A(2)**. The remaining conditions are for $\pi(2, (S, R))$. We visualise important points about them with a drawing. We assume that R is defined for $(\{s_1\}, s_2)$ if there is an arrow from s_1 into s_2.



There are five arrows. Among them, two are red, and they are not permitted in this argumentation framework. The red arrow to the left of $(a_1, 2)$ signifies that an attack would be possible without any attacking argument, which [Attack coherence] prohibits. The other red arrow indicates that there may be an argument that attacks itself. Such an argument should not be taken seriously. [No self attacks] prevents it from appearing in the argumentation framework. [Closure by set union] is perhaps intuitive enough. [Quasi-closure by subset relation] precludes the following situation: a group of arguments attacks some argument, but no subgroups of the group attack it. These two conditions may not be so adequate when a group of arguments collectively mean more (or less) than when they are taken individually [14]. We do not deal with this problem of argument accrual. The three attack monotonicity conditions reflect the following observations. Suppose $R(S_1, s)$ is defined for some $S_1 \subseteq S$ and an argument $s \in S$. Now, let us increase argument capacity of some argument $s_1 \in S_1$ into $s_1' \in \mathcal{S}$

so that we have $S_2 = (S_1 \backslash \{s_1\}) \cup \{s'_1\}$. An argument with a greater capacity is considered more meaningful, having a greater impact on attacking other arguments. For instance, if we regard the capacity of an argument representative of the number of sub-arguments of the argument, it says more with a greater capacity. Consequently:

1. If R is defined for (S_1, s) for some $S_1 \subseteq S$ and any $s \in S$, then if the capacity of some argument $s_1 \in S_1$ increases into $s'_1 \in \mathcal{S}$, then, all else unchanged, the attack strength of $S'_1 = (S_1 \backslash \{s_1\}) \cup \{s'_1\}$ on s should not be weaker, and, of course, S'_1 should still be attacking s.
2. Also, if R is defined for (S_1, s) then if R is also defined for (S_2, s) such that $S_1 \subseteq S_2$, then the attack by S_2 on s should not be weaker than by S_1 on s.

Also, if R is defined for (S_1, s) for any $S_1 \subseteq S$ and any $s \in S$, and if the argument capacity of s increases into $s' \in \mathcal{S}$, then the attack strength of S_1 on s' should not be weaker than that of S_1 on s, for an argument with a large argument capacity has more materials for other arguments to attack.

Attacks and Conflict-Eliminable Sets. Assume an abstract argumentation framework (S, R). We say that $S_1 \subseteq S$ attacks $s \in S$ iff there exists $S_2 \subseteq S_1$ such that R is defined for (S_2, s). We say that $S_1 \subseteq S$ defeats $s \in S$ iff S_1 attacks s and there exists $S_2 \subseteq S_1$ such that $R(S_2, s) \geq \pi(2, s)$. We define $\mathsf{Attacker} : 2^{\mathcal{S}} \rightarrow 2^{\mathcal{S}}$ to be such that $\mathsf{Attacker}(S_1) = \{s \in S \mid \text{there exists } s_1 \in S_1 \text{ such that } s \text{ attacks } s_1.\}$. We say that $S_1 \subseteq S$ is conflict-eliminable iff there exists no $s \in S_1$ such that S_1 defeats s.

A conflict-eliminable set is associated with its intrinsic arguments. Let $\alpha : 2^{\mathcal{S}} \rightharpoonup 2^{\mathcal{S}}$ be such that it is defined for $S_1 \subseteq S$ iff S_1 is conflict-eliminable. If α is defined for $S_1 \subseteq S$, then we define that $\alpha(S_1) = \{(\pi(1, s), n) \mid s \in S_1 \text{ and } n = \pi(2, s) - V^{\max}(S_1, s)\}$ where $V^{\max}(S_1, s)$ is either 0 in case S_1 does not attack s, or else $R(S_2, s)$ for some $S_2 \subseteq S_1$ such that (1) R is defined for (S_2, s); and (2) if R is defined for (S_x, s) for $S_x \subseteq S_1$, then $R(S_x, s) \leq R(S_2, s)$. We say that $\alpha(S_1)$ are intrinsic arguments of S_1 if α is defined for S_1.

A conflict-eliminable set of arguments has its own view of (S, R). Let $\mathsf{Del}_R(S, S_x)$ be $\{(S_y, s) \mid s \in S_x \text{ and } S_y \subseteq S_x \text{ and } R(S_y, s) \text{is defined.}\}$, which is the set of attack relations within S_x. Now, let S_1 be a subset of S. If α is defined for S_1, then we say that $((S \backslash S_1) \cup \alpha(S_1), R \backslash \mathsf{Del}(S, S_1))$ is S_1's view of S, which we denote by $\mathsf{View}_R(S, S_1)$.

Conflict-Eliminable Sets' Attacks and Admissibility. We say that $S_1 \subseteq S$ c-attacks $s \in S$ iff α is defined for S_1 and there exists some $S_2 \subseteq \alpha(S_1)$ such that $\pi(2, \mathsf{View}_R(S, S_1))$ is defined for (S_2, s). We say that S_1 c-defeats $s \in S$ iff S_1 c-attacks s and $\pi(2, \mathsf{View}_R(S, S_1))(\alpha(S_1), s) \geq \pi(2, s)$. These notions are similar to attacks and defeats, but are from the point of view of a conflict-eliminable set of arguments. Hence, if α is defined for $S_1 \subseteq S$, then for any $s \in S$, $\alpha(S_1)$ does not c-attack s, as can be straightforwardly verified. We say that $S_1 \subseteq S$ is c-admissible iff α is defined for S_1 and if $S_2 \subseteq \pi(1, \mathsf{View}_R(S, S_1))$ attacks $s \in S_1$ and if $S_x \subseteq S_2$ is such that $R(S_x, s)$ is defined, then there exists some $S_3 \subseteq \alpha(S_1)$ such that S_3 c-defeats some $s_x \in S_x$. We say that $S_1 \subseteq S$ is c-preferred iff S_1 is c-admissible and there exists no $S_1 \subset S_y \subseteq S$ such that S_y is c-admissible. Now, in order that a conflict-eliminable set S_1 be coherent in its attacks, it must only attack external arguments with its intrinsic arguments. In comparison, attacks into S_1 are not bound by the restriction: an external argument can attack the conflict-eliminable set S_1 by attacking any $s_1 \in S_1$. Let $S_1 \subseteq S$ be such that $\alpha(S_1)$ is defined. We say that a conflict-eliminable set S_1 is one-directionally attacked iff there exists $S_x \subseteq \pi(1, \mathsf{View}_R(S, S_1))$ such that S_x attacks $s \in S_1$ and S_1 does not c-attack any $s_x \in S_x$.

Coalition Profitability and Formability Semantics. We say that S_2 is in at least as good a state as S_1 is, of which we state $S_1 \preceq S_2$, iff α is defined both for S_1 and S_2 and any of the three conditions below is satisfied: (1) S_2 is c-admissible; (2) S_1 is one-directionally attacked; or (3) neither S_1 nor S_2 is c-admissible or one-directionally attacked. We say that coalition is permitted between $S_1, S_2 \subseteq S$ iff $S_1 \cap S_2 = \emptyset$ and α is defined for $S_1 \cup S_2$. We define profitability relation $\trianglelefteq : 2^{\mathsf{S}} \times 2^{\mathsf{S}}$ to be such that $S_1 \trianglelefteq S_2$ satisfies three axioms below.

1. $S_1 \subseteq S_2$ (larger set).
   Explanation: A larger set is a better set.
2. $S_1 \preceq S_2$ (better state).
   Explanation: A set that is in a better state is a better set.
3. $|\{s \in \mathsf{Attacker}(S_1) \mid S_1$ does not c-defeat s and $s \notin S_1\}| \geq$
   $|\{s \in \mathsf{Attacker}(S_1) \mid S_2$ does not c-defeat s and $s \notin S_2\}|$
   (fewer attackers).
   Explanation: A set that is attacked by a fewer number of attackers is better.

We say that S_2 is at least as profitable for S_1 as S_1 is for itself iff $S_1 \trianglelefteq S_2$. By profitability discontinuation theorem [3], satisfaction of the three conditions: (1) $S_1 \trianglelefteq S_2$; (2) $S_1 \trianglelefteq S_3$; and (3) $S_2 \subseteq S_3$, does not guarantee $S_2 \trianglelefteq S_3$.

Suppose we denote by $\mathsf{Max}(S_1)$ the set of all $S_x \subseteq S$ that satisfy the conditions: (1) $S_1 \trianglelefteq S_x$; and (2) if $S_x \subset S_y \subseteq S$, then it is not the case that $S_1 \trianglelefteq S_y$. Then, if S_1 forms a coalition with S_2 satisfying $S_1 \trianglelefteq S_1 \cup S_2$, $\mathsf{Max}(S_1) \backslash \mathsf{Max}(S_1 \cup S_2)$ is no longer reachable from $S_1 \cup S_2$. Then suppose that some $S_x \in \mathsf{Max}(S_1)$ is a maximal element in $\mathsf{Max}(S_1)$ under some criteria, it may become unreachable from $S_1 \cup S_2$ by $\trianglelefteq$, depending on which conflict-eliminable set S_2 is. From S_1, if such maximal S_x is to be formed potentially incrementally: first with some $S_y \subseteq (S_x \backslash S_1)$ such to obtain $S_1 \trianglelefteq S_1 \cup S_y$; then with some $S_z \subseteq ((S_x \backslash S_1) \backslash S_y)$ such to obtain $S_1 \cup S_y \trianglelefteq S_1 \cup S_y \cup S_z$, and so on, it is clear that S_y, S_z and so on should be such that $S_x \in \mathsf{Max}(S_1 \cup S_y)$, $S_x \in \mathsf{Max}(S_1 \cup S_y \cup S_z)$, and so on. We shall define a stronger relation: $\trianglelefteq_{\mathrm{m}}$, which is such that if $S_1 \trianglelefteq_{\mathrm{m}} S_2$, then there exists some $S_a \in \mathsf{Max}(S_2)$ which is a maximal element of $\mathsf{Max}(S_1)$. Formally, let $\leq_l, \leq_b, \leq_f$: $2^{\mathcal{S}} \times 2^{\mathcal{S}}$ be such that they satisfy all the following:

1. $S_1 \leq_l S_2$ iff $|S_1| \leq |S_2|$.
2. $S_1 \leq_b S_2$ iff S_2 is at least as good by (better state) as S_1.
3. $S_1 \leq_f S_2$ iff S_2 is at least as good by (fewer attackers) as S_1.

and let $S_1 <_\beta S_2$ for each $\beta \in \{l, b, f\}$ hold just when $S_1 \leq_\beta S_2$ but not $S_2 \leq_\beta S_1$. Then we define $\trianglelefteq_{\mathrm{m}} : 2^{\mathcal{S}} \times 2^{\mathcal{S}}$ to be such that if $S_1 \trianglelefteq_{\mathrm{m}} S_2$, then both of the following conditions satisfy:

1. $S_1 \trianglelefteq S_2$.
2. Some $S_x \in \mathsf{Max}(S_2)$ is such that, for all $S_y \in \mathsf{Max}(S_1)$, if $S_x <_\beta S_y$ for some $\beta \in \{l, b, f\}$, then there exists $\gamma \in (\{l, b, f\} \backslash \beta)$ such that $S_y <_\gamma S_x$.

The second condition is giving a definition to maximality of an element in $\mathsf{Max}(S)$. We have four coalition formability semantics as follows.

$$\mathsf{W}(S_1) = \{S_2 \subseteq S \mid S_1 \trianglelefteq S_1 \cup S_2 \text{ or } S_2 \trianglelefteq S_1 \cup S_2\}.$$
$$\mathsf{M}(S_1) = \{S_2 \subseteq S \mid S_1 \trianglelefteq S_1 \cup S_2 \text{ and } S_2 \trianglelefteq S_1 \cup S_2\}.$$
$$\mathsf{WS}(S_1) = \{S_2 \subseteq S \mid S_1 \trianglelefteq_{\mathrm{m}} S_1 \cup S_2 \text{ or } S_2 \trianglelefteq_{\mathrm{m}} S_1 \cup S_2\}.$$
$$\mathsf{S}(S_1) = \{S_2 \subseteq S \mid S_1 \trianglelefteq_{\mathrm{m}} S_1 \cup S_2 \text{ and } S_2 \trianglelefteq_{\mathrm{m}} S_1 \cup S_2\}.$$

Here and everywhere the semantics of classical logic disjunction is assumed for or. Assume three utility postulates:

I Coalition is good when it is profitable at least to one party.
II Coalition is good when it is profitable to both parties.
III Coalition is good when maximal potential future profits are expected from it.

W (, M, WS, S) respects I (, II, I + III, II + III).

2.2 Rational Contraction

We assume propositional logic. Our languages consist of: (1) a fixed number of logical symbols: $\top, \bot, \wedge, \vee, \neg$, as well as parentheses and brackets; and (2) a finite number of propositional variables, each of which is referred to by p with or without a subscript. We denote each language by $\mathbf{K}$ with or without a subscript, but dedicate $\mathbf{K}_0$ to the language having the largest number of propositional variables. We denote the class of sentences constructable in each $\mathbf{K}$ by $\mathcal{P}_{\mathbf{K}}$, and refer to a sentence by F with or without a subscript. We define $\mathsf{L} : 2^{\mathcal{P}_{\mathbf{K_0}}} \rightarrow 2^{\mathcal{P}_{\mathbf{K_0}}}$ to be such that $F \in \mathsf{L}(\{F_1, F_2, \ldots\})$ iff F is a logical consequence of a finite subset of $\{F_1, F_2, \ldots\}$. We say that a set of sentences Γ is consistent iff $\mathsf{L}(\Gamma) \neq \mathcal{P}_{\mathbf{K_0}}$. Among three common binary operations: expansion, contraction and revision [1,9] in belief revision theories, we will require just contraction $\div$ which minimally removes some set of sentences off a larger set of sentences. This operator satisfies the following axioms.

1. $\mathsf{L}(\Gamma_1) \div \Gamma_2 = \mathsf{L}(\mathsf{L}(\Gamma_1) \div \Gamma_2)$ (Closure).
2. $\mathsf{L}(\Gamma_1) \div \Gamma_2 \subseteq \mathsf{L}(\Gamma_1)$ (Inclusion).

3. $\mathsf{L}(\Gamma_1) \div \Gamma_2 = \mathsf{L}(\Gamma_1) \div \Gamma_3$ if $\mathsf{L}(\Gamma_2) = \mathsf{L}(\Gamma_3)$ (Extensionality).
4. $\mathsf{L}(\Gamma_1) \div \Gamma_2 = \mathsf{L}(\Gamma_1)$ if, for each $F \in \Gamma_2$, either $\mathsf{L}(\{F\}) = \mathsf{L}(\{\top\})$ or else $F \notin \mathsf{L}(\Gamma_1)$ (Vacuity).
5. For each $F \in \Gamma_2$, $F \notin \mathsf{L}(\Gamma_1) \div \Gamma_2$ if $\mathsf{L}(\{F\}) \neq \mathsf{L}(\{\top\})$ (Success).
6. $\mathsf{L}(\Gamma_1) \subseteq (\mathsf{L}(\Gamma_1) \div \Gamma_2) \cup \Gamma_2$ (Recovery).

The condition (Vacuity) says firstly that no tautological sentence is removed, and secondly that it is always a sentence in $\mathsf{L}(\Gamma_1)$ that is to be removed. The condition (Recovery) ensures a minimal change of $\mathsf{L}(\Gamma_1)$.

3 Rational Argumentation Frameworks

Our rational argumentation framework is a tuple $(\Gamma, \div)$ where Γ is a finite non-empty set of sentences.

Definition 1 (Coherence). We say that $(\Gamma, \div)$ is coherent iff (1) no $F \in \Gamma$ is such that $\mathsf{L}(\{F\}) = \mathsf{L}(\{\top\})$ or that $\mathsf{L}(\{F\}) = \mathcal{P}_{\mathbf{K_0}}$ and (2) no $F_1, F_2 \in \Gamma$ are such that $\mathsf{L}(\{F_1\}) = \mathsf{L}(\{F_2\})$.

A coherent rational argumentation framework $(\Gamma, \div)$ will be assumed in the rest. It is clear from the above definition that no $F \in \Gamma$ is a tautology which is a vacuous argument or an inconsistent sentence which is not to be taken seriously. Now, it is certainly possible to define conflict-freeness in a set of arguments: let Γ_1 be a non-empty subset of Γ, then Γ_1 is conflict-free iff $\mathsf{L}(\{F_1\} \cup \{F_2\}) \neq \mathcal{P}_{\mathbf{K_0}}$ for all $F_1, F_2 \in \Gamma_1$.[2] This notion, however, gives away very useful information of relative strength of attacks. We will instead rely upon conflict-eliminability.

Definition 2 (Opposition Force). Let $\mathsf{O} : 2^{\mathcal{P}_{\mathbf{K_0}}} \times \mathcal{P}_{\mathbf{K_0}} \rightarrow 2^{\mathcal{P}_{\mathbf{K_0}}}$ be such that $\mathsf{O}(\Gamma_1, F_2) = \{F_1 \in \Gamma_1 \mid F_2 \in \Gamma \text{ and } \mathsf{L}(\{F_1, F_2\}) = \mathcal{P}_{\mathbf{K_0}} \text{ and } \Gamma_1 \subseteq \Gamma\}$. We say that $\mathsf{O}(\Gamma_1, F_2)$ is the opposition force in Γ_1 against $F_2 \in \Gamma$.

$\mathsf{O}(\Gamma_1, F_2)$ may be empty. We denote $\{\neg F \mid F \in \mathsf{O}(\Gamma_1, F_2)\}$ by $\mathsf{O}^-(\Gamma_1, F_2)$.

Example 1. Suppose a set of sentences:$\{F_1, F_2, \neg F_1 \wedge \neg F_2\}$. We have $\mathsf{O}(\{F_1, F_2\}, \neg F_1 \wedge \neg F_2) = \{F_1, F_2\}$. Then, $\mathsf{O}^-(\{F_1, F_2\}, \neg F_1 \wedge \neg F_2) = \{\neg F_1, \neg F_2\}$.

Definition 3 (Attacks and Defeats). For $F \in \mathcal{P}_{\mathbf{K_0}}$, let $\mathsf{Lang}(F)$ be the smallest language $\mathbf{K}$ that recognises F. We say that $\Gamma_1 \subseteq \Gamma$ attacks $F \in \Gamma$ iff $\mathsf{O}(\Gamma_1, F) \neq \emptyset$. We say that $\Gamma_1 \subseteq \Gamma$ defeats $F \in \Gamma$ iff Γ_1 attacks F and $(\mathsf{L}(\{F\}) \div \mathsf{O}^-(\Gamma_1, F)) \cap \mathcal{P}_{\mathsf{Lang}(F)} = \mathsf{L}(\{\top\}) \cap \mathcal{P}_{\mathsf{Lang}(F)}$.

Example 2. Suppose $\{p_1, \neg p_1 \wedge p_2\}$ where p_1 and p_2 are propositional variables. Both $\mathsf{O}(\{p_1\}, \neg p_1 \wedge p_2)$ and $\mathsf{O}(\{\neg p_1 \wedge p_2\}, p_1)$ are non-empty, and so each element of the set is attacking the other. However, while $(\mathsf{L}(\{p_1\}) \div \{p_1 \vee \neg p_2\}) \cap \mathcal{P}_{\mathsf{Lang}(p_1)} = \mathsf{L}(\{\top\}) \cap \mathcal{P}_{\mathsf{Lang}(p_1)}$, $(\mathsf{L}(\{\neg p_1 \wedge p_2\}) \div \{\neg p_1\}) \cap \mathcal{P}_{\mathsf{Lang}(\neg p_1 \wedge p_2)} \neq \mathsf{L}(\{\top\}) \cap \mathcal{P}_{\mathsf{Lang}(\neg p_1 \wedge p_2)}$, as it contains p_2 for instance. Hence, $\{\neg p_1 \wedge p_2\}$ defeats p_1, but $\{p_1\}$ does not defeat $\neg p_1 \wedge p_2$.

[2] In this paper, we will focus on pairwise logical inconsistency only.

Some explanations concerning '$(\mathsf{L}(\{F\}) \div \mathsf{O}^{-}(\Gamma_1, F)) \cap \mathcal{P}_{\mathbf{K_0}}$' in the above definition may be helpful. Notice in the left operand, i.e. '$(\mathsf{L}(\{F\}) \div \mathsf{O}^{-}(\Gamma_1, F))$', that we require '$\mathsf{L}(\{F\})$' instead of $\{F\}$. If, say, F is some propositional variable p_1, then $\mathsf{L}(\{F\})$ contains $p_1 \vee p_3, p_1 \vee p_4, \ldots$. Suppose $\mathsf{O}^{-}(\Gamma_1, F) = \{p_1\}$, then certainly $p_1 \notin \mathsf{L}(\{p_1\}) \div \{p_1\}$, but it may be that some of $p_1 \vee p_3, p_1 \vee p_4, \ldots$ are in $\mathsf{L}(\{p_1\}) \div \{p_1\}$, even though $p_3, p_4, \ldots$ is not in $\mathsf{Lang}(F)$. The set intersection ensures that the result of belief contraction is relevant to $\mathsf{Lang}(F)$.

Definition 4 (Conflict-Eliminable Sets). We say that $\Gamma_1 \subseteq \Gamma$ is conflict-eliminable iff Γ_1 does not defeat any $F \in \Gamma_1$.

The following notion will come in handy. We assume that the length of a sentence is proportional to the number of symbols occurring in the sentence.

Definition 5 (Minimal Support). Let Γ_1 be such that $\Gamma_1 = \mathsf{L}(\Gamma_1)$. We say that F_1 is a support of Γ_1 iff $\Gamma_1 = \mathsf{L}(\{F_1\})$. We say that F_1 is a minimal support of Γ_1 iff there exists no support F_2 of Γ_1 that is shorter than F_1 in length. We denote some minimal support of $\Gamma_1 = \mathsf{L}(\Gamma_1)$ by $\mathsf{minS}(\Gamma_1)$. We define that $\mathsf{minS}(\Gamma_1) = \mathsf{minS}(\Gamma_2)$ if $\Gamma_1 = \mathsf{L}(\Gamma_2)$. For later convenience, we shall assume that $F = \mathsf{minS}(\mathsf{L}(\{F\}))$ for a coherent rational framework $(\Gamma, \div)$ and for any $F \in \Gamma$.

Proposition 1 (Existence of a Minimal Support). *Let F_1 be a member of Γ. Then, if $\Gamma_2 = \mathsf{L}(\{F_1\}) \div \Gamma_1$ for some $\Gamma_1 \subseteq \Gamma$, there exists some F_2 such that $F_2 = \mathsf{minS}(\Gamma_2)$.*

Proof. Γ in the assumed rational argumentation framework is finite. Also a sentence is necessarily of a finite length. □

The purpose of minS is linked closely to our need of referring to a contracted argument. Let us be specific. Suppose a non-defeating attack of $\{p_1\}$ on $\neg p_1 \wedge \neg p_2$. We have $\mathsf{L}(\{\neg p_1 \wedge \neg p_2\}) \div \{\neg p_1\}$ which is nominally an infinite set not as easily treated as a sentence. However, it is actually sufficient if we have $\neg p_2$ for the representation of the infinite set, since $\mathsf{L}(\{\neg p_1 \wedge \neg p_2\}) \div \{\neg p_1\} = \mathsf{L}(\{\neg p_2\})$. With minS, we have $\neg p_2 = \mathsf{minS}(\mathsf{L}(\{\neg p_1 \wedge \neg p_2\}) \div \{\neg p_1\})$. Generally, $\mathsf{minS}(\mathsf{L}(\{F\}) \div \{F_1\})$ is minimal information (as a sentence) about $\mathsf{L}(\{F\}) \div \{F_1\}$ we need.

Definition 6 (Intrinsic Arguments). Let $\Gamma_1 \subseteq \Gamma$ be a conflict-eliminable set. We define intrinsic arguments of Γ_1, denoted $\Gamma_1^{\star}$ with a super-script $\star$, to be $\{F \in \Gamma_1 \mid \mathsf{O}(\Gamma_1, F) = \emptyset\} \cup \{F_1 \mid F \in \Gamma_1 \text{ and } \mathsf{O}(\Gamma_1, F) \neq \emptyset \text{ and } F_1 = \mathsf{minS}(\mathsf{L}(\{F\}) \div \mathsf{O}^{-}(\Gamma_1, F))\}$.

Intrinsic arguments of a set of arguments are those sentences that are left after all non-defeating attacks have weakened their targets.

Example 3. Suppose $\Gamma_1 = \{p_1 \wedge \neg p_2, \neg p_1 \wedge p_3, p_4\}$, and suppose that each propositional letter is distinct. $\Gamma_1^{\star}$ is: $\{\neg p_2, p_3, p_4\}$.

Proposition 2 (No Conflicts). *Let $\Gamma_1 \subseteq \Gamma$ be a conflict-eliminable set. Then $\mathsf{L}(\{F_1\} \cup \{F_2\}) \neq \mathcal{P}_{\mathbf{K_0}}$ for any $F_1, F_2 \in \Gamma_1^{\star}$.*

Characterisation of Attacks by a Conflict-Eliminable Set. As we stated in Sect. 2, there is an asymmetry in attacks to and from a conflict-eliminable set. While arguments outside it could attack any argument in the conflict-eliminable set, the conflict-eliminable set of arguments may attack the external arguments at most by its intrinsic arguments. We define this coalition attack and then admissible/preferred coalition sets.

Definition 7 (C-Opposition Force). Let $\mathsf{O}_C : 2^{\mathcal{P}_{\mathbf{K_0}}} \times \mathcal{P}_{\mathbf{K_0}} \rightarrow 2^{\mathcal{P}_{\mathbf{K_0}}}$ be such that $\mathsf{O}_C(\Gamma_1, F_2)$ is: empty if Γ_1 is not conflict-eliminable; or else $\{F_1 \in \Gamma_1^\star \mid \Gamma_1 \subseteq \Gamma$ and $F_2 \in \Gamma$ and $\mathsf{L}(\{F_1\} \cup \{F_2\}) = \mathcal{P}_{\mathbf{K_0}}\}$. We say that $\mathsf{O}_C(\Gamma_1, F_2)$ is the c-opposition force in Γ_1 against $F_2 \in \Gamma$.

We denote $\{\neg F \mid F \in \mathbf{O}_C(\Gamma_1, F_2)\}$ by $\mathsf{O}_C^-(\Gamma_1, F_2)$.

Definition 8 (C-Attacks and c-Defeats). We say that $\Gamma_1 \subseteq \Gamma$ c-attacks $F \in \Gamma$ iff $\mathsf{O}_C(\Gamma_1, F) \neq \emptyset$. We say that $\Gamma_1 \subseteq \Gamma$ c-defeats $F \in \Gamma$ iff Γ_1 c-attacks F and $(\mathsf{L}(\{F\}) \div \mathbf{O}_C^-(\Gamma_1, F)) \cap \mathsf{Lang}(F) = \mathsf{L}(\{\top\}) \cap \mathsf{Lang}(F)$.

Proposition 3 (No c-Self Attacks). *Let $\Gamma_1 \subseteq \Gamma$ be conflict-eliminable, and let F be a member of $\Gamma_1^\star$. Then $\mathsf{O}_C(\Gamma_1, F) = \emptyset$.*

Proof. Obvious by the definition of c-opposition force and by Proposition 2. □

Definition 9 (One-Directional Attacks). Assume that Γ_1 is a conflict-eliminable set. We say that Γ_1 is one-directionally attacked by $F \in \Gamma \backslash \Gamma_1$ iff $\mathsf{O}_C(\Gamma_1, F) = \emptyset$ and there exists some $F_1 \in \Gamma_1$ such that $\{F\}$ attacks F_1.

Example 4. Suppose $\Gamma = \{p_1 \wedge \neg p_2, \neg p_1 \wedge p_3, p_4, p_1\}$. $\Gamma_1 = \{p_1 \wedge \neg p_2, \neg p_1 \wedge p_3, p_4\}$ is a conflict-eliminable set. And p_1 is the only external argument to the set. Because $\Gamma_1^\star = \{\neg p_2, p_3, p_4\}$, it is not c-attacking p_1. But p_1 is (c-)attacking $\neg p_1 \wedge p_3$ in the set.

Definition 10 (C-Acceptance). We say that $\Gamma_1 \subseteq \Gamma$ c-accepts $F \in \Gamma$ iff Γ_1 does not defeat F and if $\mathsf{O}(\Gamma \backslash \Gamma_1, F) \neq \emptyset$, then Γ_1 c-defeats each $F_1 \in \mathsf{O}(\Gamma \backslash \Gamma_1, F)$.

Definition 11 (C-Admissible and c-Preferred Sets). We say that $\Gamma_1 \subseteq \Gamma$ is c-admissible iff Γ_1 is conflict-eliminable and Γ_1 c-accepts all its members. We say that a c-admissible set $\Gamma_1 \subseteq \Gamma$ is also a c-preferred set iff there exists no $\Gamma_1 \subset \Gamma_2 \subseteq \Gamma$ such that Γ_2 is c-admissible.

4 Logic-Based Coalition Profitability and Formability

Let us adapt some of the notations in Sect. 2. We define $\mathsf{Attacker}(\Gamma_1)$ to be $\{F \in \Gamma \mid$ there exists some $F_1 \in \Gamma_1$ such that F attacks $F_1.\}$. We say that coalition is approved between $\Gamma_1 \subseteq \Gamma$ and $\Gamma_2 \subseteq \Gamma$ iff $\Gamma_1 \cap \Gamma_2 = \emptyset$ and $\Gamma_1 \cup \Gamma_2$ is conflict-eliminable. We define $\preceq: 2^{\mathcal{P}_{\mathbf{K_0}}} \times 2^{\mathcal{P}_{\mathbf{K_0}}}$ to be such that $\Gamma_1 \preceq \Gamma_2$ iff Γ_1 and Γ_2 are conflict-eliminable and any of the three conditions: (1) Γ_2 is c-admissible; (2) Γ_1 is one-directionally attacked; or (3) neither Γ_1 nor Γ_2 is c-admissible or one-directionally attacked, satisfies. We define $\trianglelefteq : 2^{\mathcal{P}_{\mathbf{K_0}}} \times 2^{\mathcal{P}_{\mathbf{K_0}}}$ to be such that $\Gamma_1 \trianglelefteq \Gamma_2$ satisfies the following three conditions.

1. If $\Gamma_1 \trianglelefteq \Gamma_2$, then $\Gamma_1 \subseteq \Gamma_2$ (inclusion).
2. If $\Gamma_1 \trianglelefteq \Gamma_2$, then $\Gamma_1 \preceq \Gamma_2$ (better state).
3. $|\{F \in \mathsf{Attacker}(\Gamma_1) \mid \Gamma_1$ does not c-defeat F and $F \not\in \Gamma_1\}| \geq$
$|\{F \in \mathsf{Attacker}(\Gamma_1) \mid \Gamma_2$ does not c-defeat F and $F \not\in \Gamma_2\}|$
(fewer attackers).

The meaning is the same as in abstract setting: if $\Gamma_1 \trianglelefteq \Gamma_2$ then Γ_2 is at least as profitable for Γ_1 as Γ_1 is for itself. We denote by $\mathsf{Max}(\Gamma_1)$ the set of all $\Gamma_x \subseteq \Gamma$ where $\Gamma_1 \trianglelefteq \Gamma_x$ and if $\Gamma_x \subset \Gamma_y \subseteq \Gamma$, then not $\Gamma_1 \trianglelefteq \Gamma_y$. Let $\leq_l, \leq_b, \leq_f$: $2^{\mathcal{P}_{\mathbf{K_0}}} \times 2^{\mathcal{P}_{\mathbf{K_0}}}$ be such that they satisfy the following three conditions:

1. $\Gamma_1 \leq_l \Gamma_2$ iff $|\Gamma_1| \leq |\Gamma_2|$.
2. $\Gamma_1 \leq_b \Gamma_2$ iff Γ_2 is at least as good by (better state) as Γ_1.
3. $\Gamma_1 \leq_f \Gamma_2$ iff Γ_2 is at least as good by (fewer attackers) as Γ_1.

We write $\Gamma_1 <_\beta \Gamma_2$ for each $\beta \in \{l, b, f\}$ just when $\Gamma_1 \leq_\beta \Gamma_2$ and not $\Gamma_2 \leq_\beta \Gamma_1$. We define $\trianglelefteq_m : 2^{\mathcal{P}} \times 2^{\mathcal{P}}$ to be such that if $\Gamma_1 \trianglelefteq_{\mathrm{m}} \Gamma_2$, then:

1. $\Gamma_1 \trianglelefteq \Gamma_2$.
2. Some $\Gamma_x \in \mathsf{Max}(\Gamma_2)$ is such that, for all $\Gamma_y \in \mathsf{Max}(\Gamma_1)$, if $\Gamma_x \leq_\beta \Gamma_y$ for some $\beta \in \{l, b, f\}$, then there exists $\gamma \in (\{l, b, f\} \backslash \beta)$ such that $\Gamma_y <_\gamma \Gamma_x$.

The four coalition formability semantics in this logic-based argumentation are as follows.

$$\mathsf{W}(\Gamma_1) = \{\Gamma_2 \subseteq \Gamma \mid \Gamma_1 \trianglelefteq \Gamma_1 \cup \Gamma_2 \text{ or } \Gamma_2 \trianglelefteq \Gamma_1 \cup \Gamma_2\}.$$
$$\mathsf{M}(\Gamma_1) = \{\Gamma_2 \subseteq \Gamma \mid \Gamma_1 \trianglelefteq \Gamma_1 \cup \Gamma_2 \text{ and } \Gamma_2 \trianglelefteq \Gamma_1 \cup \Gamma_2\}.$$
$$\mathsf{WS}(\Gamma_1) = \{\Gamma_2 \subseteq \Gamma \mid \Gamma_1 \trianglelefteq_{\mathrm{m}} \Gamma_1 \cup \Gamma_2 \text{ or } \Gamma_2 \trianglelefteq_{\mathrm{m}} \Gamma_1 \cup \Gamma_2\}.$$
$$\mathsf{S}(\Gamma_1) = \{\Gamma_2 \subseteq \Gamma \mid \Gamma_1 \trianglelefteq_{\mathrm{m}} \Gamma_1 \cup \Gamma_2 \text{ and } \Gamma_2 \trianglelefteq_{\mathrm{m}} \Gamma_1 \cup \Gamma_2\}.$$

4.1 The Relation Between Abstract and Logic-Based Argumentation Frameworks

There is a good reason behind the close correlation between the semantics in Sect. 2 and those for our rational argumentation frameworks. Let $\varpi : \mathcal{P}_{\mathbf{K_0}} \rightarrow (\mathbb{N} \cup \{\infty\})$ be such that:

1. $\varpi(F) = \infty$ if $\mathsf{L}(\{F\}) = \mathsf{L}(\{\bot\})$.
2. $\varpi(F) = 0$ if $\mathsf{L}(\{F\}) = \mathsf{L}(\{\top\})$.
3. $\varpi(F_1) \leq \varpi(F_2)$ if $\mathsf{L}(\{F_1\}) \subseteq \mathsf{L}(\{F_2\})$.

Let $\kappa : 2^{\mathcal{P}_{\mathbf{K_0}}} \times \{\div\} \rightharpoonup 2^{\mathsf{S}} \times \{R\}$ be such that:

1. κ is defined for $(\Gamma, \div)$ iff Γ is finite.
2. Let $\tau : 2^{\mathcal{P}_{\mathbf{K_0}}} \rightarrow 2^{2^{\mathcal{P}_{\mathbf{K_0}}}}$ be such that: $\tau(\{F\}) = \{F_1 \mid \mathsf{L}(\{F_1\}) \subseteq \mathsf{L}(\{F\})$ and $F_1 = \mathsf{minS}(\mathsf{L}(\mathsf{L}(\{F_1\}) \cap \mathsf{Lang}(F))) \neq \top\}$; and $\tau(\Gamma) = \{\{\tau(F)\} \mid F \in \Gamma\}$. Assume that $\tau(\Gamma)$ is an ordered set in the rest. If κ is defined for $(\Gamma, \div)$, then $\pi(1, \kappa((\Gamma, \div)))$ maps one-to-one to $\tau(\pi(1, (\Gamma, \div)))$ in the following way: for any sentence F in the n-th set of $\tau(\pi(1, (\Gamma, \div)))$ there is $(a_n, \varpi(F))$ in $\pi(1, \kappa((\Gamma, \div)))$. For each $s \in \pi(1, \kappa((\Gamma, \div)))$, we refer to the corresponding sentence in $\tau(\pi(1, (\Gamma, \div)))$ by $\rho(s)$.

3. If κ is defined for $(\Gamma, \div)$, then $\pi(2, \kappa(\Gamma, \div))$ - that is, R - is defined for any (S, s) as long as: (1) $S' = S \cup \{s\}$ is a subset of $\pi(1, \kappa(\Gamma, \div))$ such that $\pi(1, (a_i, n_i)) \neq \pi(1, (a_j, n_j))$ for any $(a_i, n_i), (a_j, n_j) \in S \cup \{s\}$; and (2) for each $s_1 \in S$, $\mathsf{L}(\{\rho(s_1)\} \cup \rho(s)) = \mathcal{P}_{\mathbf{K_0}}$. Further, for each such (S, s), we define $R(S, s)$ to be: $\varpi(\rho(s))$ if $\bigcup_{s_1 \in S}\{\rho(s_1)\}$ defeats $\rho(s)$; and $\varpi(\rho(s)) - \varpi(\mathsf{minS}(\mathsf{L}(\{\rho(s)\}) \div \bigcup_{s_1 \in S}\{\neg\rho(s_1)\}))$, otherwise.

Perhpas no further explanations are needed for κ and ρ. We mention that τ obtains from Γ the set of all the formulas which are some $F \in \Gamma$, or some F_1 which is logically weaker than some $F_2 \in \Gamma$ and which is a member of $\mathsf{Lang}(F_2)$.

Theorem 1 (Embedding). *For any coherent rational argumentation framework* $(\Gamma, \div)$*,* $\kappa((\Gamma, \div))$ *is an argumentation framework as defined in Sect. 2.*

Proof. [Finite arguments] holds trivially. [Positive argument capacity] and [No self attacks] hold because a coherent rational framework does not contain a tautology or an inconsistent sentence. [Unique argument identity] holds by the way ϖ is defined. [Attack coherence] holds trivially. [Quasi-closure by subset relation] and [Closure by set union] hold by the way $\pi(2, \kappa(\Gamma, \div))$ is defined. [Attack with a positive strength] holds by the way $\pi(2, \kappa(\Gamma, \div))$ is defined. Note that a contracted argument maps into a smaller integer. The three monotonicity conditions hold by the way $\kappa(\Gamma, \div)$ and $(\Gamma, \div)$ are related. □

5 Conclusion

We showed how the abstract coalition profitability and formability semantics as mentioned in Sect. 2 may be defined in a minimalistic rational argumentation framework. Theorem 1 implies that the definition of the abstract argumentation framework, i.e. the 11 conditions in 2.1 [3], are logically grounded. Our rational framework is rational, provided that (1) representations of arguments in formal logic and (2) belief contraction are rational. The minimality of assumptions directly leads to minimality of logical incoherence.

Related Work. Instantiation of abstract argumentation with formal logic [2,4, 10,11,15] is a fairly natural idea. Yet it is an important study to be undertaken if the knowledge of abstract argumentation is to be applied in practice. Dung's abstract argumentation can be misused; adequate postulates will prevent the misuse. In the above-mentioned works an argument is divided into supports and a conclusion. We showed a way of attaining fair expressiveness in the absence of the insulation by utilising belief contraction. We believe the work by Gabbay and Garcez [10] to be the most relevant work to ours. They, too, observe shifts in attack strength. Compared to [10], we are focusing: on more static pictures of logic-based argumentation as our work does not step through sequences of attacks; on minimal argumentation; and on the relation between logic-based and abstract argumentation for coalition semantics.

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References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: partial meet contraction and revision functions. J. Symb. logic **50**, 510–530 (1985)
2. Amgoud, L., Besnard, P.: Bridging the gap between abstract argumentation systems and logic. In: Godo, L., Pugliese, A. (eds.) SUM 2009. LNCS, vol. 5785, pp. 12–27. Springer, Heidelberg (2009)
3. Arisaka, R., Satoh, K.: Coalition Formability Semantics with Conflict-Eliminable Sets of Arguments. arXiv e-prints:1605.00495 (2016)
4. Caminada, M., Amgoud, L.: An axiomatic account of formal argumentation. In: AAAI, pp. 608–613 (2005)
5. Caminada, M., Amgoud, L.: On the evaluation of argumentation formalisms. Artif. Intell. **171**(5–6), 286–310 (2007)
6. Coste-Marquis, S., Devred, C., Marquis, P.: Symmetric argumentation frameworks. In: Godo, L. (ed.) ECSQARU 2005. LNCS (LNAI), vol. 3571, pp. 317–328. Springer, Heidelberg (2005)
7. Darwiche, A., Pearl, J.: On the logic of iterated belief revision. Artif. Intell. **89**, 1–29 (1997)
8. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming, and n-person games. Artif. Intell. **77**(2), 321–357 (1995)
9. Fuhrmann, A., Hansson, S.O.: A survey of multiple contractions. Logic Lang. Inf. **3**(1), 39–75 (1997)
10. Gabbay, D.M., D'Avila Garcez, A.S.: Logical modes of attack in argumentation networks. Stud. Logica. **93**(2), 199–230 (2009)
11. Gorogiannis, N., Hunter, A.: Instantiating abstract argumentation with classical logic arguments: postulates and properties. Artif. Intell. **175**(9–10), 1479–1497 (2011)
12. Kaci, S., van der Torre, L., Weydert, E.: Acyclic argumentation: attack = conflict + preference. In: ECAI, pp. 725–726 (2006)
13. Katsuno, H., Mendelzon, A.O.: Propositional knowledge base revision and minimal change. Artif. Intell. **52**(3), 263–294 (1991)
14. Prakken, H.: A study of accrual of arguments, with applications to evidential reasoning. In: ICAIL, pp. 85–94 (2005)
15. Prakken, H., Sartor, G.: Argument-based extended logic programming with defeasible priorities. J. Appl. Non-class. Logics **7**, 25–75 (1997)