Chapter 4
Adaptive Control of Switched Nonlinear Systems

4.1 Background and Motivation

It has been shown in [1–5] that the adaptive backstepping technique is a powerful tool which has been widely used to solve some complex optimization problems and applied in the fields of industry and engineering. Recently, many adaptive backstepping-based control methods have been used in switched nonlinear systems; see, for example, [6–10] and the references therein. The authors in [11] solved the problem of adaptive stabilization for a class of uncertain switched nonlinear systems whose non-switching part consists of feedback linearizable dynamics. In [12], the authors investigated the problem of adaptive stabilization for a class of switched nonlinearly parameterized systems where the solvability of the adaptive stabilization problem for subsystems is unnecessary.

It is well known that the stability of a switched system under arbitrary switching can be guaranteed if a CLF exists for all subsystems [13]. Therefore, CLF has been extensively used for control synthesis of switched linear systems [14–17]. Recently, there have been some results on the global stabilization problem for switched nonlinear systems in strict-feedback form under arbitrary switchings by using the backstepping technique [9, 18]. Meanwhile, [19] investigated the global stabilization problem for a class of switched nonlinear systems in p-normal form by the so-called power integrator backstepping design method.

In practice, uncertainties inevitably exist in many practical systems. In recent years, some attentions has been paid to both general nonlinear systems and switched nonlinear systems with uncertainties, but most of the obtained results require that the uncertainties should satisfy some additional conditions. However, in many cases, we cannot get the knowledge of system uncertainty a priori, which can only be described by completely unknown functions. In this case, the excellent approximation capability of neural networks (or fuzzy logic systems) has been explored in the literature to tackle the corresponding control problems for either switched systems or non-switched systems. Thus, many significant results have been proposed. To list a few, the authors in [20] investigated the control problem of nonlinear pure-

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X. Zhao et al., *Control Synthesis of Switched Systems*, Studies in Systems, Decision and Control 80, DOI 10.1007/978-3-319-44830-5_4

feedback systems with unknown nonlinear functions by using the implicit function theorem and NN approximation. The adaptive tracking control problem for a class of uncertain nonlinear strict-feedback systems is solved by [21] using fuzzy logic system approximation. A practical design method is developed by [22] for cooperative tracking control of higher-order nonlinear systems with a dynamic leader. For a class of switched uncertain nonlinear systems without the measurements of the system states, the problem of adaptive neural tracking control via output-feedback was solved in [23] by using a novel switched filter.

However, few results on adaptive tracking control have been developed for lower triangular switched nonlinear systems with completely unknown uncertainties. On the other hand, most system models of the above-mentioned results about adaptive control for switched nonlinear systems are in the strict-feedback form that limits applications of the results to more general switched nonlinear systems. Therefore, considering the adaptive tracking control for switched nonstrict-feedback nonlinear systems with completely unknown uncertainties is more reasonable. In this chapter, the adaptive tracking control problem is investigated for both strict-feedback and nonstrict-feedback switched nonlinear systems with completely unknown uncertainties.

Notations: In this chapter, the notations are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the notation $\|\cdot\|$ refers to the Euclidean vector norm. $\mathbb{R}^+$ is the set of all nonnegative real numbers. For positive integers $1 \le i \le n,\ 1 \le j \le m,$ we also denote $\Xi_{i,\max} = \max\{\Xi_{i,j} : 1 \le j \le m\},\ \Xi_{i,\min} = \min\{\Xi_{i,j} : 1 \le j \le m\}$. $\mathscr{C}^i$ stands for a set of functions with continuous i^{th} partial derivatives. For a given matrix A (or vector v), A^T (or v^T) denotes its transpose, and $Tr\{A\}$ denotes its trace when A is a square. $\mathscr{K}$ represents the set of functions: $\mathbb{R}^+ \to \mathbb{R}^+$, which are continuous, strictly increasing and vanishing at zero; $\mathscr{K}_\infty$ denotes a set of functions which is of class $\mathscr{K}$ and unbounded.

4.2 Adaptive Control of Switched Strict-Feedback Nonlinear Systems

4.2.1 Problem Formulation and Preliminaries

Consider a class of switched nonlinear systems in the following form,

$$\begin{aligned} \dot{x}_i &= g_{i,\sigma(t)} x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i),\ i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= g_{n,\sigma(t)} u_{\sigma(t)} + f_{n,\sigma(t)}(\bar{x}_n), \\ y &= x_1, \end{aligned} \tag{4.1}$$

where $\bar{x}_i := (x_1, x_2, \ldots, x_i)^T \in \mathbb{R}^i$, $i = 1, 2, \ldots, n$ is the system state, y is the system output; $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal;

$u_k \in R$ is the control input of the k^{th} subsystem, For any $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, m$, $f_{i,k}(\bar{x}_i)$ is an unknown smooth nonlinear function representing the system uncertainty, and $g_{i,k}$ is a positive constant.

Our control objective is to design state-feedback controllers such that the output of system (4.1) tracks a given time-varying signal $y_d(t)$ within a bounded error and all the signals of the closed-loop systems remain bounded under arbitrary switchings.

Assumption 4.1 The tracking target $y_d(t)$ and its time derivatives up to the n^{th} order are continuous and bounded.

In the controller design and stability analysis procedure, fuzzy logic systems will be used to approximate the unknown functions. Therefore, the following useful concept and lemma are first recalled.

Fuzzy logic systems include some IF-THEN rules, and the i^{th} IF-THEN rule is written as

$$\mathbb{R}_i : \text{ If } x_1 \text{ is } F_1^i \text{ and ... and } x_n \text{ is } F_n^i \text{ then } y \text{ is } B^i,$$

where $\mathbf{x} = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$, and $y \in \mathbb{R}$ are the input and output of the fuzzy logic systems, respectively. $F_1^i, F_2^i, ..., F_n^i$ and B^i are fuzzy sets in $\mathbb{R}$. By using the strategy of singleton fuzzification, the product inference and the center-average defuzzification, the fuzzy logic system can be formulated as

$$y(\mathbf{x}) = \frac{\sum_{i=1}^{N} w_i \prod_{j=1}^{n} \mu_{F_j^l}(x_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} \mu_{F_j^l}(x_j) \right]},$$

where N is the number of IF-THEN rules; w_i is the point at which fuzzy membership function $\mu_{B^i}(w_i) = 1$. Let

$$s_i(\mathbf{x}) = \prod_{j=1}^{n} \mu_{F_j^l}(x_j) / \sum_{i=1}^{N} \left[\prod_{j=1}^{n} \mu_{F_j^l}(x_j) \right], \; S(\mathbf{x}) = [s_1(\mathbf{x}), \ldots, s_N(\mathbf{x})]^T$$

and $W = [w_1, w_2, \ldots, w_N]^T$. Then the fuzzy logic system can be rewritten as

$$y = W^T S(\mathbf{x}), \tag{4.2}$$

If all memberships are chosen as Gaussian functions, the following lemma holds.

Lemma 4.1 [24] *Let $f(\mathbf{x})$ be a continuous function defined on a compact set Ω. Then, for a given desired level of accuracy $\varepsilon > 0$, there exists a fuzzy logic system (4.2) such that*

$$\sup_{x \in \Omega} \left| f(\mathbf{x}) - W^T S(\mathbf{x}) \right| \leq \varepsilon.$$

Remark 4.1 Lemma 4.1 plays a key role in the following design procedure and it indicates that any given real continuous function $f(x)$ can be represented by the linear combination of the basis function vector $S(\mathbf{x})$ within a bounded error ε. That is, $f(\mathbf{x}) = W^T S(\mathbf{x}) + \delta(\varepsilon)$, $|\delta(\varepsilon)| \le \varepsilon$. It is noted that $0 < S^T S \le 1$.

4.2.2 Main Results

In this section, we present an adaptive fuzzy control scheme for system (4.1) via the backstepping technique. In Sect. 3.1, a detailed design procedure was given. In each step, a common virtual control function α_i should be designed by using an appropriate common Lyapunov function V_i, and the control law u_k is finally designed.

4.2.2.1 Adaptive Control Design Under Multiple Adaptive Laws

In this subsection, a systemic control design procedure under multiple adaptive laws is presented. Design the control laws as

$$u_k = -\frac{1}{g_{n,k}}\left(\frac{\hat{\theta}_n}{2\zeta_{n,\min}^2} z_n + \lambda_n z_n + \frac{z_n}{2}\right), \tag{4.3}$$

where $\zeta_{n,k}$ and λ_n are positive design parameters, $\zeta_{n,\min} = \min\{\zeta_{n,k} : k \in M\}$, $\hat{\theta}_n$ is the estimation of $\theta_n = \left\| W_{n,\max} \right\|^2$, $W_{n,\max} = \max\{W_{n,k} : k \in M\}$ and $W_{n,k}$ is used in fuzzy logic system $W_{n,k}^T S_{n,k}(\mathbf{x})$ to approximate the unknown function $\hat{f}_{n,k}(\mathbf{x})$. $\hat{f}_{n,k}(\mathbf{x})$ is specified in the proof of Theorem 4.1.

The adaptive laws are defined as the solution to the following differential equations,

$$\dot{\hat{\theta}}_i = \frac{r_i}{2\zeta_{i,\min}^2} z_i^2 - \beta_i \hat{\theta}_i, \tag{4.4}$$

where r_i, $\zeta_{n,k}$ and β_i are positive design parameters, $\zeta_{n,\min} = \min\{\zeta_{n,k} : k \in M\}$, and the choice of $\hat{\theta}_j(0)$, $j = 1, 2, \ldots, n$ are required to satisfy $\hat{\theta}_j(0) \ge 0$ such that $\hat{\theta}_j \ge 0$. Now, we state one of our main results as follows.

Theorem 4.1 *Consider the closed-loop system (4.1) with the controllers (4.3) and the adaptive laws (4.4). For* $1 \le i \le n$, $k \in M$, *there exists* $W_{i,k}^T S_{i,k}(\mathbf{x})$ *such that* $\sup_{\mathbf{x}\in\Omega} \left| \hat{f}_{i,k}(\mathbf{x}) - W_{i,k}^T S_{i,k}(\mathbf{x}) \right| \le \varepsilon_{i,k}$ *in the sense that the approximation error* $\varepsilon_{i,k}$ *is bounded, and all the initial values of* $\hat{\theta}_i$ *satisfy* $\hat{\theta}_i(0) \ge 0$. *Then, the tracking error and closed-loop signals are bounded.*

Proof For $1 \le i \le n-1$, we define the common virtual control functions as α_i which are required to be in the form:

$$\alpha_i(X_i) = -\frac{1}{g_{i,\min}}\left(\frac{\hat{\theta}_i}{2\zeta_{i,\min}^2} + \lambda_i + \frac{1}{2}\right)z_i, \tag{4.5}$$

where $\zeta_{i,k}$ is a positive design parameter, $\zeta_{i,\min} = \min\{\zeta_{i,k} : k \in M\}$, $g_{i,\min} = \min\{g_{i,k} : k \in M\}$, $\lambda_i = g_{i,\max} + c_i$, $g_{i,\max} = \max\{g_{i,k} : k \in M\}$ and c_i is a positive constant. $\hat{\theta}_i$ is the estimation of $\theta_i = \|W_{i,\max}\|^2$ where $W_{i,\max} = \max\{W_{i,k} : k \in M\}$ and $W_{i,k}$ is used in fuzzy logic system $W_{i,k}^T S_{i,k}(\mathbf{x})$ to approximate the unknown function $\hat{f}_{i,k}(\mathbf{x})$. $X_i = [\bar{x}_i^T, \bar{\hat{\theta}}_i, \bar{y}_d^{(i)}]^T$ with $\bar{x}_i^T = [x_1, x_2, \ldots, x_i]^T$, $\bar{\hat{\theta}}_i = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i]^T$, $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^T$ and $\bar{y}_d^{(i)}$ being the i^{th} derivative of y_d.

Step 1. Denote $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$. Consider a Lyapunov function candidate as

$$V_1 = \frac{1}{2}z_1^2. \tag{4.6}$$

For any $k \in M$, the derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= z_1(g_{1,k}\alpha_1 + g_{1,k}z_2 + f_{1,k} - \dot{y}_d)\\ &= z_1(g_{1,k}\alpha_1 + g_{1,k}z_2 + \hat{f}_{1,k}),\end{aligned} \tag{4.7}$$

where $\hat{f}_{1,k} = f_{1,k} - \dot{y}_d$. By Lemma 4.1, the following equation can be obtained,

$$\hat{f}_{1,k} = W_{1,k}^T S_{1,k}(X_1) + \delta_{1,k}(X_1), \ \left|\delta_{1,k}(X_1)\right| \le \varepsilon_{1,k}. \tag{4.8}$$

Remark 4.2 It should be pointed out that the fuzzy logic system is used to approximate the redefined unknown nonlinear function $\hat{f}_{1,k}$ that includes the unknown function $f_{1,k}$ and the derivative of the desired output rather than the unknown function $f_{1,k}$ only.

Substituting (4.8) into (4.7), one gets that

$$\begin{aligned}\dot{V}_1 &= g_{1,k}z_1\alpha_1 + g_{1,k}z_1z_2 + z_1W_{1,k}^T S_{1,k}(z_1) + z_1\delta(z_1)\\ &\le g_{1,k}z_1\alpha_1 + g_{1,k}z_1z_2 + \frac{1}{2\zeta_{1,k}^2}z_1^2\left\|W_{1,k}\right\|^2\\ &\quad + \frac{\zeta_{1,k}^2 + \varepsilon_{1,k}^2}{2} + \frac{1}{2}z_1^2,\end{aligned} \tag{4.9}$$

where $\zeta_{1,k}$ is a positive design parameter.

A feasible virtual control function can be constructed as

$$\alpha_1 = -\frac{1}{g_{1,\min}}\left(\frac{\hat{\theta}_1}{2\zeta_{1,\min}^2} + \lambda_1 + \frac{1}{2}\right)z_1, \tag{4.10}$$

where $\lambda_1 = g_{1,\max} + c_1$ with c_1 being a positive constant.

By substituting (4.1) into (4.9), one has

$$\dot{V}_1 \le -\lambda_1 z_1^2 + \left(\frac{\left\| W_{1,k} \right\|^2}{2\zeta_{1,k}^2} - \frac{g_{1,k}\hat{\theta}_1}{2g_{1,\min}\zeta_{1,\min}^2} \right) z_1^2 + \frac{\zeta_{1,k}^2 + \varepsilon_{1,k}^2}{2} + g_{1,k} z_1 z_2. \quad (4.11)$$

Step 2. Let $z_3 = x_3 - \alpha_2$, and choose

$$V_2 = V_1 + \frac{1}{2} z_2^2. \quad (4.12)$$

For any $k \in M$, the time derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2(g_{2,k}\alpha_2 + g_{2,k} z_3 + f_{2,k} - \dot{\alpha}_1) \\ &= \dot{V}_1 + z_2(g_{2,k}\alpha_2 + g_{2,k} z_3 + \hat{f}_{2,k}), \end{aligned} \quad (4.13)$$

where $\hat{f}_{2,k} = f_{2,k} - \dot{\alpha}_1$, $\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{i=0}^{1} \frac{\partial \alpha_1}{\partial y_d^{(i)}} y_d^{(i+1)}$.

By Lemma 4.1, the following equation can be obtained,

$$\hat{f}_{2,k} = W_{2,k}^T S_{2,k}(X_2) + \delta_{2,k}(X_2), \ \left| \delta_{2,k}(X_2) \right| \le \varepsilon_{2,k}. \quad (4.14)$$

Substituting (4.14) into (4.13), yields that

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + g_{2,k} z_2 \alpha_2 + g_{2,k} z_2 z_3 + z_2(W_{2,k}^T S(z_2) + \delta_{2,k}(z_2)) \\ &\le \dot{V}_1 + g_{2,k} z_2 \alpha_2 + g_{2,k} z_2 z_3 + \frac{1}{2\zeta_{2,k}^2} z_2^2 \left\| W_{2,k} \right\|^2 + \frac{\zeta_{2,k}^2 + \varepsilon_{2,k}^2}{2} + \frac{1}{2} z_2^2, \end{aligned} \quad (4.15)$$

where $\zeta_{2,k}$ is a positive design parameter.

Design the virtual control function α_2 as

$$\alpha_2 = -\frac{1}{g_{2,\min}} \left(\frac{\hat{\theta}_2}{2\zeta_{2,\min}^2} + \lambda_2 + \frac{1}{2} \right) z_2, \quad (4.16)$$

where $\lambda_2 = g_{2,\max} + c_2$ with c_2 being a positive constant.

Then, one can get from (4.11), (4.15) and (4.16) that

$$\dot{V}_2 \le \sum_{j=1}^{2} \left\{ -\lambda_j z_j^2 + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} + g_{j,k} z_j z_{j+1} + \left(\frac{\left\| W_{j,k} \right\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}_j}{2g_{j,\min}\zeta_{j,\min}^2} \right) z_j^2 \right\}. \quad (4.17)$$

Step i. Let $z_{i+1} = x_{i+1} - \alpha_i$, and assume that we have finished the first $i - 1$ $(2 \le i \le n)$ steps. That is, for the following collection of auxiliary $(z_1, \ldots, z_{i-1})$-equations

$$\dot{z}_j = g_{j,k} x_{j+1} + \phi_{j,k}(X_j),\ \ j = 1, \ldots, i-1, \tag{4.18}$$

where

$$\phi_{j,k}(X_j) = f_{j,k}(\bar{x}_j) - \sum_{l=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_l} \dot{x}_l - \sum_{l=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}_l} \dot{\hat{\theta}}_l - \sum_{l=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial y_d^{(l)}} y_d^{(l+1)}. \tag{4.19}$$

We have a set of common virtual control functions as (4.5). A common Lyapunov function can be designed as

$$V_{i-1} = \frac{1}{2} \sum_{j=1}^{i-1} z_j^2. \tag{4.20}$$

For any $k \in M$, the time derivative of V_{i-1} satisfies

$$\begin{aligned} \dot{V}_{i-1} \le \sum_{j=1}^{i-1} \Bigg\{ & -\lambda_j z_j^2 + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} + g_{j,k} z_j z_{j+1} \\ & + \left(\frac{\|W_{j,k}\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}_j}{2g_{j,\min}\zeta_{j,\min}^2} \right) z_j^2 \Bigg\}, \end{aligned} \tag{4.21}$$

where $\zeta_{j,k}$ is a positive design parameter.

Choose

$$V_i = V_{i-1} + \frac{1}{2} z_i^2. \tag{4.22}$$

Analogous to the procedures above, the following inequality can be obtained

$$\begin{aligned} \dot{V}_i \le \sum_{j=1}^{i} \Bigg\{ & -\lambda_j z_j^2 + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} + g_{j,k} z_j z_{j+1} \\ & + \left(\frac{\|W_{j,k}\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}_j}{2g_{j,\min}\zeta_{j,\min}^2} \right) z_j^2 \Bigg\}. \end{aligned} \tag{4.23}$$

Step n. By repeatedly using the inductive argument above, a common Lyapunov function, a common virtual control function and state-feedback controllers are chosen, respectively, as

$$V_n = \sum_{j=1}^{n} \left\{ \frac{1}{2} z_l^2 + \frac{1}{2r_j} \tilde{\theta}_j^2 \right\}, \tag{4.24}$$

$$\alpha_{n-1} = -\frac{1}{g_{n-1,\min}} \left(\frac{\hat{\theta}_{n-1}}{2\zeta_{n-1,\min}^2} z_{n-1} + \lambda_{n-1} z_{n-1} + \frac{z_{n-1}}{2} \right), \tag{4.25}$$

$$u_k = -\frac{1}{g_{n,k}}\left(\frac{\hat{\theta}_n}{2\zeta_{n,\min}^2}z_n + \lambda_n z_n + \frac{z_n}{2}\right), \tag{4.26}$$

where $\theta_j = \left\| W_{j,\max} \right\|^2$, $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$ $(j = 1, 2, \ldots, n)$ are the error between θ_j and its estimation $\hat{\theta}_j$.

For any $k \in M$, the time derivative of V_n satisfies

$$\begin{aligned}\dot{V}_n \le & \sum_{j=1}^{n-1}\Bigg\{-\lambda_j z_j^2 + g_{j,k}z_j z_{j+1} + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} - \frac{1}{r_j}\tilde{\theta}_j\dot{\hat{\theta}}_j \\ & + \left(\frac{\left\| W_{j,k} \right\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}_j}{2g_{j,\min}\zeta_{j,\min}^2}\right)z_j^2\Bigg\} - \lambda_n z_n^2 \\ & + \frac{\zeta_{n,k}^2 + \varepsilon_{n,k}^2}{2} - \frac{1}{r_n}\tilde{\theta}_n\dot{\hat{\theta}}_n + \left(\frac{\left\| W_{n,k} \right\|^2}{2\zeta_{n,k}^2} - \frac{\hat{\theta}_n}{2\zeta_{n,\min}^2}\right)z_n^2,\end{aligned} \tag{4.27}$$

where $\lambda_j = g_{j,\max} + c_j$, and c_j is a positive constant.

Substituting (4.4) into (4.27) gives that

$$\begin{aligned}\dot{V}_n \le & \sum_{j=1}^{n-1}\Bigg\{-\lambda_j z_j^2 + g_{j,k}z_j z_{j+1} + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} + \frac{1}{r_j}\beta_j\tilde{\theta}_j\hat{\theta}_j \\ & + \left(\frac{\left\| W_{j,k} \right\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}_j}{2g_{j,\min}\zeta_{j,\min}^2} - \frac{\tilde{\theta}_j}{2\zeta_{j,\min}^2}\right)z_j^2\Bigg\} \\ & - \lambda_n z_n^2 + \frac{\zeta_{n,k}^2 + \varepsilon_{n,k}^2}{2} + \frac{1}{r_n}\beta_n\tilde{\theta}_n\hat{\theta}_n + \left(\frac{\left\| W_{n,k} \right\|^2}{2\zeta_{n,k}^2} - \frac{\theta_n}{2\zeta_{n,\min}^2}\right)z_n^2 \\ \le & \sum_{j=1}^{n}\left\{-\lambda_j z_j^2 + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2} + \frac{1}{r_j}\beta_j\tilde{\theta}_j\hat{\theta}_j\right\} + \sum_{j=1}^{n-1} g_{j,k}z_j z_{j+1}.\end{aligned} \tag{4.28}$$

It is not difficult to see that

$$\sum_{j=1}^{n-1} g_{j,k}z_j z_{j+1} \le g_{j,\max}\sum_{j=1}^{n} z_j^2, \tag{4.29}$$

and

$$\tilde{\theta}_j\hat{\theta}_j = \tilde{\theta}_j(\theta_j - \tilde{\theta}_j) \le -\frac{1}{2}\tilde{\theta}_j^2 + \frac{1}{2}\theta_j^2. \tag{4.30}$$

One can get from (4.28), (4.29) and (4.30) that

$$\dot{V}_n \leq \sum_{j=1}^{n}\left\{-c_j z_j^2 - \frac{1}{2r_j}\beta_j\tilde{\theta}_j^2\right\} + \sum_{j=1}^{n}\left\{\frac{\zeta_{j,\max}^2 + \varepsilon_{j,\max}^2}{2} + \frac{1}{2r_j}\beta_j\theta_j^2\right\}. \tag{4.31}$$

Let $a_0 = \min\{2c_j, \beta_j : 1 \leq j \leq n\}$, $b_0 = \sum_{j=1}^{n}\{\frac{1}{2r_j}\beta_j\theta_j^2 + \frac{\zeta_{j,\max}^2+\varepsilon_{j,\max}^2}{2}\}$. One has

$$\dot{V}_n \leq -a_0 V_n + b_0. \tag{4.32}$$

According to the comparison principle, one gets

$$V_n(t) \leq \left(V_n(0) - \frac{b_0}{a_0}\right)e^{-a_0 t} + \frac{b_0}{a_0},\ t \geq 0. \tag{4.33}$$

Inequality (4.33) indicates that all the signals in the closed-loop system are bounded. In particular, we have

$$\lim_{t\to\infty}|z_1| \leq \sqrt{\frac{2b_0}{a_0}}. \tag{4.34}$$

The proof is completed here. □

4.2.2.2 Adaptive Control Design Under One Adaptive Law

In this subsection, a controller design approach with one adaptive law is presented.

The control laws are chosen as

$$u_k = -\frac{1}{g_{n,k}}\left(\frac{\hat{\theta}}{2\zeta_{n,\min}^2}z_n + \lambda_n z_n + \frac{z_n}{2}\right), \tag{4.35}$$

where $\zeta_{n,k}$ and λ_n are positive design parameters, $\zeta_{n,\min} = \min\{\zeta_{n,k} : k \in M\}$, $\hat{\theta}$ is the estimation of $\theta = \sum_{i=1}^{n}\left\|W_{i,\max}\right\|^2$, $W_{i,\max} = \max\{W_{i,k} : k \in M\}$ and $W_{i,k}$ is used in fuzzy logic system $W_{i,k}^T S_{i,k}(x)$ to approximate the unknown function $\hat{f}_{i,k}(x)$.

The adaptive law is defined as the solution to the following differential equation:

$$\dot{\hat{\theta}} = \sum_{j=1}^{n}\frac{r}{2\zeta_{j,\min}^2}z_j^2 - \beta\hat{\theta}, \tag{4.36}$$

where r, $\zeta_{j,k}$ and β are positive design parameters, $\zeta_{j,\min} = \min\{\zeta_{j,k} : k \in M\}$ and the choice of $\hat{\theta}(0)$ is required to satisfy $\hat{\theta}(0) \geq 0$ such that $\hat{\theta} \geq 0$.

Next, we give another main result of the chapter.

Theorem 4.2 *Consider the closed-loop system (4.1) with the controllers (4.35) and the adaptive laws (4.36). For* $1 \le i \le n$, $k \in M$, *there exists* $W_{i,k}^T S_{i,k}(\mathbf{x})$ *such that* $\sup_{x\in\Omega}\left|\hat{f}_{i,k}(\mathbf{x}) - W_{i,k}^T S_{i,k}(\mathbf{x})\right| \le \varepsilon_{i,k}$ *in the sense that the approximation error* $\varepsilon_{i,k}$ *is bounded, and the initial value of* $\hat{\theta}$ *satisfies* $\hat{\theta}(0) \ge 0$. *Then, the tracking error and closed-loop signals are bounded.*

Proof For $1 \le i \le n-1$, define the common virtual control functions α_i as:

$$\alpha_i(X_i) = -\frac{1}{g_{i,\min}}\left(\frac{\hat{\theta}}{2\zeta_{i,\min}^2} + \lambda_i + \frac{1}{2}\right)z_i, \tag{4.37}$$

where $\zeta_{i,k}$ is a positive design parameter, $\zeta_{i,\min} = \min\{\zeta_{i,k} : k \in M\}$, $g_{i,\min} = \min\{g_{i,k} : k \in M\}$, $\lambda_i = g_{i,\max} + c_i$, $g_{i,\max} = \max\{g_{i,k} : k \in M\}$ and c_i is a positive constant. $\hat{\theta}$ is the estimation of $\theta = \sum_{i=1}^n \left\| W_{i,\max}\right\|^2$, $X_i = [\bar{x}_i^T, \hat{\theta}, \bar{y}_d^{(i)}]^T$ where $\bar{x}_i^T = [x_1, x_2, \ldots, x_i]^T$, $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^T$ and $\bar{y}_d^{(i)}$ being the i^{th} derivative of y_d.

Consider a common Lyapunov function

$$V = \sum_{j=1}^{n}\frac{1}{2}z_j^2 + \frac{1}{2r}\tilde{\theta}^2, \tag{4.38}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the error between θ and its estimation $\hat{\theta}$.

For any $k \in M$, the time derivative of V satisfies

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n-1} z_i(g_{i,k}\alpha_i + g_{i,k}z_{i+1} + f_{i,k} - \dot{\alpha}_{i-1}) \\ &\quad + z_n(g_{n,k}u_k + f_{n,k} - \dot{\alpha}_{n-1}) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} \\ &= \sum_{i=1}^{n-1} z_i(g_{i,k}\alpha_i + g_{i,k}z_{i+1} + \hat{f}_{i,k}) \\ &\quad + z_n(g_{n,k}u_k + \hat{f}_{n,k}) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}}\end{aligned} \tag{4.39}$$

where $\hat{f}_{i,k} = f_{i,k} - \dot{\alpha}_{i-1}$, $\dot{\alpha}_{i-1} = \sum_{l=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_l}\dot{x}_l + \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + \sum_{l=0}^{j-1}\frac{\partial\alpha_{j-1}}{\partial y_d^{(l)}}y_d^{(l+1)}$.

For $1 \le i \le n$, the following equation can be obtained by using Lemma 4.1.

$$\hat{f}_{i,k} = W_{i,k}^T S_{i,k}(X_i) + \delta_{i,k}(X_i),\ \left|\delta_{2,k}(X_i)\right| \le \varepsilon_{i,k}. \tag{4.40}$$

Substituting (4.36) and (4.37) into (4.39), one has

$$\begin{aligned}\dot{V} \leq& \frac{\beta}{r}\tilde{\theta}\hat{\theta} + \sum_{j=1}^{n-1}\left\{-\lambda_j z_j^2 + g_{j,k}z_j z_{j+1} + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2}\right.\\ &\left. + \left(\frac{\left\|W_{j,k}\right\|^2}{2\zeta_{j,k}^2} - \frac{g_{j,k}\hat{\theta}}{2g_{j,\min}\zeta_{j,\min}^2} - \frac{\tilde{\theta}}{2\zeta_{j,\min}^2}\right)z_j^2\right\}\\ & - \lambda_n z_n^2 + \frac{\zeta_{n,k}^2 + \varepsilon_{n,k}^2}{2} + \left(\frac{\left\|W_{n,k}\right\|^2}{2\zeta_{n,k}^2} - \frac{\theta}{2\zeta_{n,\min}^2}\right)z_n^2\\ \leq& \frac{\beta}{r}\tilde{\theta}\hat{\theta} + \sum_{j=1}^{n}\left\{-\lambda_j z_j^2 + \frac{\zeta_{j,k}^2 + \varepsilon_{j,k}^2}{2}\right\} + \sum_{j=1}^{n-1} g_{j,k}z_j z_{j+1}. \end{aligned} \tag{4.41}$$

The rest of proof is omitted here as it is similar to (4.29)–(4.34). □

4.2.3 Simulation Results

In this section, an example is provided to demonstrate the effectiveness of our main results.

Consider the following switched nonlinear system

$$\begin{aligned}\dot{x}_1 &= g_{1,\sigma(t)}x_2 + f_{1,\sigma(t)},\\ \dot{x}_2 &= g_{2,\sigma(t)}u_{\sigma(t)} + f_{2,\sigma(t)},\\ y &= x_1,\\ y_d &= \sin t,\end{aligned} \tag{4.42}$$

where $g_{1,1} = 2,\ g_{1,2} = 1,\ f_{1,1} = x_1,\ f_{1,2} = \sin x_1,\ g_{2,1} = 2,\ g_{2,2} = 1,\ f_{2,1} = x_1 x_2,\ f_{2,2} = x_1 x_2^2$. First, the controllers under multiple adaptive laws are designed by Theorem 4.1. The initial conditions are $x_1(0) = 0.05,\ x_2(0) = 0.05$, and $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$. We choose $c_1 = 2,\ c_2 = 1,\ r_1 = 10,\ r_2 = 3,\ \beta_1 = \beta_2 = 0.02,\ \varsigma_{1,1} = 0.25,\ \varsigma_{1,2} = 3,\ \varsigma_{2,1} = 0.5,\ \varsigma_{2,2} = 1.8$. Second, the controllers under one adaptive law is designed by Theorem 2, and the initial conditions are $x_1(0) = 0.05,\ x_2(0) = 0.05,\ \hat{\theta}(0) = 0$. We choose $c_1 = 2,\ c_2 = 1,\ r = 12,\ \beta = 0.025,\ \varsigma_{1,1} = 0.25,\ \varsigma_{1,2} = 3,\ \varsigma_{2,1} = 1.5,\ \varsigma_{2,2} = 1.8$. The objective is to design the controllers u_k such that y can track a desired trajectory y_d under arbitrary switchings.

According to Theorem 4.1, the adaptive laws $\hat{\theta}_1,\ \hat{\theta}_2$ and the control law u_k are chosen, respectively, as

$$\dot{\hat{\theta}}_1 = \frac{r_1}{2\zeta_{1,1}^2} z_1^2 - \beta_1 \hat{\theta}_1,\ \dot{\hat{\theta}}_2 = \frac{r_2}{2\zeta_{2,1}^2} z_2^2 - \beta_2 \hat{\theta}_2,$$

$$u_1 = -\frac{1}{g_{2,1}} \left(\frac{\hat{\theta}_2}{2\zeta_{2,1}^2} z_2 + \lambda_2 z_2 + \frac{z_2}{2} \right),$$

$$u_2 = -\frac{1}{g_{2,2}} \left(\frac{\hat{\theta}_2}{2\zeta_{2,1}^2} z_2 + \lambda_2 z_2 + \frac{z_2}{2} \right),$$

where $z_1 = x_1 - y_d,\ z_2 = x_2 - \alpha_1,\ \lambda_2 = c_2 + g_{2,1}$. The virtual control function α_1 is given by

$$\alpha_1 = -\frac{1}{g_{1,2}} \left(\frac{\hat{\theta}_1}{2\zeta_{1,1}^2} z_1 + \lambda_1 z_1 + \frac{z_1}{2} \right),$$

where $\lambda_1 = c_1 + g_{1,1}$. The controller design based on Theorem 4.1 is completed here. In the next, another design according to Theorem 2 is presented.

According to Theorem 4.2, an adaptive law $\hat{\theta}$ and the control law $u_1,\ u_2$ are chosen, respectively, as

$$\dot{\hat{\theta}} = \frac{r}{2\zeta_{1,1}^2} z_1^2 + \frac{r}{2\zeta_{2,1}^2} z_2^2 - \beta \hat{\theta},$$

$$u_1 = -\frac{1}{g_{2,1}} \left(\frac{\hat{\theta}}{2\zeta_{2,1}^2} z_2 + \lambda_2 z_2 + \frac{z_2}{2} \right),$$

$$u_2 = -\frac{1}{g_{2,2}} \left(\frac{\hat{\theta}}{2\zeta_{2,1}^2} z_2 + \lambda_2 z_2 + \frac{z_2}{2} \right),$$

where $z_1 = x_1 - y_d,\ z_2 = x_2 - \alpha_1,\ \lambda_2 = c_1 + g_{2,1}$.

The virtual control function α_1 is given as

$$\alpha_1 = -\frac{1}{g_{1,2}} \left(\frac{\hat{\theta}}{2\zeta_{1,1}^2} z_1 + \lambda_1 z_1 + \frac{z_1}{2} \right),$$

where $\lambda_1 = c_1 + g_{1,1}$.

The simulation results are shown in Figs. 4.1, 4.2, 4.3 and 4.4, respectively. Figure 4.1 shows the system output y and reference signal y_d. Figure 4.2 depicts the response of the tracking error $y - y_d$. Figure 4.3 illustrates the trajectory of the adaptive law. Figure 4.4 demonstrates the evolution of the switching signal. From Figs. 4.1, 4.2 and 4.3, it can be seen that the output y of both controllers can track the target signal y_d well, and all the closed-loop signals remain bounded.

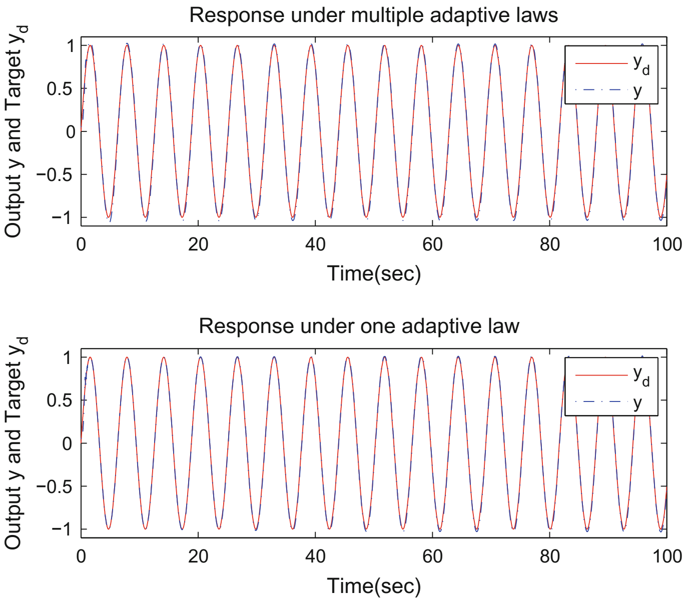


Fig. 4.1 Tracking performances

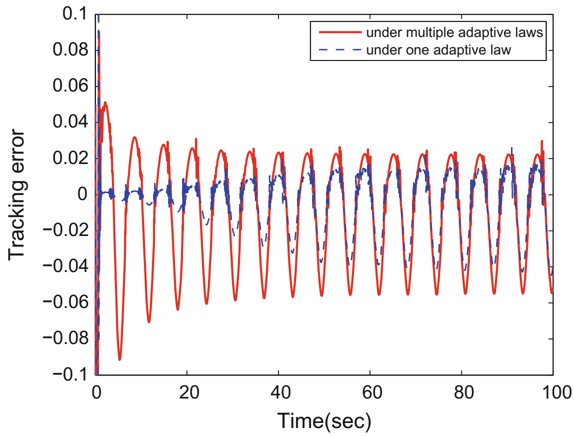


Fig. 4.2 Responses of the tracking error $y - y_d$

4.2.4 Conclusions

The tracking control problem for switched strict-feedback nonlinear systems with completely unknown nonlinear functions is given. The application of the adaptive backstepping technique is extended to a class of switched nonlinear systems with unknown uncertainties. The stability analysis shows that the designed controllers can

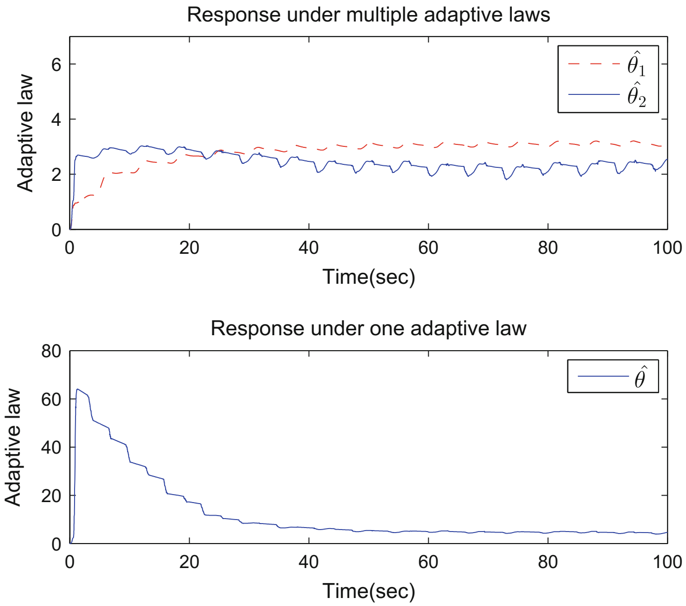
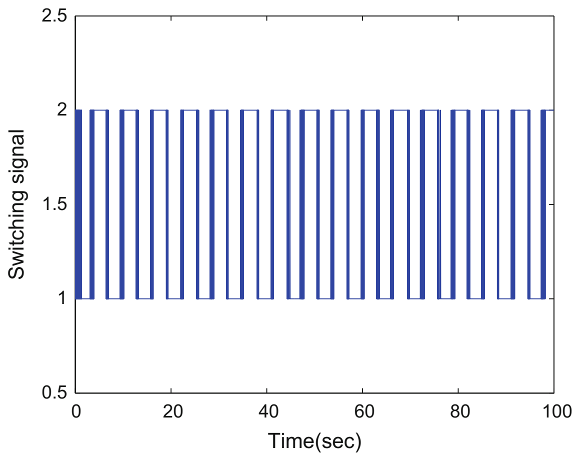


Fig. 4.3 Responses of the adaptive laws

Fig. 4.4 Switching signal



ensure all the closed-loop signals remain bounded, and the system output converges to a small neighborhood of the reference signal.

4.3 Adaptive Control of Switched Nonstrict-Feedback Nonlinear Systems

4.3.1 Problem Formulation and Preliminaries

In this section, the following nonlinear switched system in nonstrict-feedback form is considered:

$$\begin{aligned} \dot{x}_i &= g_{i,\sigma(t)}x_{i+1} + f_{i,\sigma(t)}(x) + w_{i,\sigma(t)},\ 1 \le i \le n-1 \\ \dot{x}_n &= g_{n,\sigma(t)}u_{\sigma(t)} + f_{n,\sigma(t)}(x) + w_{n,\sigma(t)} \\ y &= x_1 \end{aligned} \tag{4.43}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is the system state, y is the system output; $\sigma(t) : [0, \infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal; $u_k \in \mathbb{R}$ is the control input of the k-th subsystem. For any $i = 1, 2, \ldots, n$ and $k \in M$, $f_{i,k}(x)$ are unknown smooth nonlinear functions satisfying locally Lipschitz conditions, $g_{i,k}$ are positive constants, and $w_{i,k}$ is the bounded external disturbance of the system.

Our control objective is to design state-feedback controllers such that the output of system (4.43) tracks a given time-varying signal $y_d(t)$ and all the signals of the closed-loop systems remain bounded under arbitrary switchings.

Assumption 4.2 The tracking target $y_d(t)$ and its time derivatives up to the n^{th} order are continuous and bounded. It is further assumed that there exists a positive constant d such that $|y_d| \le d$.

Assumption 4.3 There exist strictly increasing smooth functions $\phi_{i,k}(.)s : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi_{i,k}(0) = 0$ such that for $i = 1, 2, \ldots, n-1,\ k \in M$,

$$|f_{i,k}(x)| \le \phi_{i,k}(\|x\|).$$

Remark 4.3 The increasing property of $\phi_{i,k}(.)$ means that if $a_i \ge 0$, $i = 1, 2, \ldots, n$, then $\phi_{i,k}(\sum_{i=1}^n a_i) \le \sum_{i=1}^n \phi_{i,k}(na_i)$. Note that $\phi_{i,k}(s)$ is a smooth function, and $\phi_{i,k}(0) = 0$. Therefore, there exists a smooth function $p_{i,k}(s)$ such that $\phi_{i,k}(s) = sp_{i,k}(s)$, which gives that

$$\phi_{i,k}\left(\sum_{i=1}^n a_i\right) \le \sum_{i=1}^n na_i p_{i,k}(na_i). \tag{4.44}$$

In the control design procedure, radial basis function (RBF) neural networks are used to approximate a continuous function $f(X)$ on a compact set $\Omega \in R^q$. For any $\varepsilon > 0$, there exists a neural network $\Phi^T P(X)$ such that

$$\sup_{x\in\Omega} \left| f(X) - \Phi^T P(X) \right| \le \varepsilon, \tag{4.45}$$

where $P(X) = [p_1(X), p_2(X), \ldots, p_l(X)]^T$ is the basis function vector, $\Phi = [\phi_1, \phi_2, \ldots, \phi_l]^T$ is the ideal constant weight vector with $l > 1$ being the number of the neural network nodes and $p_i(X)$ are chosen as the form:

$$p_i(X) = \exp\left(\frac{-(X-\mu_i)^T(X-\mu_i)}{\zeta_i^2}\right), \tag{4.46}$$

where ζ_i is the width of the Gaussian function, and $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center vector.

Remark 4.4 The readers may refer to [25] for more details about neural networks. Inequality (4.45) indicates that any given real continuous function $f(X)$ can be represented by the linear combination of the basis function vector $P(X)$ within a bounded error ε.

Lemma 4.2 *For any $\xi \in R$ and $\varpi > 0$, the following inequality holds,*

$$0 \leq |\xi| - \xi \tanh\left(\frac{\xi}{\varpi}\right) \leq \delta\varpi \tag{4.47}$$

where $\delta = 0.2785$.

4.3.2 Adaptive Control Design Based on Neural Networks

In this section, a backstepping-based adaptive control design procedure is presented. For the i^{th} subsystem, define a common virtual control function α_i as

$$\alpha_i(X_i) = -\frac{z_i}{\underline{g}_i}\left(\lambda_i + l_i^2 + \eta_i^2\hat{\theta}P_i^T(X_i)P_i(X_i)\right). \tag{4.48}$$

where λ_i, l_i and η_i are positive design parameters; $\underline{g}_i = \min\{g_{i,k},\ k \in M\}$; $\hat{\theta}$ is the estimation of θ which is an unknown constant and is specified later; $X_i = (\bar{x}_i^T, y_d, \dot{y}_d, \ldots, y_d^{(i)}, \hat{\theta})^T$, $\bar{x}_i = (x_1, x_2, \ldots, x_i)^T$; $P_i(X_i)$ represents the basis function of the ith neural network system. Subsequently, a set of the variable change of coordinates is defined as $z_i = x_i - \alpha_{i-1}$. Then, the z-system after coordinate transform is that

$$\begin{aligned} \dot{z}_i &= g_{i,k}x_{i+1} + f_{i,k}(x) + w_{i,k} - \dot{\alpha}_{i-1},\ 1 \leq i \leq n-1 \\ \dot{z}_n &= g_{n,k}u_k + f_{n,k}(x) + w_{n,k} - \dot{\alpha}_{n-1} \end{aligned} \tag{4.49}$$

where $\alpha_0 = y_d$.

For $i = 1, 2, \ldots, n-1$, the time derivative of α_{i-1} is given by

$$\dot{\alpha}_{i-1} = \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s}(f_{s,k} + g_{s,k}x_{s+1} + w_{s,k}) + \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} \tag{4.50}$$

where $\sum_{s=1}^{0} \frac{\partial \alpha_{i-1}}{\partial x_s}(f_{s,k} + g_{s,k}x_{s+1} + w_{s,k}) = 0$, and $\sum_{s=1}^{0} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} = 0$.

The controller can be chosen as

$$u = -\frac{z_n}{\underline{g}_n}\left(\lambda_n + l_n^2 + \eta_n^2 \hat{\theta} P_n^T(X_n)P_n(X_n)\right), \tag{4.51}$$

where λ_n, l_n and η_n are positive design parameters; $\underline{g}_n = \min\{g_{n,k},\ k \in M\}$; $X_n = (\bar{x}_i^T, y_d, \dot{y}_d, \ldots, y_d^{(n)}, \hat{\theta})^T$; $P_n(X_n)$ represents the basis function vector of the n^{th} neural network system.

The adaptive law is designed as

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} r\eta_i^2 z_i^2 P_i^T P_i - \beta\hat{\theta} \tag{4.52}$$

where r and β are positive design parameters.

Lemma 4.3 *For the variable transformations $z_i = x_i - \alpha_{i-1}$, $i = 1, 2, \ldots, n$, the following inequality holds,*

$$\|x\| \leq \sum_{i=1}^{n} |z_i|\,\varphi_i(\hat{\theta}) + d \tag{4.53}$$

where $\alpha_0 = y_d$, $\varphi_i(\hat{\theta}) = \frac{1}{\underline{g}_i}\left(-(\lambda_i + l_i^2) - \eta_i^2\hat{\theta} P_i^T(X_i)P_i(X_i)\right) + 1$, $i = 1, 2, \ldots, n-1$, and $\varphi_n = 1$.

The main result is given in the following theorem.

Theorem 4.3 *Consider the closed-loop system (4.43) with the controller (4.51) and the adaptive law (4.52). For $1 \leq i \leq n, k \in M$, assume that all the unknown nonlinear functions $\bar{f}_{i,k}(x)$ are approximated by neural networks in the sense that the approximation error $\varepsilon_{i,k}$ is bounded. Then, for bounded initial conditions, the target signal can be tracked within a small bounded error and other closed-loop signals remain bounded.*

Proof Consider the common Lyapunov function candidate as

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2r}\tilde{\theta}^2 \tag{4.54}$$

where $r > 0$ is a design parameter.

The time derivative of V is given by

$$\dot{V} = \sum_{i=1}^{n-1} z_i (f_{i,k} + g_{i,k} x_{i+1} + w_{1,k} - \dot{\alpha}_{i-1}) + z_n (f_{n,k} + g_{n,k} u + w_{n,k} - \dot{\alpha}_{n-1}) - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}} \tag{4.55}$$

where $\alpha_0 = y_d$.

By using (4.50), the following inequality can be obtained,

$$\begin{aligned}
\dot{V} =& \sum_{i=1}^{n-1} z_i \Bigg\{ f_{i,k} + g_{i,k} x_{i+1} + w_{i,k} - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} \\
& - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (f_{s,k} + g_{s,k} x_{s+1} + w_{s,k}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg\} \\
& + z_n \Bigg\{ f_{n,k} + g_{n,k} u + w_{n,k} - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(s)}} y_d^{(s+1)} \\
& - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (f_{s,k} + g_{s,k} x_{s+1} + w_{s,k}) - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg\} - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}} \\
=& \sum_{i=1}^{n} z_i \Bigg\{ f_{i,k} - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} f_{s,k} \Bigg\} + \sum_{i=1}^{n-1} z_i \Big\{ g_{i,k} x_{i+1} + w_{i,k} \\
& - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} (g_{s,k} x_{s+1} + w_{s,k}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg\} \\
& + z_n \Bigg\{ g_{n,k} u + w_{n,k} - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(s)}} y_d^{(s+1)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\
& - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s} (g_{s,k} x_{s+1} + w_{s,k}) \Bigg\} - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}}
\end{aligned} \tag{4.56}$$

By using Assumption 4.3, Lemma 4.3 and Remark 4.3, one has

$$\begin{aligned}
& z_i \left(f_{i,k} - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s} f_{s,k}(x) \right) \\
& \quad = -z_i \sum_{s=1}^{i} \frac{\partial \alpha_{i-1}}{\partial x_s} f_{s,k}(x) \leq \sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}| \left| f_{s,k}(x) \right|
\end{aligned}$$

$$
\begin{aligned}
&\leq \sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}|\phi_{s,k}(\|x\|) \\
&\leq \sum_{s=1}^{i}\sum_{j=1}^{n} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}||z_j|\bar{\phi}_{s,k}(|z_j|\varphi_j(\theta)) + \sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}|\phi_{s,k}(d) \\
&\leq \sum_{s=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial x_s}\right)^2 + \sum_{s=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_{s,k}^2(|z_j|\varphi_j(\theta)) \\
&\quad + \sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}|\phi_{s,k}(d)
\end{aligned}
\tag{4.57}
$$

where $\bar{\phi}_{s,k}(|z_j|\varphi_j(\theta)) = \varphi_j(\theta) h_{s,k}(|z_j|\varphi_j(\theta))$.

Substituting (4.57) into (4.56) yields that

$$
\begin{aligned}
\dot{V} \leq& \sum_{i=1}^{n}\sum_{s=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial x_s}\right)^2 \\
&+ \sum_{i=1}^{n}\sum_{s=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_{s,k}^2(|z_j|\varphi_j(\theta)) + \sum_{i=1}^{n}\sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}|\phi_{s,k}(d) \\
&+ \sum_{i=1}^{n-1} z_i \Bigg\{ g_{i,k} x_{i+1} + w_{i,k} - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} \\
&- \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s}(g_{s,k} x_{s+1} + w_{s,k}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg\} \\
&+ z_n \Bigg\{ g_{n,k} u + w_{n,k} - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(s)}} y_d^{(s+1)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\
&\qquad - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s}(g_{s,k} x_{s+1} + w_{s,k}) \Bigg\} - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}}
\end{aligned}
\tag{4.58}
$$

One can obtain that

$$
\sum_{i=1}^{n}\sum_{s=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_{s,k}^2(|z_j|\varphi_j(\theta)) = \sum_{i=1}^{n} z_i^2 \sum_{s=1}^{n} q(n,s) \bar{\phi}_{s,k}^2(|z_j|\varphi_j(\theta)) \tag{4.59}
$$

where $q(n,s) = \frac{(n-(s-1))}{2}$.

By using Lemma 4.2, the following inequality holds for $\varpi_{i,k} > 0$,

$$\sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}| \phi_{s,k}(d) \leq z_i Z_i \tanh\left(\frac{z_i Z_i}{\varpi_{i,k}}\right) + \delta \varpi_{i,k} \tag{4.60}$$

where $Z_i = \sum_{s=1}^{i} |z_i \frac{\partial \alpha_{i-1}}{\partial x_s}| \phi_{s,k}(d)$.

It follows from (4.58)–(4.60) that

$$\begin{aligned}
\dot{V} \leq & \sum_{i=1}^{n} z_i^2 \sum_{s=1}^{i} \frac{n}{2}\left(\frac{\partial \alpha_{i-1}}{\partial x_s}\right)^2 + \sum_{i=1}^{n} z_i^2 \sum_{s=1}^{n} q(n,s) \bar{\phi}_{s,k}^2(|z_j| \varphi_j(\theta)) \\
& + \sum_{i=1}^{n} z_i Z_i \tanh\left(\frac{z_i Z_i}{\varpi_{i,k}}\right) + \sum_{i=1}^{n} \delta \varpi_{i,k} + \sum_{i=1}^{n-1} z_i \Bigg\{ g_{i,k} x_{i+1} + w_{i,k} \\
& - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} - \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s}(g_{s,k} x_{s+1} + w_{s,k}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg\} \\
& + z_n \Bigg\{ g_{n,k} u + w_{n,k} - \sum_{s=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(s)}} y_d^{(s+1)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\
& - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s}(g_{s,k} x_{s+1} + w_{s,k}) \Bigg\} - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}} \\
= & \sum_{i=1}^{n} z_i \Bigg\{ z_i \sum_{s=1}^{i} \frac{n}{2}\left(\frac{\partial \alpha_{i-1}}{\partial x_s}\right)^2 + z_i \sum_{s=1}^{n} q(n,s) \bar{\phi}_{s,k}^2(|z_j| \varphi_j(\theta)) \\
& + Z_i \tanh\left(\frac{z_i Z_i}{\varpi_{i,k}}\right) - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s}(g_{s,k} x_{s+1} + w_{s,k}) \\
& - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} + w_{i,k} \Bigg\} \\
& + \sum_{i=1}^{n-1} z_i g_{i,k} x_{i+1} + z_n g_{n,k} u - \frac{1}{r} \tilde{\theta} \dot{\hat{\theta}} + \sum_{i=1}^{n} \delta \varpi_{i,k}
\end{aligned} \tag{4.61}$$

Note that

$$\sum_{i=1}^{n-1} z_i g_{i,k} x_{i+1} = \sum_{i=1}^{n-1} z_i g_{i,k} z_{i+1} + \sum_{i=1}^{n-1} z_i g_{i,k} \alpha_i \tag{4.62}$$

and define

$$\bar{f}_{i,k} = z_i \sum_{s=1}^{i} \frac{n}{2}\left(\frac{\partial \alpha_{i-1}}{\partial x_s}\right)^2 + z_i \sum_{s=1}^{n} q(n,s) \bar{\phi}_{s,k}^2(|z_j| \varphi_j(\theta))$$

$$+ Z_i \tanh\left(\frac{z_i Z_i}{\varpi_{i,k}}\right) - \sum_{s=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_s}(g_{s,k}x_{s+1} + w_{s,k})$$
$$- \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} + w_{i,k} + g_{i-1,k}z_{i-1} \tag{4.63}$$

where $g_0 = 0$ and $z_0 = 0$.

Substituting (4.62) and (4.63) into (4.61) gives that

$$\dot{V} \le \sum_{i=1}^{n-1} z_i(\bar{f}_{i,k} + g_{i,k}\alpha_i) + z_n(\bar{f}_{n,k} + g_{n,k}u_k) + \sum_{i=1}^{n} \delta\varpi_{i,k} - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} \tag{4.64}$$

The neural network $\Phi_{i,k}^T P_{i,k}$ is utilized to approximate the unknown function $\bar{f}_{i,k}$ such that for any given $\bar{\varepsilon}_{i,k} > 0$,

$$\bar{f}_{i,k} = \Phi_{i,k}^T P_{i,k}(X_i) + \varepsilon_{i,k}(X_i) \tag{4.65}$$

where $X_i = (\bar{x}_i^T, y_d, \dot{y}_d, \ldots, y_d^{(i)}, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i)^T$, $\left|\varepsilon_{i,k}\right| \le \bar{\varepsilon}_{i,k}$, $\varepsilon_{i,k}$ denotes the approximation error. Thus, for $i = 1, 2, \ldots, n$,

$$\begin{aligned} z_i \bar{f}_{i,k} &= z_i \Phi_{i,k}^T P_{i,k}(X_i) + z_i \varepsilon_{i,k}(X_i) \\ &\le \frac{\eta_i^2}{2} z_i^2 \left\|\Phi_{i,k}\right\|^2 P_{i,k}^T P_{i,k} + \frac{1}{2\eta_i^2} + \frac{l_{i,k}^2}{2} z_i^2 + \frac{\varepsilon_{i,k}^2}{2l_{i,k}^2} \\ &\le \eta_i^2 z_i^2 \theta_i P_i^T P_i + l_i^2 z_i^2 + \frac{\bar{\varepsilon}_i^2}{l_i^2} + \frac{1}{\eta_i^2} \end{aligned} \tag{4.66}$$

where $\eta_i, l_i > 0$, $\theta_{i,k} = \left\|\Phi_{i,k}\right\|^2$, $\theta_i = \max\{\theta_{i,k} : k \in M\}$, $P_i(X_i)$ and $\bar{\varepsilon}_i(X_i)$ represent the basis function vector and the estimation error belongs to θ_i.

The feasible virtual control functions, adaptive laws and controllers are designed, respectively, as

$$\alpha_i = -\frac{z_i}{\underline{g}_i}\left(\lambda_i + l_i^2 + \eta_i^2 \hat{\theta}_i P_i^T P_i\right) \tag{4.67}$$
$$\dot{\hat{\theta}}_i = r_i \eta_i^2 z_i^2 P_i^T P_i - \beta_i \hat{\theta}_i \tag{4.68}$$
$$u_k = -\frac{z_n}{\underline{g}_n}\left(\lambda_n + l_n^2 + \eta_n^2 \hat{\theta}_n P_n^T P_n\right) \tag{4.69}$$

where for $i = 1, 2, \ldots, n$, λ_i, r_i, β_i are positive design parameters, and $\hat{\theta}_i$ is the estimation of θ_i.

Consider that too many adaptive parameters $(\hat{\theta}_1, \ldots, \hat{\theta}_n)$ can cause the problem of over-parameterization. Set $r_1 = r_2 = \cdots = r_n = r$, $\beta_1 = \beta_2 = \cdots = \beta_n = \beta$, and define $\theta = \sum_{i=1}^{n} \theta_i$, $\hat{\theta} = \sum_{i=1}^{n} \hat{\theta}_i$. The adaptive laws (4.68) can be changed as follows

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \dot{\hat{\theta}}_i = \sum_{i=1}^{n} \left(r\eta_i^2 z_i^2 P_i^T P_i - \beta\hat{\theta}_i \right) = \sum_{i=1}^{n} r\eta_i^2 z_i^2 P_i^T P_i - \beta\hat{\theta}. \tag{4.70}$$

Then, the stabilizing functions, the adaptive law and controllers can be designed as (4.48), (4.52) and (4.51) respectively.

Substituting (4.48), (4.51) and (4.52) into (4.62) one has

$$\dot{V} \leq -\sum_{i=1}^{n} \lambda_i z_i^2 + \sum_{i=1}^{n} \frac{\beta}{r}\tilde{\theta}\hat{\theta} + \sum_{i=1}^{n} \left(\frac{\bar{\varepsilon}_i^2}{l_i^2} + \frac{1}{\eta_i^2} + \delta\varpi_i \right) \tag{4.71}$$

where $\varpi_i = \max\{\varpi_{i,k}, k \in M\}$.

It is true that

$$\tilde{\theta}\hat{\theta} = \tilde{\theta}(\theta - \tilde{\theta}) \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2 \tag{4.72}$$

Then, (4.71) can be rewritten as

$$\dot{V} \leq -\frac{1}{2}\sum_{i=1}^{n} \left(2\lambda_i z_i^2 + \frac{\beta}{r}\tilde{\theta}^2 \right) + \sum_{i=1}^{n} \left(\frac{\bar{\varepsilon}_i^2}{l_i^2} + \frac{1}{\eta_i^2} + \delta\varpi_i + \frac{\beta}{2r}\theta^2 \right) \leq -a_0 V + b_0 \tag{4.73}$$

where $a_0 = \min\{2\lambda_i, \beta : 1 \leq i \leq n\}$ and $b_0 = \sum_{i=1}^{n} (\bar{\varepsilon}_i^2 / l_i^2 + 1/\eta_i^2 + \delta\varpi_i + \frac{\beta}{2r}\theta^2)$.

Furthermore

$$V(t) \leq \left(V(0) - \frac{b_0}{a_0} \right) e^{-a_0 t} + \frac{b_0}{a_0} \tag{4.74}$$

which means that all the signals in the closed-loop system are bounded. In particular, we have

$$\lim_{t\to\infty} |z_1| \leq \sqrt{\frac{2b_0}{a_0}}$$

The proof is completed here. □

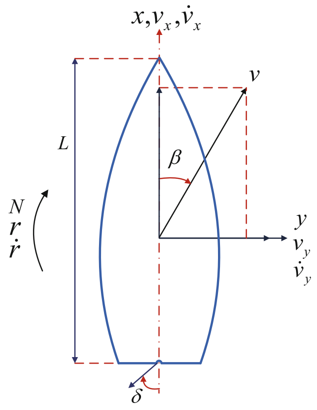


Fig. 4.5 Ship manoeuvring system

4.3.3 Simulation Results

In this section, the simulation studies for the ship manoeuvring systems shown in Fig. 4.5 are carried out to illustrates the effectiveness of our results.

L : length of ship

N : moment component on body relative to z-axis

r : yaw rate

v : speed of ship

v_x : forward velocity in x-axis

v_y : drift velocity along y-axis

β : drift angle

x, y : force components on body

ψ : yaw angle

δ : rudder angle

The ship maneuvering system can be described by the following Norrbin nonlinear model,

$$T\dot{h} + h + \tau h^3 = K\delta + \omega, \tag{4.75}$$

where T is the time constant, $h = \dot{\psi}$ denotes the yaw rate, ψ stands for the heading angle, τ is the Norrbin coefficient, K represents the rudder gain, δ is the rudder angle and ω stands for the outside disturbances. The value of τ can be determined via a spiral test. The ship's dynamic parameters are basically determined by its size and shape, and may vary with operational conditions such as ship speed, draft, trim, and water depth.

A simplified mathematical model of the rudder system can be described as:

$$T_E\dot{\delta} + \delta = K_E\delta_E, \tag{4.76}$$

where T_E represents the rudder time constant, δ stands for the actual rudder angle, K_E denotes the rudder control gain and δ_E is the rudder order.

Let $x_1 = \psi$, $x_2 = h$, $x_3 = \delta$; one has

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f + bx_3 + \omega, \\ \dot{x}_3 &= -\frac{1}{T_E}x_3 + \frac{K_E}{T_E}\delta_E, \end{aligned} \tag{4.77}$$

where $f = -\frac{1}{T}x_2 - \frac{\tau}{T}x_2^3$ is an unknown nonlinear function, $b = \frac{K}{T}$.

Note that some parameters of the aforementioned system will change when the speed of the ship changes. We adopt the following switched model to depict the dynamic behavior when the ship is at low speed, medium speed and high speed, respectively.

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f_{\sigma(v)}(x_2) + b_{\sigma(v)}x_3 + \omega_{\sigma(v)}, \\ \dot{x}_3 &= -\frac{1}{T_{E,\sigma(v)}}x_3 + \frac{K_{E,\sigma(v)}}{T_{E,\sigma(v)}}\delta_{E,\sigma(v)}, \end{aligned} \tag{4.78}$$

where $f_{\sigma(v)}(x_2) = -\frac{1}{T_{\sigma(v)}}x_2 - \frac{\tau_{\sigma(v)}}{T_{\sigma(v)}}x_2^3$, $b_{\sigma(v)} = \frac{K_{\sigma(v)}}{T_{\sigma(v)}}$ and $\sigma(v)$ is the switching signal that satisfies:

$$\sigma(v) = \begin{cases} 1, \ 0 < v \le v_L \\ 2, \ v_L < v \le v_M \\ 3, \ v_M < v \le v_T \end{cases}$$

where v_L, v_M, v_T represent the value of low speed, medium speed and top speed, respectively.

The vessel data comes from a ship that is listed in Table 4.1. The controller parameters are chosen as those in Table 4.2. Furthermore, the outside disturbances are: $w_1 = 0.01\sin t$; $w_2 = 0.015\cos t$; $w_3 = 0.013\sin t$. We construct the basis function vectors P_1, P_2 and P_3 using 7, 15 and 27 nodes, the centers μ_1, μ_2 and μ_3 evenly spaced on $[-3, 3] \times [-4, 1] \times [-2, 2]$, $[-0.5, 3.5] \times [-4, 4] \times [-8, 8]$ and $[-4, 4] \times [-30, 10] \times [-0.5, 3.5]$, and the widths $\zeta_1 = 2$, $\zeta_2 = 2.5$, and $\zeta_3 = 2$,

Table 4.1 Model parameters of ship maneuvering system

$v = 3.72$ m/s (low speed)		$v = 7.5$ m/s (medium speed)		$v = 15.3$ m/s (high speed)	
Parameter	Value	Parameter	Value	Parameter	Value
L (m)	160.9	L	160.9	L	160.9
K_1 (s^{-1})	0.32	K_2	0.114	K_3	0.051
T_1 (s)	30	T_2	63.69	T_3	80.47
τ_1 (s^2)	40	τ_2	30	τ_3	25
$T_{E,1}$ (s)	4	$T_{E,2}$	2.5	$T_{E,3}$	1
$K_{E,1}$	2	$K_{E,2}$	1	$K_{E,3}$	0.72

Table 4.2 Controller parameters

Parameter	λ_1	λ_2	λ_3	l_1	λ_2	λ_3	r
Value	2	3	5	12	14	10	0.01
Parameter	η_1	η_2	η_3	$\underline{g}_1$	$\underline{g}_2$	$\underline{g}_3$	β
Value	8	10	12	1	6.3×10^{-4}	0.4	0.1

respectively. The initial conditions are $x_1(0) = x_2(0) = x_3(0) = 0.02$, $\hat{\theta}(0) = 1$ and the target signal is $y_d = 10 \sin 0.05t$.

To illustrate the effectiveness of the proposed controller, comparison results are presented. The first one uses existing results in [26] and our results, respectively, to control the system when the ship is at a constant speed: low speed. The other one uses existing results [26] and our results respectively to control the system when the ship switches among different speeds.

According to (4.51) and (4.52), the adaptive law $\hat{\theta}$ and the control law u_k are chosen, respectively, as

$$\begin{aligned}\dot{\hat{\theta}} &= \sum_{i=1}^{3} r\eta_i^2 z_i^2 P_i^T P_i - \beta\hat{\theta} \\ &= 0.64 z_1^2 P_1^T P_1 + z_2^2 P_2^T P_2 + 1.44 z_3^2 P_3^T P_3 - 0.1\hat{\theta} \\ u_k &= -\frac{z_3}{\underline{g}_3}\left(\lambda_3 + l_3^2 + \eta_3^2 \hat{\theta} P_3^T P_3\right) \\ &= -2.5 z_3 \left(105 + 144 P_3^T P_3\right)\end{aligned}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$.

The virtual control functions α_1 and α_2 are given by

$$\begin{aligned}\alpha_1 &= -\frac{z_1}{\underline{g}_1}\left(\lambda_1 + l_1^2 + \eta_1^2 \hat{\theta} P_1^T P_1\right) \\ &= -z_1(146 + 64\hat{\theta} P_1^T P_1)\end{aligned}$$

$$\alpha_2 = -\frac{z_2}{\underline{g}_2}\left(\lambda_2 + l_2^2 + \eta_2^2 \hat{\theta} P_1^T P_1\right)$$
$$= -1.6 \times 10^3 \times z_2 \left(199 + 100\hat{\theta} P_2^T P_2\right)$$

Figures 4.6, 4.7 and 4.8 show the comparison results by using the existing method in [26] and our approach, respectively. It can be seen that both methods can ensure the target signal is tracked within a small bounded error.

Figures 4.6, 4.7, 4.8, 4.9, 4.10 and 4.11 depict the comparison results by using the existing method in [26] and our method under different speeds, and Fig. 4.12 gives the switching evolution among different speeds. From Figs. 4.9 and 4.10, it can be seen that the existing results in [26] cannot guarantee a good tracking performance

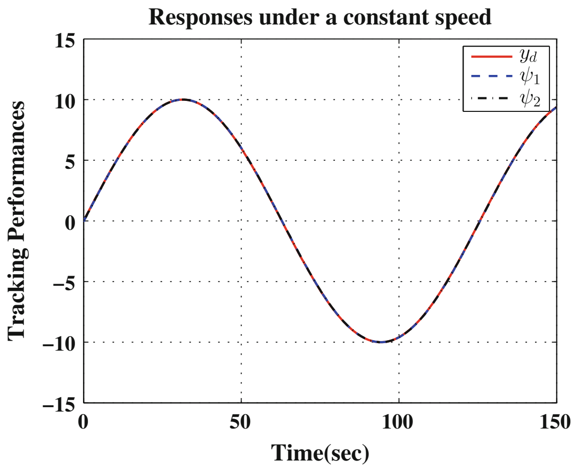


Fig. 4.6 Tracking performances under a constant speed. y_d is the target signal; ψ_1 and ψ_2 represent the outputs by using existing results in [26] and our results respectively

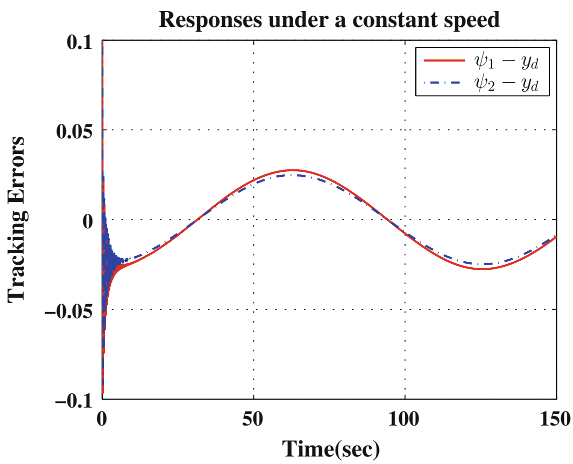


Fig. 4.7 Responses of tracking errors under a constant speed. $\psi_1 - y_d$ and $\psi_2 - y_d$ stand for the tracking error by using existing results in [26] and our results respectively

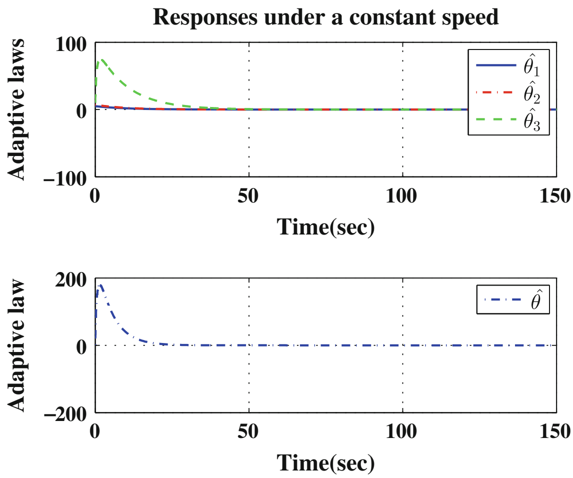


Fig. 4.8 Responses of adaptive laws under a constant speed. $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ denote the adaptive laws by existing results in [26]; $\hat{\theta}$ represents the adaptive law by our results

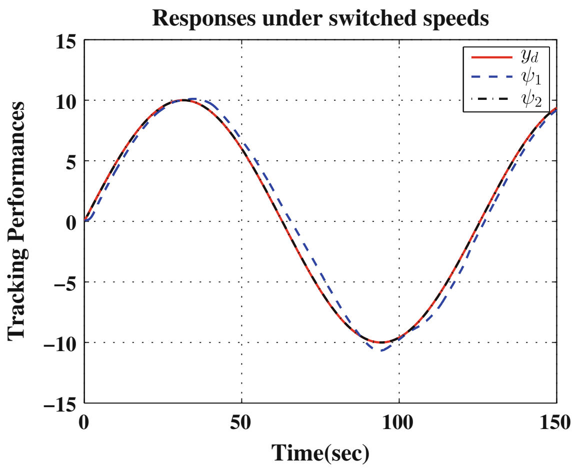


Fig. 4.9 Tracking performances under switched speeds. y_d is the target signal; ψ_1 and ψ_2 represent the outputs by using existing results in [26] and our results respectively

under switched speeds. However, our method can still ensure the target signal is tracked within a small bounded error. Figure 4.11 indicates that the adaptive law's number in our results is less than the one in [26].

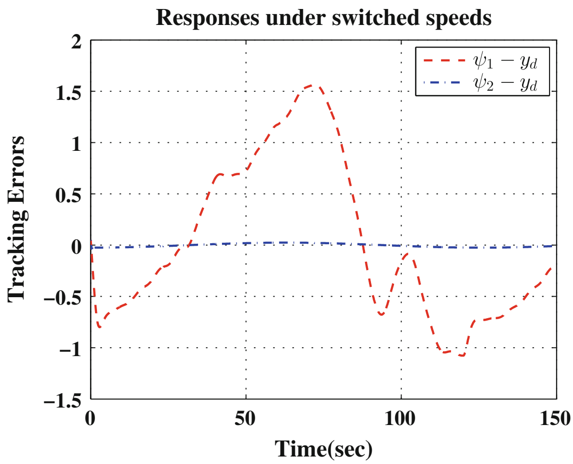


Fig. 4.10 Responses of tracking errors under switched speeds. $\psi_1 - y_d$ and $\psi_2 - y_d$ stand for the tracking error by using existing results in [26] and our results respectively

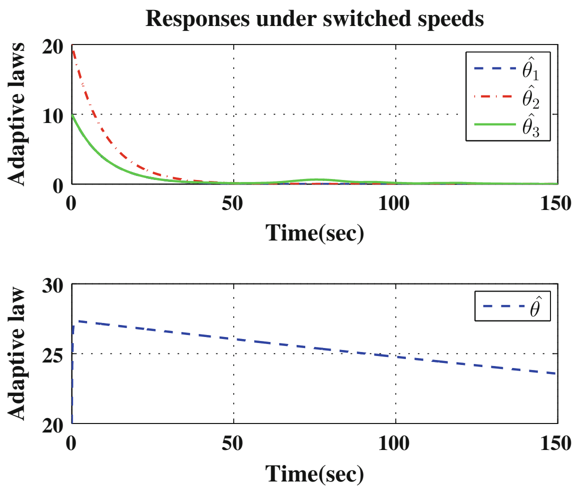


Fig. 4.11 Responses of adaptive laws under switched speeds. $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ denote the adaptive laws by existing results in [26]; $\hat{\theta}$ represents the adaptive law by our results

4.3.4 Conclusions

The problem of adaptive neural tracking control for a class of switched uncertain nonlinear systems in nonstrict-feedback form is investigated. The stability analysis in indicates that the designed controllers can ensure that the target signal can be tracked with a small bounded error and the stability of the system can be kept under arbitrary switchings.

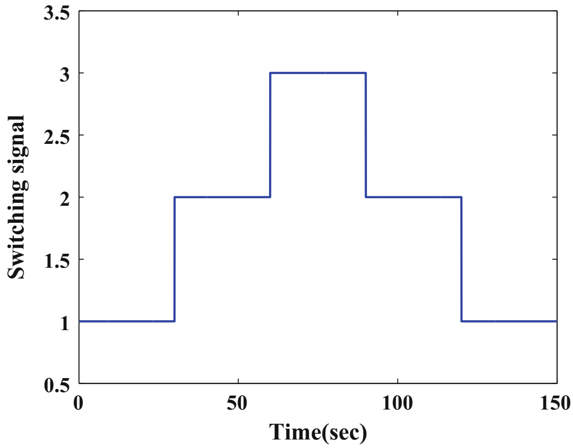


Fig. 4.12 Responses of switching signal

References

1. Miao B, Li T, Luo W (2013) A DSC and MLP based robust adaptive NN tracking control for underwater vehicle. Neurocomputing 111:184–189
2. Peng Z, Wang D, Zhang H, Lin Y (2015) Cooperative output feedback adaptive control of uncertain nonlinear multi-agent systems with a dynamic leader. Neurocomputing 149, Part A(0):132–141
3. Xu B, Zhang Y (2015) Neural discrete back-stepping control of hypersonic flight vehicle with equivalent prediction model. Neurocomputing 154:337–346
4. Ye J (2008) Tracking control for nonholonomic mobile robots: Integrating the analog neural network into the backstepping technique. Neurocomputing 71(16C18):3373–3378
5. Yu J, Chen B, Yu H, Lin C, Ji Z, Cheng X (2015) Position tracking control for chaotic permanent magnet synchronous motors via indirect adaptive neural approximation. Neurocomputing 156:245–251
6. Cai M, Xiang Z (2015) Adaptive neural finite-time control for a class of switched nonlinear systems. Neurocomputing 155:177–185
7. Li Y, Tong S, Li T (2015) Adaptive fuzzy backstepping control design for a class of pure-feedback switched nonlinear systems. Nonlinear Anal Hybrid Syst 16:72–80
8. Long L, Zhao J (2015) Adaptive fuzzy tracking control of switched uncertain nonlinear systems with unstable subsystems. Fuzzy Sets Syst. doi:10.1016/j.fss.2015.01.006
9. Ma R, Zhao J (2010) Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings. Automatica 46:1819–1823
10. Yang W, Tong S (2015) Output feedback robust stabilization of switched fuzzy systems with time-delay and actuator saturation. Neurocomputing. doi:10.1016/j.neucom.2015.02.072
11. Chiang M, Fu L (2014) Adaptive stabilization of a class of uncertain switched nonlinear systems with backstepping control. Automatica 50(8):2128–2135
12. Long L, Wang Z, Zhao J (2015) Switched adaptive control of switched nonlinearly parameterized systems with unstable subsystems. Automatica 54:217–228
13. Vu L, Liberzon D (2005) Common Lyapunov functions for families of commuting nonlinear systems. Syst control lett 54:405–416
14. Briat C, Seuret A (2012) Convex dwell-time characterizations for uncertain linear impulsive systems. IEEE Trans Autom Control 57:3241–3246
15. Briat C, Seuret A (2013) Affine minimal and mode-dependent dwell-time characterization for uncertain switched linear systems. IEEE Trans Autom Control 58:1304–1310

16. Liberzon D (2003) Switching in systems and control. Springer, Heidelberg
17. Margaliot M, Langholz G (2003) Necessary and sufficient conditions for absolute stability: the case of second-order systems. IEEE Trans Circuits Syst I Fundam Theory Appl 50:227–234
18. Wu JL (2009) Stabilizing controllers design for switched nonlinear systems in strict-feedback form. Automatica 45:1092–1096. doi:10.1016/j.automatica.2008.12.004
19. Long L, Zhao J (2012) Control of switched nonlinear systems in p-normal form using multiple Lyapunov functions. IEEE Trans Autom Control 57:1285–1291
20. Ge S, Wang C (2002) Adaptive NN control of uncertain nonlinear pure-feedback systems. Automatica 38(4):671–682
21. Chen B, Liu X, Liu K, Lin C (2009) Direct adaptive fuzzy control of nonlinear strict-feedback systems. Automatica 45(6):1530–1535
22. Zhang H, Lewis FL (2012) Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. Automatica 48(7):1432–1439
23. Long L, Zhao J (2014) Adaptive output-feedback neural control of switched uncertain nonlinear systems with average dwell time. IEEE Trans Neural Networks Learn Syst PP(99):1
24. Wang L, Mendel JM (1992) Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. IEEE Trans Neural Networks 3:807–814
25. Liu X, Ho DW, Yu W, Cao J (2014) A new switching design to finite-time stabilization of nonlinear systems with applications to neural networks. Neural Networks 57:94–102
26. Yuan L, Wu H (2011) Simulation and design of fuzzy sliding-mode controller for ship heading-tracking. J Marine Sci Appl 10(1):76–81