Chapter 2
Stabilization of Switched Linear Systems with Stable Subsystems

2.1 Background and Motivation

In a certain sense, switching signals in systems can be classified into autonomous (uncontrolled) or controlled ones [1, 2], that respectively, result from the system itself and the designers' intervention [3]. The stabilization problems of switched systems with both classes of switching signals, have always been the hottest topic in the studies of switched systems. Relatively, plenty of theoretical results have been available for systems under the uncontrolled switching signals, in both the continuous-time domain [4], and discrete-time domain [5]. However, for the switched systems with controlled switching signals, the corresponding stabilization problem is complicated in finding suitable switching signals to ensure system stability and improve system performances.

In practice, the time-constrained switching signals [6] with restrictions on switching instants are frequently encountered, and have drawn considerable attention. A minimum time interval called dwell time (DT) is first introduced for switched systems. By using multiple Lyapunov functions, it has been proved in [7] that the switched linear systems with stable subsystems are exponentially stable if the dwell time τ is sufficiently large. However, in many practical switched systems, specifying a fixed dwell time may be restrictive. The concept of average dwell time (ADT) extending the concept of DT allows the possibility of dwell time being less than a fixed constant. The ADT switching signal has been found important in not only theory but also in practice, and many sound and pioneered results have been obtained for analysis and synthesis of switched systems by using ADT switching signal [8–12].

However, the property in the ADT switching that the average time interval between any two consecutive switchings is not smaller than a constant independent of the system modes, is probably still not anticipated. In addition, it has been well shown in the literature that, the minimum of admissible ADT is computed by two mode-independent parameters. It is straightforward that such a setup of the two *common* parameters for all subsystems in a mode-independent manner will give rise to a certain conservativeness.

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Furthermore, controller failures, uncontrollable/unobservable modes, and sensor faults are often encountered in real plants, which may lead to switched system models with unstable modes. Therefore, it is of fundamental importance to numerous applications but theoretically challenging to carry out studies of switched systems with unstable subsystems [13–15].

A new class of switching signals called mode-dependent average dwell time (MDADT) switching is proposed in this chapter. Then, the stabilization problems of switched systems composed of stable subsystems are discussed via MDADT switching. Furthermore, the results are extended to the systems comprising unstable subsystems.

Notations: In this chapter, the used notations are standard. $\mathbb{R}$ and $\mathbb{R}^n$ denote the set of the real numbers and n-dimensional Euclidean space, respectively; $\mathbb{Z}^+$ represents the set of positive integers; the notation $\|\cdot\|$ refers to the Euclidean norm. $\mathscr{C}^1$ denotes the set of continuously differentiable functions, and a function α: $[0, \infty) \rightarrow [0, \infty)$ is said to be of class $\mathscr{K}$ if it is continuous, strictly increasing, and $\alpha(0) = 0$. Class $\mathscr{K}_\infty$ denotes the subset of $\mathscr{K}$ consisting of all those functions that are unbounded. In addition, the notation $P > 0 (\geq 0)$ means that P is a real symmetric and positive definite (semi-positive definite) matrix.

2.2 Stabilization for Switched Systems Composed of Stable Subsystems

2.2.1 Problem Formulation and Preliminaries

Consider a class of switched linear systems given by

$$\delta \mathbf{x}(t) = A_{\sigma(t)}\mathbf{x}(t) + B_{\sigma(t)}\mathbf{u}(t) \tag{2.1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, the symbol δ denotes the derivative operator in the continuous-time context ($\delta\mathbf{x}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t)$) and the shift forward operator in the discrete-time case ($\delta\mathbf{x}(t) = \mathbf{x}(t+1)$). $\sigma(t)$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $S = \{1, \ldots, M\}$; M is the number of subsystems. Also, for a switching sequence $0 < t_1 < \cdots < t_i < t_{i+1} < \cdots$, $\sigma(t)$ is continuous from the right everywhere and may be either autonomous or controlled. When $t \in [t_i, t_{i+1})$, we say the $\sigma(t_i)^{th}$ subsystem is active. The two-matrix pair (A_p, B_p), $\forall \sigma(t) = p \in S$, represents the p^{th} subsystem or p^{th} mode of (2.1).

The following stability definition of system (2.1) is first introduced for later developments, and we denote time by k in the discrete-time case.

Definition 2.1 ([2]) The equilibrium $x = 0$ of system (2.1) is globally uniformly exponentially stable (GUES) under a certain switching signal $\sigma(t)$ if for $\mathbf{u}(t) = 0$ (or

$\mathbf{u}(k) = 0$) and initial conditions $\mathbf{x}(t_0)$ (or $\mathbf{x}(k_0)$), there exist constants $\alpha > 0$, $\delta > 0$ (respectively, $0 < \varsigma < 1$) such that the solution of the system satisfies $\|\mathbf{x}(t)\| \leq \alpha e^{-\delta(t-t_0)} \|\mathbf{x}(t_0)\|$, $\forall t \geq t_0$ (respectively, $\|\mathbf{x}(k)\| \leq \alpha \varsigma^{(k-k_0)} \|\mathbf{x}(k_0)\|$, $\forall k \geq k_0$).

The control input $\mathbf{u}(t)$ (or $\mathbf{u}(k)$) in (2.1) is used to achieve system stability or certain performances for certain switching signals. The state feedback is considered with $\mathbf{u}(t) = K_{\sigma(t)}\mathbf{x}(t)$ (or $\mathbf{u}(k) = K_{\sigma(k)}\mathbf{x}(k)$), where K_p, $\forall \sigma(t) = p \in S$, is the controller gain to be determined. Then, the resulting closed-loop system is given by

$$\delta \mathbf{x}(t) = \bar{A}_p \mathbf{x}(t) \tag{2.2}$$

where,

$$\bar{A}_p = A_p + B_p K_p \tag{2.3}$$

Next, we aim at finding a more general set of admissible switching signals and the corresponding state-feedback controllers, such that the resulting closed-loop system (2.2) is GUES. For this purpose, let us first revisit the definition of the ADT property and the stability results for switched nonlinear systems with ADT.

Definition 2.2 ([16]) For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_2, t_1)$ denote the number of discontinuities of $\sigma(t)$ in the open interval (t_1, t_2). We say that $\sigma(t)$ has an average dwell time τ_a if there exist two positive numbers N_0 (we call N_0 the chatter bound here) and τ_a such that

$$N_\sigma(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \quad \forall t_2 \geq t_1 \geq 0$$

Lemma 2.1 ([16]) *Consider the continuous-time switched system* $\dot{\mathbf{x}}(t) = \mathbf{f}_{\sigma(t)}(\mathbf{x}(t))$, $\sigma(t) \in S$ *and let* $\lambda > 0$, $\mu > 1$ *be given constants. Suppose that there exist* $\mathscr{C}^1$ *functions* $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, *and two class* $\mathscr{K}_\infty$ *functions* κ_1, κ_2 *such that,* $\forall p \in S$

$$\kappa_1(\|\mathbf{x}(t)\|) \leq V_p(\mathbf{x}(t)) \leq \kappa_2(\|\mathbf{x}(t)\|) \tag{2.4}$$

$$\dot{V}_p(\mathbf{x}(t)) \leq -\lambda V_p(\mathbf{x}(t)) \tag{2.5}$$

and $\forall(\sigma(t_i) = p, \sigma(t_i^-) = q) \in S \times S,\ p \neq q,$

$$V_p(\mathbf{x}(t_i)) \leq \mu V_q(\mathbf{x}(t_i)) \tag{2.6}$$

then the system is globally uniformly asymptotically stable (GUAS) for any switching signal with ADT

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\lambda} \tag{2.7}$$

Lemma 2.2 ([10]) *Consider the discrete-time switched system* $\mathbf{x}(k+1) = f_{\sigma(k)}(\mathbf{x}(k))$, $\sigma(k) \in S$ *and let* $0 < \lambda < 1$ *and* $\mu > 0$, $\forall p \in S$ *be given constants.*

Suppose that there exists positive definite $\mathscr{C}^1$ *functions* $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in S$ *and two class* $\mathscr{K}_\infty$ *functions* κ_1, κ_2 *such that,*

$$\kappa_1(\|\mathbf{x}(k)\|) \leq V_p(\mathbf{x}_k) \leq \kappa_2(\|\mathbf{x}(k)\|) \tag{2.8}$$

$$\Delta V_p(\mathbf{x}(k)) \leq -\lambda V_p(\mathbf{x}(k)) \tag{2.9}$$

and $\forall(\sigma(k_i) = p, \sigma(k_{i-1}) = q) \in S \times S,\ p \neq q,$

$$V_p(\mathbf{x}(k_i)) \leq \mu V_q(\mathbf{x}(k_i)) \tag{2.10}$$

then the system is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^* = -\frac{\ln \mu}{\ln(1-\lambda)}. \tag{2.11}$$

2.2.2 Main Results

The definition of the MDADT property used to restrict a new class of switching signals is first given in the following.

Definition 2.3 For a switching signal $\sigma(t)$ and any $T \geq t \geq 0$, let $N_{\sigma p}(T, t)$ be the switching numbers that the p^{th} subsystem is activated over the interval $[t, T]$ and $T_p(T, t)$ denote the total running time of the p^{th} subsystem over the interval $[t, T]$, $p \in S$. We say that $\sigma(t)$ has a mode-dependent average dwell time τ_{ap} if there exist positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_{ap} such that

$$N_{\sigma p}(T, t) \leq N_{0p} + \frac{T_p(T, t)}{\tau_{ap}}, \quad \forall T \geq t \geq 0$$

Remark 2.1 Definition 2.3 constructs a new set of switching signals with a MDADT property. If there exist positive scalars τ_{ap}, $p \in S$ such that a switching signal has the MDADT property, it only requires the average time among the intervals associated with the p^{th} subsystem is larger than τ_{ap}.

The following lemmas present the stability results for the switched nonlinear systems with MDADT.

Lemma 2.3 (Continuous-Time Version) *Consider the continuous-time switched system*

$$\dot{\mathbf{x}}(t) = \mathbf{f}_{\sigma(t)}(\mathbf{x}(t)), \sigma(t) \in S \tag{2.12}$$

and let $\lambda_p > 0$, $\mu_p > 1$, $p \in S$ *be given constants. Suppose that there exist* $\mathscr{C}^1$ *functions* $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, *and class* $\mathscr{K}_\infty$ *functions* κ_{1p}, κ_{2p}, $p \in S$ *such that,* $\forall p \in S$,

$$\kappa_{1p}(\|\mathbf{x}(t)\|) \leq V_p(\mathbf{x}(t)) \leq \kappa_{2p}(\|\mathbf{x}(t)\|) \tag{2.13}$$

$$\dot{V}_p(\mathbf{x}(t)) \leq -\lambda_p V_p(\mathbf{x}(t)) \tag{2.14}$$

and $\forall(\sigma(t_i) = p, \sigma(t_i^-) = q) \in S \times S, \ p \neq q,$

$$V_p(\mathbf{x}(t_i)) \leq \mu_p V_q(\mathbf{x}(t_i)) \tag{2.15}$$

then the system is GUAS for any switching signal with MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln \mu_p}{\lambda_p} \tag{2.16}$$

Proof For any $T > 0$, let $t_0 = 0$ and denote $t_1, t_2 \cdots t_i, t_{i+1}, \ldots t_{N_\sigma(T,0)}$ the switching times on the interval $[0, T]$, where $N_\sigma(T, 0) = \sum_{p=1}^{M} N_{\sigma p}(T, 0)$.

Then, we set

$$\phi(t) := \mathrm{e}^{\lambda_{\sigma(t)} t} V_{\sigma(t)}(x(t)) \tag{2.17}$$

Function (2.17) is piecewise differentiable along solution (2.12). For any $t \in [t_i, t_{i+1}]$, we have:

$$\dot{\phi}(t) = \lambda_{\sigma(t_i)}\phi(t) + \mathrm{e}^{\lambda_{\sigma(t_i)} t} \dot{V}_{\sigma(t_i)}(x(t))$$

By (2.14), we obtain that $\dot{\phi}(t) \leq 0$. This, together with (2.15) and (2.17), implies that

$$\begin{aligned}
\phi(t_{i+1}) &= \mathrm{e}^{\lambda_{\sigma(t_{i+1})} t_{i+1}} V_{\sigma(t_{i+1})}(x(t_{i+1})) \\
&\leq \mu_{\sigma(t_{i+1})} \mathrm{e}^{\lambda_{\sigma(t_{i+1})} t_{i+1}} V_{\sigma(t_i)}(x(t_{i+1})) \\
&= \mu_{\sigma(t_{i+1})} \mathrm{e}^{\lambda_{\sigma(t_{i+1})} t_{i+1} - \lambda_{\sigma(t_i)} t_{i+1}} \phi(t_{i+1}^-) \\
&\leq \mu_{\sigma(t_{i+1})} \mathrm{e}^{(\lambda_{\sigma(t_{i+1})} - \lambda_{\sigma(t_i)}) t_{i+1}} \phi(t_i) \\
&\leq \mu_{\sigma(t_i)} \mu_{\sigma(t_{i+1})} \mathrm{e}^{(\lambda_{\sigma(t_{i+1})} - \lambda_{\sigma(t_i)}) t_{i+1} + (\lambda_{\sigma(t_i)} - \lambda_{\sigma(t_{i-1})}) t_i} \phi(t_{i-1}) \\
&\leq \prod_{j=0}^{i} \mu_{\sigma(t_{j+1})} \mathrm{e}^{\sum_{j=0}^{i} (\lambda_{\sigma(t_{j+1})} - \lambda_{\sigma(t_j)}) t_{j+1}} \phi(t_0)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\phi(T^-) &\leq \phi(t_{N_\sigma}) \\
&\leq \prod_{j=0}^{N_\sigma - 1} \mu_{\sigma(t_{j+1})} e^{\sum_{j=0}^{N_\sigma - 1} (\lambda_{\sigma(t_{j+1})} - \lambda_{\sigma(t_j)}) t_{j+1}} \phi(0)
\end{aligned}$$

Then, it follows from (2.17) that:

$$\exp(\lambda_{\sigma(T^-)}T)V_{\sigma(T^-)}(\mathbf{x}(T)) \le \prod_{j=0}^{N_\sigma-1}\mu_{\sigma(t_{j+1})}\mathrm{e}^{\sum_{j=0}^{N_\sigma-1}(\lambda_{\sigma(t_{j+1})}-\lambda_{\sigma(t_j)})t_{j+1}}V_{\sigma(0)}(\mathbf{x}(0))$$

This implies that

$$\begin{aligned}
V_{\sigma(T^-)}(\mathbf{x}(T)) &\le \prod_{j=0}^{N_\sigma-1}\mu_{\sigma(t_{j+1})}\exp\left\{\sum_{j=0}^{N_\sigma-1}(\lambda_{\sigma(t_{j+1})}-\lambda_{\sigma(t_j)})t_{j+1}\right.\\
&\quad\left.-\lambda_{\sigma(t_{N_\sigma})}T+\lambda_{\sigma(t_0)}t_0\right\}V_{\sigma(0)}(\mathbf{x}(0))\\
&\le \prod_{p=1}^{M}\mu_p^{N_{\sigma p}}\exp\left\{-\sum_{p=1}^{M}\left[\lambda_p\sum_{s\in\psi(p)}(t_{s+1}-t_s)\right]\right.\\
&\quad\left.-\lambda_{\sigma(t_{N_\sigma})}(T-t_{N_\sigma})\right\}V_{\sigma(0)}(\mathbf{x}(0))\\
&\le \exp\left\{\sum_{p=1}^{M}N_{0p}\ln\mu_p\right\}\exp\left\{\sum_{p=1}^{M}\frac{T_p}{\tau_{ap}}\ln\mu_p-\sum_{p=1}^{M}\lambda_pT_p\right\}V_{\sigma(0)}(x(0))\\
&= \exp\left\{\sum_{p=1}^{M}N_{0p}\ln\mu_p\right\}\exp\left\{\sum_{p=1}^{M}\left(\frac{\ln\mu_p}{\tau_{ap}}-\lambda_p\right)T_p\right\}V_{\sigma(0)}(x(0))
\end{aligned}$$

where $\psi(p)$ denotes the set of s satisfying $\sigma(t_s)=p$, $t_s\in\{t_0,t_1\cdots t_i,t_{i+1},\ldots t_{N_\sigma-1}\}$. Therefore, if there exist constants $\tau_{ap},\ p\in S$ satisfying (2.16), one has:

$$V_{\sigma(T^-)}(x(T)) \le \exp\left\{\sum_{p=1}^{M}N_{0p}\ln\mu_p\right\}\exp\left\{\max_{p\in S}\left(\frac{\ln\mu_p}{\tau_{ap}}-\lambda_p\right)T\right\}V_{\sigma(0)}(\mathbf{x}(0))$$

Thus, one can conclude that $V_{\sigma(T^-)}(\mathbf{x}(T))$ convergences to zero as $T\to\infty$ if the MDADT satisfies (2.16). Then, the asymptotic stability can be deduced with the aid of (2.13). □

Lemma 2.4 (Discrete-Time Version) *Consider the discrete-time switched system*

$$\mathbf{x}(k+1)=\mathbf{f}_{\sigma(k)}(\mathbf{x}(k)),\sigma(k)\in S \tag{2.18}$$

and let $0<\lambda_p<1$ *and* $\mu_p\ge 1,\ p\in S$ *be given constants. Suppose that there exist* $\mathscr{C}^1$ *functions* $V_{\sigma(k)}:\mathbb{R}^n\to\mathbb{R}$, $\sigma(k)\in S$, *and class* $\mathscr{K}_\infty$ *functions* κ_{1p} *and* κ_{2p}, $p\in S$, *such that* $\forall\sigma(k)=p\in S$

$$\kappa_{1p}(\|\mathbf{x}(k)\|)\le V_p(\mathbf{x}_k)\le\kappa_{2p}(\|\mathbf{x}(k)\|) \tag{2.19}$$

$$\Delta V_p(\mathbf{x}(k)) \leq -\lambda_p V_p(\mathbf{x}(k)) \tag{2.20}$$

and $\forall(\sigma(k_i) = p, \sigma(k_{i-1}) = q) \in S \times S,\ p \neq q,$

$$V_p(\mathbf{x}(k_i)) \leq \mu_p V_q(\mathbf{x}(k_i)) \tag{2.21}$$

then the system is GUAS for any switching signal with MDADT

$$\tau_{ap} > \tau_{ap}^* = -\frac{\ln \mu_p}{\ln(1-\lambda_p)} \tag{2.22}$$

Proof For any $K > 0$, let $k_0 = 0$ and denote $k_1, k_2, \ldots k_i, k_{i+1}, \ldots k_{N_\sigma(K,0)}$ the switching times on interval $[0, K]$, where $N_\sigma(K, 0) = \sum_{p=1}^{M} N_{\sigma p}(K, 0)$.

One can get from (2.20) that, $\forall p \in S$:

$$V_p(\mathbf{x}(k+1)) - V_p(\mathbf{x}(k)) < 0 \tag{2.23}$$

$$V_p(\mathbf{x}(k+1)) \leq (1-\lambda_p)V_p(\mathbf{x}(k)) \tag{2.24}$$

This together with (2.21) means that

$$\begin{aligned}
V_{\sigma(k_{i+1})}(\mathbf{x}(k_{i+1})) &\leq \mu_{\sigma(k_{i+1})} V_{\sigma(k_{i+1}-1)}(\mathbf{x}(k_{i+1})) \\
&\leq \mu_{\sigma(k_{i+1})} V_{\sigma(k_{i+1}-1)}(\mathbf{x}(k_{i+1}-1))(1-\lambda_{\sigma(k_{i+1}-1)}) \\
&= \mu_{\sigma(k_{i+1})}(1-\lambda_{\sigma(k_i)}) V_{\sigma(k_i)}(\mathbf{x}(k_{i+1}-1)) \\
&\leq \mu_{\sigma(k_{i+1})}(1-\lambda_{\sigma(k_i)})^{k_{i+1}-k_i} V_{\sigma(k_i)}(\mathbf{x}(k_i)) \\
&\quad \cdots \\
&\leq \prod_{j=0}^{i} \mu_{\sigma(k_{j+1})} \prod_{j=0}^{i} (1-\lambda_{\sigma(k_j)})^{k_{j+1}-k_j} V_{\sigma(k_0)}(\mathbf{x}(k_0))
\end{aligned}$$

Then, by (2.24), one gets that

$$\begin{aligned}
V_{\sigma(K)}(\mathbf{x}(K)) &\leq (1-\lambda_{\sigma(k_{N_\sigma})})^{K-k_{N_\sigma}} V_{\sigma(k_{N_\sigma})}(\mathbf{x}(k_{N_\sigma})) \\
&\leq (1-\lambda_{\sigma(k_{N_\sigma})})^{K-k_{N_\sigma}} \prod_{j=0}^{N_\sigma-1} \mu_{\sigma(k_{j+1})} \prod_{j=0}^{N_\sigma-1} (1-\lambda_{\sigma(k_j)})^{k_{j+1}-k_j} V_{\sigma(0)}(\mathbf{x}(0)) \\
&= \prod_{p=1}^{M} \mu_p^{N_{\sigma p}} \prod_{p=1}^{M} (1-\lambda_p)^{T_p} V_{\sigma(0)}(\mathbf{x}(0)) \\
&= \prod_{p=1}^{M} \mu_p^{N_{\sigma p}} \exp\left\{\sum_{p=1}^{M} \left[T_p \ln(1-\lambda_p)\right]\right\} V_{\sigma(0)}(\mathbf{x}(0))
\end{aligned}$$

$$\leq \exp\left\{\sum_{p=1}^{M} N_{0p} \ln \mu_p\right\} \exp\left\{\sum_{p=1}^{M} \frac{T_p}{\tau_{ap}} \ln \mu_p + \sum_{p=1}^{M} \ln(1-\lambda_p) T_p\right\} V_{\sigma(0)}(\mathbf{x}(0))$$

Thus, if there exist constants $\tau_{ap},\ p \in S$ satisfying (2.22), one has:

$$V_{\sigma(K)}(\mathbf{x}(K)) \leq \exp\left\{\sum_{p=1}^{M} N_{0p} \ln \mu_p\right\} \exp\left\{\max_{p \in S}\left[\frac{\ln \mu_p}{\tau_{ap}} + \ln(1-\lambda_p)\right] K\right\} V_{\sigma(0)}(\mathbf{x}(0))$$

Then, one can conclude that $V_{\sigma(K)}(\mathbf{x}(K))$ converges to zero as $K \to \infty$ if the MDADT satisfies (2.22). Subsequently, the asymptotic stability can be obtained by resorting to (2.19). □

Remark 2.2 It can be seen from Lemmas 2.1 and 2.2 that the parameters λ and μ are mode-independent for all subsystems. However, the parameters λ_p, μ_p prescribed in Lemmas 2.3 and 2.4 are mode-dependent, therefore, we can conclude that $\tau_{ap}^* \leq \tau_a^*$, $\forall p \in S$ from (2.5)–(2.7) and (2.14)–(2.16), and the mode-dependent features would reduce the conservativeness existing in Lemmas 2.1 and 2.2.

Remark 2.3 It is clear that Lemma 2.3 (or Lemma 2.4 in the discrete-time case) presents a more general switching signal than Lemma 2.1 (respectively, Lemma 2.2) which corresponds to the special case of $\lambda = \lambda_p$, $\mu = \mu_p$, $\tau_a = \tau_{ap}$, $\forall p \in S$. In fact, we note that if $\tau_a = \tau_{ap}$, $\forall p \in S$, one readily knows from Definition 2.3 that

$$\sum_{p \in S} N_{\sigma p}(T,t) \leq \sum_{p \in S} N_{0p} + \sum_{p \in S} \frac{T_p}{\tau_a}, \quad \forall T \geq t \geq 0$$

Thus, there exist positive numbers $N_0 = \sum\limits_{p \in S} N_{0p}$ and $\tau_a = \tau_{ap}$ such that

$$N_\sigma(T,t) \leq N_0 + \frac{T-t}{\tau_a}, \quad \forall T \geq t \geq 0$$

Based on the results obtained above, we present the stability conditions for system (2.1) with MDADT.

Theorem 2.1 (Continuous-Time Case) *Consider the switched linear system (2.1) when $\mathbf{u}(t) \equiv 0$ and let $\lambda_p > 0,\ \mu_p > 1,\ p \in S$ be given constants. If there exist matrices $P_p > 0$, $\forall p \in S$, such that, $\forall (p,q) \in S \times S,\ p \neq q$,*

$$A_p^T P_p + P_p A_p + \lambda_p P_p \leq 0 \tag{2.25}$$

$$P_p - \mu_p P_q \leq 0 \tag{2.26}$$

then, the switched linear system (2.1) is GUES with MDADT satisfying (2.16).

Proof Here, we choose the Lyapunov function candidate as follows,

$$V_p(\mathbf{x}(t)) = \mathbf{x}^T(t)P_p\mathbf{x}(t), \quad \forall\sigma(t) = p \in S \tag{2.27}$$

where P_p, $\forall p \in S$ is a positive definite matrix satisfying (2.25) and (2.26). Then, from (2.1), (2.14), (2.15) and (2.27), we have, $\forall(p, q) \in S \times S,\ p \neq q$,

$$\dot{V}_p(\mathbf{x}(t)) + \lambda_p V_p(\mathbf{x}(t)) = \lambda_p \mathbf{x}^T(t)P_p\mathbf{x}(t) + \mathbf{x}^T(t)P_pA_p\mathbf{x}(t) + \mathbf{x}^T(t)A_p^TP_p\mathbf{x}(t)$$

$$V_p(\mathbf{x}(t_i)) - \mu_p V_q(\mathbf{x}(t_i)) = \mathbf{x}^T(t_i)P_p\mathbf{x}(t_i) - \mu_p\mathbf{x}^T(t_i)P_q\mathbf{x}(t_i)$$

Thus, if (2.25) and (2.26) hold, system (2.1) is GUAS for any switching signal with MDADT (2.16). In addition, by denoting $\delta = -\frac{1}{2}[\max\limits_{p\in S}(\frac{\ln\mu_p}{\tau_{ap}} - \lambda_p)]$, we can obtain from (2.13) and (2.27) that the system state satisfies $\|\mathbf{x}(t)\| \leq \alpha \mathrm{e}^{-\delta(t-t_0)}\|\mathbf{x}(t_0)\|$, $\forall t \geq t_0$ for a certain $\alpha > 0$,; that is the underlying system is GUES. □

Theorem 2.2 (Discrete-Time Case) *Consider the switched linear system (2.1) when* $\mathbf{u}(t) \equiv 0$ *and let* $0 < \lambda_p < 1$ *and* $\mu_p \geq 1$, $p \in S$ *be given constants. If there exist matrices* $P_p > 0$, $\forall p \in S$, *such that,* $\forall(p, q) \in S \times S,\ p \neq q$,

$$A_p^TP_pA_p + \lambda_pP_p - P_p \leq 0 \tag{2.28}$$

$$P_p - \mu_pP_q \leq 0 \tag{2.29}$$

then, the switched linear systems (2.1) is GUES with MDADT satisfying (2.22).

Proof We establish the Lyapunov function

$$V_p(\mathbf{x}(k)) = \mathbf{x}^T(k)P_p\mathbf{x}(k), \quad \forall\sigma(k) = p \in S \tag{2.30}$$

where P_p, $\forall p \in S$ is a positive definite matrix satisfying (2.28) and (2.29). Then, together with (2.1), (2.20), (2.21) and (2.30), we can get, $\forall(p, q) \in S \times S,\ p \neq q$,

$$\Delta V_p(\mathbf{x}(k)) + \lambda_p V_p(\mathbf{x}(k)) = \lambda_p\mathbf{x}^T(k)P_p\mathbf{x}(k) - \mathbf{x}^T(k)P_p\mathbf{x}(k) + \mathbf{x}^T(k)A_p^TP_pA_p\mathbf{x}(k)$$

$$V_p(\mathbf{x}(k_i)) - \mu_pV_q(\mathbf{x}(k_i)) = \mathbf{x}^T(k_i)P_p\mathbf{x}(k_i) - \mu_p\mathbf{x}^T(k_i)P_q\mathbf{x}(k_i)$$

Therefore, if (2.28) and (2.29) hold, system (2.1) is GUAS for any switching signal with MDADT (2.22) in the light of Lemma 2.4. Subsequently, by denoting $\varsigma = \sqrt{\exp\{\max\limits_{p\in S}[\frac{\ln\mu_p}{\tau_{ap}} + \ln(1-\lambda_p)]\}}$, we can obtain from (2.19) and (2.30) that $\|\mathbf{x}(k)\| \leq \alpha\varsigma^{(k-k_0)}\|\mathbf{x}(k_0)\|$, $\forall k \geq k_0$ for a certain $\alpha > 0$, that is, the underlying system is GUES. □

Now, we give a stabilizing controller design approach for system (2.1) with the MDADT switching.

Theorem 2.3 (Continuous-Time Case) *Consider the switched linear systems (2.2) and let $\lambda_p > 0,\ \mu_p > 1,\ p \in S$ be given constants. If there exist matrices $U_p > 0$, and T_p, $\forall p \in S$, such that, $\forall (p, q) \in S \times S,\ p \neq q$,*

$$A_p U_p + B_p T_p + U_p A_p^T + T_p^T B_p^T + \lambda_p U_p \leq 0 \tag{2.31}$$

$$U_q \leq \mu_p U_p \tag{2.32}$$

then there exists a set of stabilizing controllers such that system (2.2) is GUES for any switching signal with MDADT satisfying (2.16). Moreover, if (2.31) and (2.32) are feasible, the controller gains can be provided by

$$K_p = T_p U_p^{-1} \tag{2.33}$$

Proof Theorem 2.1 implies that if

$$\bar{A}_p^T P_p + P_p \bar{A}_p + \lambda_p P_p \leq 0 \tag{2.34}$$

$$P_p - \mu_p P_q \leq 0 \tag{2.35}$$

system (2.2) is GUES for any switching signal with MDADT (2.16). Replacing $\bar{A}_p$ in (2.34) by (2.3), setting $U_p = P_p^{-1}$ and $T_p = K_p P_p^{-1}$, we can see that, if (2.31) holds, (2.34) is satisfied. Moreover, if (2.32) holds, we can obtain that $U_q - \mu_p U_p \leq 0$. By Schur complement, we note that $U_q - \mu_p U_p \leq 0$ is equivalent to

$$\Lambda = \begin{bmatrix} -\mu_p U_p & I \\ I & -U_q^{-1} \end{bmatrix} \leq 0.$$

Furthermore, by Schur complement, one has that $\Lambda \leq 0$ is equivalent to $-U_q^{-1} - I^T(\mu_p U_p)^{-1} I \leq 0$; that is, (2.35) holds. In addition, if the inequalities (2.31) and (2.32) have feasible solutions, the admissible controller gains can be given by (2.33) because $T_p = K_p P_p^{-1}$, which ends the proof. □

Theorem 2.4 (Discrete-Time Case) *Consider the switched linear systems (2.2) and let $0 < \lambda_p < 1$ and $\mu_p \geq 1,\ p \in S$ be given constants. If there exist matrices $U_p > 0$, and T_p, $\forall p \in S$, such that, $\forall (p, q) \in S \times S,\ p \neq q$,*

$$\begin{bmatrix} -U_p & A_p U_p + B_p T_p \\ * & -(1 - \lambda_p) U_p \end{bmatrix} \leq 0 \tag{2.36}$$

$$U_q \leq \mu_p U_p \tag{2.37}$$

then there exists a set of controllers such that system (2.2) is GUES for any switching signal with MDADT satisfying (2.22). Moreover, if (2.36) and (2.4) have a solution, the admissible controllers can be given by (2.33).

Proof By Theorem 2.2 we have that if

$$\bar{A}_p^T P_p \bar{A}_p + \lambda_p P_p - P_p \leq 0 \tag{2.38}$$

$$P_p - \mu_p P_q \leq 0 \tag{2.39}$$

system (2.2) is GUES for any switching signal with MDADT (2.22). Substituting $\bar{A}_p$ in (2.38) and by Schur complement, we have

$$\begin{bmatrix} -P_p & P_p B_p K_p + P_p A_p \\ * & -(1-\lambda_p)P_p \end{bmatrix} \leq 0 \tag{2.40}$$

setting $U_p = P_p^{-1}$ and $T_p = K_p P_p^{-1}$ and performing a congruence transformation to (40) via $diag\{U_p, U_p\}$, we can obtain (2.36). Therefore, (2.36) and (2.4) ensure (2.38) and (2.39). In addition, if the inequalities (2.36) and (2.4) have feasible solutions, the admissible controller gains can be given by (2.33), which ends the proof. □

2.2.3 Simulation Results

An example in the continuous-time domain is presented to demonstrate the potential and validity of the results obtained above.

Example 2.1 Consider the switched linear systems consisting of three subsystems described by:

$$A_1 = \begin{bmatrix} 3.9 & 1.5 \\ 2.5 & 2.3 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1.4 & 0.3 \\ 1 & -2.7 \end{bmatrix}, \; A_3 = \begin{bmatrix} -2.2 & 0.1 \\ -2 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

Here, we aim to design a set of mode-dependent stabilizing controllers and find corresponding switching signals with MDADT property such that the resulting closed-loop system is stable.

To illustrate the advantages of the proposed MDADT switching, we shall also present the design results of both controllers and switching signals for the systems with ADT switching. By different approaches and setting the relevant parameters appropriately, the computation results for the system with two different switching schemes are listed in Table 2.1.

Table 2.1 Computation results for the system under two different switching schemes

Switching schemes	ADT switching	MDADT switching
Criteria for controller design	Corollary 2.1 in [12]	Theorem 2.3 in the chapter
Controller gains	Γ_1 : $K_1 = \begin{bmatrix} 73.66 & 66.14 \end{bmatrix}$ $K_2 = \begin{bmatrix} -19.94 & -2.75 \end{bmatrix}$ $K_3 = \begin{bmatrix} 3.25 & -15.24 \end{bmatrix}$	Γ_2 : $K_1 = \begin{bmatrix} 93.79 & 69.75 \end{bmatrix}$ $K_2 = \begin{bmatrix} -59.81 & -34.25 \end{bmatrix}$ $K_3 = \begin{bmatrix} -53.91 & -63.58 \end{bmatrix}$
Switching signals	$\tau_{\mathrm{a}}^* = 0.99$ $(\mu = 2, \lambda \leq 0.7)$	$\tau_{\mathrm{a}1}^* = 0.22, \tau_{\mathrm{a}2}^* = 0.49, \tau_{\mathrm{a}3}^* = 0.99$ $(\mu_1 = \mu_2 = \mu_3 = 2,$ $\lambda_1 \leq 3.1, \lambda_2 \leq 1.4, \lambda_3 \leq 0.7)$

It can be seen from Table 2.1 that the minimal MDADT are reduced to $\tau_{a1}^* = 0.22$, $\tau_{a2}^* = 0.49$, $\tau_{a3}^* = 0.99$, for given $\mu = \mu_1 = \mu_2 = \mu_3 = 2$, and one special case of MDADT switching is $\tau_{\mathrm{a}}^* = \tau_{\mathrm{a}1}^* = \tau_{\mathrm{a}2}^* = \tau_{\mathrm{a}3}^* = 0.99$ by setting $\lambda = \lambda_1 = \lambda_2 = \lambda_3 = 0.7$, which corresponds to minimal ADT.

To further show the merit of MDADT switching, let us now consider the resulting closed-loop system performances. Applying the obtained controller, under the scheme of ADT switching and MDADT switching, respectively, we can obtain the state responses for each closed-loop subsystem as shown in Fig. 2.1. It is seen that there are some fluctuations with larger amplitude in the state response of closed-loop subsystem 1.

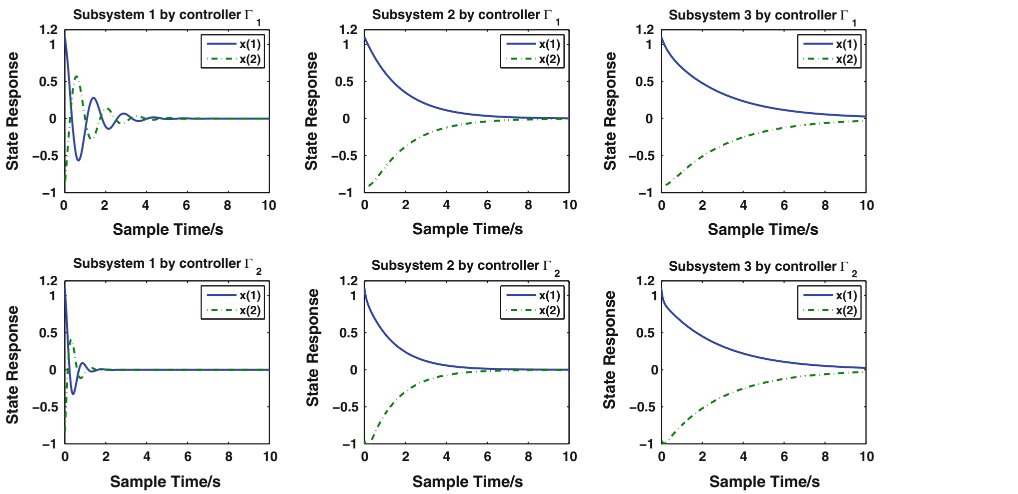

Fig. 2.1 The state response comparisons of the closed-loop subsystems by controllers Γ_1 and Γ_2

Now, generating one possible switching sequences with the ADT property and the MDADT property, one can obtain the corresponding state responses of the closed-loop system as shown in Figs. 2.2 and 2.3, respectively, for the same initial state condition. It can be seen from the curves that the state response of the closed-loop system fluctuates under the ADT switching scheme, but is smooth under the MDADT switching scheme.

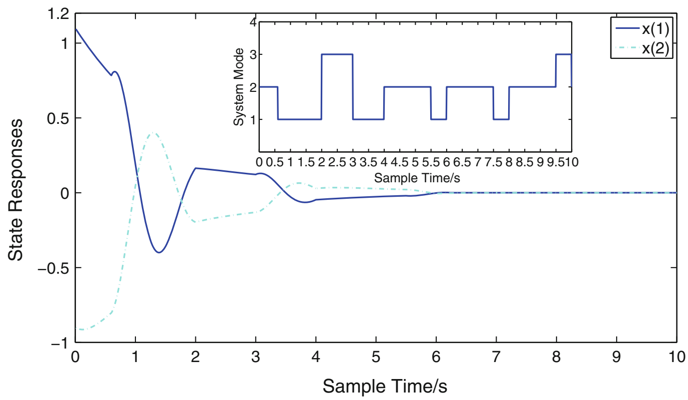

Fig. 2.2 State response of the closed-loop systems by controllers Γ_1 under switching signal σ with $\tau_a = 1.0$

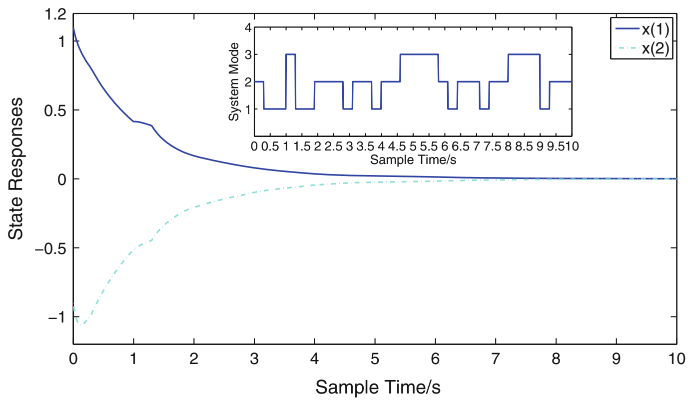

Fig. 2.3 State response of the closed-loop systems by controllers Γ_2 under switching signal σ with $\tau_{a1} = 0.3$, $\tau_{a2} = 0.6$, $\tau_{a3} = 1.0$

2.2.4 Conclusions

The MDADT switching stabilization problems for switched linear systems with stable subsystems are investigated. First, the stability results for a class of switched systems with MDADT are derived in both linear and nonlinear contexts. The minimal MDADT for admissible switching signals and the corresponding state feedback controller are designed for switched linear systems in both continuous-time and discrete-time cases. Finally, a numerical example is given to demonstrate the validity and effectiveness of the developed results.

2.3 Stabilization of Switched Linear Systems with Unstable Subsystems

2.3.1 Problem Formulation and Preliminaries

Consider the following switched linear systems,

$$\delta\mathbf{x}(t) = A_{\sigma(t)}\mathbf{x}(t) + B_{\sigma(t)}\mathbf{u}(t), \mathbf{x}(t_0) = x_0, t \geq t_0, \tag{2.41}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^m$, x_0 and $t_0 \geq 0$ denote the state vector, control input, initial state and initial time, respectively; the symbol δ denotes the derivative operator in the continuous-time case ($\delta\mathbf{x}(t) = \dot{\mathbf{x}}(t)$) and the shift forward operator in the discrete-time case ($\delta\mathbf{x}(t) = \mathbf{x}(t+1)$); $\sigma(t)$ represents a switching signal which is a piecewise constant function from the right of time and takes its values in the finite set $\mathfrak{L} = \{1, 2, \ldots, m\}$, where $m > 1$ is the number of subsystems. Moreover, the A_r, $\forall r \in \mathfrak{L}$ is either a Hurwitz stable or unstable subsystem matrix. Without lose of generality, we assume that $\mathfrak{L} = \mathfrak{S} \bigcup \mathfrak{U}$, where $\mathfrak{S} = \{1, 2, \ldots, s\}$ and $\mathfrak{U} = \{s+1, \ldots, m\}$; that is, there are s stable subsystems and $m - s$ unstable subsystems. When $t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{Z}^+$, the $\sigma(t_k)^{th}$ mode is activated. Let $\{A_r \in \mathbb{R}^{n\times n}, B_r \in \mathbb{R}^{n\times m}, r \in \mathfrak{L}\}$ be a family of constant matrices describing subsystems.

Next, some definitions are introduced for later developments of the main results in this chapter.

Definition 2.4 ([17]) The equilibrium $x = 0$ of switched system (2.41) is globally uniformly exponentially stable (GUES) under a certain switching signal $\sigma(t)$, if for $\mathbf{u}(t) = 0$ there exists positive numbers $\lambda > 0, \alpha > 0$, (resp., $0 < \nu < 1$) such that $\|\mathbf{x}(t)\| \leq \lambda e^{-\alpha(t-t_0)} \|\mathbf{x}(t_0)\|$, (resp., $\|\mathbf{x}(t)\| \leq \lambda \nu^{-(t-t_0)} \|\mathbf{x}(t_0)\|$), $\forall t \geq t_0$ with any initial conditions $\mathbf{x}(t_0)$.

Definition 2.5 For any time interval $[t_1, t_2]$, denote $N_{\sigma p}(t_2, t_1)$ as the numbers of the p^{th} subsystem being activated, and $T_p(t_2, t_1)$ as the overall running time of the p^{th} subsystem, $p \in \mathfrak{S}$. We can find two constants N_{0p} and τ_{ap} satisfying

$$N_{\sigma p}(t_2, t_1) \leq N_{0p} + \frac{T_p(t_2, t_1)}{\tau_{ap}}, \quad \forall t_2 \geq t_1 \geq 0. \tag{2.42}$$

where τ_{ap} is called the mode-dependent average dwell time of the switching signal $\sigma(t)$.

In this chapter, we also define another type of MDADT called fast MDADT in the following.

Definition 2.6 For any time interval $[t_1, t_2]$, denote $N_{\sigma q}(t_2, t_1)$ as the numbers of the q^{th} subsystem being activated, and $T_q(t_2, t_1)$ as the overall running time of the q^{th} subsystem, $q \in \mathfrak{U}$. We can find two constants N_{0q} and τ_{aq} satisfying

$$N_{\sigma q}(t_2, t_1) \geq N_{0q} + \frac{T_q(t_2, t_1)}{\tau_{aq}}, \quad \forall t_2 \geq t_1 \geq 0. \tag{2.43}$$

where τ_{aq} is called the mode-dependent average dwell time of the switching signal $\sigma(t)$.

Remark 2.4 The MDADT in Definition 2.5 requiring $N_{\sigma p}(t_2, t_1) \leq N_{0p} + \frac{t_2 - t_1}{\tau_a} \Longleftrightarrow \frac{T_p(t_2,t_1)}{N_{\sigma p(t_2,t_1)} - N_{0p}} \geq \tau_{ap}$, $\forall t_2 \geq t_1 \geq 0$ can be called slow switching (in average sense), which means that average time among the intervals associated with the p^{th} subsystem is larger than τ_{ap}. By resorting to this MDADT to achieve stabilization, the basic idea is to allow the transient effect to dissipate after each switching. In this framework, the energy decrement of the Lyapunov function during dwelling on stable subsystems can compensate possible energy at the switching instance and/or during dwelling at unstable subsystems. However, Definition 2.6 requires $N_{\sigma q}(t_2, t_1) \geq N_{0q} + \frac{t_2 - t_1}{\tau_a} \Longleftrightarrow \frac{T_q(t_2,t_1)}{N_{\sigma q(t_2,t_1)} - N_{0q}} \leq \tau_{aq}$, $\forall t_2 \geq t_1 \geq 0$. It is called fast switching (in average sense), because the average time among the intervals associated with the q^{th} subsystem is no more than τ_{aq}. The basic idea of using the fast MDADT is to compensate the state divergence via dwelling at appropriate unstable subsystems, but obviously the dwell time cannot be too big. Therefore, in order to achieve stabilization, we apply the slow MDADT to stable subsystems and fast MDADT to unstable subsystems in the following.

2.3.2 Main Results

In this section, we consider the problems of stability and stabilization for switched linear systems described in the previous section.

2.3.2.1 Stability Analysis

We first introduce a class of quasi-alternative switching signals.

Definition 2.7 Suppose that a switching law $\sigma(t)$ satisfies the following conditions.
(1) If $\sigma(t_k) \in \mathfrak{S}$, then $\sigma(t_{k+1}) \in \mathfrak{L}$,
(2) If $\sigma(t_k) \in \mathfrak{U}$, then $\sigma(t_{k+1}) \in \mathfrak{S}$,

The switching signal $\sigma(t)$ satisfying the above conditions is called a quasi-alternative switching signal.

Remark 2.5 Definition 2.7 implies that a switched system cannot directly switches from an unstable mode to another unstable mode. If condition (1) is changed as: "If $\sigma(t_k) \in \mathfrak{S}$, then $\sigma(t_{k+1}) \in \mathfrak{U}$;", Definition 2.7 implies that $\sigma(t)$ is a alternative switching signal, that is, stable subsystems and unstable subsystems alternately switch to each other.

Next, stability conditions for switched nonlinear system

$$\delta \mathbf{x}(t) = f_{\sigma(t)}(\mathbf{x}(t)). \tag{2.44}$$

are first presented in the following lemmas by designing quasi-alternative switching signals with MDADT property.

Lemma 2.5 *Consider continuous-time switched nonlinear system (2.44), $\sigma(t) \in \mathfrak{L}$, and let $\eta_p < 0$, $\mu_p > 1$, $p \in \mathfrak{S}$ and $\eta_q > 0$, $0 < \mu_q < 1, q \in \mathfrak{U}$. Suppose that there exist two sets of $\mathscr{C}^1$ non-negative functions $V_p(\mathbf{x}(t)) : \mathbb{R}^n \to \mathbb{R}$, $p \in \mathfrak{S}$ and $V_q(x(t)) : \mathbb{R}^n \to \mathbb{R}$, $q \in \mathfrak{U}$, two class $\mathscr{K}_\infty$ functions α_1 and α_2, such that*

$$\alpha_1(\|\mathbf{x}(t)\|) \le V_p(\mathbf{x}(t)) \le \alpha_2(\|\mathbf{x}(t)\|), \forall p \in \mathfrak{S}, \tag{2.45}$$
$$\alpha_1(\|\mathbf{x}(t)\|) \le V_q(\mathbf{x}(t)) \le \alpha_2(\|\mathbf{x}(t)\|), \forall q \in \mathfrak{U}, \tag{2.46}$$
$$\dot{V}_p(\mathbf{x}(t)) \le \eta_p V_p(\mathbf{x}(t)), \quad \forall p \in \mathfrak{S}, \tag{2.47}$$
$$\dot{V}_q(\mathbf{x}(t)) \le \eta_q V_q(\mathbf{x}(t)), \quad \forall q \in \mathfrak{U}, \tag{2.48}$$
$$V_p(\mathbf{x}(t_k)) \le \mu_p V_r(\mathbf{x}(t_k^-)), \quad \forall p \in \mathfrak{S}, \forall r \in \mathfrak{L}, \quad p \ne r, \tag{2.49}$$
$$V_q(\mathbf{x}(t_k)) \le \mu_q V_p(\mathbf{x}(t_k^-)), \quad \forall p \in \mathfrak{S}, \forall q \in \mathfrak{U}. \tag{2.50}$$

Then switched system (2.44) is GUES for any quasi-alternative switching signals with MDADT

$$\begin{cases} \tau_{ap} \ge \frac{-ln\mu_p}{\eta_p}, \forall p \in \mathfrak{S}, \\ \tau_{aq} \le \frac{-ln\mu_q}{\eta_q}, \forall q \in \mathfrak{U}. \end{cases} \tag{2.51}$$

Proof Without loss of generality, we denote $t_1, t_2 \ldots t_k, t_{k+1} \ldots t_{N_{N_{\sigma(T,0)}}}$ as the switching times on time interval $[0, T]$. Then we consider the function

$$W(t) = e^{-\eta_{\sigma(t)} t} V_{\sigma(t)}(\mathbf{x}(t)). \tag{2.52}$$

It is clear that this function is piecewise differentiable along solutions of (2.44). When $t \in [t_k, t_{k+1})$, we get from (2.47), (2.48), (2.52) that

$$\begin{aligned}\dot{W}(t) &= -\eta_{\sigma(t_k)}e^{-\eta_{\sigma(t_k)}t}V_{\sigma(t_k)}(\mathbf{x}(t)) + e^{-\eta_{\sigma(t_k)}t}\dot{V}_{\sigma(t_k)}(\mathbf{x}(t)) \\ &\le -\eta_{\sigma(t_k)}e^{-\eta_{\sigma(t_k)}t}V_{\sigma(t_k)}(\mathbf{x}(t)) + e^{-\eta_{\sigma(t_k)}t}\eta_{\sigma(t_k)}V_{\sigma(t_k)}(\mathbf{x}(t)) \\ &= 0. \end{aligned} \tag{2.53}$$

Thus $W(t)$ is non-increasing when $t \in [t_k, t_{k+1})$. This together with (2.49), (2.50), (2.52) gives that

$$\begin{aligned}W(t_{k+1}) &= e^{-\eta_{\sigma(t_{k+1})}t_{k+1}}V_{\sigma(t_{k+1})}(\mathbf{x}(t_{k+1})) \\ &\le \mu_{\sigma(t_{k+1})}e^{-\eta_{\sigma(t_{k+1})}t_{k+1}}V_{\sigma(t_k)}(\mathbf{x}(t_{k+1})) \\ &= \mu_{\sigma(t_{k+1})}e^{-\eta_{\sigma(t_{k+1})}t_{k+1}+\eta_{\sigma(t_k)}t_{k+1}}W(\mathbf{x}(t_{k+1}^-)) \\ &\le \mu_{\sigma(t_{k+1})}e^{-(\eta_{\sigma(t_{k+1})}-\eta_{\sigma(t_k)})t_{k+1}}W(\mathbf{x}(t_k)) \\ &\le \mu_{\sigma(t_{k+1})}\mu_{\sigma(t_k)}e^{-[(\eta_{\sigma(t_{k+1})}-\eta_{\sigma(t_k)})t_{k+1}+(\eta_{\sigma(t_k)}-\eta_{\sigma(t_{k-1})})t_k]}W(\mathbf{x}(t_{k-1})) \\ &\quad \cdots \\ &\le \prod_{i=0}^{k}\mu_{\sigma(t_{i+1})}\exp\{-[(\eta_{\sigma(t_{k+1})}-\eta_{\sigma(t_k)})t_{k+1}+(\eta_{\sigma(t_k)}-\eta_{\sigma(t_{k-1})})t_k \\ &\quad +\cdots+(\eta_{\sigma(t_1)}-\eta_{\sigma(t_0)})t_1]\}W(\mathbf{x}(t_0)). \end{aligned} \tag{2.54}$$

Then, from (2.52) and (2.54), one can obtain that

$$e^{-\eta_{\sigma(T^-)}T}W(\mathbf{x}(T^-)) \le \prod_{i=0}^{N_\sigma-1}\mu_{\sigma(t_{i+1})}e^{\sum_{i=0}^{N_\sigma-1}-(\eta_{\sigma(t_{i+1})}-\eta_{\sigma(t_i)})t_i}V_{\sigma(t_0)}(\mathbf{x}(t_0)). \tag{2.55}$$

Moreover, it can be derived from (2.42), (2.43) and (2.55) that

$$\begin{aligned}V_{\delta(T^-)}(\mathbf{x}(T)) &\le \prod_{p=1}^{s}\mu_p^{N_{\sigma p}}\prod_{q=s+1}^{m}\mu_q^{N_{\sigma q}}e^{(\sum_{p=1}^{s}\eta_pT_p(T,0)+\sum_{q=s+1}^{m}\eta_qT_q(T,0))}V_{\sigma(0)}(\mathbf{x}(0)) \\ &\le \prod_{p=1}^{s}\mu_p^{(N_{0p}+\frac{T_p(T,0)}{\tau_{ap}})}\prod_{q=s+1}^{m}\mu_q^{(N_{0q}+\frac{T_q(T,0)}{\tau_{aq}})} \\ &\quad \times e^{(\sum_{p=1}^{s}\eta_pT_p(T,0)+\sum_{q=s+1}^{m}\eta_qT_q(T,0))}V_{\sigma(0)}(\mathbf{x}(0)) \\ &= e^{(\sum_{p=1}^{s}(N_{0p}+\frac{T_p(T,0)}{\tau_{ap}})\ln\mu_p+\sum_{q=s+1}^{m}(N_{0q}+\frac{T_q(T,0)}{\tau_{aq}})\ln\mu_q)} \\ &\quad \times e^{(\sum_{p=1}^{s}\eta_pT_p(T,0)+\sum_{q=s+1}^{m}\eta_qT_q(T,0))}\times V_{\sigma(0)}(\mathbf{x}(0)) \\ &\le e^{(\sum_{p=1}^{s}N_{0p}\ln\mu_p+\sum_{q=s+1}^{m}N_{0q}\ln\mu_q)} \end{aligned}$$

$$\times e^{(\sum_{p=1}^{s}(\eta_p+\frac{\ln\mu_p}{\tau_{ap}})T_p(T,0)+\sum_{q=s+1}^{m}(\eta_q+\frac{\ln\mu_q}{\tau_{aq}})T_q(T,0))} V_{\sigma(0)}(\mathbf{x}(0)). \tag{2.56}$$

By (2.56), it can be got that, if τ_{ap}, $p \in \mathfrak{S}$ and τ_{aq}, $q \in \mathfrak{U}$ satisfy the conditions in (2.51), then

$$V_{\delta(T^-)}(\mathbf{x}(T)) \le \lambda e^{-\alpha(T-t_0)} V_{\sigma(0)}(\mathbf{x}(0)),$$

where $\lambda = e^{(\sum_{p=1}^{s} N_{0p}\ln\mu_p + \sum_{q=s+1}^{m} N_{0q}\ln\mu_q)}$, $-\alpha = \max_{(p,q)\in(S\times U)}\left\{\left(\eta_p + \frac{\ln\mu_p}{\tau_{ap}}\right), \left(\eta_q + \frac{\ln\mu_q}{\tau_{aq}}\right)\right\}$, which associated with Definition 2.4 verifies that $V_{\delta(T^-)}(\mathbf{x}(T))$ exponentially converges to zero as $T \to \infty$.

Finally, we conclude that switched nonlinear system (2.44) is GUES under quasi-alternative switching signals satisfying (2.51) if the conditions (2.45)–(2.50) hold. This completes the proof. □

Lemma 2.6 *Consider discrete-time switched nonlinear system (2.44), $\sigma(t) \in \mathfrak{L}$, and let $-1 < \eta_p < 0$, $\mu_p > 1$, $p \in \mathfrak{S}$ and $\eta_q > 0$, $0 < \mu_q < 1$, $q \in \mathfrak{U}$. Suppose that there exist two sets of $\mathscr{C}^1$ non-negative functions $V_p(\mathbf{x}(t)) : \mathbb{R}^n \to \mathbb{R}$, $p \in \mathfrak{S}$ and $V_q(\mathbf{x}(t)) : \mathbb{R}^n \to \mathbb{R}$, $q \in \mathfrak{U}$, two class $\mathscr{K}_\infty$ functions α_1 and α_2, such that*

$$\alpha_1(\|\mathbf{x}(t)\|) \le V_p(\mathbf{x}(t)) \le \alpha_2(\|\mathbf{x}(t)\|), \forall p \in \mathfrak{S}, \tag{2.57}$$
$$\alpha_1(\|\mathbf{x}(t)\|) \le V_q(\mathbf{x}(t)) \le \alpha_2(\|\mathbf{x}(t)\|), \forall q \in \mathfrak{U}, \tag{2.58}$$
$$\triangle V_p(\mathbf{x}(t)) \le \eta_p V_p(\mathbf{x}(t)), \quad \forall p \in \mathfrak{S}, \tag{2.59}$$
$$\triangle V_q(\mathbf{x}(t)) \le \eta_q V_q(\mathbf{x}(t)), \quad \forall p \in \mathfrak{U}, \tag{2.60}$$
$$V_p(\mathbf{x}(t_k)) \le \mu_p V_r(\mathbf{x}(t_k^-)), \quad \forall q \in \mathfrak{S}, \forall r \in \mathfrak{L}, \quad p \ne r, \tag{2.61}$$
$$V_q(\mathbf{x}(t_k)) \le \mu_q V_p(\mathbf{x}(t_k^-)), \quad \forall p \in \mathfrak{S}, \forall q \in \mathfrak{U}. \tag{2.62}$$

Then switched system (2.44) is GUES for any quasi-alternative switching signals with MDADT

$$\begin{cases} \tau_{ap} \ge \frac{-ln\mu_p}{1+\eta_p}, \forall p \in \mathfrak{S}, \\ \tau_{aq} \le \frac{-ln\mu_q}{1+\eta_q}, \forall q \in \mathfrak{U}. \end{cases} \tag{2.63}$$

Proof The proof of Lemma 2.6 is similar to that of Lemma 2.5. We omit it here. □

Remark 2.6 Different from Lemma 2.3 (or Lemma 2.4 in the discrete-time case), unstable subsystems are considered in Lemma 2.5 (resp., Lemma 2.6). For stable subsystems, it also follows the slow switching scheme (Definition 2.5). But for unstable subsystems, it adopts the fast switching scheme (Definition 2.6). Such a switching strategy can guarantee to dwell on stable subsystems long enough to compensate possible energy increments at the switching instance and during dwelling on unstable subsystems, and avoid dwelling on unstable subsystems too long. Anyway, it should be pointed out that the dwell time on stable subsystems is not required to be bigger than that on unstable subsystems. In fact, if a switched system is composed of stable subsystems, Lemma 2.5 (Lemma 2.6 in the discrete-time case) will reduce to Lemma 2.3 (resp., Lemma 2.4).

Next, the following two theorems for switched linear system (2.41) can be given on the basis of the Lemmas 2.5 and 2.6. Theorem 2.6 corresponds to the continuous-time version and Theorem 2.7 corresponds to the discrete-time version.

Theorem 2.5 *Consider switched linear system (2.41) when* $u(t) = 0$*, and let* $\eta_p < 0$*,* $\mu_p > 1$*,* $p \in \mathfrak{S}$ *and* $\eta_q > 0$*,* $0 < \mu_q < 1$*,* $q \in \mathfrak{U}$ *be given constants. If there exists a set of matrices* $P_p > 0$*,* $P_q > 0$*,* $p \in \mathfrak{S}, q \in \mathfrak{U}$*, such that*

$$A_p^T P_p + P_p A_p \leq \eta_p P_p, \quad \forall p \in \mathfrak{S}, \tag{2.64}$$

$$A_q^T P_q + P_q A_q \leq \eta_q P_q, \quad \forall q \in \mathfrak{U}, \tag{2.65}$$

$$P_p \leq \mu_p P_r, \quad \forall q \in \mathfrak{S}, \ \forall r \in \mathfrak{L}, \quad p \neq r, \tag{2.66}$$

$$P_q \leq \mu_q P_p, \quad \forall p \in \mathfrak{S}, \ \forall q \in \mathfrak{U}. \tag{2.67}$$

Then, the system is GUES for any quasi-alternative switching signals with MDADT satisfying (2.51).

Proof Construct a multiple Lyapunov function for continuous-time switched system (2.41) in the form of

$$V_{\sigma(t)}(\mathbf{x}(t)) = \begin{cases} \mathbf{x}(t)^T P_p \mathbf{x}(t), & \sigma(t) = p \in \mathfrak{S} \\ \mathbf{x}(t)^T P_q \mathbf{x}(t) & \sigma(t) = q \in \mathfrak{U}, \end{cases} \tag{2.68}$$

where $P_p > 0, P_q > 0, \ \ p \in \mathfrak{S}, q \in \mathfrak{U}$ are positive definite matrices satisfying (2.64)–(2.67).

In the sequel, one can obtain from (2.64)–(2.67) that $\forall (p, q) \in \mathfrak{S} \times \mathfrak{U}$,

$$\begin{aligned}
\dot{V}_p(\mathbf{x}(t)) - \eta_p V_p(\mathbf{x}(t)) &= \mathbf{x}^T(t)(A_p^T P_p + P_p A_p - \eta_p P_p)\mathbf{x}(t), \\
&\leq 0, \ p \in \mathfrak{S}, \\
\dot{V}_q(\mathbf{x}(t)) - \eta_q V_p(\mathbf{x}(t)) &= \mathbf{x}^T(t)(A_q^T P_q + P_q A_q - \eta_q P_q)\mathbf{x}(t), \\
&\leq 0, \ q \in \mathfrak{U}. \\
V_p(\mathbf{x}(t)) - \mu_p V_r(\mathbf{x}(t)) &= \mathbf{x}^T(t)(P_p - \mu_p P_r)\mathbf{x}(t), \\
&\leq 0, \ p \in \mathfrak{S}, \ r \in \mathfrak{L}, \ p \neq r. \\
V_q(\mathbf{x}(t)) - \mu_q V_p(\mathbf{x}(t)) &= \mathbf{x}^T(t)(P_q - \mu_q P_p)\mathbf{x}(t), \\
&\leq 0, \ p \in \mathfrak{S}, \ q \in \mathfrak{U}.
\end{aligned}$$

Finally, one can readily conclude by Lemma 2.5 that switched system (2.41) is GUES for any quasi-alternative switching signals with MDADT satisfying (2.51). □

Theorem 2.6 *Consider switched linear system (2.41) when* $\mathbf{u}(t) = 0$*, and let* $-1 < \eta_p < 0$*,* $\mu_p > 1$*,* $p \in \mathfrak{S}$ *and* $\eta_q > 0$*,* $0 < \mu_q < 1$*,* $q \in \mathfrak{U}$ *be given constants. If there exists a set of matrices* $P_p > 0$*,* $P_q > 0$*,* $p \in \mathfrak{S}, q \in \mathfrak{U}$*, such that*

$$A_p^T P_p A_p - P_p \le \eta_p P_p, \quad \forall p \in \mathfrak{S}, \tag{2.69}$$

$$A_q^T P_q A_q - P_q \le \eta_q P_q, \quad \forall q \in \mathfrak{U}, \tag{2.70}$$

$$P_p \le \mu_p P_r, \quad \forall p \in \mathfrak{S},\ \forall r \in \mathfrak{L},\quad p \neq r, \tag{2.71}$$

$$P_q \le \mu_q P_p, \quad \forall p \in \mathfrak{S},\ \forall q \in \mathfrak{U}. \tag{2.72}$$

then, the system is GUES for any quasi-alternative switching signals with MDADT satisfying (2.63).

Proof The proof of Theorem 2.6 is similar to that of Theorem 2.5. We omit it here. □

2.3.2.2 Controller Design

In this subsection, the problem of controller design for switched system (2.41) with MDADT switching is presented. Unlike some control methods requiring all subsystems be controllable, we only require the existence of at least one controllable subsystem. Without loss of generality, we assume that $\{A_p \in \mathbb{R}^{n\times n}, B_p \in \mathbb{R}^{n\times m}, p \in \mathfrak{C}\}$ are controllable subsystems, where $\mathfrak{C} = \{1, 2, \ldots, s\}$, and $\{A_q \in \mathbb{R}^{n\times n}, q \in \mathfrak{B}\}$ are subsystems that can not be stabilized, where $\mathfrak{B} = \{s+1, s+2, \ldots, m\}$. Our objective is to design p controllers to ensure switched system (2.41) to be GUES with MDADT switching. In this subsection, the state feedback is considered with $u(t) = K_p x(t)$, $p \in \mathfrak{C}$, where K_p is the controller gain to be determined. Then the closed-loop system (3.1) can be obtained as follows,

$$\delta \mathbf{x}(t) = \begin{cases} A_p\mathbf{x}(t) + B_p K_p \mathbf{x}(t), & \forall p \in \mathfrak{C}, \\ A_q \mathbf{x}(t), & \forall q \in \mathfrak{B}. \end{cases} \tag{2.73}$$

However, it should be pointed out that if the A_p, $\forall p \in \mathfrak{C}$ itself is a Hurwitz matrix, the controller gain K_p is chosen as 0.

Theorem 2.7 *Consider switched linear system (2.73), and let* $\eta_p < 0$*,* $\mu_p > 1$*,* $p \in \mathfrak{C}$ *and* $\eta_q > 0$*,* $0 < \mu_q < 1$*,* $q \in \mathfrak{P}$ *be given constants. If there exists a set of matrices* $Q_r > 0$*,* $r \in \mathfrak{L}$*, and* R_p*,* $p \in \mathfrak{C}$ *such that*

$$Q_p A_p^T + A_p Q_p + R_p^T B_p^T + B_p R_p \le \eta_p Q_p, \quad \forall p \in \mathfrak{C}, \tag{2.74}$$

$$Q_q A_q^T + A_q Q_q \le \eta_q Q_q, \quad \forall q \in \mathfrak{B}, \tag{2.75}$$

$$Q_r \le \mu_p Q_p, \quad \forall p \in \mathfrak{C},\ \forall r \in \mathfrak{L},\quad p \neq r, \tag{2.76}$$

$$Q_p \le \mu_q Q_q, \quad \forall p \in \mathfrak{C},\ \forall q \in \mathfrak{B}. \tag{2.77}$$

then, there is a set of stabilizing controllers such that the system is GUES for any quasi-alternative switching signals with MDADT satisfying (2.51). Moreover, if a feasible solution of (2.74)–(2.77) exists, the controller gains are given by

$$K_p = R_p Q_p^{-1}. \tag{2.78}$$

Proof When $\sigma(t) \in \mathfrak{S}$, perform a congruence transformation to (2.74) via Q_p^{-1}. Then by (2.78), one can obtain that

$$A_p^T Q_p^{-1} + Q_p^{-1} A_p + K_p^T B_p^T Q_p^{-1} + Q_p^{-1} B_p K_p \leq \eta_p Q_p^{-1}, \quad \forall p \in \mathfrak{C}, \tag{2.79}$$

which is equivalent to

$$(A_p + B_p K_p)^T Q_p^{-1} + Q_p^{-1}(A_p + B_p K_p) \leq \eta_p Q_p^{-1}, \quad \forall p \in \mathfrak{C}. \tag{2.80}$$

Then, by the Schur complement theorem, we can get that (2.76) is equivalent to

$${Q_p}^{-1} \leq \mu_p {Q_r}^{-1}, \quad \forall p \in \mathfrak{C}, \ \forall r \in \mathfrak{L}, \ p \neq r. \tag{2.81}$$

Similarly, when $\sigma(t) \in \mathfrak{U}$, it can be derived that (2.76) and (2.78) are also equivalent to the following inequalities, respectively,

$$A_q^T Q_q^{-1} + Q_q^{-1} A_q \leq \eta_q {Q_q}^{-1}, \quad \forall p \in \mathfrak{B}, \tag{2.82}$$

$${Q_q}^{-1} \leq \mu_q {Q_p}^{-1}, \quad \forall p \in \mathfrak{C}, \ \forall q \in \mathfrak{B}. \tag{2.83}$$

Finally, by Theorem 2.5 and letting $P_p = Q_p^{-1}$, we can conclude that, if (2.80)–(2.83) hold, switched system (2.73) is GUES for any quasi-alternative switching signal with MDADT satisfying (2.51). This completes the proof. □

Theorem 2.8 *Consider switched linear system (2.73), and let $-1 < \eta_p < 0$, $\mu_p > 1$, $p \in \mathfrak{C}$ and $\eta_q > 0$, $0 < \mu_q < 1$, $q \in \mathfrak{B}$ be given constants. If there exists a set of matrices $Q_r > 0$, $r \in \mathfrak{L}$, and R_p, $p \in \mathfrak{C}$ such that*

$$\begin{bmatrix} -Q_p & A_p Q_p + B_p R_p \\ * & -(1+\eta_p) Q_p \end{bmatrix} \leq 0, \quad \forall p \in \mathfrak{C}, \tag{2.84}$$

$$\begin{bmatrix} -Q_q & A_q Q_q \\ * & -(1+\eta_q) Q_q \end{bmatrix} \leq 0, \quad \forall q \in \mathfrak{B}, \tag{2.85}$$

$$Q_r \leq \mu_p Q_p, \quad \forall p \in \mathfrak{C}, \ \forall r \in \mathfrak{L}, \ \ p \neq r, \tag{2.86}$$

$$Q_p \leq \mu_q Q_q, \quad \forall p \in \mathfrak{C}, \ \forall q \in \mathfrak{B}. \tag{2.87}$$

Then, there is a set of stabilizing controllers such that the system is GUES for any quasi-alternative switching signals with MDADT satisfying (2.63). Moreover, if a feasible solution of (2.84)–(2.87) exists, the controller gains are given by

$$K_p = R_p Q_p^{-1}. \tag{2.88}$$

Proof The proof of Theorem 2.8 is similar to that of Theorem 2.7. We omit it here. □

2.3.3 Simulation Results

The following numerical example is given in this section to verify our main results developed above.

Example 2.2 Consider the continuous-time switched linear system (2.41) consisting of four subsystems and assume that the third and fourth are uncontrollable subsystems. The corresponding subsystem matrices are

$$A_1 = \begin{bmatrix} -10.11 & 10.32 \\ -8.60 & 8.81 \end{bmatrix}, B_1 = \begin{bmatrix} -2.2 \\ 0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 11.12 & -13.32 \\ 11.10 & -13.30 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 3.4 \\ -1.2 \end{bmatrix}, A_3 = \begin{bmatrix} 9.72 & -9.69 \\ 12.92 & -12.89 \end{bmatrix}, A_4 = \begin{bmatrix} 10.24 & -10.23 \\ 13.64 & -13.63 \end{bmatrix}.$$

The eigenvalues of A_1 are $\lambda_{11} = -1.51$ and $\lambda_{12} = 0.21$, eigenvalues of A_2 are $\lambda_{21} = 0.02$ and $\lambda_{22} = -2.2$, eigenvalues of A_3 are $\lambda_{31} = 0.03$ and $\lambda_{32} = -3.2$ and eigenvalues of A_4 are $\lambda_{41} = 0.01$ and $\lambda_{42} = -3.4$, It can be seen that none of these matrices is Hurwitz stable. In addition, one can easily check that $\{A_p \in \mathbb{R}^{2\times 2}, B_p \in \mathbb{R}^{2\times 1}, p = 1, 2\}$ are controllable.

Next, we are interested in designing a set of controllers and a kind of quasi-alternative switching signal $\sigma(t)$ with properties (2.42) and (2.43) to asymptotically

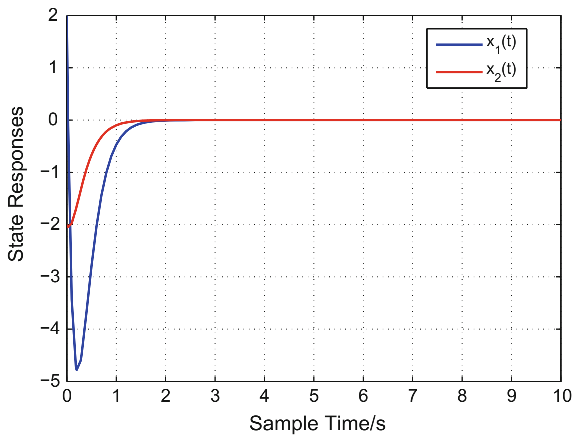

Fig. 2.4 State responses of the first subsystem

stabilize the system. By using Theorem 2.7, if we choose $\mu_1 = 2.9,\ \eta_1 = -1.0, \mu_2 = 2.3,\ \eta_2 = -3.1, \mu_3 = 0.44,\ \eta_3 = 3.0, \mu_4 = 0.51,\ \eta_4 = 1.3$, the feasible solutions are obtained as follows,

$$Q_1 = \begin{bmatrix} 77.0146 & 69.8370 \\ 69.8370 & 65.1947 \end{bmatrix}, Q_2 = \begin{bmatrix} 83.2764 & 77.1246 \\ 77.1246 & 73.6036 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 190.6688 & 176.4114 \\ 176.4114 & 168.1891 \end{bmatrix}, Q_4 = \begin{bmatrix} 180.3970 & 169.1126 \\ 169.1126 & 163.0650 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 6.5871 & -13.5623 \end{bmatrix}, R_2 = \begin{bmatrix} -18.2149 & -3.7398 \end{bmatrix},$$

$$K_1 = R_1 Q_1^{-1} = \begin{bmatrix} 9.5768 & -10.4667 \end{bmatrix}, K_2 = R_2 Q_2^{-1} = \begin{bmatrix} -5.8056 & 6.0325 \end{bmatrix}.$$

Applying the obtained controllers to the first and second subsystems, respectively, the corresponding state responses of the subsystems under initial state condition $\mathbf{x}(0) = [2\ \ -2]^T$ are shown in Figs. 2.4 and 2.5, in which we can see that the closed-loop subsystems are asymptotically stable. Then, one can obtain that the requirements of MDADT for subsystem $A_i, i = 1, 2, 3, 4$ are:

$$\tau_{a1} \geq \frac{\ln \mu_1}{\eta_1} = \frac{-\ln 2.9}{-1.0} = 1.065,$$

$$\tau_{a2} \geq \frac{\ln \mu_2}{\eta_2} = \frac{-\ln 2.3}{-3.1} = 0.269,$$

$$\tau_{a3} \leq \frac{\ln \mu_3}{\eta_3} = \frac{-\ln 0.44}{3.0} = 0.274,$$

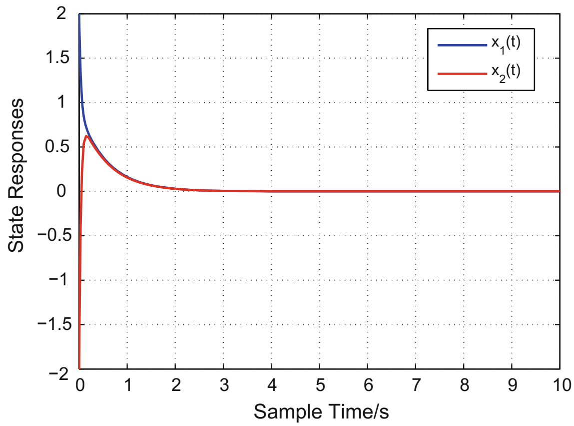

Fig. 2.5 State responses of the second subsystem

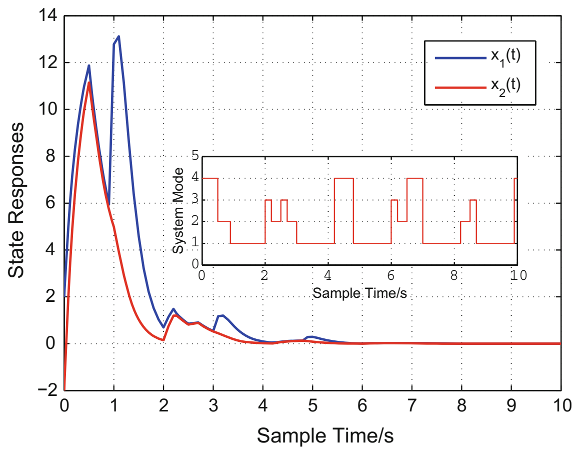

Fig. 2.6 State responses of switched linear system under quasi-alternative switching signal with MDADT

$$\tau_{a4} \leq \frac{\ln \mu_4}{\eta_4} = \frac{-\ln 0.51}{1.3} = 0.518.$$

Furthermore, we generate one possible quasi-alternative switching sequence (4, 2, 1, 3, 2, 3, 2, 1, 4, 1, 3, 2, 4…) with the MDADT property ($\tau_{a1} = 1.2 > 1.065$, $\tau_{a2} = 0.3 > 0.269$, $\tau_{a3} = 0.2 < 0.274$, $\tau_{a4} = 0.5 < 0.518$). The corresponding state responses of the system under initial state condition $x(0) = [2 \ -2]^T$, are shown in Fig. 2.6, from which we can see that the switched linear system is stable under MDADT switching.

2.3.4 Conclusions

In the above, the problems of stability and stabilization for switched systems comprising unstable subsystems are studied in both continuous-time and discrete-time contexts by using a new defined class of switching signal. The proposed switching signal is very efficient for analysis and design for switched systems comprising unstable subsystems. The stability results for switched systems comprising unstable subsystems are first derived on the basis of our proposed switching signals. Moreover, based on the obtained results, improved stabilization conditions are also established, which are concerned with uncontrollable subsystems. Finally, a numerical example is provided to verify the advantages of the proposed approach.

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