# **Improved Chaotic Multidirectional Associative Memory**

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**Abstract.** In this paper, we propose an Improved Chaotic Multidirectional Associative Memory (ICMAM). The proposed model is based on the Chaotic Multidirectional Associative Memory (CMAM) which can realize one-to-many associations. In the conventional CMAM, the oneto-many associative ability is very sensitive to chaotic neuron parameters. Moreover, although the Chaotic Multidirectional Associative Memory with adaptive scaling factor of refractoriness can select appropriate scaling factor of refractoriness  $\alpha$  based on internal states of neurons automatically, their one-to-many association ability is lower than that of well-tuned Chaotic Multidirectional Associative Memory with variable scaling factor of refractoriness when the number of layers is large. In the proposed model, one-to-many association ability which does not depend on the number of layers is realized by dividing internal states of neurons by the number of layers. We carried out a series of computer experiments in order to demonstrate the effectiveness of the proposed model, and confirmed that the one-to-many association ability of this model almost equals to that of well-tuned Chaotic Multidirectional Associative Memory with variable scaling factor of refractoriness even when the number of layers is large.

## **1 Introduction**

In the field of neural networks, a lot of associative memories have been proposed. However, most of these models can deal with only one-to-one associations [\[1](#page-7-0),[2\]](#page-7-1). In contrast, as the model which can realize one-to-many associations, some models which are based on the chaotic neuron models [\[3](#page-7-2)] or chaotic neuron-based models  $[4,5]$  $[4,5]$  have been proposed  $[6-11]$  $[6-11]$ . However, the association ability of neural networks composed of chaotic neuron models or chaotic neuron-based models are very sensitive to chaotic neuron parameters such as scaling factor of refractoriness  $\alpha$  and damping factor k and so on. And, in these models, appropriate parameters have to determined by trial and error. Although the Chaotic Multidirectional Associative Memory with adaptive scaling factor of refractoriness [\[12\]](#page-7-7) can select appropriate scaling factor of refractoriness  $\alpha$  based on internal states of neurons automatically, their one-to-many association ability is lower than that of well-tuned Chaotic Multidirectional Associative Memory with variable scaling factor of refractoriness when the number of layers is large.



<span id="page-1-0"></span>**Fig. 1.** Structure of proposed ICMAM.

In this paper, we propose an Improved Chaotic Multidirectional Associative Memory (ICMAM). In the proposed model, one-to-many association ability which does not depend on the number of layers is realized by dividing internal states of neurons by the number of layers.

# **2 Improved Chaotic Multidirectional Associative Memory**

Here, we explain the proposed Improved Chaotic Multidirectional Associative Memory (ICMAM). The proposed ICMAM is based on the conventional Chaotic Multidirectional Associative Memory [\[7\]](#page-7-8), and can realize one-to-many association of M-tuple binary patterns.

#### $2.1$ Structure

The proposed model has three or more layers as similar as the conventional Chaotic Multidirectional Associative Memory. Figure [1](#page-1-0) shows the structure of the proposed model which has three layers. Each layer consists of two parts; (1) Key Input Part composed of conventional neuron models and (2) Context Part composed of chaotic neuron models [\[3\]](#page-7-2). Since chaotic neuron models in the Context Part change their states by chaos, plural patterns corresponding to the input common term can be recalled, that is, one-to-many association can be realized.

#### $2.2$ **2.2 Learning Process**

In the proposed model, pattern sets are memorized by the orthogonal learning. In the proposed model which has  $M$  layers, the connection weights from the layer x to the layer  $y$  is given by

$$
\boldsymbol{w}^{yx} = \boldsymbol{X}_y (\boldsymbol{X}_x^T \boldsymbol{X}_x)^{-1} \boldsymbol{X}_x^T \tag{1}
$$

$$
\boldsymbol{w}^{xy} = \boldsymbol{X}_x (\boldsymbol{X}_y^T \boldsymbol{X}_y)^{-1} \boldsymbol{X}_y^T \tag{2}
$$

and  $\boldsymbol{X}_x$  and  $\boldsymbol{X}_y$  are given by

$$
\mathbf{X}_{x} = \{ \mathbf{X}_{x}^{(1)}, \cdots, \mathbf{X}_{x}^{(p)}, \cdots, \mathbf{X}_{x}^{(P)} \}
$$
(3)

$$
\mathbf{X}_y = \{ \mathbf{X}_y^{(1)}, \cdots, \mathbf{X}_y^{(p)}, \cdots, \mathbf{X}_y^{(P)} \}
$$
(4)

where P is the number of the training pattern sets, and  $\mathbf{X}_x^{(p)}$  is the pattern p which is stored in the layer x,  $\mathbf{X}_y^{(p)}$  is the pattern p which is stored in the layer y. Each element of training patterns takes −1 or 1.

In the orthogonal learning, since the stored common pattern causes superimposed pattern in the recall process, the pattern sets including one-to-many relation can not be memorized. In the proposed model, each learning pattern is memorized together with its own contextual information in order to memorize the training set including one-to-many relations as similar as the conventional CMAM. Here, the contextual information patterns are generated randomly.

#### 2.3 **Recall Process 2.3 Recall Process**

In the recall process of the proposed model, only neurons in the Key Input Part receives input in the first step. This is because we assume that contextual information is usually unknown for users. In the proposed model, since the chaotic neurons in the Context Part change their states by chaos, plural patterns corresponding to the input common pattern can be recalled.

### **Step 1: Input to Layer** <sup>x</sup>

The input pattern is given to the key input part in the layer  $x$ .

## **Step 2: Propagation from Layer** <sup>x</sup> **to Other Layers**

The information in the layer  $x$  is propagated to the key input part in other layers. The output of the neuron k in the key input part of the layer  $y (y \neq x)$ at the time  $t$ ,  $x_k^y(t)$  is calculated by

$$
x_k^y(t) = f\left(\sum_{j=1}^{N^x} w_{kj}^{yx} x_j^x(t)\right)
$$
\n<sup>(5)</sup>

where  $N^x$  is the number of neurons in the layer x,  $w_{ki}^{yx}$  is the connection weight from the neuron j in the layer x to the neuron k in the layer y, and  $x_i^x(t)$  is the output of the neuron  $j$  in the layer  $x$  at the time  $t$ .

#### **Step 3: Propagation from Other Layers to Layer** <sup>x</sup>

The information in other layers is propagated to the layer  $x$ . The output of the neuron j in the Key Input Part of the layer x,  $x_i^x(t+1)$ , is given by

$$
x_j^x(t+1) = f\left(\frac{1}{M-1} \sum_{y \neq x}^{M} \left(\sum_{k=1}^{n^y} w_{jk}^{xy} x_k^y(t)\right) + vA_j^x\right)
$$
(6)

where M is the number of layers,  $n<sup>y</sup>$  is the number of neurons in the key input part of the layer y,  $w_{jk}^{xy}$  is the connection weight from the neuron k in the layer y to the neuron  $j$  in the layer  $x$ , and  $v$  is the connection weight from the external input.

 $A_j^x$  is the external input to the neuron j in the layer x and is given by

$$
A_j^x = \begin{cases} 0 & (t < t_{in})\\ \hat{x}_j^x(t_{in}) & (t_{in} \le t) \end{cases} \tag{7}
$$

$$
t_{in} = \min\left\{t \Big| \sum_{j=1}^{n^x} (\hat{x}_j^x(t) - \hat{x}_j^x(t-1)) = 0\right\}
$$
 (8)

$$
\hat{x}_j^x(t) = \begin{cases} 1 & (0 \le x_j^x(t)) \\ -1 & (x_j^x(t) < 0) \end{cases}
$$
\n(9)

where  $\hat{x}_j^x(t)$  is the quantized output of the neuron j in the layer x at the time t.

The output of the neuron j of the Context Part in the layer x,  $x_i^x(t+1)$  is given by

$$
x_j^x(t+1) = f\left(\frac{1}{M-1}\sum_{y\neq x}^M \left(\sum_{k=1}^{n^y} w_{jk}^{xy} \sum_{d=0}^t k_m^d x_k^d(t-d)\right) -\alpha(t) \sum_{d=0}^t k_r^d x_j^x(t-d)\right)
$$
(10)

where  $k_m$  and  $k_r$  are damping factors. And,  $\alpha(t)$  is the scaling factor of refractoriness at the time  $t$ , and it is given by

$$
\alpha(t) = a + b \sin\left(c \cdot \frac{\pi}{12} \cdot t\right) \tag{11}
$$

Steps 2 and 3 are repeated.

## **3 Computer Experiment Results**

Here, we show the computer experiment results in order to demonstrate of effectiveness of the proposed ICMAM. The experimental conditions is shown in Table [1.](#page-6-0) In the experiments, the  $N$  binary random pattern sets which have 1-to- $N$  relation were memorized, and the common pattern is given to the network.

#### **3.1 One-to-Many Association Ability**

Here, we compared the one-to-many association ability in the 3∼7-layered proposed ICMAM with the well-turned 3∼7-layered conventional Chaotic Multidirectional Associative Memory with variable scaling factor of refractoriness

(Adjusted Model) and conventional Chaotic Multidirectional Associative Memory with adaptive scaling factor [\[12](#page-7-7)] (Conventional Model).

Figure [2](#page-4-0) shows the one-to-many association ability of the proposed model, the adjusted model and the conventional model. As shown in this figure, the oneto-many association ability of the proposed model almost equals to that of the adjusted model. Moreover, the one-to-many association ability of the proposed model is superior to that of the adjusted model when the number of stored patterns are large.



<span id="page-4-0"></span>**Fig. 2.** One-to-many association ability.

#### $3.2$ **3.2 One-to-Many Association Ability in Various Size Networks**

Figure [3](#page-5-0) shows the one-to-many association ability of the various size proposed model. In this experiments, we used the network composed of 300 or 400 or 500 neurons in the Key Input Part and 100 neurons in the Context Part.

From these results, we confirmed that the proposed model in various size has good one-to-many association ability as similar as in the result shown in Fig. [2.](#page-4-0)

Figure [4](#page-6-1) shows the one-to-many association ability in the network which has 8 or 9 layers. In the conventional model, when the number of layers is large,



<span id="page-5-0"></span>**Fig. 3.** Relation between one-to-many association ability and the number of neurons in key input part.



<span id="page-6-1"></span>**Fig. 4.** One-to-many association ability in 8 or 9-layered network.

**Table 1.** Experimental conditions

<span id="page-6-0"></span>

The number of neurons in key input part		400
The number of neurons in context part		100
Damping factor	$k_m$	0.86
Damping factor	$k_r$	0.89
Coefficient in scaling factor	$\alpha$	0.9
Coefficient in scaling factor	h	0.47
Coefficient in scaling factor	$\epsilon$	2
Steepness parameter	$\epsilon$	0.013
Connection weight from external input	$\eta$	10

one-to-many association ability decreases. In contrast, as shown in Fig. [4,](#page-6-1) oneto-many association ability of the proposed ICMAM which has 8 or 9 layers is almost similar as that of the proposed ICMAM when the number of layers are small.

## **4 Conclusion**

In this paper, we have proposed the Improved Chaotic Multidirectional Associative Memory (ICMAM). The proposed model is based on the Chaotic Multidirectional Associative Memory (CMAM) [\[7\]](#page-7-8) which can realize one-to-many associations. In the proposed model, one-to-many association ability which does not depend on the number of layers is realized by dividing internal states of neurons by the number of layers.

We carried out a series of computer experiments and confirmed that the proposed model has following features.

- (1) One-to-many association ability of the proposed model is almost equal to that of the well-tuned Chaotic Multidirectional Associative Memory with variable scaling factor of the refractoriness.
- (2) The parameters can be determined appropriately in various size networks even when the number of layers is large.

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