Fuzzy Soft Set Decision Making Algorithms: Some Clarifications and Reinterpretations

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Abstract. We do two things in relation with fuzzy soft set decision making in this paper. Both in the score-based and fuzzy choice values approaches to decision making, the modifications that account for the model with positive and negative attributes are put forward and discussed for the most common fuzzy negation. We also provide a reinterpretation of the fuzzy choice values solution in terms of choice values associated with fuzzy opportunity costs.

Keywords: Soft set · Fuzzy soft set · Resultant fuzzy soft set · Comparison table · Decision making

1 Introduction

Since the introduction of fuzzy sets by Zadeh [1], a huge literature on their properties and applications to decision making has been produced. However in some practical problems, imprecise individual or group knowledge cannot be suitably represented by fuzzy sets (FSs): cf., Bustince et al. [2,3]. To name but a few models, Atanassov [4,5] proposes the concept of intuitionistic fuzzy set, which coincides with the notion of vague set (Bustince and Burillo [6]). Preference structures in group decision making problems under uncertainty appear in the form of the fuzzy preference relation (cf., Castro et al. [7], which provides an application to consensus-driven group recommender systems; see Alcantud et al. [8] for a different approach to consensus analysis, also Alcantud and de Andrés [9] for a fuzzy viewpoint).

Within the fuzzy framework there are also cases where the practitioner cannot proceed with a unique membership degree because she receives a set of possible input values (e.g., when several experts supply their own membership degrees). To model these situations, Torra [10] introduces hesitant fuzzy sets (HFSs) which incorporate many-valued sets of memberships (cf., Herrera et al. [11], Rodríguez et al. [12], and Xu [13]). Alcantud et al. [14] give real applications that validate the model by hesitant fuzzy sets. Alcantud and de Andrés [15] propose a segment-based approach to evaluate HFSs. Zhan and Zhu [16] give a summary of decision making methods based on (fuzzy) soft sets and rough soft sets.

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Molodtsov [17] initiates the theory of soft sets. His results are complemented e.g., by Aktaş and Çağman [18], Alcantud [19] (who proves formal relationships among soft sets, fuzzy sets, and their extensions) and Maji et al. [20].

Among the most successful extensions of the soft set model we can cite fuzzy soft sets (Maji et al. [21], also Alcantud et al. [22], Li et al. [23] and Tang [24] for applications to decision making for medical diagnosis) and incomplete soft sets (Alcantud and Santos-García [25], Han et al. [26], Zou and Xiao [27]).

Subsection 2.2 below reviews fundamental approaches to fuzzy soft set decision making. Then we analyze this problem with desirable and undesirable parameters, both in the cases of Roy and Maji [28] and Alcantud [29]. To normalize the information, the complement of the fuzzy set associated with each non-desirable attribute may be used, although the choice of the complement is not evident (cf., Klir and Yuan [30, Sect. 3.2]). In this paper we explore the decision making situation under Zadeh's fuzzy negation $c(x) = 1 - x$. Finally, we reinterpret the notion that provides the solution in Kong et al. [31].

This paper is organized as follows. Section 2 recalls some terminology and definitions. Section 3 contains our results. We conclude in Sect. 4.

2 Basic Definitions: Soft Sets, Fuzzy Soft Sets

We adopt the usual description and terminology for soft sets and fuzzy soft sets. U denotes a universe of objects and E denotes a universal set of parameters.

2.1 Soft Sets and Fuzzy Soft Sets

Definition 1 (Molodtsov [17]). A pair (F, A) is a *soft set* over U when $A \subseteq E$ and $F : A \longrightarrow \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U.

A soft set over U is interpreted as a parameterized family of subsets of U, and A represents a set of parameters. Then for any $e \in A$, we say that $F(e)$ is the subset of U approximated by the parameter e or the set of e-approximate elements of the soft set. To put an example, if $U = \{f_1, f_2, f_3, f_4\}$ is a universe of films and A contains the parameter e that describes "3D image" and the parameter e' that describes "suitable for children aged under 7" then $F(e) = \{f_2\}$ means that the only 3D film is f_2 and $F(e') = \{f_1, f_3\}$ means that the only suitable for children aged under 7 films are f_1 and f_3. For soft set based decision making, the reader may consult Maji et al. [32], Çağman and Enginoğlu's [33] and Feng and Zhou [34].

The following notion is a natural extension of the concept of soft set:

Definition 2 (Maji et al. [21]). A pair (F, A) is a *fuzzy soft set* over U when $A \subseteq E$ and $F : A \longrightarrow \mathbf{FS}(U)$, where $\mathbf{FS}(U)$ denotes the set of all fuzzy sets on U.

Any soft set can be considered as a fuzzy soft set with the natural identification of subsets of U with FSs of U. Following with our film example above, fuzzy soft sets permit to deal with other properties like "funny" or "scary" for which partial memberships are almost compulsory.

Henceforth we assume that there are k options and n properties. In that case a soft set or fuzzy soft set can be represented both by a matrix $T = (t_{ij})_{k\times n}$ and in tabular form. Rows correspond to the k objects in U, and columns correspond to the n parameters in A (see Examples 1 and 2 below). In the case of a soft set, all cells are either 0 or 1 (this is to say, its representation is binary).

2.2 Fuzzy Soft Sets and Decision Making

The most distinctive approaches to fuzzy soft set based decision making are probably Roy and Maji [28], Kong et al. [31], Feng et al. [35] and Alcantud [29].

Roy and Maji [28] pioneered this research. Alcantud [29] is closely related to their successful proposal. This article develops and discusses a novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set. It improves the performance of Roy and Maji's algorithm at the two stages of their proposal. The following comparison between both approaches helps to introduce their structure:

Stage 1. Roy and Maji [28] propose to begin with an aggregation procedure that yields a single resultant fuzzy soft set from preliminary multi-source information. Alcantud [29] shows that their approach very often results into a large loss of information and henceforth generates uncertainty. Accordingly, [29] develops an alternative proposal that avoids such situation to a great extent.

Stage 2. In order to evaluate the alternatives from the information in the resultant fuzzy soft set, Roy and Maji [28] propose to construct a Comparison matrix that permits to compute scores for the alternatives. In [29] it is argued that a new *relative* Comparison matrix improves the performance of the algorithm of solution, because it contributes both to ensure a higher power of discrimination and to produce a well-determined solution.

As a result of these innovations the procedure in Alcantud [29] is considerably less inconclusive than existing solutions which produce ties on a regular basis (as shown in [29] both by arguments and many examples from the literature).

Remark 1. Kong et al. [31] propose a different procedure at Stage 2 without examining Stage 1. Feng et al. [35] explain that the difference between [31] and [28] is whether the criterion for making a decision should use scores (see definition in Sect. 3.1) or fuzzy choice values (that is, the sum of all membership values across attributes) attached with each option. We concur with their argument that the redesigned approach by scores in Roy and Maji [28], and afterwards in Alcantud [29], is more suitable for making decisions in an imprecise environment than fuzzy choice values. We return to the discussion about [31] in Sect. 3.2.

Concerning their own contribution, Feng et al. [35] introduce an adjustable method based on level soft sets at Stage 2. In their flexible decision mechanism the optimal choice is dependent upon the selected level soft sets.

3 Results

3.1 Comments on Score-Based Solutions

Roy and Maji [28] and Alcantud [29] share the spirit that scores are a better tool than fuzzy choice values in order to evaluate options characterized by fuzzy soft sets, and also that a resultant fuzzy soft set can be produced from more primitive data at an earlier stage.

It is worth insisting that in both cases *it is implicitly assumed that the attributes that are being examined are desirable or not negative.* The reason is that in the comparisons that produce the scores in both models, it is always better to have higher amounts. In order to fully grasp the importance of this overtone, henceforth we analyze the following situation:

Example 1. Let $U' = \{o_1, o_2\}$ be a universe of two cars, and $A = \{e_1, e_2, e_3, e_4, e_5\}$. The tabular representation of the fuzzy soft set that describes the options in terms of the parameters is given by Table 1.

Alcantud [29, Example 7] shows that both the algorithms proposed in [29] and [28] suggest that option o_2 should be selected.

However *if the parameters are some positive and some negative, this conclusion could be challenged even when the choice procedure has been fixed.*

Table 1. Tabular representation of the fuzzy soft set in Example 1.

	e_1	e_2	e_3	e_4	e_5
o_1	0.9	0.1	0.2	0.1	0.3
o_2	0.19	0.3	0.4	0.3	0.4

We proceed to discuss the latter statement with an attention to the two fundamental score-based approaches [29] and [28] (v., Sect. 3.2 below for the fuzzy choice values approach). Henceforth we suppose that in Example 1, the parameters $\{e_4, e_5\}$ are 'negative', e.g., describe attributes like "being expensive" or "pollutes above legal limits". At the same time, parameters $\{e_1, e_2, e_3\}$ are 'desirable', e.g., describe attributes like "security appliances", "efficient fuel consumption" or "fashionably designed". For simplicity this general case is labelled *mixed properties* henceforth.

In order to normalize the information in a fuzzy soft set representation, it seems only natural that *the complement of the fuzzy set associated with each non-desirable attribute should be used.* However in operational terms this is not as direct as apparent, since there is not a unique fuzzy complement or negation (cf., Klir and Yuan [30, Sect. 3.2]). In this paper we explore the case when *we fix $c(x) = 1 - x$ as the fuzzy complement.* Then one needs to subtract from 1 the membership values when the attributes are undesirable. In this fashion we produce the *normalized* matrix or tabular form of the fuzzy soft set, to which the

Table 2. Normalized tabular representation of the fuzzy soft set in Example 1. The fuzzy complement $c(x) = 1 - x$ is applied.

	e_1	e_2	e_3	e'_4	e'_5
o_1	0.9	0.1	0.2	0.9	0.7
o_2	0.19	0.3	0.4	0.7	0.6

standard versions of the algorithms can be applied. Hence in Example 1, Table 2 should replace Table 1 before implementing any solution.

Let us now discuss what adjustments the aforementioned consideration introduces in the Algorithms by [29] and [28] under mixed properties, when they are applied to the original sources of information (in our example, to Table 1).

Comments on Roy and Maji's Score-Based Solution. After normalizing the fuzzy soft set representation of Example 1 (cf., Table 2) with our selected fuzzy complement $c(x)$, Roy and Maji's standard algorithm computes a Comparison table (cf., Table 3) in a way that echoes the classical aggregation procedure due to Marquis de Condorcet.We recall that to produce cell $1, 2$ in Table 3 we count for how many characteristics option o_1 performs at least as well as o_2 (especifically, $\{e_1, e_4, e_5\}$),and in its cell $2, 1$ we count for how many characteristics option o_2 performs at least as well as o_1 (especifically, $\{e_2, e_3\}$). Also, R_i is row i's sum, and T_i is column i's sum, for each i. We are lead to conclude that o_1 is a better option because the scores associated with Table 2 are $S_1 = R_1 - T_1 = 3 - 2 = 1$ and $S_2 = R_2 - T_2 = 2 - 3 = -1$.

Table 3. Comparison table for the application of [28] in Example 1, when attributes e_4, e_5 are undesirable while the others are desirable.

	o_1	o_2	R_i
o_1	0	3	3
o_2	2	0	2
T_i	2	3	

In order to apply a Roy and Maji's inspired algorithm to the original Table 1 directly, the only caution we must make is that the Comparison table that captures their idea should be Table 3. Hence when we count for how many properties an option performs at least as well as another one and there are mixed properties as in Table 1, we must distinguish the case where the property is undesirable (and in all such cases inequalities are reversed: the smaller a membership value the better).

This is a universal feature of our framework which ensures that the conclusion by our alternative Roy-and-Maji's-type solution coincides with the solution through complements implemented above.

Comments on Alcantud's Score-Based Solution. We have identified what simple modification in Roy and Maji's [28] algorithm is needed under mixed properties in order to comply with their spirit and at the same time, coincide with the simple solution that applies [28] to normalized information. Here we replicate the analysis with solution [29]. The following algorithm is needed:

Algorithm 1 - Alcantud [29] modified for mixed properties.

Input: a general fuzzy soft set represented by a matrix $T = (t_{ij})_{k\times n}$.

Without loss of generality we reorder the n properties so that the first q properties are desirable whereas the remaining $n-q$ are not.

1. For $j = 1, \ldots, n$, let M_j be the maximum membership value of any object, i.e., $M_j = \max_{i=1,\ldots,k} t_{ij}$. For $j = q+1, \ldots, n$, let m_j be the minimum membership value of any object, i.e., $m_j = \min_{i=1,\ldots,k} t_{ij}$.
   Now construct a $k \times k$ Comparison matrix $A' = (a'_{ij})_{k\times k}$ where for each i, j, we let a'_{ij} be the sum of the non-negative values in the finite sequence
   $$\frac{t_{i1} - t_{j1}}{M_1}, \frac{t_{i2} - t_{j2}}{M_2}, \ldots, \frac{t_{iq} - t_{jq}}{M_q}, \frac{t_{jq+1} - t_{iq+1}}{1 - m_{q+1}}, \ldots, \frac{t_{jn+1} - t_{in+1}}{1 - m_{n+1}}.$$
2. Continue exactly as in [29].

Observe that in addition to changing the sign of the comparison between membership values for the non-positive properties as in the adapted Roy and Maji's algorithm, the denominator at the quotient has been changed: the relative Comparison matrix $A = (a_{ij})_{k\times k}$ in [29] is computed by summing up the non-negative values in the sequence

$$\frac{t_{i1} - t_{j1}}{M_1}, \frac{t_{i2} - t_{j2}}{M_2}, \ldots\ldots, \frac{t_{in} - t_{jn}}{M_n}$$

in order to obtain cell a_{ij}. Here is the reason for our modified algorithm:

Proposition 1. *The ranking solution in Algorithm* 1 *coincides with the application of the original algorithm in* [29] *to the normalized tabular or matrix representation of the fuzzy soft set through the fuzzy complement* $c(x) = 1 - x$.

Proof. Let $T = (t_{ij})_{k\times n}$ be the $k \times n$ matrix representation of a fuzzy soft set for which the first q properties are desirable whereas the remaining $n-q$ are not. Therefore the normalized matrix representation of the fuzzy soft set is

$$T' = \begin{pmatrix} t_{11} & \ldots & t_{1q} & 1 - t_{1q+1} & \ldots & 1 - t_{1n} \\ \vdots & & \vdots & \vdots & & \vdots \\ t_{k1} & \ldots & t_{kq} & 1 - t_{kq+1} & \ldots & 1 - t_{kn} \end{pmatrix}$$

We proceed to check that in both the procedures explained in the statement, the relative Comparison matrix is the same. In the second procedure, for each $i, j \in \{1, \ldots, k\}$, a'_{ij} is the sum of the non-negative values in

$$\frac{t_{i1} - t_{j1}}{M_1}, \ldots, \frac{t_{iq} - t_{jq}}{M_q}, \frac{t_{jq+1} - t_{iq+1}}{1 - m_{q+1}}, \ldots, \frac{t_{jn+1} - t_{in+1}}{1 - m_{n+1}}$$

This sequence coincides with

$$\frac{t_{i1} - t_{j1}}{M_1}, \ldots, \frac{t_{iq} - t_{jq}}{M_q}, \frac{(1 - t_{iq+1}) - (1 - t_{jq+1})}{M'_{q+1}}, \ldots, \frac{(1 - t_{in+1}) - (1 - t_{jn+1})}{M'_{n+1}}$$

where $M'_j = \max_{i=1,\ldots,k} t'_{ij} = \max_{i=1,\ldots,k}(1 - t_{ij}) = 1 - \min_{i=1,\ldots,k} t_{ij}$ for each $j = q+1, \ldots, n$. Hence $a'_{ij} = a_{ij}$ where $A = (a_{ij})_{k \times k}$ is the relative Comparison matrix that arised from the application of [29] to T'. Now both procedures continue in exactly the same fashion. ■

3.2 A Comment on Kong et al.'s Solution

In this section we provide a different interpretation of the proposal in Kong et al. [31] (which has been criticized e.g., in Feng et al. [35] or Alcantud [29]). It relies on the economic concept of "opportunity cost".

In the context of decision under uncertainty, Savage early introduced the "minimax regret" criterion that produces an associated loss table. In this table every original value is subtracted to the maximum value that any object achieves under the property that it is linked to. Operationally: for each column, we first compute the maximum value in the original decision table and then every value at that column is replaced by such maximum minus it.

By the recourse to this new opportunity cost table we are measuring for each option and attribute, how much we are losing by not choosing the option with highest membership value for the attribute. Let us give an example:

Example 2. Kong et al. [31] used the fuzzy soft set (S, P) represented by Table 4 as an example that shows the disparity of conclusions when choice values (i.e., row sums) are used instead of scores (cf., Roy and Maji [28]).

As they explain, in such example the algorithm in Roy and Maji [28] shows that option o_3 should be selected because when their scores s_i are computed, one obtains $s_3 > s_2 > s_5 > s_1 > s_6 > s_4$. However a fuzzy-choice-value-based decision produces a different ranking and optimal selection, because $c_6 > c_2 > c_3 > c_1 = c_4 = c_5$ and therefore o_6 is the suggested alternative.

If we compute the opportunity cost table associated with the fuzzy soft set (S, P) we obtain the data in Table 5. Since opportunity costs are negative, the comparisons $Op_6 < Op_2 < Op_3 < Op_1 = Op_4 = Op_5$ permit us to observe that the ranking of alternatives is identical to Kong et al.'s conclusion, i.e., $o_6 \succ o_2 \succ o_3 \succ o_1 \sim o_4 \sim o_5$. In Proposition 2 below we prove that the coincidence observed in Example 2 holds in general.

Table 4. Tabular representation of the fuzzy soft set (S, P) in Kong et al. [31, Table 1]. The input at i, j is t_{ij}, c_i is the sum of the amounts in row i (fuzzy choice value).

	p_1	p_2	p_3	p_4	p_5	Fuzzy choice value
o_1	0.1	0.5	0.3	0.4	0.3	$c_1 = 1.6$
o_2	0.3	0.5	0.2	0.3	0.6	$c_2 = 1.9$
o_3	0.1	0.7	0.4	0.5	0.1	$c_3 = 1.8$
o_4	0.7	0.2	0.2	0.2	0.3	$c_4 = 1.6$
o_5	0.2	0.6	0.3	0.2	0.3	$c_5 = 1.6$
o_6	0.9	0.2	0.1	0.1	0.8	$c_6 = 2.1$

Table 5. Opportunity cost table associated with Table 4: at each cell i, j we introduce $M_j - t_{ij}$. Op_i is the sum of the amounts in row i (opportunity cost values).

	p_1	p_2	p_3	p_4	p_5	Opportunity cost value
o_1	0.8	0.2	0.1	0.1	0.5	$Op_1 = 1.7$
o_2	0.6	0.2	0.2	0.2	0.2	$Op_2 = 1.4$
o_3	0.8	0	0	0	0.7	$Op_3 = 1.5$
o_4	0.2	0.5	0.2	0.3	0.5	$Op_4 = 1.7$
o_5	0.7	0.1	0.1	0.3	0.5	$Op_5 = 1.7$
o_6	0	0.5	0.3	0.4	0	$Op_6 = 1.2$
M_i	0.9	0.7	0.4	0.5	0.8	

Proposition 2. *The ranking solution in Kong et al.* [31] *does not change if we use opportunity cost tables instead of fuzzy soft set representations.*

Proof. Let $T = (t_{ij})_{k \times n}$ be the $k \times n$ matrix representation of a fuzzy soft set (F, A) over U. For each column i we define $M_i = \max\{t_{1i}, \ldots, t_{ki}\}$. Then the opportunity cost table associated with (F, A) is $T_O = (M_j - t_{ij})_{k \times n}$. Kong et al.'s fuzzy choice values are $c_i = t_{i1} + t_{i2} + \ldots + t_{in}$ for each $i = 1, \ldots, k$.

The fuzzy choice value associated with option i and the opportunity cost table is $Op(o_i) = M_1 - t_{i1} + M_2 - t_{i2} + \ldots + M_n - t_{in}$ hence if we let $M = M_1 + \ldots + M_n$ then $Op(o_i) = M - (t_{i1} + t_{i2} + \ldots + t_{in}) = M - c_i$. This justifies that a ranking of the alternatives by non-increasing fuzzy choice values coincides with a ranking of the alternatives by non-decreasing opportunity cost fuzzy values. ∎

Remark 2. Kong et al. [31] implicitly assume that the attributes are positive, because the fuzzy choice value sums up all amounts. Therefore the practitioner should exercise the same cautions as in Sect. 3.1 before applying their criterion.

4 Concluding Remarks

The attributes in a fuzzy soft set decision making analysis must be carefully examined to check if they are all positive or not. When there are both desirable and undesirable attributes, fuzzy complements or negations should be applied to one of the cases (typically, negative attributes). We discuss which modifications to the standard version of the decision algorithms in Roy and Maji [28] and Alcantud [29] permit to implement that feature when $c(x) = 1 - x$ is the fuzzy complement. A possibility for future research is the investigation when other fuzzy complements like the Sugeno class $s_\lambda(x) = \frac{1-x}{1+\lambda x}$, $\lambda \in (-1, +\infty)$, or the Yager class $c_\omega(x) = (1 - x^\omega)^{\frac{1}{\omega}}$, $\omega \in (0, +\infty)$, are preferred.

We have also provided a reinterpretation of the controversial solution by Kong et al. [31] stated in terms of choice values associated with fuzzy opportunity costs.

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