Chapter 9
A Spatial-Semiotic Framework in the Context of Information and Communication Technologies (ICTs)

Melih Turgut

9.1 Introduction

Let me begin this chapter with a personal anecdote. When I was in primary school, I was always interested in the notion of *space*. I tried to imagine it through my own experiences from textbooks, models, classroom discussions and television; a dark, gas-free environment, with shining stars, a visible earth, other planets, and also positions and movements of the planets and the sun. Now, looking back at my imaginative efforts, I am able to see my struggle created an *amalgam* of existing *mental images* (i.e., *mental pictures*) taken as input from previous processes and from nested figures to construct *dynamic images* of space. How did I create such mental and dynamic images, and what kind of processes was I involved in? It is clear that I was navigating in the solar system myself, and manipulating the objects in my mind and imagining them from different viewpoints, but, at the same time, I was unconsciously moving my body, in particular my head and my hands. In other words, I was using a specific ability to construct new mental and dynamic images, my *spatial ability*, but interestingly, through my body's specific movements.

Referring to my own thinking processes described above, it was only one experience that remember; what about for our entire life? As individuals living in three-dimensional space, we are always using our spatial ability in daily life. For instance, not only limited to learning 2D or 3D geometry, but also to finding locations through our mental map, while placing self-assembly furniture around the house or while playing computer games. The last example is important in emphasizing spatial ability's capacity to enable a user to take mental images as input and to produce dynamic images quickly. You can think of such a game player as an individual looking at a flat monitor, which is providing the user an immediate (sequence of) *transformation*

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that can be assumed as specific transformations from space $\mathbf{R}^2$ to $\mathbf{R}^3$ by showing the user 3D objects throughout. However, I would like to differentiate the playing a game process from watching a movie, and yet their construction source is dependent on the user's movements or acts. I identify the construction of dynamic images through ICT into two areas: *dependent* or *independent* from the user. Playing a computer game, including the production of dynamic images is dependent on the user acts. However, at the same time, the user's body movements may also appear in the process. What is the process do users follow while they pursue a similar dependent process whilst thinking spatially? What would happen if the subjects use 3D modeling software? What kinds of *signs* emerge in this process? Combining these with the earlier questions, in this chapter, I will try to elaborate a framework describing subjects' spatial thinking (i.e., visualization) processes in a multimodal perspective, while they are using 3D modeling software in the exploration of spatial ability tasks. I will try to analyze and discuss such a framework in terms of two case studies, mirroring a multimodal paradigm, in particular, with a *semiotic lens* of different source of *signs*, based not only on discourse, but also on extra-linguistic acts, such as *gestures*, *sketches*, *graphs* and so on. I will consider the Action, Production and Communication space framework (Arzarello 2008; Arzarello et al. 2009; Arzarello and Sabena 2014) to address a piece of recent extensive research data, where partial results are presented through the *instrumental approach* (Turgut and Uygan 2015).

This chapter is distinguished into the introduction and five main sections. In the second section, I try to review, summarize and describe certain concepts and processes; mental and dynamic images, visualization, and spatial thinking, specifically in the context of 3D modeling software. In the third section, I provide the reader with the notion of semiotics in the sense of C.S. Peirce and its relation to mathematics education, with an embodied cognition perspective. In the fourth section, I try to elaborate a spatial-semiotic framework for spatial thinking through the theoretical underpinnings from previous sections. In the fifth section, I approach two case studies to discuss the proposed framework using a multimodal paradigm, completing the chapter with a conclusion. Finally, the sixth section covers a discussion of the results and limitations of the study.

9.2 Mental and Dynamic Images, Process of Visualization and Spatial Thinking

Generally speaking, spatial ability can be defined as a combination of different (sometimes, *nested*) sub-skills creating various visual-spatial images and generating information, such as, visualizing 2D or 3D stimuli, rotating them mentally, and visualizing them from different viewpoints (Hegarty and Waller 2004; Linn and Petersen 1985; McGee 1979; Olkun 2003). Although there are several definitions of sub-skills in the related literature (Linn and Petersen 1985; Lohman 1996; Maier

1998), as a general reference, spatial ability can be categorized into two areas (McGee 1979); *spatial visualization* that refers to several sub-skills, such as projection skills from 2D to 3D (or vice versa), imagining and manipulating the objects mentally, which includes rotating objects mentally, and *spatial orientation* that refers to imagining objects from different viewpoints.

Certain questions relating to the core aim in this chapter arise. How do those specific images, belonging to spatial visualization or spatial orientation, occur in our minds and why learning about the spatial thinking process is really important? As Dreyfus (1995) emphasizes, a researcher studying the learning of mathematics within a psychological perspective has to be aware of the *internal representations* of learners, as well as associated *external representations*, to possibly understand the learners' interpretations of figures, diagrams and suchlike. This is because we are interested in how internal representations are created, and how they affect an individual's creation of *visual-spatial images*, which also affect his/her mathematical reasoning while solving context-specific spatial tasks. However, inductively, the creation of visual-spatial images may be in relation to an individual's phenomenological experiences, i.e., their interaction with associated *objects* and their *interpretants* as 'internal spatial abilities', mediated by *signs*, which may be discussed by the seminal *semiotic triad* of C.S. Peirce (Buchler 2015; Yeh and Nason 2004, p. 4). I will return to this semiotic perspective in later sections to interpret students' spatial thinking processes, in particular, while they are using 3D modeling software.

Before outlining spatial framework of this chapter, I will look at certain specific terms that researchers use while elaborating the aforementioned internal representations in the spatial reasoning process (in the context of learning geometry). These include *mental images*, *mental pictures*, *spatial images*, *visual images*, *visual imagery*, and *dynamic images* or *dynamic imagery*. In addition, there are also references to the spatial reasoning process; *visualization*, *visual thinking*, and *spatial thinking*. I agree with Gutiérrez (1996), who notes that researchers express (i.e., through visualization or spatial thinking) such terms interchangeably, but at the time synonymously.

Hauptman (2010, p. 124) proposes that a *spatial image* 'is the end product of a mental process that uses various aspects of a concrete object (or objects) to create a picture of that object in our mind'. However, this definition apparently has a general viewpoint and includes several kinds of processes. A complete classification for such images is provided by Presmeg (1986, 2006) from the point of view of learning mathematics (as a general reference, not only belonging to the learning of geometry) in terms of qualitative analyses that she conducted. She defines *visual image*s as 'mental constructs depicting visual or spatial information' (Presmeg 2006, p. 207), and describes how, in the *visualization* process (i.e., construction and/or manipulation of visual images), an individual can use and/or need one or more of the following kinds of imageries (Presmeg 1986, pp. 43–44):

- *concrete*, *pictorial imagery*, which is related to real-life contexts, already created 'pictures in the mind'.

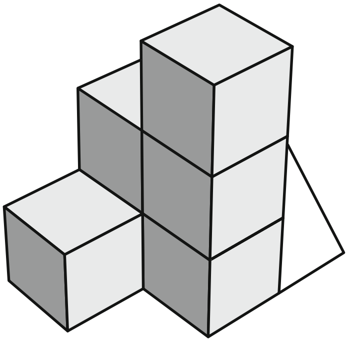
- *pattern imagery*, which refers to visual mathematics-ready patterns in the mind, for instance, 30-60-90 triangle and associated lengths.
- *memory images of formulae*, remembering and/or seeing formulae in the mind.
- *kinaesthetic imagery*, muscular and physical acts and movements attached to the process of describing figures, objects or mathematics.
- *dynamic imagery*, which refers to the creation and transformation of dynamic mental images in the mind in the process of visualization.

After a careful analysis and elaboration of definitions and theoretical constructs on spatial ability, mental-visual-spatial images in psychology and mathematics education literature, Gutiérrez (1996) expresses that, ‘a mental image is any kind of cognitive representation of a mathematical concept or property by means of visual or spatial elements’. He also considers *visualization* as a general reference to spatial reasoning or spatial thinking, saying that it is, ‘a kind of reasoning activity based on the use of visual or spatial elements, mental or physical, performed to solve problems’ (p. 9). Referring to the same term, Bishop (1983) proposes two kinds of spatial-mathematical *abilities* in an individual’s process of *visualization*; the ability to *interpret figural information* (IFI) and the ability to *visually process* (VP). He summarizes as follows:

> … IFI involves knowledge of the visual conventions and spatial ‘vocabulary’ used in geometric work, graphs, charts, and diagrams of all types. Mathematics abounds with such forms and IFI includes the ‘reading’ and interpretation of these. …VP on the other hand, involves the ideas of visualization, the translation of abstract relationships and non-figural data into visual terms, the manipulation and extrapolation of visual imagery, and the transformation of one visual image into another. (Bishop 1983, p. 177)

Since, VP refers to the creation and manipulation of mental and dynamic images, and IFI refers not only to converting, using and interpreting such mental and dynamic images to solve a certain mathematical task, but also to using spatial vocabulary in pursuit of the task. Because these abilities provide information about *actions* to be performed, not on skills or sub-skills that one requires for the completion of such an action, as Gutiérrez (1996) discusses, abilities IFI and VP can be described as a ‘category of *processes* to be performed’ (p. 7). For example, see the following open-ended task (Fig. 9.1).

In this task, a possible way of thinking for subjects to solve the task can be summarized. First, a student uses ready-made geometric figures, cubes and triangle(s), and, thereafter, she/he has to imagine the rear of a building to answer how many cubes could be placed there, for which there are several possibilities. Secondly, the subject can follow reasoning to consider the total area of the building. Consequently, the first process refers to VP and the second refers to IFI. Here, it should be noted that certain specific visual images of Presmeg (1986), in particular, cubes and triangles as *concrete images*, a subject’s possible gestures or physical movements of hands and so on, as *kinaesthetic images*, and imagining the rear of the building as a *dynamic image*, could be used and/or created in the VP of the subject. I will consider such visualization-spatial thinking processes to construct a viewpoint looking into subjects’ use of 3D modeling software while they are using it as an *artefact*.



Task: *How many unit squares can be the total area of this building*?

Fig. 9.1 A sample spatial task

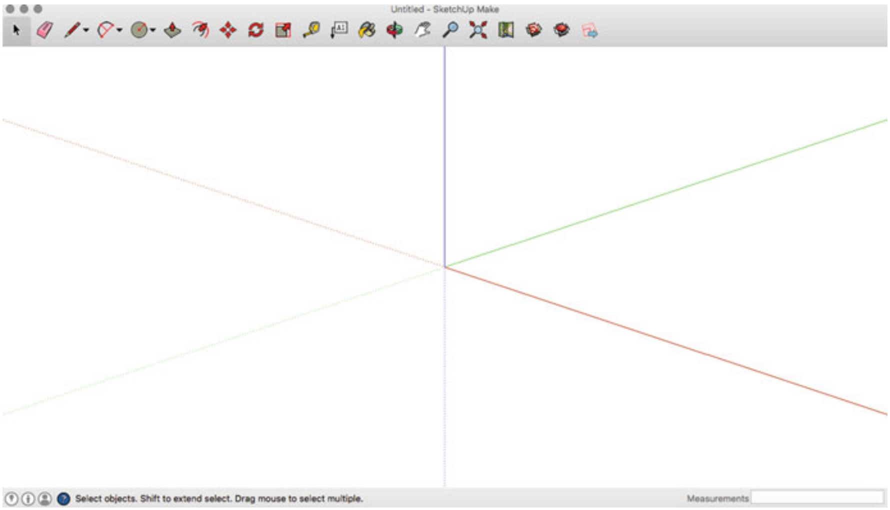


Fig. 9.2 SketchUp® Make interface

9.2.1 Individuals' Use of SketchUp® Whilst Exploring Spatial Tasks

SketchUp® is 3D modeling software developed for graphic design, the design of architectural environments and engineering, and also for 3D graphics education. The freeware version is SketchUp® Make, which includes several tools and functions to create various kinds of figures, shapes or models, both in 2D and 3D in its interface (Fig. 9.2).

A full discussion of the functions and tools of the software, with respect to spatial thinking and learning geometry, would be outside the theme of this chapter, but

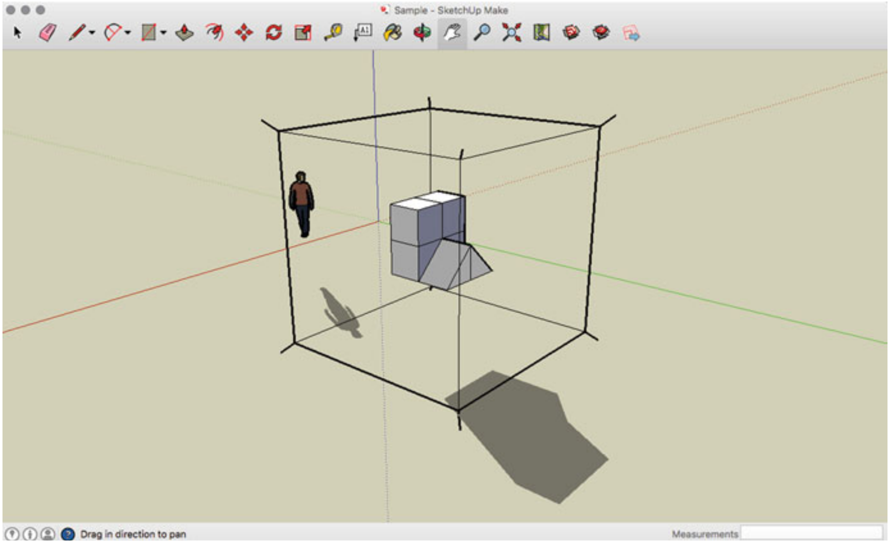


Fig. 9.3 Figures and shadows

I can point out some papers to readers (Fleron 2009; La Ferla et al. 2009; Turgut and Uygan 2013). If one provides users an environment, the tools and functions of the software provide users with an immediate interrelated *imagination* and *visualization* process (Turgut and Uygan 2014). For example, in Fig. 9.3, the user has several functions and tools to analyze where the light is coming from. It is obvious that, in order to explore the proposed task, the user will also join the VP and IFI processes that I postulate and describe above.

Experimental studies indicate the positive effects of educational designs through use of this software on students' spatial skills (Erkoc et al. 2013; Kurtulus and Uygan 2010). I am not interested in such potential in this chapter, but am interested in discussion of a semiotic perspective on how students (as well as the teacher) are using this software. For instance, what kinds of semiotic resources attached to students' use of tools and functions appear while they are thinking spatially? This question brings us to another point; the framing and investigation of a phenomenon relating to signs, spatial thinking and mathematics education.

9.3 Semiotics, Spatial Thinking and Mathematics Education

Semiotics, in the sense of C.S. Peirce, can be labeled as a discipline of science, dealing with *signs*, which mediates a dialectic relationship between an *object* and its *interpretant*. According to Peirce, an interacting triangle of signs or *representamens*, object and interpretant, forms *semiosis*, which is defined by him as:

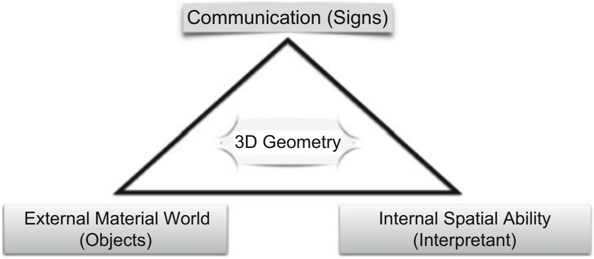


Fig. 9.4 Semiotic triad of 3D geometry (Adopted from Yeh and Nason 2004, p. 4)

> … a sign, or *representamen*, which is something which means something to somebody in some respect or capacity. It addresses somebody, that is, it creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. (Buchler 2015, p. 99)

For example, a (red) traffic light can be considered as the *sign* or *representamen*, stopped vehicles as the *object*, and the idea that when people see the red light they must stop as the *interpretant* (Chandler 2014). From the cognitive perspective, Peirce notes that, for a meaningful comprehension, *such semiosis process must occur*. Therefore, from a didactical point of view, semiosis gives clues about how meaning-making occurs, which is of great importance, for us as mathematics educators, for an understanding of how the learning of mathematics occurs.

Peirce classifies the signs into three; *icon*, *index*, and *symbol*. An icon(ic) sign refers to physically resembling or imitating the object through a list of equations, graphs, portraits, maps, metaphors, or *gestures* such as imitating hand movements and so on. An index refers to signs that are directly related to the object, but without any similarity or analogy to the object, by which a person (of course *an animal*) can infer an immediate link between the sign and object in terms of sensorial functions. For example, a smiling face may be considered to be an index sign referring to happiness. Symbols refer to signs driven by conventional or virtual rules, such as language, words, numbers, traffic lights and so on.

In context of learning 3D geometry, Yeh and Nason (2004) propose a semiotic framework considering Peircean semiotics, to design a virtual teaching-learning environment, consisting of three main components; *external material world*, *internal spatial ability*, and *communication* (Fig. 9.4).

The external material world refers to all geometric objects, such as patterns and their relationships to shapes, triangles, cubes, a shell or a growing tree, and to their properties. Internal spatial ability refers to the mental process of human potential to know, perceive and manipulate external geometric objects. The communication component refers to signs at large, which includes not only spoken and written language, but also includes 'mathematical notation, pictures, diagrams, kinaesthetic body movements, and even geometric objects themselves' (ibid., p. 5). It is apparent that this framework considers and also exploits the semiosis process in the design of a virtual environment, to construct mathematical meanings through students' exploration of the proposed context in different semiotic representations. However, this framework does not directly focus on the emergence of signs in the spatial thinking process.

A focus on *signs* in spatial thinking (or also in learning geometry) would provide details from both a cognitive and didactic point of view, and also glimpse how students *gesture*, as an *icon sign*, while thinking spatially. There is extensive literature on the elaboration of the relationship between gestures and spatial thinking. I can refer to interesting papers on the subject (Alibali 2005; Atit et al. 2013; Chu and Kita 2008, 2011; Ehrlich et al. 2006; Logan et al. 2014; Ng and Sinclair 2013). In her excellent review, Alibali (2005) concludes that individuals produce gestures more often when talking about spatial topics than when talking about verbal or abstract topics. It can, therefore, be concluded that gestures often accompany spatial language and have a functional role in the process of spatial thinking. As she notes, McNeill (1992) hypothesizes that '… gestures reflect mental images …' and according to this approach '… spatial thinking is integral to gesture production …' (ibid., p. 308). Experimental research studies seem to confirm this notion. For example, Chu and Kita (2011) found that individuals spontaneously produce gestures when they have difficulty with mental rotation tasks, to help themselves solve the task. Chu and Kita also conclude that gestures contribute to performance in spatial visualization tasks, since they improve 'internal computation' of dynamic images while thinking spatially. Recently, the use of gestures by preschool children to communicate mathematically, and to internalize their thinking while they are solving spatial transformation tasks, was also observed (Ng and Sinclair 2013).

Finally, as a general conclusion in the related literature, researchers agree that gestures provide useful information about individuals' ways of spatial thinking as an *internal reference*. However, what about other signs that appear in a teaching-learning mathematics process, such as words, sketches and so on, or signs produced by the teacher? What is the meaning of a student's pre-drawing in solving a spatial task? To my knowledge, there is no framework which looks at the spatial thinking picture in a holistic way of produced signs in the classroom. As mentioned earlier, the framework of Yeh and Nason (2004) does not look at all the signs that are expected to emerge in the spatial thinking process. However, there is a general perspective from the didactics of mathematics (not specifically to spatial thinking) focusing on different kinds of semiotic resources produced in the classroom. I will focus on Arzarello's notion of Action, Production and Communication space (APC-space), which is an extension of other semiotic approaches (e.g., Duval 2006; Ernest 2005); and Arzarello and his colleagues' notion of *semiotic bundle* to cover all the signs that appear in the classroom.

9.3.1 Embodied Cognition, Signs and the Notion of APC-Space

Embodied cognition perspectives hypothesize 'that cognitive processes are rooted in interactions of the *human body* and the *physical world*' (Alibali et al. 2014, p. 150). Our body's interaction with the physical world, while we are trying to express meanings (we have) is in a *multimodal* way, not only limited to gestures, but also includes words, sounds, mimics, sketches and so on, as well as our sensory-motor functions' products (Arzarello and Robutti 2008). Arzarello and Sabena (2014) note that in order to interpret the process of the interaction of human body and ICTs, such multimodal perspective receives increasing relevance in the elaboration of both thinking and communication.

Following a multimodal approach within the embodied cognition, Arzarello (2008) considers a socio-cultural dimension of the teaching-learning process in the sense of Vygotsky and his followers, to frame a new viewpoint looking at processes developed in the classroom and shared by both teacher and students; the *APC—space*. This APC—space has three main components; (i) *the body*, (ii) *the physical world*, and (iii) *the cultural environment*. Inductively, an intersection of such elements in a mathematics classroom will yield a variety of signs, including students' gestures, communications, teacher's discourse, teacher's gestures (also gestures with chalk), graphs (in our case the *use of artefacts*) and suchlike. In order to frame such a variety of signs, Arzarello (2006) introduces the notion of the semiotic bundle, which is re-elaborated by Arzarello et al. (2009):

> A semiotic bundle is a system of signs (with Peirce's comprehensive notion of sign) that is produced by one or more interacting subjects and that evolves over time. Typically, a semiotic bundle is made up of signs that are produced by a student or by a group of students while solving a problem and/or while discussing a mathematical question. Possibly, the teacher also participates in this production, and so the semiotic bundle may also include signs produced by teacher. (Arzarello et al. 2009, p. 100)

The notions of APC-space and the semiotic bundle are powerful frames to describe semiotic activities in the classroom. However, as these paradigms hypothesize, there would appear to be several kinds of semiotic resources in this process. How can we look at such resources to understand mathematical phenomena as a cognitive process? Two different, but complementary analysis units can be employed to the data; (i) a *synchronic analysis*, and (ii) a *diachronic analysis*. The first focuses on 'the relationships among different semiotic resources simultaneously activated by the subjects at a certain moment', and the second analysis looks at 'the evolution of signs activated by the subject in successive moments' (ibid., p. 100). I will try to explain why I consider such notions to examine the spatial thinking process.

9.4 Toward a Spatial-Semiotic Framework in the Context of ICT

Within the context of the chapter, I try to express three main components to describe a viewpoint for the spatial thinking process while students (and the teacher) interact with 3D modeling software; the type of mental image, two interrelated processes of VP and IFI, and the emergence of signs with a multimodal perspective. The related literature promises a link between spatial thinking and gestures. However, I believe that this cannot be limited solely to gestures. For instance, think of a student using SketchUp® to solve and explore the task in Fig. 9.1 (also in Fig. 9.3). I postulate that, first of all, the creation and manipulation of mental images occur in VP. However, in this process, students would consider certain tools and functions of the software; for instance, the ‘rotate’ or the ‘move’ tools. The use of these artefacts is an index sign, i.e., an indicator of his/her VP process, and would also possibly be an indicator of the existence of *dynamic images*. While exploration with the artefact, if the student uses specific words, belongs to his/her experience and background in geometry or mathematics, i.e., ‘cube’, ‘triangle’ or suchlike, these would be *symbolic signs* referring to *concrete images*. If he/she uses his/her fingers to trace or point something out, this gesture would be a kind of *icon sign* referring to *kinaesthetic images*. Moreover, in such steps, the user may prefer to use his/her spatial vocabulary to read and interpret the visual images that he/she imagines/creates. This process refers to IFI, where the user may also further become involved in a reasoning process. The student may use a paper-pencil form or his/her analyses through the screen, or prefer to use the tools and functions of the software, which are also index signs belonging to his/her reasoning. Also, in this process, the student (also the teacher whilst lecturing) will possibly use certain *specific words*, *gestures*, *sketches* or *specific tools* while interacting with his/her partner or with the teacher. Such interactions would yield different kinds of semiotic resources, i.e., *signs* indicating the existence and creation of concrete, kinaesthetic or dynamic images. Therefore, in summary, my hypothesis is that, *thinking spatially in 3D modeling software is also multimodal*.

The phenomenological analysis above promises to consider a multimodal approach of the notions of APC-space and the semiotic bundle to explain different kinds of signs that appear in the classroom, when tasks and problems are investigated by the community of the classroom, and when the students and the teacher are thinking spatially. I illustrate my hypothesis with the following figure (Fig. 9.5), describing the relationship among VP and IFI processes, the type of mental images and the emergence of signs when the community pursue a task in 3D modeling software.

In this framework, analyses of signs belonging to spatial thinking can be elaborated on through a *synchronic analysis*, and a *diachronic analysis* as Arzarello (2006) and Arzarello et al. (2009) propose. Two examples evaluate and discuss the framework in the following sections.

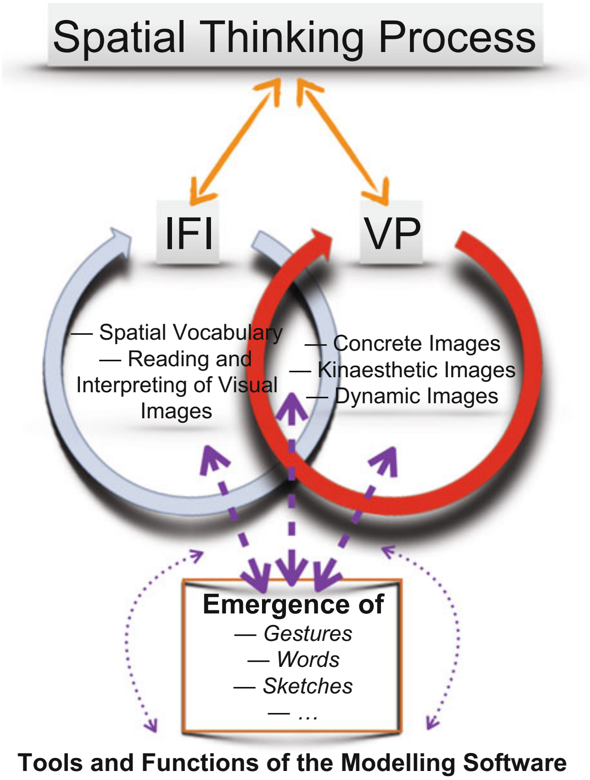


Fig. 9.5 A spatial-semiotic framework in the context of 3D modeling software

9.5 Case Studies

Data for the examples is provided from extensive research carried out with two middle students and a teacher, aiming to analyze the students' instrumental genesis (Vérillon and Rabardel 1995) and the teacher's role in this process. I now consider an item of data from this research to interpret one of the student's and the teacher's spatial thinking processes, while they are interacting with 3D modeling software. The student in this research was selected considering two main criteria; (i) The student's geometry-mathematics performance as evaluated by the teacher, and (ii) his spatial skills analyzed by two spatial tests (for details, see Turgut and Uygan 2015). Following this, Davut (a pseudonym), a seventh grader, was selected to participate. Within the context of the Turkish middle school mathematics curriculum, it should be noted that the student was familiar with basic geometric concepts and their properties, such as triangles, rectangles, squares and so on. The teacher conducting this research had the role of teacher/researcher, when the research was carried out. The teacher had both bachelor and master degrees, and was also a doctoral student in

Fig. 9.6 The first task

mathematics education with a background in the integration of ICT to mathematics education in practice.

With regard to the instrumental genesis window, initially the students involved received training in the use of the SketchUp® for the exploration of the tasks. The teacher introduced basic tools and functions of the software (e.g., top view, pan, select, move, rotate, lines, eraser, rectangle, paint bucket and measurement box) in order to prepare the students for the main application. The students practised two tasks, and thereafter, five main tasks applied, with designs inspired by a doctoral thesis on geographical information systems (Lee 2005). The first task of the main application is outlined in Fig. 9.6. The other four tasks had the same or similar aims, and were designed for the student to use two main sub-skills; mental rotation and mental integration as sub-skills of spatial visualization, according to McGee (1979).

My conviction is that, to introduce 3D modeling software from the window of spatial thinking also needs immediate spatial thinking. First of all, I focus on one element of the teacher's training process, showing how he is thinking spatially while describing the software's functions and tools. Secondly, I look at an interview between one of the students and the teacher to analyze the emergence of signs through a multimodal paradigm. In an synchronic analysis, I only address specific signs belonging to both teacher's and student's spatial thinking processes during the training of the student regarding the software, and with a diachronic analysis, I attempt to elaborate the evolution of signs under VP and IFI processes, i.e., use of spatial vocabulary, reading and interpreting visual images, and the existence of concrete, kinaesthetic and dynamic images. In the overall analyses, I underline specific signs in speech, and will use brackets for gestures or other kinds of signs.

9.5.1 Synchronic and Diachronic Analyses of the Teacher's Training Process

The teacher began to introduce the software and its specific tools in terms of his experience. He expressed that SketchUp® is modeling software is used in engineering fields, but also by practitioners (not professionals) to design 3D models. Thereafter he tried to introduce middle school students to the *x*, *y*, and *z*-axes by

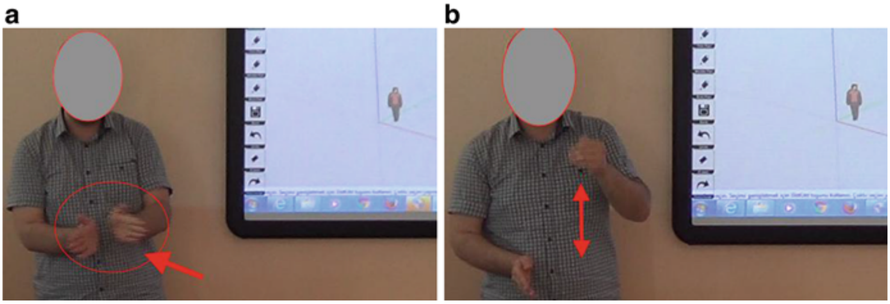


Fig. 9.7 (**a**, **b**) A teacher's gestures describing the coordinate axes

following the interface of the software. Roughly speaking, the semiotic bundle consisted of gestures, sketches, and words. Certain specific signs appear in the introduction to the software, in the process of the teacher's spatial thinking:

Teacher: "… you know the *x* and *y* as ordered pairs, right?"
Response: "Yes, …"
Teacher: "Here we have one more … on the plane, while working on the maps, we use latitudes and longitudes to determine exact positions [*gesturing* see Fig. 9.7a], here we have height [*gesturing* see Fig. 9.7b] as the *z* axis …"

The teacher tries to connect Cartesian geometry and latitudes and longitudes, and for this, he exploits applications of the ordered pair notion using specific gestures, but the description is in relation to the software's interface. He exploits certain references that he (and the students) know; 'maps', 'latitudes and longitudes', 'exact positions' and 'height', which are signs of concrete images in (his) VP. These phrases can also be considered as signs of *specific* (spatial) *vocabulary*, or of his *interpreting the visual images* in his mind, which appears to be related to IFI. Specific gestures also appear that correspond to the *xy* axes (Fig. 9.7a) and the *z*-axis (Fig. 9.7b), which can be considered as an attachment of concrete and dynamic images in his VP. To make connection, he also uses his finger by tracing the axes on the software's interface, which is sign of a *kinaesthetic image* in his VP. Later, he begins to introduce one particular tool of the software, the *orbit*, expressing:

Teacher: "… when we look at the interface, it looks two-dimensional, however, it is three-dimensional software, and we will transform the two-dimensional to a three-dimensional working space through the orbit tool" … "In fact, we rotate the whole screen [gesturing Fig. 9.8a] through this, all the objects [gesturing Fig. 9.8b] …"

The teacher tries to introduce students to the notion of 3D rotation on the interface of the software. He uses basic notions of elementary geometry, related to the notion of dimension saying 'two-dimensional' and 'three-dimensional', with particular emphasis on an advanced notion, the transformation ('transform'). These phrases can be considered a manifestation of the teacher's concrete and dynamic images in VP that are related to 2D and 3D thinking in his mind. Moreover, such

Fig. 9.8 (**a**, **b**) A teacher's gestures describing 3D rotation

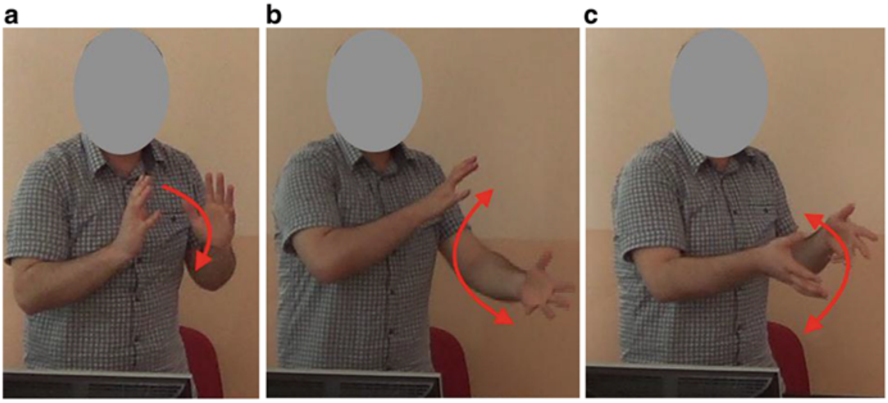


Fig. 9.9 (**a**–**c**) A teacher's gestures during a description of the 'views' function

expressions can also be considered as signs of IFI, because they point out spatial vocabulary. His bodily movements, beyond gesturing, appear to describe this process (see Fig. 9.8 a, b), i.e., the plane view transforms into a 'three-dimensional working space' as a sign of his description of 3D rotation. After the 'orbit' tool, the teacher introduces the 'views' function of the software.

Teacher: "… in terms of the views function, here we can change our viewpoint to top [gesturing Fig. 9.9a], left or right [gesturing Fig. 9.9b] and front or back [gesturing Fig. 9.9c]. For example, looking at a tall building from the outside …"

The teacher relates the views function of the software and, at this moment, gestures appear (Fig. 9.9) that are related to an amalgam of his *spatial orientation* and *mental rotation* skills, which are attached signs of existence of immediate dynamic images in VP. While describing different viewpoints, the teacher uses different

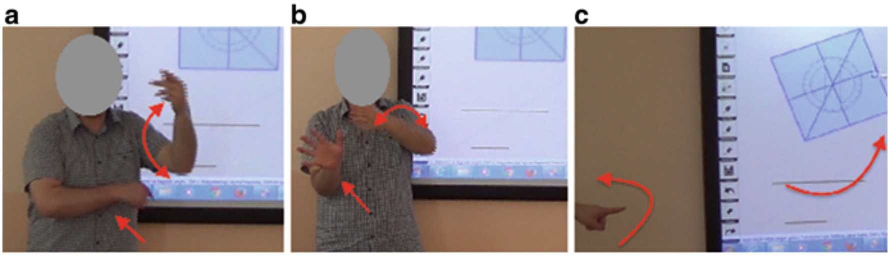


Fig. 9.10 (**a**–**c**) A teacher's gestures describing rotations according to specific axes

gestures to interpret his mental pictures associated with spatial orientation. He also uses underpinning phrases such as, 'our viewpoint', and 'a tall building' as spatial vocabulary referring to IFI. 'A tall building', here, can also be a sign of a concrete image belonging to VP. As a next step, he introduces the 'rotate' tool saying:

Teacher: "… you may only know how to rotate something on a plane, for example, the rotation of a rectangular piece of paper [imitating with a paper]. Here, we have several rotations, for example, with respect to a square's lengths [gesturing Fig. 9.10a, b] or its centre [imitating with the paper]. … What about the angle of rotation? …"

After introduction of the tool, the teacher shows the role *protractor* on the screen by gesturing with his finger and describing the rotation (Fig. 9.10c). He also connects the rotation angle of the objects with 'reflections' of the objects and, using the orbit tool, he shows the students how 3D rotation can be achieved with respect to different axes. As can be seen in Fig. 9.10a, b, the teacher uses his right hand as the rotation axes, with his left hand imitating the rotation. All these gesture signs may be evidence of his dynamic images of rotations, while 'rotating something on the plane' and 'rotation of a rectangular piece of paper' can be considered as signs of concrete images in VP. These, as well as the phrase 'angle of rotation' seem related to the spatial vocabulary of IFI. The teacher's imitation of the paper and immediate sketches on the screen can be also considered as signs of his VP and IFI.

9.5.2 Synchronic and Diachronic Analyses of the Student's Exploration of the Tasks

The interview begins with the teacher introducing the task to the student. At first, Davut (interestingly) uses the mouse pointer to describe what he should do:

Davut: "… the rotation axis is fixed? … This square [*pointing to the right figure with the mouse cursor*] should be rotated 90° clockwise, and this square [*making a rotation imitation with the mouse cursor clockwise*] should be rotated 180° … [*he completes the steps with the tools*]"

…

Davut: "… this [*gesturing* Fig. 9.11a] should be rotated 180° [*gesturing* Fig. 9.11b], and this [*gesturing*] will not change; it will remain itself. Finally I must overlap them …"

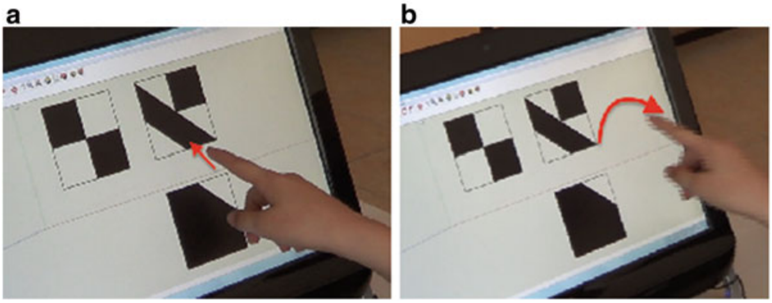


Fig. 9.11 (**a**, **b**) The student Davut's pointing gestures during the second task

Fig. 9.12 (**a**–**c**) The student Davut's pointing cursor and finger gestures

Davut has dynamic images associated with the positions of the figures, and completes the rotations in his mind, apparent by the signs he makes by mimicking through the mouse cursor as evidence of VP to complete mental rotation and mental integration. In addition, the underlined phrases seem to confirm this. Besides, he is now aware the given objects can be rotated with respect to different axes, he asks about the 'rotation axis', which can be considered a sign of IFI. After this, he completes the task using mental images. In the second task, Davut's gestures appear to describe the rotation, and this can be considered as a sign of attachment to his dynamic images, as in the previous task. Interestingly, in the fourth task which is similar to the others, he uses anti-clockwise rotations, although in the task a clockwise rotation is stated, with these signs providing the existence of dynamic images in his VP. He is interpreting the visual image, which is also a sign of IFI.

In the fifth task, the teacher provides three figures and asks how he should manipulate two, to obtain the final figure. However, this task can be tricky, because the final task cannot be achieved in terms of given two's manipulations. Davut explores the task, with the following excerpt drawn from this discussion:

Davut: "… the right figure should be rotated 180º … the left one …"
Teacher: "How did you decide this?"
Davut: "I figured it out from this part [*pointing with the mouse cursor*] of the given figure"
Teacher: "What about the other figure?"

Davut: "… there may be some missing parts here [gesture *pointing to the figure*, see Fig. 9.12a], there may be a problem?"

Teacher: "Why are you thinking like that?"

Davut: "There is a missing part here [*rotating the left figure*], these figures do not form [*pointing with fingers* see Fig. 9.12b] the final figure whatever I do, even if I try different angles or different rotation axis"

The dynamic images that Davut created provide him with a glimpse of tricky. He uses the concrete images in his mind to disintegrate parts of the figure. He also uses the mouse cursor instead of his finger, which can be accepted as the existence of a kinaesthetic image. With the underlined phrases, all these signs, they together confirm his VP in solving the task. Moreover, he also expresses all possibilities, such as 'different angle' and 'different rotation axis', which can be considered as an IFI process.

9.6 Conclusions

In this chapter, I propose a spatial-semiotic framework based on a hypothesis that *thinking spatially in a 3D modeling software environment is multimodal*, i.e., thinking spatially is a complex process including different kinds of semiotic resources, such as gestures, sketches, words, imitations, and suchlike. The theoretical underpinnings of the framework were due to VP and IFI processes (Bishop 1983), concrete, kinaesthetic and dynamic images in spatial thinking (Presmeg 1986, 2006), and Peircean semiotics, multimodal paradigm, notions of APC-space and the semiotic bundle in mathematics education (Arzarello 2006, 2008; Arzarello et al. 2009). The results of the case studies were promising regarding the framework. The cases reveal that the emergence of signs was in coordination, i.e., the signs were in relation to the software's tools and functions (see Fig. 9.8a, the teacher's body movements for the orbit tool) as expected. With respect to the signs in such a process, there was a semiotic bundle consisting of three main components; words, gestures and sketches, which confirm (but not always) the framework's hypothesis. Evidently, in the first case, the emergence of signs that confirm the framework's hypothesis was clearer compared to the second case. This may be due to several reasons. The first may be the teacher's background in mathematics, as a mathematics teacher. He used different careful examples showing his thinking with his experience of the software. However, in the student's case, some gestures were interlaced with the mouse's cursor (see Fig. 9.12a). This could be a gesture describing a semi-circle embedded in the given figure. This could also be related to the student's communication skills.

The second reason could the fact that I looked at the data myself. I analyzed the data through my phenomenological experience in the spatial thinking field based on two interrelated viewpoints; (i) my experience in SketchUp®, and (ii) my participation to the training process and interviews. A cross analysis between an insider's and an outsider's viewpoints could provide more detail for a clearer understanding of the observed phenomenon.

The third reason may be the limitation of the tasks with 2D spatial thinking including mental rotation and mental integration. Although the teacher's training includes 3D interface, in this chapter, I could not provide 3D spatial tasks to focus the students' spatial thinking processes within the context of 3D modeling software. In addition, 2D tasks were apparently not related to a type of reasoning in mathematics. Therefore, an elaboration of spatial thinking in solving 3D geometry tasks could provide different kinds of semiotic resources to evaluate or ameliorate the proposed framework. For this purpose, dynamic geometry environments (DGEs), for example Cabri, GeoGebra, Sketchpad and suchlike, with dragging functions to provide the user various kinds of epistemic situations to explore the provided tasks, could be used (Leung 2008; Leung et al. 2013; Lopez-Real and Leung 2006). Such DGEs could be designed as a tool of semiotic mediation (Bartolini Bussi and Mariotti 2008) to also look at the construction of mathematical (meanings) signs in the solution of tasks. More information attached to the IFI process of students could be helpful in reanalyzing the function of the framework, in the interpretation of the obtained data in depth. Consequently, I note that this fresh framework requires more elaboration. For example, one might discuss the role of geometric reasoning (Duval 1995, 1998) or geometric space work (Kuzniak 2014) in the functioning of the framework. This may be possible through strategies of networking theoretical frameworks elaborated by Prediger and Bikner-Ahsbahs (2014).

It is clearly stated in the related literature that spatial thinking predicts achievement in Science, Technology, Engineering and Mathematics (STEM) fields (Kell and Lubinski 2013; Shea et al. 2001; Wai et al. 2009). Therefore one can conclude that development of spatial ability—because it is malleable (Stieff and Uttal 2015; Uttal et al. 2013)—might contribute the improvement of achievement in STEM fields. In this respect, how the present spatial—semiotic framework will function in the analyzing the role of spatial ability in STEM fields can be summarized as follows.

This fresh framework enables researchers not only look at the emergence of signs when the subjects commence ICTs and while they are thinking spatially, but also provides a detailed understanding of the teacher's spatial thinking process. Such kind of analyses in the teaching–learning process could be a potential tool to investigate the subjects' *internalization* and/or *externalization* of spatial images. Such variety of specific signs such as gestures, sketches and/or words that attached to *creation* and *manipulation of spatial images* could provide an understanding of the subjects' way of spatial thinking in the use of ICTs. More specifically, framework has two interrelated dimensions: IFI and VP, and emergence of the attached signs in the use of ICT tools. In the use of this framework, researchers could gain a lens for a better understanding the subjects' thinking process in depth, because it provides the subjects' use of *spatial language*, their *interpretation* of the visual–spatial images, and their use of concrete, kinaesthetic and dynamic images together with the functions and tools of ICTs. In the light of this, researchers could create affective pedagogical designs to improve spatial ability and therefore a better achievement in STEM fields.

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