# A CA-Based Model of Dyads in Pedestrian Crowds: The Case of Counter Flow

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**Abstract.** The calibration and validation of pedestrian dynamics simulation require the acquisition of empirical evidences of human behaviour. In this framework, this paper firstly presents the results of an experimental study focused on the negative impact of counter flow and grouping on pedestrian speed. In particular, we focused on two member groups (dyads) as the most frequently observed and basic interacting element of crowds. On the basis of the behavioural effects observed with the experiment, a novel cellular automaton is proposed to represent the different behaviour of individuals and dyads, with particular reference to the group spatial alignment and the dynamic leader-follower structure. This has been demonstrated to modulate the speed of dyads by maintaining the spatial cohesion among the two members. In addition, the model is also able to reproduce the significant impact of flow ratio observed in the experiment results.

**Keywords:** Cellular automata · Pedestrian dynamics · Groups · Experiment

## 1 Introduction

The role of advanced computer-based systems for the simulation of the dynamical behaviour of pedestrians is becoming a consolidated and successful field of research and application, thanks to the possibility to test the efficiency, comfort and safety of urban crowded facilities evaluating key performance indicators (e.g., travel time, perceived density, waiting time).

The development of simulation systems requires a cross-disciplinary methodology to calibrate and validate the model according to benchmark information about pedestrian behaviours and wayfinding decisions. In particular, the validation of simulations requires to test the model itself and the related simulation results by means of empirical studies about pedestrian dynamics. In this framework, this paper firstly introduces the results of an experiment focused on the impact of flow ratio and grouping (*dyad*) on pedestrian dynamics in a controlled laboratory setting. The final aim is the design and validation of a CA-based model focused on the reproduction of the observed effects of


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flow ratio and grouping, with particular reference to heterogeneous speed profiles and group cohesion mechanism.

Considering groups in the simulation models has been generally neglected until the last decade, in which microscopic modelling of individual behaviour represented the main focus of this research field. Recent empirical studies on pedestrian groups [8,22], on the other hand, brought much interests on this element and many approaches considering its presence in the simulations have been proposed [13,20]. In particular, the granulometric distribution of pedestrian flows is strongly affected by two-members groups, which represent the most frequently observed and basic interacting elements of crowds [9]. Analyses of pedestrian dynamics not considering this aspect have a reduced accuracy, since grouping was found to impact the overall dynamics of the crowd movements [19] and evacuation dynamics [13]. This is due to the need to maintain spatial cohesion among members to communicate while walking. In particular, group proxemic behaviour generates different spatial configuration while walking at variable density conditions [12,22]: *line abreast* pattern at low density, *diagonal* pattern at medium density and *river-like* pattern at high density.

The proposed methodological approach represents a complete *analysis-synthesis* cycle [2], employing the results of an empirical study on the impact of groups in pedestrian counter flow for the design and calibration of a CA-based model. Furthermore, a first novel contribution of this work sees the employment of a simulation system for pedestrian and crowd dynamics (ELIAS38) to help the design of the experiment itself, using thus results from the synthesis side to support the analysis. The observed quantitative data from the experiment allowed the configuration of a set of ad-hoc rules that are able to reproduce a valid behaviour of the simulated pedestrians and groups.

In the literature the issue of defining formal computational models about pedestrian dynamics has been tackled from different perspectives:

- *Physical* approach [5,10] represents pedestrians as particles subject to forces (e.g., attraction and repulsive forces), in analogy with fluid dynamics;
- *Cellular Automata* approach [3,6,14] represents pedestrians as occupied states of the cells. Pedestrian interactions are based on the *floor field* method: a virtual traces that influence pedestrian transitions and movements;
- *Agent-based* approach [7,17] represents pedestrians as heterogeneous, situated and autonomous entities moving according to behavioural rules and specifications. Higher level aspects of behaviour, such as wayfinding, are also considered.

The hereby presented CA model is specifically tailored for the simulation of the experiment procedures, towards the extension of the simulation platform ELIAS38 for a validated simulation of pedestrian groups. In the experiment singles and dyads of participants walked in a corridor at different counter flow ratio. Both types of pedestrian are represented in the CA model by basic behavioural specifications which allow them to reach their destination. In addition, groups members interact through refined proxemic rules, by defining in the model a dynamic (*leader-follower*) structure.

The paper is structured as the following: the case study and the experiment results are proposed in Sect. 2. The CA model is presented in Sect. 3, with results in Sect. 4. The paper concludes with final remarks about the validation process of the CA model.

**Fig. 1.** A video frame of the analysed measurement area of the experiment (procedure No. 2).

## 2 Experimental Study

The experiment was performed on June 13, 2015 at the Research Center for Advance Science and Technology of The University of Tokyo (Tokyo, JAPAN). The objective of the study was to test the following hypotheses:

– Hp 1: The increase of flow ratio negatively impacts the speed;
– Hp 2: Dyads walk slower than singles, to maintain spatial alignment.

The experiment has been designed with the support of the ELIAS38 simulation tool, analysing different counter flow scenarios in order to avoid excessive level of density in the measurement area. The simulation results allowed to correct critical aspects of the procedures, like the dimension of the measurement area and flow ratio configuration.

In this paper we denote the *flow ratio* as the rate between the minor flow and the total flow in bidirectional scenarios. Flow ratio was managed as independent variable among four different experimental procedures: a unidirectional flow (flow ratio = 0) and three different configurations of bidirectional flow (flow ratio = .167, .333, .5). The setting was designed as a corridor-like scenario, composed of: (*i*) the measurement area in the centre ($10\,m \times 3\,m$); (*ii*) two side buffer zones to allow pedestrians reaching a stable speed ($2\,m \times 3\,m$); (*iii*) two starting areas ($12\,m \times 3\,m$).

The starting positions were drawn on the floor with coloured cross signs (lateral distance 0.5 m, longitudinal distance 1.5 m). The number of dyads per line and the positions of members (e.g., line-abreast, river-like patter) were homogeneously distributed among the starting positions. At the start signal, all participants were asked to walk toward the opposite side of the corridor. Each procedure was repeated four times, asking participants to change their starting positions.

The sample was composed of 54 male students of The University of Tokyo (from 18 to 25 years old). 24 participants were randomly paired (44 % of the total, as observed in [9]) and asked to walk close to each other during the experiment reproducing the locomotion behaviour of dyad members (no instructions were given to them about the spatial pattern). The rest of participants walked as individual pedestrians. The recorded video images have been analysed by using the PeTrack software [4], which allowed to automatically track pedestrians' trajectories. A screenshot from the video footages of the experiment is shown in Fig. 1.

**Table 1.** The experiment and simulation results about pedestrian speeds and alignment of dyads.

| Flow ratio | Grouping | EXP speed [m/s] | EXP X-align | EXP Y-align | SIM speed [m/s] | SIM X-align | SIM Y-align |
|---|---|---|---|---|---|---|---|
| 0 | Total | 1.31 ± .10 | - | - | 1.26 ± .07 | - | - |
| | Singles | 1.32 ± .11 | - | - | 1.29 ± .06 | - | - |
| | Dyads | 1.30 ± .09 | .53 ± .21 | .23 ± .25 | 1.23 ± .07 | .51 ± .05 | .18 ± .07 |
| 0.167 | Total | 1.20 ± .13 | - | - | 1.22 ± .09 | - | - |
| | Singles | 1.22 ± .15 | - | - | 1.26 ± .07 | - | - |
| | Dyads | 1.16 ± .10 | .42 ± .13 | .23 ± .24 | 1.17 ± .08 | .50 ± .04 | .25 ± .07 |
| 0.333 | Total | 1.07 ± .09 | - | - | 1.17 ± .12 | - | - |
| | Singles | 1.08 ± .10 | - | - | 1.23 ± .09 | - | - |
| | Dyads | 1.05 ± .09 | .35 ± .17 | .31 ± .22 | 1.10 ± .11 | .51 ± .03 | .29 ± .10 |
| 0.5 | Total | 1.08 ± .10 | - | - | 1.18 ± .12 | - | - |
| | Singles | 1.10 ± .10 | - | - | 1.22 ± .09 | - | - |
| | Dyads | 1.07 ± .11 | .41 ± .13 | .26 ± .20 | 1.09 ± .12 | .50 ± .02 | .30 ± .05 |

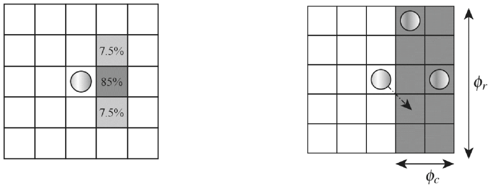
## 2.1 Experimental Results

A two-factors analyses of variance[1] (two-way ANOVA) was conducted to test the impact of *flow ratio* and *grouping* on speed (see Table 1 and Fig. 5). Results showed a significant effect for the flow ratio factor [F(3,856) = 242.777, p value < .000] and a significant effect for the grouping factor [F(1,856) = 26.946, p value < .000]. No significant interaction among the two factors was found [F(3,856) = 1.008, p = .388]. A linear regression showed that the overall speed of participants was affected by the increase of flow ratio [F(1,862) = 546.039, p value < .000, R-square of .388; speed = 1.287 − .489 * flow ratio]. A post hoc Tukey test showed that the speed of participants at flow ratio = .333 and flow ratio = .5 did not differ significantly at p = .406. In conclusion, results partially confirmed the Hp 1, showing that the increase of flow ratio from 0 to .333 significantly affected the overall speed of participants.

A post hoc Tukey test showed that the speed of singles and dyads among the procedures No. 1 (flow ratio = 0) did not differ significantly at p = .05056. The average speed of singles (1.13 m/s ± .08) and dyads (1.09 m/s ± .06) among procedures No. 2, No. 3 and No. 4 differed significantly at p value < .000. In conclusion, results partially confirmed the Hp 2, showing that in case of counter flow dyads walked in average the 4 % slower than singles, due to the need of group members to preserve spatial alignment in case of local situations of density.

The alignment of dyads on the X-axis (.43 m ± .17 SD) and the Y-axis (.26 m ± .23 SD) was measured as the gap between the positions of members and the geometrical centre of the group (*centroid*) (see Table 1). A series of one factor analyses of variance (one-way ANOVA) showed a significant impact of the flow ratio on the X-alignment of dyads [F(3,188) = 9.531, p value < .000]. A non significant impact of the flow ratio on the Y-alignment was found [F(3,188) = 1.197, p value = .312]. A linear regression analysis showed a significant impact of the X-alignment on the speed of dyads [F(1,190) = 32.180, p value < .000, with an R-square of .145; speed of dyads = 1.017 + .381 * X-alignment]. A non significant impact was found for the Y-alignment on speed

[1] All the analyses presented in this work have been conducted at the p < .01 level.




(a) (b)

**Fig. 2.** (a) Configuration of the probabilities of movement for the free flow case. (b) The grey area is considered for the evaluation of movement: the central pedestrian, moving towards the right, will much probably perform a movement towards south-east, due to the other pedestrians belonging to the counter flow.

[F(1,190) = 4.344, p value = .038, with an R-square of 0.022; speed of dyads = 1.165 − .150 * Y-alignment]. In conclusion, the increase of flow ratio negatively affected the alignment of dyads on the X-axis and consequently their speed.

In case of counter flow, dyads walked in average the 4 % slower than singles, due to the need to maintain spatial cohesion. As generally observed (see e.g., [9]), in condition of relatively low local density dyads walk side by side with a line abreast pattern. Results of the experiment showed that dyads split due the local condition of density in counter flow situations; consequently they slow down to regroup and maintain cohesion, arranging their spatial pattern with detriment of speed (*leader-follower* structure).

## 3 A Simulation Model Considering Dyads

A Cellular Automaton (CA) based on ad-hoc rules has been designed to simulate the experiment procedures. As generally applied in the field of pedestrian simulation [3, 14], this CA is based on a grid of square cells of $40 \times 40\,\text{cm}^2$ size that reproduce the environment and the space occupation of a person. Each cell can be occupied by one pedestrian at most and this generates a maximum density of 6.25 ped/m$^2$, covering the range of densities usually observed [21]. The assumed duration of the discrete time step is 0.3 s. In this way, a unique desired speed of 1.333 m/s is simulated, in accordance with the average speed observed with the experiment procedures. Given the objective of this work to analyse the impact of flow ratio and proxemic behaviour on the speed of dyads, possible approaches from the literature for the management of heterogeneous speeds [1] have not been considered.

The movement of the simulated pedestrians is modelled with simple rules leading them towards the other extreme of the corridor. In particular, at each time-step all pedestrians can choose between the three forward cells next to its position or alternatively to maintain the current position. The set of forward cells is configured ad-hoc for the two direction of flows. To allow the pedestrians to produce more smooth trajectories and avoid sudden changes of direction, the probability distribution for choosing the next cell in absence of other nearby persons is configured as shown in Fig. 2(a). The probability to move in the forward cell is configured as 85 % for these tests, while the remaining 15 % is equally distributed among the two other closer cell to the target.

To model a plausible dynamics in case of counter flow, the behaviour of pedestrians is modelled to let them understand where to move next, by possibly minimising conflicts with the surrounding persons. The choice is performed according to a *perception* phase: the pedestrian evaluates the situation in rectangular areas of $\phi_r \times \phi_c$ cells in front of the possible movements by counting the number of persons inside. Cell closer to counter flow pedestrians are considered less convenient, thus during the simulation pedestrians tend to choose cells behind other persons moving in the same direction. This allows the generation of the well-known *lane formation* effect of pedestrian dynamics [16] and also to achieve realistic average speed of pedestrians comparing to the observed experiment scenarios (see results in the next section). For the time being, quantitative analysis on the capability of the model to generate and maintain lanes in counter flow situations [15] has not been performed, but it will be subject of future works.

This mechanism is exemplified in Fig. 2(b), where the couple of calibration parameters $(\phi_r, \phi_c)$ of this component is set to $(10, 2)$ as for the executed tests. After the evaluation, a probabilistic choice is performed, making choices to converge towards less dense zones. This perception component is inspired by the model proposed in [18].

For simplicity, in this model we adopted a *shuffled sequential* update rule, avoiding a-priori the generation of conflicts. It is known in the literature that this update strategy can lead to a higher and unrealistic simulation of flows at high pedestrian density in the scene [11]. To improve the simulated dynamics, we introduced a probability $p_c$ for which a pedestrian can choose as well an already occupied position, generating a conflict. If this happens, the pedestrian will hold its position for the current step. For our tests, this probability has been set to about 0.6.

The objective of the experiment procedures performed and described in this paper was to verify the impact of counter flow ratio and grouping on speed. The behaviour of dyads was therefore a primary element of this model, representing its novel part. To maintain the observed *cohesion* of the dyad members, their behaviour is modelled with a dynamic *leader-follower* structure, by means of a set of ad-hoc rules defined for each member type. The structure of one group varies over time and the leader is temporary defined as the person which is forward to the other, with respect to their direction. In

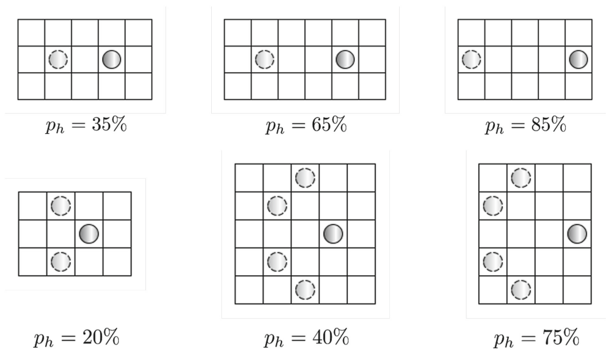



**Fig. 3.** Configuration of the probability $p_h$ of the leader according to the possible positions of the follower (dashed circle).

case of line abreast pattern (much frequent in this model due to the space discretisation) the mechanism is inhibited and the two members move according to the general rule of movement, to allow a better usage of the space.

The *follower* has a simpler behaviour and it is much probably driven by the position of the other member. At each step, the distance between the two members is calculated with the *manhattan* metrics. If this value is $\leq 2$ cells, then the follower has a 80 % probability of moving in the closer cell to the leader and 20 % of choosing a movement with the base rule. The probability to follow rises to 100 % if the leader is over the 2 cells threshold of distance.

Differently, the behaviour of the leader is more aimed at reaching the target, but it is also described by a tendency to hold its position and wait for the follower to allow a re-composition. This tendency is managed with a probability $p_h$ to not move at a given step, varied according to the positions of the two members. The configuration of this probability is graphically explained by Fig. 3. The possible cases are divided as: (*i*) the dyad walk with a river-like pattern and (*ii*) the two members are not aligned. This allowed to reproduce the experiment results and to maintain the distances among dyads members within the achieved range.

## 4 Simulation Results

A simulation campaign was performed to test and validate the model mechanism described in the previous section. The simulated environment has been designed with a grid of $85 \times 8$ cells (34 m $\times$ 3.2 m), describing the complete experimental setting (comprising the measurement area, the buffer zones and starting areas). As for the experiment, data about speed and trajectories of singles and dyads have been recorded only from the measurement area. The same number and proportion of singles and dyads was reproduced with the simulations. 10 iterations were run per each procedure.

The same statistics have been calculated on simulation results (see Table 1 and Fig. 5). Results showed a significant effect for the flow ratio factor [F(3,3014) = 3.788, p value $<$ .000] and a significant effect for the grouping factor [F(1,3014) = 6.643, p value $<$ .000]. Moreover, results showed a significant interaction among the two factors [F(3,3014) = 3.788, p value $<$ .000]. A linear regression showed a significant and negative impact of the flow ratio on speed [F(1,3020) = 277.131, p value $<$ .000, R-square of 0.084; speed = 1.252 $-$ 0.165 * flow ratio]. A post hoc Tukey test showed that the speed among the simulated procedures No. 3 (flow ratio = .333) and No. 4 (flow ratio = .5) did not differ significantly at p = .993. In conclusion, the simulations confirmed the experiment results, showing that the increase of flow ratio from 0 to .333 negatively affected the overall speed of the simulated pedestrians.

The average simulated speed of singles (1.24 m/s $\pm$ .09) and dyads (1.14 m/s $\pm$ .11) differed significantly at p value $<$ .000 among all procedures. In conclusion, the simulation results confirmed the ones achieved by means of the experiment, showing the effectiveness of the implemented group cohesion mechanism among dyad members, which moved in average the 8 % slower than singles.

A series of one factor analysis of variance (one-way ANOVA) were conducted to test the impact of flow ratio on dyads X-alignment (.51 m $\pm$ .04 SD) and Y-alignment

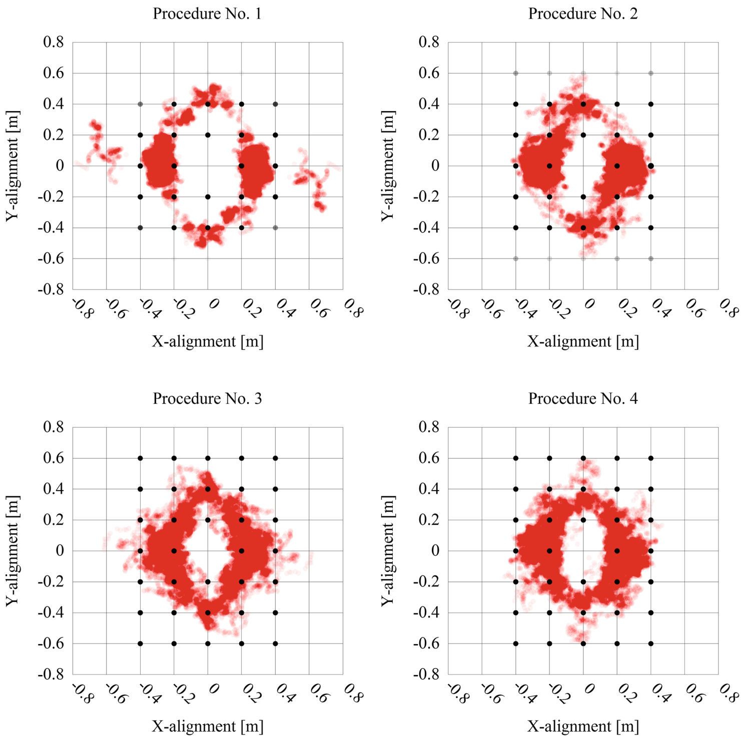



**Fig. 4.** The alignment of dyad among the experiment (red dots) and simulation (black dots) results. The charts report the aggregated positions of dyads, with a transparent effect to emphasise their distribution. (Color figure online)

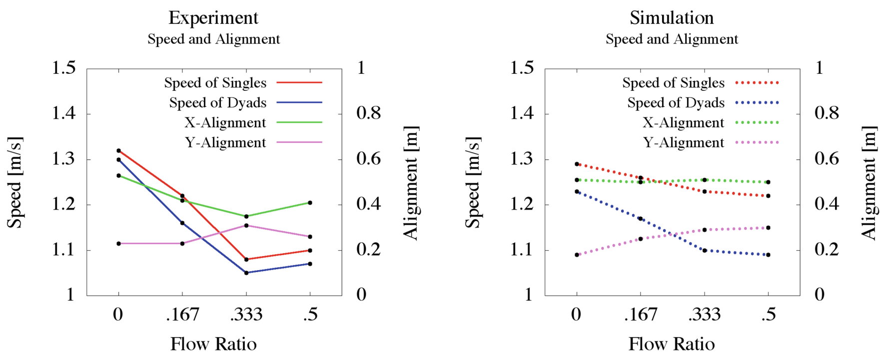



**Fig. 5.** A comparison among the experiment and simulation results.

(.25 m ± .09 SD) (see Table 1 and Fig. 4). Results showed a non significant impact of flow ratio on the X-alignment [F(3,147) = .574, p value = .633]. A significant impact of flow ratio on the Y-alignment was found [F(3,147) = 21.630, p value $<$ .000]. In conclusion, the introduced cohesion mechanism was quite effective in simulating the group proxemic behaviour: the average distance between members on the X and Y-alignment and their relative positions have been plausibly reproduced. However, the dynamic arrangement of spatial patterns of dyads according to the counter flow ratio did not completely fit with the experiment results, due to space discretisation by itself.

## 5 Conclusions and Future Works

According to the proposed methodological framework, an experimental study on the impact of counter flow and grouping on pedestrian dynamics was performed for sake of the design and validation of a CA-based model. The results of the experiment showed that the two hypotheses Hp 1 and Hp 2 were partially confirmed: the increase of flow ratio from 0 to .333 negatively impacts the overall speed of pedestrians; the difference between the speeds of singles and dyads is confirmed in case of counter flow and it is explained by the need of maintaining the spatial cohesion by the group members.

On the basis of the achieved results and observed behaviour, a CA model focused on the reproduction of the group proxemic behaviour has been designed. The model reproduces the dynamics by means of simple ad hoc rules, which can be calibrated by means of several parameters. In particular, individuals choose their movement direction according to the position of their target and the perception of nearby pedestrians. In this way, they try to avoid conflicts with counter flow. Dyad members, instead, are modelled with a dynamic leader-follower structure that aims at preserving the observed cohesion and spatial patterns during the simulations.

A set of simulations, reproducing the experiment procedures, was performed to validate the proposed mechanisms of the model. Results showed that the model is able to reproduce the observed effects of flow ratio and grouping on the speed of pedestrians (as shown in Fig. 5). Furthermore, the spatial patterns reproduced by the model are also similar to the observed results, demonstrating the effectiveness of the leader-follower behavioural rules.

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## References

1. Bandini, S., Crociani,L., Vizzari, G.: Heterogeneous pedestrian walking speed in discrete simulation models. In: Proceedings of Traffic and Granular Flow 2013. pp. 273–279. Springer International Publishing (2015)
2. Bandini, S., Gorrini, A., Vizzari, G.: Towards an integrated approach to crowd analysis and crowd synthesis: a case study and first results. Pattern Recogn. Lett. **44**, 16–29 (2014)
3. Blue, V.J., Adler, J.L.: Cellular automata microsimulation for modeling bi-directional pedestrian walkways. Transp. Res. Part B **35**(3), 293–312 (2001)

4. Boltes, M., Seyfried, A.: Collecting pedestrian trajectories. Neurocomputing **100**, 127–133 (2013)
5. Chraibi, M., Seyfried, A., Schadschneider, A.: Generalized centrifugal-force model for pedestrian dynamics. Phys. Rev. E **82**(4), 46111 (2010)
6. Crociani, L., Lämmel, G.: Multidestination pedestrian flows in equilibrium: a cellular automaton-based approach. Comput. Aided Civil Infrastruct. Eng. **31**(2016), 432–448 (2016)
7. Crociani, L., Piazzoni, A., Vizzari, G., Bandini, S.: When reactive agents are not enough: tactical level decisions in pedestrian simulation. Intelligenza Artificiale **9**(2), 163–177 (2015)
8. Duives, D., Daamen, W., Hoogendoorn, S.: Influence of group size and group composition on the adhered distance headway. Transp. Res. Procedia **2**, 183–188 (2014)
9. Gorrini, A., Bandini, S., Vizzari, G.: Empirical investigation on pedestrian crowd dynamics and grouping. In: Proceedings of Traffic and Granular Flow 2013. pp. 83–91. Springer International Publishing (2015)
10. Helbing, D., Molnar, P.: Social force model for pedestrian dynamics. Phys. Rev. E **51**(5), 4282 (1995)
11. Kirchner, A., Nishinari, K., Schadschneider, A.: Friction effects and clogging in a cellular automaton model for pedestrian dynamics. Phys. Rev. E **67**(5), 56122 (2003)
12. Moussaïd, M., Perozo, N., Garnier, S., Helbing, D., Theraulaz, G.: The walking behaviour of pedestrian social groups and its impact on crowd dynamics. PloS One **5**(4), e10047 (2010)
13. Müller, F., Wohak, O., Schadschneider, A.: Study of influence of groups on evacuation dynamics using a cellular automaton model. Transp. Res. Procedia **2**, 168–176 (2014)
14. Nishinari, K., Kirchner, A., Namazi, A., Schadschneider, A.: Extended floor field CA model for evacuation dynamics. IEICE Trans. Inf. Syst. **87**(3), 726–732 (2004)
15. Nowak, S., Schadschneider, A.: Quantitative analysis of pedestrian counterflow in a cellular automaton model. Phys. Rev. E **85**(6), 066128-10 (2012)
16. Schadschneider, A., Klingsch, W., Klüpfel, H., Kretz, T., Rogsch, C., Seyfried, A.: Evacuation dynamics: empirical results, modeling and applications. In: Meyers, R.A. (ed.) Encyclopedia of Complexity and Systems Science, pp. 3142–3176. Springer, New York (2009)
17. Shao, W., Terzopoulos, D.: Autonomous pedestrians. Graph. Models **69**(5–6), 246–274 (2007)
18. Suma, Y., Yanagisawa, D., Nishinari, K.: Anticipation effect in pedestrian dynamics: modeling and experiments. Physica A **391**(1–2), 248–263 (2012)
19. Templeton, A., Drury, J., Philippides, A.: From mindless masses to small groups: conceptualizing collective behavior in crowd modeling. Rev. Gen. Psychol. **19**, 215–229 (2015)
20. Was, J., Lubas, R., Mysliwiec, W.: Proxemics in discrete simulation of evacuation. In: Proceedings of the 10th International Conference on Cellular Automata for Research and Industry, ACRI 2012, pp. 768–775 (2012)
21. Weidmann, U.: Transporttechnik der fussgänger - transporttechnische eigenschaftendes fussgängerverkehrs (literaturstudie). Literature Research 90, Institut füer Verkehrsplanung, Transporttechnik, Strassen- und Eisenbahnbau IVT an der ETH Zürich (1993)
22. Zanlungo, F., Ikeda, T., Kanda, T.: Potential for the dynamics of pedestrians in a socially interacting group. Phys. Rev. E **89**(1), 012811 (2014)