The Elastic Compression in the Contact Region of a Cam Mechanism General Kinematic Pair

J. Ondrášek

Abstract The paper deals with the elastic compression in the contact region of a general kinematic pair. In the case of a general cam mechanism, the general kinematic pair is formed of the contact of the working surfaces of a cam and a follower. In technical practice, the contact of the cam and the follower is created by the cylinders in contact with the parallel axes for one thing and for another, by the cylinder and the three-dimensional body, which is described locally in contact with the orthogonal radii of curvature. This issue is solved on the basis of the contact mechanics. For both cases, the relations for calculating the elastic compression in the contact regions were defined. This mathematical model is verified through the experimental identification, the general kinematic pair is carried out using a testing device. The aim of the tests was to determine the mutual approach of distant points in the two three-dimensional solids in contact.

Keywords Elastic compression · Contact region · General kinematic pair · Cam mechanism

1 Introduction

In some computational analyses of cam mechanisms, an interaction of a cam contour with a follower has to be considered as a real kinematic pair, i.e. with a backlash and a friction. This occurs, for example, when bearing rating life of the cam mechanism roller is determined. In the case of a model creation of a cam mechanism general kinematic pair, the contact of two cylinders with the parallel axes or a cylindrical body and a non-conforming body may be substituted for the interaction of a cam contour with a follower. The cylinder represents a cam or a cylindrical cam roller and the non-conforming body a crowned cam roller.

J. Ondrášek (✉)
VÚTS, a.s., Liberec, Czech Republic
e-mail: jiri.ondrasek@vuts.cz

J. Beran et al. (eds.), *Advances in Mechanism Design II*,
Mechanisms and Machine Science 44, DOI 10.1007/978-3-319-44087-3_8

Three ways exist for the creation of mathematical models: theoretical, experimental and combination of both. A theoretical mathematical model of the general kinematic pair requires knowledge of some parameters, i.e. a normal displacement and a normal stiffness in a contact region. These parameters may be determined on the basis of a Hertzian contact theory or FEM. In this case, FEM is ineffective.

2 Hertzian Contact Theory

This section gives only basic information on how to determine the compression of two solids in contact. The Hertzian contact stress theory deals with this issue and this theory is described in detail in [1].

When two three-dimensional body are brought into contact they touch initially at a single point or along a line. Under the action of a slightest load, they will deform and contact is made over a finite area which is small compared with the dimensions of both bodies. On the basis of a theory of contact, the shape of the contact area is predicted and the components of deformation and stress in both bodies are calculated in the vicinity of the contact region. For the Hertzian contact stress theory, the fundamental assumptions are, see [1]:

- The shape of each surface in the contact region can be described by a homogeneous quadratic polynomial in two variables:

$$z_i = \mp\frac{1}{2}\left(\frac{1}{\rho_{xi}}x^2 + \frac{1}{\rho_{yi}}y^2\right), \quad i = 1, 2, \tag{1}$$

  where ρ_{xi} and ρ_{yi} are the principal radii of curvature of the surface at the rectangular coordinate system origin, see Fig. 1.
- Contact stresses and deformations satisfy the differential equations for stress and strain of homogeneous, isotropic, and elastic bodies in equilibrium. The pressure distribution on the contact area is given by equation:

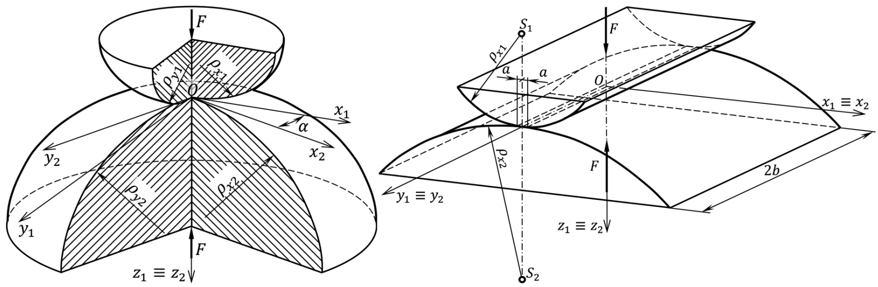

Fig. 1 Contact of two non-conforming bodies and contact of two cylindrical bodies

$$p(x,y) = p_H\sqrt{1-(x/a)^2-(y/b)^2}, \tag{2}$$

where a and b are respective major and minor semi-axes of the elliptical contact area and a maximum value p_H is called Hertzian pressure. In the case of the contact of two cylindrical bodies, the variable y is constant and Eqs. (1) and (2) are the functions just of the variable x.

- Both contacting surfaces are smooth and frictionless.
- The size of the contact area is small compared with the size of both bodies.

The compressive contact of two bodies is caused by application of a normal load F and a contact area is formed. Remote parts T_1 and T_2 of the bodies are approached each other by a distance δ. Points S_1 and S_2 on the approaching contact surfaces are elastically displaced by amount u_{z1} and u_{z2}, as shown in Fig. 2.

On the basis of the Hertzian contact stress theory, the equation of the normal displacement of two non-conforming bodies is derived in the following form:

$$\delta = \delta_1 + \delta_2 = \frac{3F}{2\pi a E^*}K(e), \quad e = \sqrt{1-(b/a)^2}, \quad a > b, \tag{3}$$

where $K(e)$ is complete elliptic integral of the argument e [2], which expresses the eccentricity of the contact ellipse and E^* is the effective modulus of elasticity. The normal stiffness of the contact region is obtained by differentiating the deflection with respect to load F to get compliance, then inverting:

$$k = \left(\frac{\partial\delta}{\partial F}\right)^{-1} = \frac{K(e)}{\pi a E^*}, \quad E^* = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}, \tag{4}$$

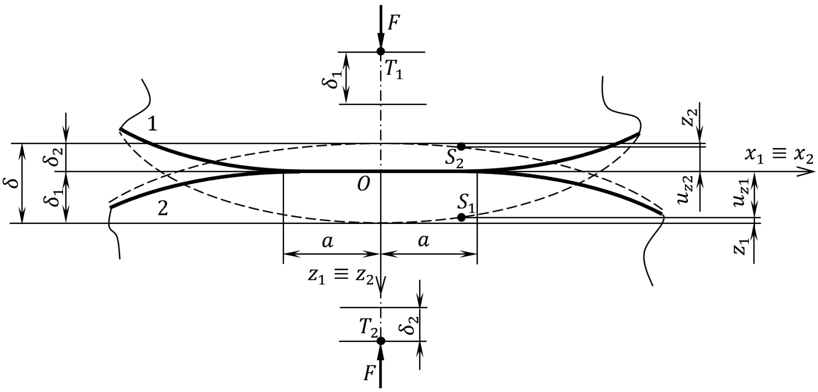

Fig. 2 Contact of two non-conforming bodies after elastic deformation

where E_i and μ_i are the respective Young's modulus of elasticity and Poisson's ratio of the individual solids. In the case of the two cylindrical bodies contact, the normal displacement δ and the normal stiffness k are expressed as:

$$\delta = \delta_1 + \delta_2, \quad \delta_i = \frac{N}{2b} \cdot \frac{1-\mu_i^2}{\pi E_i} \cdot \left[2\ln\left(\frac{4L_i}{a}\right) - 1\right], \quad i = 1, 2, \tag{5}$$

$$k = \left(\frac{\partial\delta}{\partial F}\right)^{-1} = \frac{2b\pi}{\left\{\frac{1-\mu_1^2}{\pi E_1}\left[2\ln\left(\frac{4L_1}{a}\right) - 1\right] + \frac{1-\mu_2^2}{\pi E_2}\left[2\ln\left(\frac{4L_2}{a}\right) - 1\right] - \frac{1}{E^*}\right\}}, \tag{6}$$

where variable a denotes the half width of the contact area and the constant L_i is the width of the individual bodies, which may be not equal to the length of the contact area $2b$.

3 Testing jig

To the verification of the validity of the Eqs. (3) and (5) and thus the verification of a general kinematic pair mathematical model, a testing jig has been designed and implemented, see Fig. 3. The aim of the experiments was the determination of the vertical distance change of two fitting pins, which are approached under the action of a normal load F. The lower pin is assigned to the stationary cylindrical body which represents a cam with the radii of curvature ρ_{x2} = 50 mm and the width L_2 = 25 mm and the upper pin to the body which represents a cylindrical or crowned cam rollers with the set of diameters d = {30, 35, 47, 62, 80, 90}.

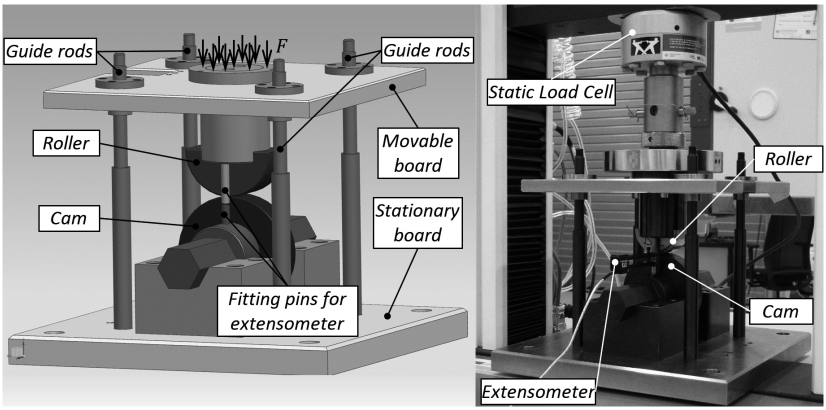

Fig. 3 Testing jig of contact zone deformation of two solids

The vertical displacement of the upper pin was determined by using the *Instron 3369* universal testing machine. The testing jig with the appropriate cam roller and with the connected uniaxial extensometer *Instron 2620-601* to pins was placed between its compressive jaws. Load cycles carried out between compressive forces F_1 and F_2, then strain values were recorded in the interval $\varepsilon_e \in \langle \varepsilon_{e1}; \varepsilon_{e2} \rangle$ [–]. The measured deformation of the contact region of the cam and the roller is defined by the equation: $\delta_e = l_0 \varepsilon_e$, where $l_0 = 12.5$ mm denotes the distance of extensometer contact tips. The percentage deviation between measured δ_e and theoretical δ values of the deformation is defined by the equation:

$$p = \left(\frac{\Delta\delta_e}{\Delta\delta} - 1 \right) \cdot 100\,\% = \left(\frac{\delta_{e2} - \delta_{e1}}{\delta_2 - \delta_1} - 1 \right) \cdot 100\,\% = \left(\frac{\delta_e(F_2) - \delta_e(F_1)}{\delta(F_2) - \delta(F_1)} - 1 \right) \cdot 100\,\%. \quad (8)$$

The typical load cycles are shown in Figs. 4 and 5 where arrows show the process of the loading. From these two examples, the hysteretic characteristic of the load cycle is evident. This is due to the fact that the contacting surfaces are not smooth and frictionless. In the case of the cylindrical cam rollers, the shape of the load cycle lower section is caused by the sum of a manufacturing inaccuracy of individual parts, see Fig. 4. Examples of calculated and measured data are presented in Table 1.

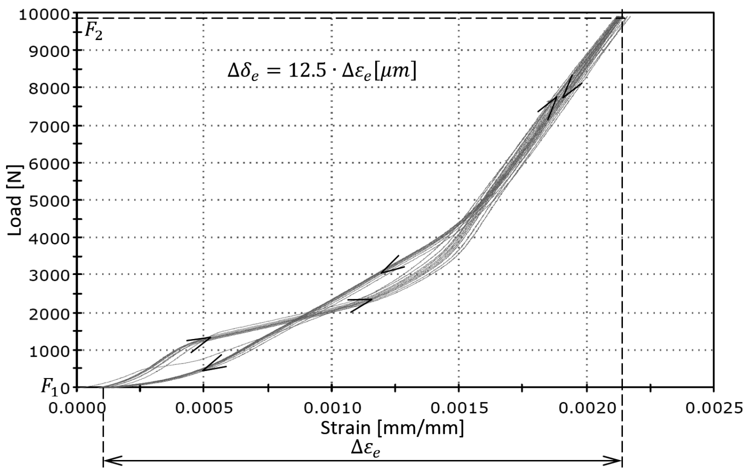

Fig. 4 Loading cycles of two cylindrical bodies

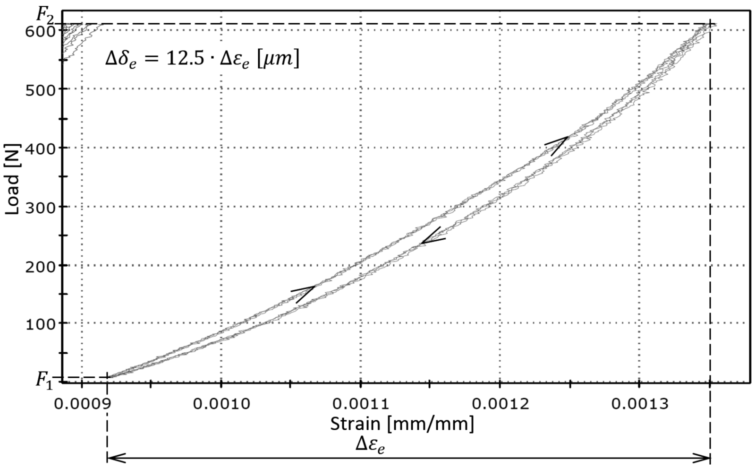

Fig. 5 Loading cycles of three-dimensional body and cylindrical body

Table 1 Contact region deformations of cam and rollers

Dimensions		Cylindrical cam roller				Crowned cam roller: R = 500 mm			
d (mm)	L_1 (mm)	F (kN)	Δδ (μm)	$\Delta\delta_e$ (μm)	p (%)	F (N)	Δδ (μm)	$\Delta\delta_e$ (μm)	p (%)
30	14	1.5–3.0	3.145	3.806	21.2	200–450	1.711	1.794	4.8
35	18	2.0–4.0	3.286	5.050	53.7	200–500	1.989	2.706	36.1
47	24	3.0–6.5	4.276	5.525	29.2	200–700	3.025	3.988	31.8
62	29	2.0–8.5	7.622	10.750	41.0	200–900	3.933	4.825	22.7
80	35	5.0–10.0	5.655	6.706	18.6	200–1000	4.325	5.969	38.0
90	35	3.5–10.0	7.398	9.094	22.9	200–1100	4.726	4.725	0.0

4 Conclusions

The Hertzian contact theory is an important tool for analysis in cam mechanism design. It allows for the prediction of compression and stiffness of a contact region in a general kinematic pair under a load. Then they are used in the definition of a model of the real general kinematic pair of the cam mechanism using the software MSC.ADAMS.

To the verification of the general kinematic pair mathematical model, a testing jig has been designed and implemented. On the basis of measurements, it is evident that the measured data are characterized by a greater value than the calculated data because the real contact area is formed smaller than the theoretical contact area under the action of a normal load. This is due to the fact, that the contacting surfaces are not smooth and frictionless and the real parts of the testing jig are characterized by a certain degree of manufacturing inaccuracies.

Acknowledgments This paper was created within the work on the project FR-TI4/801—Project supported by the Ministry of Industry and Trade.

References

1. Johnson, K.L.: Contact Mechanics. Cambridge University Press, Cambridge (1985). ISBN 0 521 34796 3
2. Arfken, G.B., Weber, H.J., Harris, F.E.: Mathematical Methods for Physicists, 7th edn., Section 18.8, “Elliptic Integrals”, pp. 927–933, Elsevier, Waltham (2013). ISBN 978-0-12384654-9