Offset of Natural Frequencies of Toothed Wheel by Means of Parametric Optimisation

J. Stadnicki and M. Głąbek

Abstract During the final stage of designing a toothed wheel which is exploited at different rotational speeds, it is occasionally necessary to offset natural frequencies from frequencies of excitations. The way of solving this problem by means of parametric optimisation of the toothed wheel profile, assuming small changes of its shape, is discussed in the paper. The problem is solved using finite element model with regard to monolithic toothed wheel of an aircraft gear.

Keywords Modal analysis · FEM · Parametric optimisation · Toothed wheel

1 Introduction

In high-speed aircraft gears, toothed wheels are often monolithic and, owing to the need to reduce the deadweight, shafts of such wheels are hollowed. An exemplary wheel form is shown in Fig. 1.

When designing a toothed wheel, gear teeth parameters, shape and dimensions of the envelope and the shaft are determined on the basis of the results of verification calculations, with consideration mainly to the criteria of safety, durability and design considerations. Compliance with these requirements sometimes results in the design of a toothed wheel whose natural frequencies are within the ranges of excitation frequencies which result from the ranges of the wheel rotational speeds occurring during operation, i.e. they lie in the areas of resonance. This applies particularly to gears that operate at different working speeds, for example, aircraft gears. Due to the fact that the wheel meets the requirements of safety, durability and design, there arises a problem of how to change the natural frequencies which lie in the areas of resonance, in other words, to offset some natural frequencies from excitation frequencies. Since solving this problem inevitably leads to changes in the shape and dimensions of the toothed wheel, whose project is already well advanced,

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Fig. 1 An exemplary
monolithic toothed with a
hollowed shaft

such a solution is being looked for that minimally changes the design of the wheel. In practice, these parts of the wheel are indicated whose change does not affect the already fulfilled requirements of safety, durability and accepted design considerations.

2 Modal Analysis of Toothed Wheel of Aircraft Gear

Modal analysis of a toothed wheel of an aircraft gear (Fig. 1) provides a set of frequencies and forms of free vibrations, called a modal model. The analysis was carried out using the finite element method in the NX Nastran 9 environment [1].

2.1 Finite Element Model of a Toothed Wheel

Figure 2 shows the FEM model of a toothed wheel. The FEM model of a toothed wheel (Fig. 2a) consists of CHEXA 8-node solid elements. Support of the shaft in rolling bearings (Fig. 2b) was modelled with the use of RBE3 elements with dependent nodes on the circuit of the shaft cross section in the plane of the bearing rolling elements and with an independent node on the symmetry axis of the shaft. In points, where dependent nodes for the left and right bearing are located, coincident node pairs were formed and linked by zero-dimensional CELAS1 elements with three translational stiffnesses (in X, Y, Z direction) and one rotational stiffness (around X axis). Numerical values of the stiffnesses were assumed as large enough (respectively 1×10^{10} N/mm and 1×10^{10} N mm) to be treated as rigid when
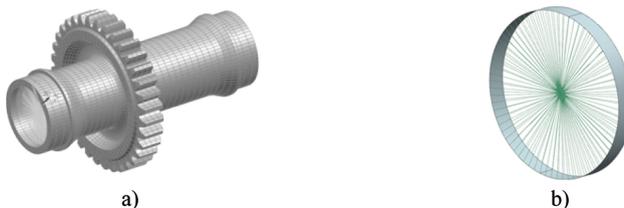

a)   b)

Fig. 2 **a** FE model of a toothed wheel of an aircraft gear, **b** the model of support

$$\omega_1^0 = 2816 Hz \qquad \omega_2^0 = 2816 Hz \qquad \omega_3^0 = 3610 Hz \qquad \omega_4^1 = 3934 Hz$$

$$\omega_5^1 = 3937 Hz \qquad \omega_6^0 = 5063 Hz \qquad \omega_7^2 = 5747 Hz \qquad \omega_8^2 = 5750 Hz$$
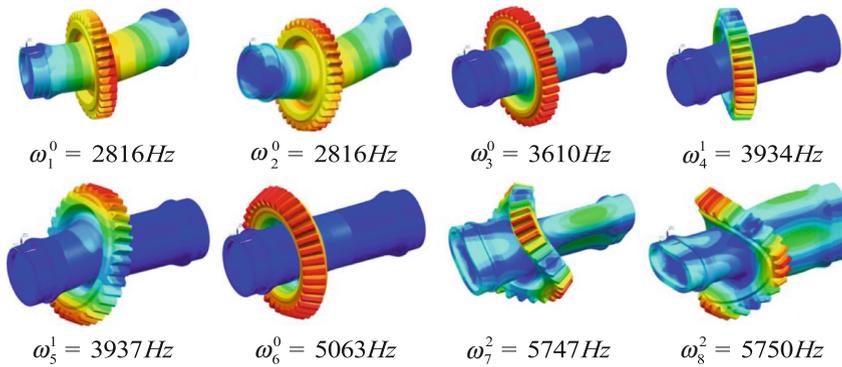
Fig. 3 Forms of free vibrations

compared to the shaft. Afterwards, one of the nodes of CELAS1 elements was deprived of all degrees of freedom. This way of support properly reflects real working conditions of a toothed wheel and eliminates over-rigidity of the FEM model. The developed model of a toothed wheel has 46 095 degrees of freedom.

The result of modal analysis of a toothed wheel FEM model is a set of eigenfrequencies and eingenforms of free vibrations. The values of a few of the first natural frequencies, important due to the possibility of resonance, and corresponding forms of vibration are shown in Fig. 3.

Natural frequencies and firms were determined according to ω_i^k—the convention where i is the ordinal number, whereas k is the number of nodal diameters around which there occurs the motion of the toothed wheel rim during vibrations. It can be noticed that the pairs of frequencies ω_1^0 and ω_2^0, ω_1^4 and ω_1^5, ω_7^2 and ω_8^2 are the same natural frequencies with the same corresponding forms shifted in a phase. At the same time, the phase shift in pairs ω_1^4 and ω_1^5, and ω_7^2 and ω_8^2 causes rotation of the form of vibrations of the toothed wheel rim around the axis of the wheel rotation, which is important for further analysis.

2.2 Campbell's Diagram

Engineering practice uses Campbell's diagram to compare the results of modal analysis of a toothed wheel—rotor with its rotational speeds which are the excitations of vibrations [2]. In the diagram, natural frequencies ω_i^k in Hz are given on the y-axis whereas rotations of excitation n are given in *rpm* on the x-axis. The analysed toothed wheel of an aircraft gear is operated at rotations corresponding to the following stages of operation: (minimum) $n_{\min} = 5500$ rpm, (flight) $n_p = 8500$ rpm, (climb) $n_w = 9500$ rpm (maximum) $n_{\max} = 11,000$ rpm (limiting 105 % max)

n_{gr} = 11,550 rpm. Campbell's diagram, shown in Fig. 4, shows the line of excitation by toothed wheel rotations (rotations growing from 0 to n_{gr} in Hz) and a line of excitation from meshing (for 35 wheel teeth, growing from 0 to 35 · n_{gr} in Hz). The lines on the diagram corresponding to natural frequencies ω_4^1, and ω_7^2 are not horizontal like the other lines but include a certain range of natural frequencies (there are pairs of straight lines inclined at small angles). This is caused by the rotation of the toothed wheel—the rotor around its own axis. If the toothed wheel remained motionless, the lines of natural frequency would remain horizontal in the diagram. However, due to rotations of the toothed wheel, centrifugal force caused by the rotations of the wheel around its own axis affects the wheel movement relative to the nodal diameter. Depending on whether these actions are consistent or opposite in phase, the result is an increase or decrease of natural frequency. This phenomenon is observed by analysing the result of modal analysis. If different phase-shifted eingenforms, for which one can indicate nodal diameters, correspond to the same frequency of free vibrations of the toothed wheel (rim), we are dealing with the change of natural frequency caused by rotations of the wheel (this is called the stiffening effect cased by centrifugal forces). Then, in Campbell's diagram such an eigenfrequency is represented by two straight lines diverging from one point, with equation:

$$\omega_i^k(n) = \omega_i^k(0) \pm k \cdot n \tag{1}$$

In Campbell's diagram in Fig. 4, two points marked □ were identified, where there is a potential risk of resonance. More specifically, the lower straight line of resonance frequency ω_7^2 intersects the straight line of excitation from meshing in point 1 whose abscissa differs from the rotational speed during the climb n_w by 1.89 % of its value. However, the straight line of resonance frequency ω_6^0 intersects the straight line of excitation from meshing in point 2 whose abscissa differs from the rotational speed during the flight n_p by 2.12 % of its value.
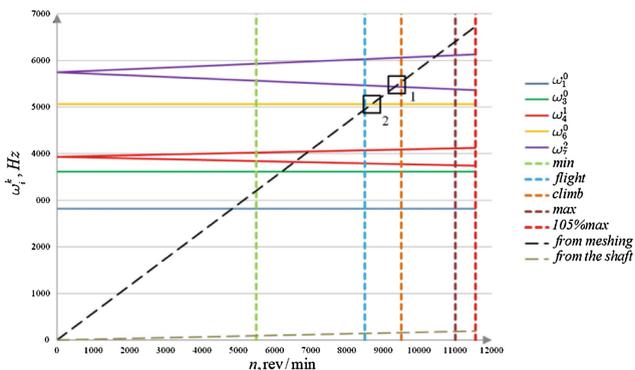

Fig. 4 Campbell's diagram of a toothed wheel

3 Parametric Optimization of a Toothed Wheel Shape—Offset of Resonance Frequencies

Since according to the adopted assumption, the offset of resonance frequencies must be performed with the least possible interference in the wheel envelope shape, the problem was solved using NX *Optimizer* for parametric optimization of the NX 9 programme. The toothed wheel, with an accuracy of gear teeth that cannot be changed, is axisymmetric. Parameters defining the profile in radial section of the wheel were assumed as decision variables of optimization problem. Figure 5 shows these profile parameters that will be subject to changes during optimization ($p51 = x_1$, $p52 = x_2$, $p53 = x_3$, $p82 = x_4$, $p55 = x_5$) and their initial values in mm. It should be noted that in order to maintain the profile integrity during the changes, suitable constraints are necessary. They are indicated in Fig. 5.

Since the eigenform for frequency ω_7^2 and the forms for frequencies ω_6^0 and ω_4^1 are the result of deformation of the toothed wheel rim, one can expect that these eigenfrequencies will be linked more strongly to one another than to the other ones. The offset of frequency ω_7^2 from the climb rotations n_w can be obtained by increasing or decreasing this frequency. Since the decrease causes the risk of resonance with maximum rotations $n_{\max}$, a better solution is to reduce ω_7^2, which also gives the chance to offset the frequency ω_6^0 from the flight rotations n_p. Thus, the criterion for optimization problem can be minimization of frequency ω_7^2.

$$Q_1(\bar{\mathbf{x}}) = \omega_7^2(\bar{\mathbf{x}}) \rightarrow \min, \quad \text{where} \quad \bar{\mathbf{x}} = [x_1, x_2, \ldots, x_5]^T \tag{2}$$

Solution to the problem (2) gave the following resonance frequencies: $\omega_1^0 = 2830\,\text{Hz}$, $\omega_3^0 = 3608\,\text{Hz}$, $\omega_4^1 = 3381\,\text{Hz}$, $\omega_6^0 = 4417\,\text{Hz}$, $\omega_7^2 = 5420\,\text{Hz}$. After having placed these values in Campbell's diagram (Fig. 4), one can notice that the adverse effect of the profile optimisation is the risk of resonance between frequency ω_4^1 (bottom line) and minimum rotations $n_{\min}$. In addition, optimisation
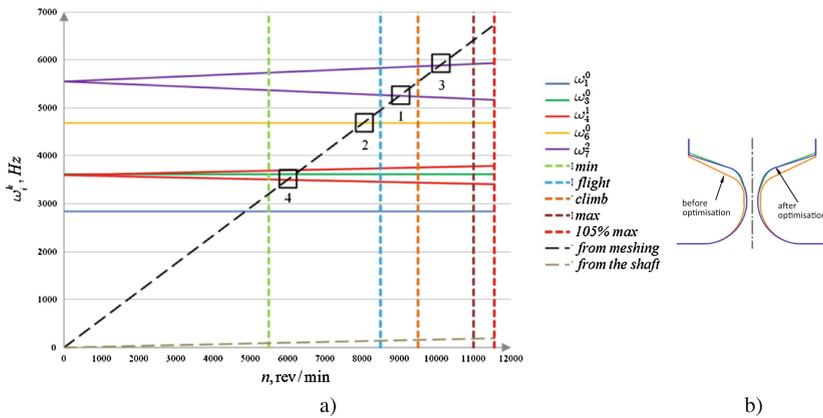
Fig. 5 Profile parameterization
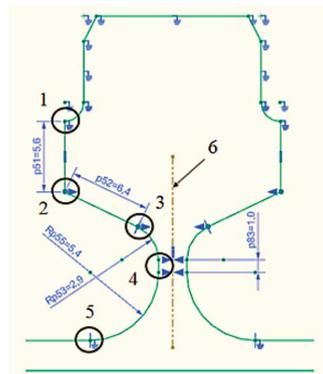
Fig. 6 a Campbell's of a toothed wheel after optimisation, **b** the profile—before and after optimisation

results confirmed the thesis that frequencies ω_4^1, ω_6^0 and ω_7^2, corresponding to eigenforms caused by the deformation of the rim, are interrelated. Taking into account the above conclusions, the following optimization problem was formulated

$$Q_2(\bar{\mathbf{x}}) = \omega_4^1(\bar{\mathbf{x}}) \rightarrow \max, \text{ with limitations} \tag{3}$$

$5540 \leq \omega_7^2 \leq 5550\,\text{Hz}$, □ 1 (Fig. 4) halfway between the flight and the climb, $\omega_6^0 \leq 4700\text{Hz}$, □ 2 (Fig. 4) sufficiently offset from flight rotations.

Figure 6a shows Campbell's diagram after the second optimisation whereas Fig. 6b presents the comparison of the toothed wheel profiles before and after optimisation. Numerical values of decision variables—profile parameters after optimisation and their percentage changes (%) in relation to the initial values are as follows: $x_1 = \text{p51} = 5.34\,\text{mm}$ (−4.6 %), $x_2 = \text{p52} = 5.76\,\text{mm}$ (−10 %), $x_3 = \text{p53} = 2.61\,\text{mm}$ (−10 %), $x_4 = \text{p83} = 0.98\,\text{mm}$ (−2 %), $x_5 = \text{p55} = 5.78\,\text{mm}$ (+7 %).

The toothed wheel with optimum profile dimensions met the requirements of security and stability as well as design requirements and in no range of operating rotations is there a risk of resonance.

4 Conclusions

The problem described in the article allows to draw the following conclusions:

– if the implemented project is in its final stage of development and yet it does not meet a particular requirement, such design parameters of the project should be

looked for that can be changed in a certain (usually small) range in order to meet all the requirements of the project,

– surfaces that are not dimensioned with fits and tolerances—free surfaces have the best chance to solve the problem of a small shape change in the final stage development of the project,

– a good way of introducing minor changes in the shape of the part in the final stage of the project development is parameterization of the part profile and then formulating and solving the problem of the optimal selection of the profile parameter according to the adopted criteria,

– modern tools for computer-aided design provide powerful tools for solving a parametric optimisation problem with the use of the FEM models of the project.

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