Influence of Bubbles in the Shock Liquid at Its Compressibility

M. Sivčák and T. Hruš

Abstract The article elaborates simplified thoughts of the effect of the size of the air bubbles and their interactions on the shock fluid compressibility. When fluid flows through the throttle hole, pressure drop occurs and the dissolved air is released from the liquid in the form of bubbles. These bubbles can influence the stiffness of the liquid. If such effect occurs e.g. in a hydraulic shock absorbers, than the damper stops perform its function. This phenomenon is called the delay. Releasing the air into the liquid can be observed even at low piston speed of the damper. However for low piston speeds the delay does not occur, the liquid and containing air bubbles behave as incompressible.

Keywords Air bubbles · Shock liquid · Cavitation · Delay

1 Introduction

The aim of this work is to verify the possibility of formation of the delay [1] in the damper using elementary physical laws. We assume that the delay is created by the compression of free air bubbles in the shock liquid. Releasing the air into the liquid can be observed even at low piston speed of the damper. However for low piston speeds the delay does not occur, the liquid and containing air bubbles behave as incompressible. We want verify whether the fluid stiffness depends on bubble radius and whether the bubbles are grouping into the bigger assemblies.

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J. Beran et al. (eds.), *Advances in Mechanism Design II*,
Mechanisms and Machine Science 44, DOI 10.1007/978-3-319-44087-3_27

2 Volume-Stiffness of the Bubbles

The set of n equal bubbles (each with radius R_n, pressure p_n and volume V_n) is described by three simple equations

1. the ideal gas law

$$p_n n V_n = p_0 V_0; \tag{1}$$

2. the relation between surface tension, radius and pressure

$$(p_n - p_k) R_n = 2\sigma; \tag{2}$$

3. the equation for volume of the bubble

$$V_n = \frac{4}{3}\pi R_n^3. \tag{3}$$

By substituting p_n from (2) and V_n from (3) into (1) we can obtain the ideal gas law in this form

$$3n\mathrm{p}_k \mathrm{V}_n + 2 \cdot 6^{\frac{2}{3}} \sqrt[3]{\pi} n \sigma V_n^{\frac{2}{3}} - 3p_0 V_0 = 0. \tag{4}$$

This enables us to find volume of bubble V_n as a function of pressure p_n. Volume stifness k_n of the bubble is given by

$$k_n = \left(\frac{\mathrm{d}V_n}{\mathrm{d}p_k}\right)^{-1}. \tag{5}$$

A set of n bubbles has a lower stiffness given by equation

$$K_n = \frac{1}{n\frac{1}{k_n}} = \frac{k_n}{n}. \tag{6}$$

An important question is *the change* of volume stiffness by changing the number of bubbles n

$$RATIO = \frac{K_n}{K_1}. \tag{7}$$

Exact forms of Eqs. (5), (6) and (7) are very complicated and not "human-readable".

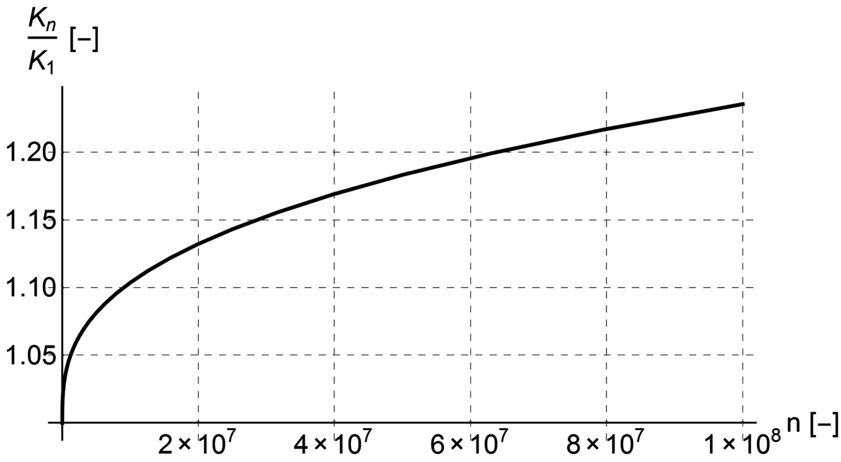

Fig. 1 Stiffness of a set of n equal bubbles (K_n) compared with stiffness of one bubble (K_1). The mass of gas in the bubbles is the same

Assuming values

$$\sigma = 40 \times 10^{-3}\,\text{N/m}, \tag{8}$$

$$p_k = 10^5\,\text{Pa}, \tag{9}$$

$$R_0 = 3\,\text{mm}, \tag{10}$$

we get the numerical result shown in Fig. 1. By decreasing number of bubbles, the volume stiffness of the gas is growing.

3 The Equilibrium State of Two Bubbles

The equilibrium position of two connected bubbles occurs if their potential energy is minimal. For this expression it is necessary to derive dependency of the potential energy on the distance between midpoints of the bubbles s12. The minimum must be subsequently applied as follows

$$\frac{\mathrm{d}Ep(s_{12})}{\mathrm{d}s_{12}} = 0. \tag{11}$$

Potential energy of two connected bubbles will be equal to sum of surface and pressure energy.

$$Ep = S\sigma + p_1 V_1 + p_2 V_2, \tag{12}$$

where S is total surface of the bubbles. Gas pressure energy inside the bubble is constant (we assume the validity of the equation of state) and thus irrelevant for searching of the energy minimum, just as surface tension.

Therefore the relation is determined by the equation

$$\frac{\mathrm{d}S}{\mathrm{d}s_{12}} = 0. \tag{13}$$

Let's introduce the marking (Fig. 2) where index 1 is spherical cap of the small bubble, index 2 for spherical cap of the bigger bubble and index 3 for spherical cap belonging to both bubbles.

Provided that formula (1) is valid, the following is true for our bubbles:

$$p_{i0} V_{i0} = p_i V_i. \tag{14}$$

The initial state of the two bubbles is determined by their size before merging

$$V_{i0} p_{i0} = \frac{4\pi R_{i0}^3}{3} \cdot \left(p_k + \frac{2\sigma}{R_{i0}} \right), \quad i = 1, 2 \tag{15}$$

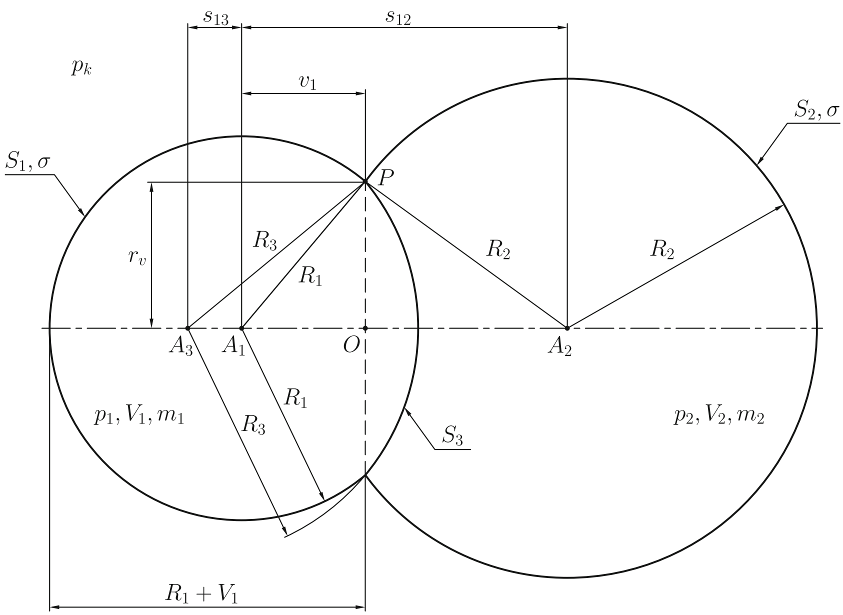

Fig. 2 Interaction between two bubbles

Pressure inside the bubble p_i is sum of pressure of surrounding fluid and addition due to surface tension

$$p_i = p_k + \frac{2\sigma}{R_i}, \tag{16}$$

Volume V_i is dependent on volumes of individual spherical caps.

$$V_i = \frac{\pi v_i\left(3r_v^2 + h_i^2\right)}{6} - \frac{\pi v_3\left(3r_v^2 + h_3^2\right)}{6}(-1)^i, \tag{17}$$

where h_i, h_3 are heights of the spherical caps and r_v is spherical cap radius

$$h_1 = R_1 + v_1, \quad h_2 = R_2 + s_{12} - v_1, \quad h_3 = R_3 - s_{13} - v_1 \tag{18}$$

and

$$r_v = \sqrt{R_1^2 - v_1^3}. \tag{19}$$

On the basis of force equilibrium it is possible to derive that radius R_3 is function of radii of outer spherical caps.

$$R_3 = \frac{2R_1R_2}{R_2 - R_1}, \tag{20}$$

distances s_{13} and v_1 are derived from the geometry on the Fig. 2 as follows

$$s_{13} = -v_1 + \sqrt{v_1^2 - R_1^2 + R_3^2}, \tag{21}$$

$$v_1 = \frac{R_1^2 - R_2^2 + s_{12}^2}{2s_{12}}. \tag{22}$$

The bubble surface will be calculated as a sum of surfaces of individual spherical caps, where the surface of the third cap must be counted twice.

$$S = 2\pi R_1h_1 + 2\pi R_2h_2 + 4\pi R_3h_3. \tag{23}$$

By substituting Eqs. (15)–(22) in the Eqs. (14) and (23) we get a system of three equations which are depending only on R_1, R_2 and s_{12}.

$$p_{i0}V_{i0} = p_i(R_1, R_2, s_{12}) \cdot V_i(R_1, R_2, s_{12}), \tag{24}$$

$$S = S(R_1, R_2, s_{12}), \tag{25}$$

Differentiating (25) by s_{12} with respect to (13) we get

$$\frac{\mathrm{d}S}{\mathrm{d}s_{12}} = \frac{\partial S}{\partial s_{12}} + \frac{\partial S}{\partial R_1}\frac{\mathrm{d}R_1}{\mathrm{d}s_{12}} + \frac{\partial S}{\partial R_2}\frac{\mathrm{d}R_2}{\mathrm{d}s_{12}} = 0. \tag{26}$$

We express unknown derivatives of radius by distance implicitly using derivatives of expressions (24). Because the solutions are difficult to read we call them f_1 and f_2.

$$\frac{\mathrm{d}(23)}{\mathrm{d}s_{12}} = 0 = f_i\left(R_1, R_2, s_{12}, \frac{\mathrm{d}R_1}{\mathrm{d}s_{12}}, \frac{\mathrm{d}R_2}{\mathrm{d}s_{12}}\right). \tag{27}$$

Equations (24), (26) and (27) form a system of five nonlinear algebraic equations with solution of equilibrium state

$$s_{12} = R_1 + R_2. \tag{28}$$

4 Conclusions

It is theoretically confirmed (Fig. 1) that stiffness of fluid with smaller bubbles is higher than the same amount of air in a smaller number of larger bubbles. The assumption that bubbles in fluid are clustering and, therefore, the stiffness of fluid is decreased, was not confirmed. The result (28) means that two bubbles located in fluid tend not to merge. Because bubbles in fluid do merge, the reason for drop of stiffness will be more complicated and we will focus on this phenomenon further.

Acknowledgments This publication was written at the Technical University of Liberec, Faculty of Mechanical Engineering with the support of the Institutional Endowment for the Long Term Conceptual Development of Research Institutes, as provided by the Ministry of Education, Youth and Sports of the Czech Republic in the year 2016.

Reference

1. Šklíba, J., Svoboda, R.: On the problem of dependence of damper force on the concentration of free air in working liquid. In: Springer Proceedings in Physics, Vibrations Problems, ICOVP-2007, 2008. ISBN 978-1-4020-9091-2