Nonlinear Vibration of Nuclear Fuel Rods

Š. Dyk and V. Zeman

Abstract From mechanical point of view, nuclear fuel rod (FR) is a complex system consisting of two subsystems—fuel rod cladding (thin walled zirconium tube) and a fuel pellets stack placed inside the cladding placed with a small radial clearance. Both subsystems are beam-type continua that possibly impact-interact during the vibration caused by fuel assembly support plates motion. The FR is supported at eight levels by prestressed spacer grid cells. The paper focuses on complex mathematical modelling of such a system including all the nonlinearities given by possible impacts between cladding and fuel pellets stack and possible loose of contact between cladding and spacer grid cells due to low prestress. In all the contact points, friction-vibration interactions respect three possible phases—stick, slip and separation—depending on the slip velocity and normal contact force. The model is used for estimation of a fretting wear of the cladding in contact with spacer grids.

Keywords Nuclear fuel rods · Nonlinear vibration · Impact · Friction forces · Wear

1 Introduction and Model Description

A fuel rod (FR) is a key part of nuclear fuel assembly as it is the component where the nuclear reaction occurs. Basically, there are two types of FAs—hexagonal and square-type, where the type refers to shape of FA's cross section. The paper deals with modelling of hexagonal-type FAs (see Fig. 1) that consists of 312 FRs, 18 guide thimbles, one centre tube and six angle-pieces that stiffen the FA's construction. At eight levels, regularly placed on the axis of the FA, there are spacer

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J. Beran et al. (eds.), *Advances in Mechanism Design II*,
Mechanisms and Machine Science 44, DOI 10.1007/978-3-319-44087-3_24

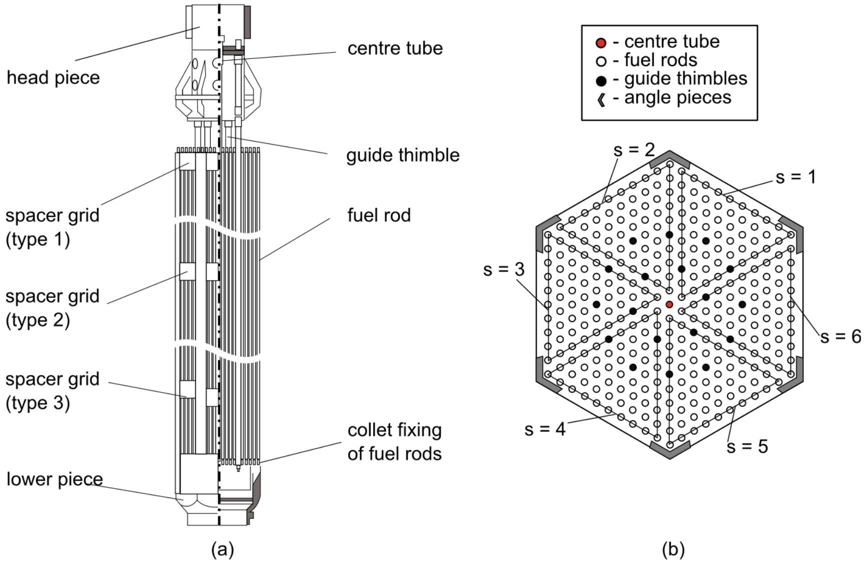

Fig. 1 A hexagonal-type fuel assembly—side view (**a**) and cross-section (**b**)

grids (SGs) that fix all the components in hexagonal cross section. Whole FA is submerged into coolant liquid (in case of VVER reactors, there is water as a coolant). In VVER-1000 reactors, there are 163 FAs in a reactor core that are fixed in lower and upper mounting plates. Due to pressure pulsations of coolant, the mounting plates are excited and their motion can be computed using global reactor model [1]. A SGs motion is investigated using linearized model of the chosen FA [2]. Obtained kinematical excitation is used in a detailed nonlinear model of one chosen FR.

A fretting wear of Zr FR cladding has been analysed in [3], but the modelling of FR vibration did not include friction forces in contact points whose effect was approximated by proportional damping. Friction forces can significantly influence results of numerical simulation. In the model described in this paper, friction and impact forces are fully considered.

A mechanical scheme of a nuclear fuel rod is shown in the Fig. 2. It consists of a FR cladding (C) in the form of long thin-walled Zr tube and fuel pellets stack (P) placed inside the C. The subsystem P is axially coupled with the C by hold-down fixation spring. Between both subsystems, there is a little radial clearance δ, that is considered to be constant through whole length of the FR. Both subsystems C and P are fixed at the bottom-end to the lower piece that is fixed in the moving lower FA support plate. At eight levels g, the cladding is supported by three spacer grid cells as shown in the Fig. 2.

Both subsystems are modelled using finite element method (FEM) for Euler-Bernoulli type one-dimensional continua. In each discrete node, all the six degrees of freedom are respected (axial displacement and two lateral displacements,

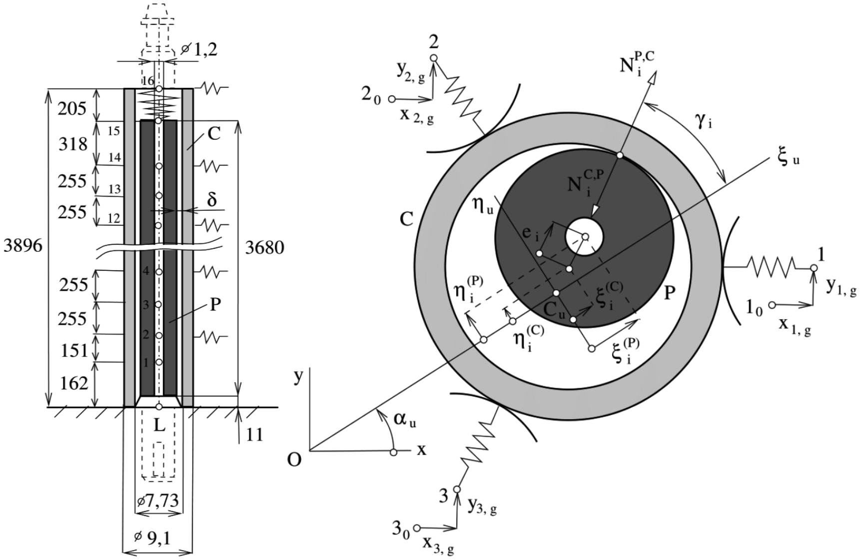

Fig. 2 Schematic mechanical model of the FR in side-view and a cross-section of the FR at the level of spacer grid *g*

torsional rotational angle and flexural bending angles). The mathematical model of the system including all above mentioned nonlinearities can be written in the form of set of second order ordinary differential equations

$$\begin{aligned}&\begin{bmatrix}\mathbf{M}_F^{(C)} & 0\\ 0 & \mathbf{M}_F^{(P)}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}_F^{(C)}\\ \ddot{\mathbf{q}}_F^{(P)}\end{bmatrix}+\begin{bmatrix}\mathbf{B}_F^{(C)} & 0\\ 0 & \mathbf{B}_F^{(P)}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}_F^{(C)}\\ \dot{\mathbf{q}}_F^{(P)}\end{bmatrix}\\ &\quad+\left(\begin{bmatrix}\mathbf{K}_F^{(C)} & 0\\ 0 & \mathbf{K}_F^{(P)}\end{bmatrix}\begin{bmatrix}\mathbf{q}_F^{(C)}\\ \mathbf{q}_F^{(P)}\end{bmatrix}+\mathbf{K}_{fix}\right)=\begin{bmatrix}\mathbf{f}_L^{(C)}(t)\\ \mathbf{f}_L^{(P)}(t)\end{bmatrix}+\begin{bmatrix}\mathbf{f}_{SG}\left(\mathbf{q}_F^{(C)},\dot{\mathbf{q}}_F^{(C)},t\right)\\ 0\end{bmatrix}\\ &\quad+\begin{bmatrix}\mathbf{f}_{P,C}\left(\mathbf{q}_F^{(C)},\dot{\mathbf{q}}_F^{(C)},\mathbf{q}_F^{(P)},\dot{\mathbf{q}}_F^{(P)}\right)\\ \mathbf{f}_{C,P}\left(\mathbf{q}_F^{(C)},\dot{\mathbf{q}}_F^{(C)},\mathbf{q}_F^{(P)},\dot{\mathbf{q}}_F^{(P)}\right)\end{bmatrix},\end{aligned}\tag{1}$$

where $\mathbf{q}_F^{(X)}, X = C, P$ are vectors of generalized coordinates of the free nodes $i = 1, 2, \ldots 16$, see Fig. 2, matrices $\mathbf{M}_F^{(X)}, \mathbf{B}_F^{(X)}, \mathbf{K}_F^{(X)}, X = C, P$, are mass, damping and stiffness matrices, respectively, of both subsystems and $\mathbf{K}_{fix}$ is stiffness matrix of fixation spring. At the right hand side of (1), there are vectors $\mathbf{f}_L^{(X)}(t), X = C, P$ of kinematical excitation by motion of the lower node *L*, and vector $\mathbf{f}_{SG}\left(\mathbf{q}_F^{(C)}, \dot{\mathbf{q}}_F^{(C)}, t\right)$ express forces caused by all the SGs. Last vector in (1) is vector of impact forces between subsystems P and C. The latter two vectors include both generalized displacements and generalized velocities of the subsystems which expresses nonlinear character of the contact forces. The model (1) can be formally rewritten into standard matrix form

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{B}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(\mathbf{q}(t), \dot{\mathbf{q}}(t), t), \tag{2}$$

that can be solved numerically after transformation to the form of the set of double number of first order equations.

2 Application and Results

The computational model of the system was implemented in MATLAB software. To describe a qualitative and quantitative changes in fuel rod vibration during an operational cycle, five states were determined, see Table 1. The initial state (I) is characterized by maximal radial clearance, maximal prestress of SG cells and minimal force of fixation spring. During the operational-cycle, the fuel pellets swell and thus the radial clearance is getting smaller and at the end (state V), there is a zero clearance. Due to swell effect, the fixation spring force is getting larger as the prestress grows. The prestress of SG cells is getting smaller due to wear in contact points. Between these two states, the parameters are considered to change linearly, see Table 1.

The fretting wear of the fuel rod cladding in the cell $j = 1, 2, 3$ and at the level of the spacer grid $g = 1, 2, \ldots 8$ can be estimated in the form [3]

$$\Delta m_{j,g} = \mu \frac{f(\omega)}{f_0} W_{j,g} \tag{3}$$

where μ is loss of FR cladding mass in one contact surface generated by the work of friction force 1 J at the excitation frequency $\omega, f(\omega)$ is experimentally obtained friction coefficient at the same frequency [4], f_0 is calculated friction coefficient and $W_{j,g}$ is the work of friction forces.

Figure 3 shows the fretting wear per a steady state part of simulation domain (0,3 s) and its evolution during an operational cycle of the reactor. The work of friction forces in (3) is obtained using nonlinear response of the system described by (1).

Table 1 Combination of system parameters considered for different states during reactor operational cycle

State number	Radial clearance between C and P (μm)	Prestress of SG cells (N)	Fixation spring hold-down force (N)
I	65	20	5
II	48.75	16.25	6.25
III	32.5	12.5	7.5
IV	16.5	8.75	8.75
V	0	5	10

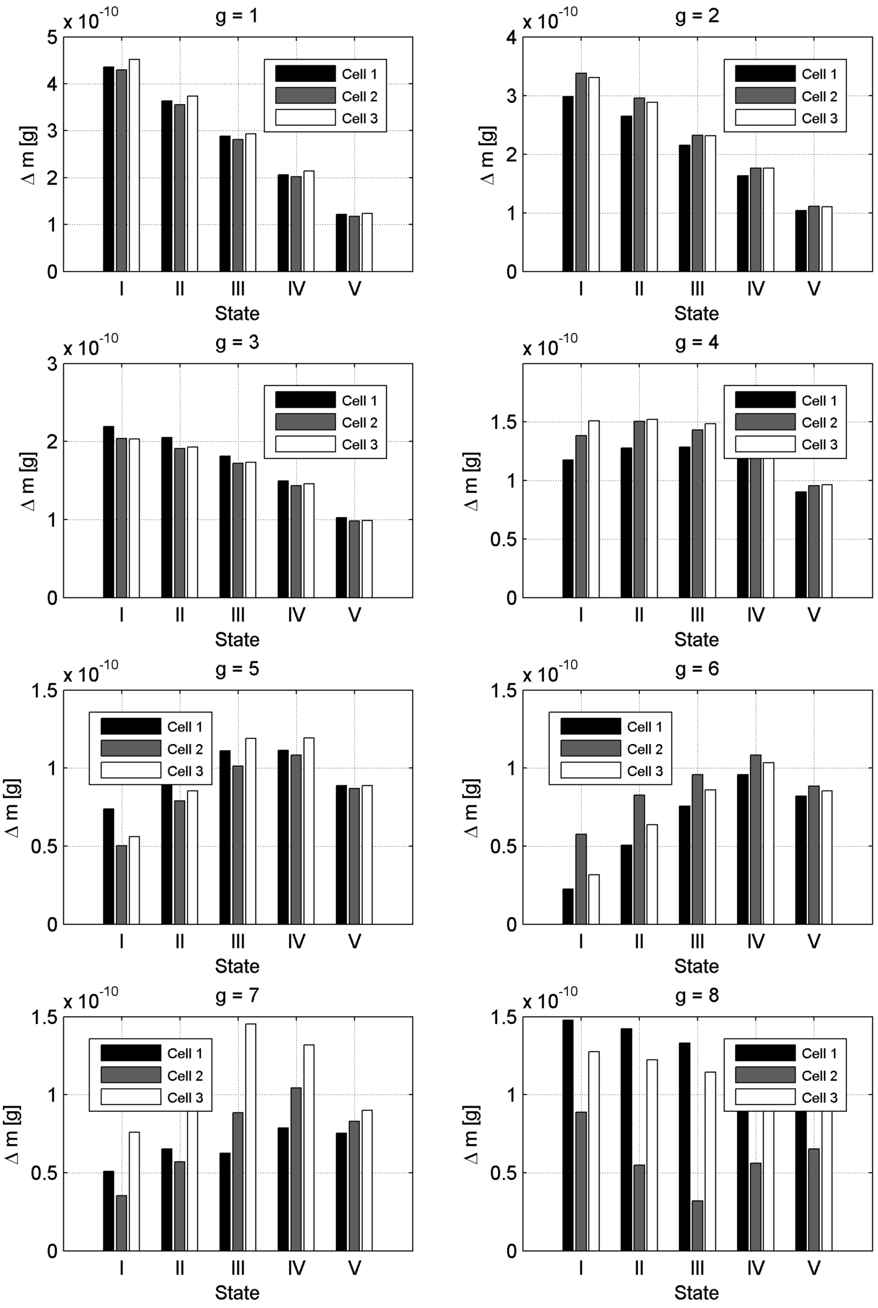

Fig. 3 Fretting wear per the simulation time at all the SG levels and in all the contact points

3 Conclusions

The contribution describes mathematical model of fuel rod in hexagonal-type fuel assembly that respects all the mechanical nonlinearities; impact and friction forces between fuel pellets stack and fuel rod cladding and between fuel rod cladding and spacer grid cells. Numerical simulation in time domain was performed to estimate the fretting wear of Zr cladding and its wall thickness that is very important for judgement of fuel operational-life left. The analysis shows the evolution of the wear during an operational cycle of the reactor based on estimated evolution of mechanical parameters such as a clearance between P and C, fixation spring force and prestress of spacer grid cells.

Acknowledgments This publication was supported by the project LO1506 of the Czech Ministry of Education, Youth and Sports.

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