Inertia Matrix and Vehicle Driveability

P. Brabec, M. Malý and R. Voženílek

Abstract This paper summarizes the evidence and results of the research focused on determining the inertia matrix (ellipsoid of inertia) of a passenger car aggregate. In the first part of the paper, the measurement results are introduced which are subsequently used for simulation of the whole vehicle dynamic behaviour.

Keywords Measurement · Inertia matrix · Ellipsoid of inertia · Vehicle dynamics

1 Introduction

For determining moments of inertia, various methods are used. Generally, they are based on a principle of dependence between the moment of inertia of a body and the natural oscillation frequency. The basic methods for determining moments of inertia are based on the principle of a physical pendulum, a torsion suspension or a bifilar suspension, eventually a trifilar or a quadrifilar one. When measuring it is supposed that the oscillation is undamped and the time of oscillation is measured [1, 2]. The aim of this research was to suggest a measurement methodology and pursuant to this methodology it would be possible to identify the main weight parameters of the system combustion engine and gearbox (aggregate). When choosing the method for determining the ellipsoid of inertia of the aggregate, the emphasis was placed particularly on the accuracy of measurement. Well specified inertia matrix of the aggregate is very important for further use in computer simulations, such as crash tests. The aggregate (or eventually motor) weight and centre-of-gravity position determination was performed on the basis of measuring the tensile force in

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individual fibres on the trifilar suspension. For determining the moments of inertia, the indirect method of measuring the time of oscillation of the body hung on the unifilar (torsion) suspension was used. For providing the moments of inertia of bodies of more complex shapes, special preparations were used. These preparations enabled to hung the body in various positions onto the torsion suspension. Therefore, a subframe for gripping the aggregate was used. This aggregate was gripped into the subframe in a defined way so that the axes of the chosen system of coordinates of the aggregate were parallel with the axes of the system of coordinates of the subframe. Because the moments of inertia of the subframe are not insignificant, it was necessary to perform the measuring twice. The first one was performed for the assembly of the subframe with the aggregate. In the second case only the subframe itself was measured by reason of separate parameters evaluation and determination of elements of inertia matrix of the aggregate and the subframe. For determining the inertia matrix, at least six measurements are needed. In our case, at least ten measurements towards an arbitrary axis passing through the beginning were performed so that we would reach the highest accuracy and subsequent calculation of the size and position of the ellipsoid of inertia. In general, for n measurements we will get the matrix of n equations with six unknowns. In this case, these unknowns can be determined from the set of measured data by using the method of least squares [3]. These unknowns are 3 moments of inertia towards the relevant axes: J_x, J_y and J_z and similarly 3 deviation moments: D_{xy}, D_{xz} a D_{yz}.

The methodology was verified when measuring several aggregates or motors, in some cases for more variants of motor accessories layout (thirteen aggregates and one motor designed for passenger vehicles, one motor designed for a commercial vehicle). The measurement was always performed without any filling, which means without (engine) oil and coolant.

In the graphs above, values of axial moments of inertia used for passenger vehicles are illustrated. These were ordinary multi-cylinder combustion engines (three- and four-cylinder ones, spark ignition and diesel ones, supercharged and not-supercharged ones) with both manual and automatic gearboxes. The values are presented in relation to a referential aggregate which was an ordinary light three-cylinder spark ignition engine with the volume of 1.2 dm^3 with a manual five-gear gearbox. The system of coordinates to which the values are attributable is expressed as follows: axis Y is identical with the axis of the crankshaft; axis Z is in plane of axes of cylinders and vertical to axis Y; axis X is vertical to the plane YZ.

From the graphs in Fig. 1 is obvious that the values sizes of axial moments of inertia of the aggregate are not linear dependent on the aggregate weight. The resulting values of moments of inertia and the centre-of-gravity position of the aggregate influence both the weight and also the position of additional devices and motor accessories. Neither relatively low physical parts which (regarding relatively big remoteness) can also influence the size of the overall moment of inertia of the aggregate cannot be neglected. The influence of the accessories was examined with careful attention; therefore, variant measuring of aggregates in different configuration with different accessories was also performed.

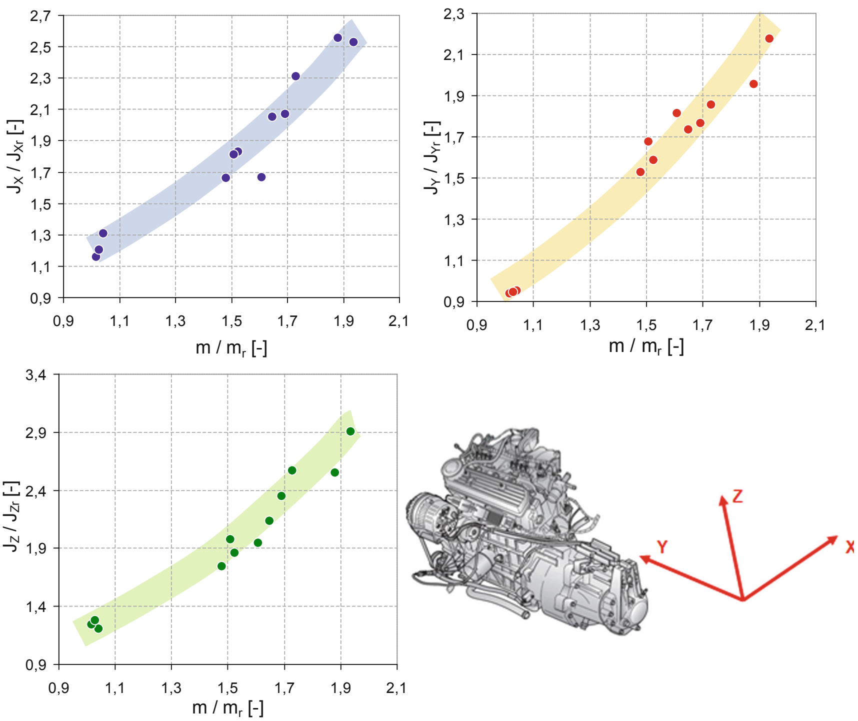

Fig. 1 Values of axial moments of inertia of passenger vehicles aggregate, introduced in relation to the referential aggregate

2 Aggregate and Dynamic Behaviour of a Vehicle

Vehicle driveability—manoeuvrability, comfort, and especially driving stability belong to the main criteria when designing a vehicle. When buying a vehicle, the customer chooses both equipment and accessories, and significant role is also drive unit parameters and the material whole which we have already identified as an aggregate.

The aim of the performed analyses with a number of laboratory measurements was to determine what influence this choice will have on the overall value of the inertia matrix (ellipsoid of inertia) of the whole vehicle and thus on the dynamic behaviour of the vehicle. For comparing behaviour of two variants of a small passenger car, two aggregates were chosen: an ordinary light atmospheric three-cylinder spark ignition engine with the volume of 1.2 dm^3 with a manual five-gear gearbox and an ordinary heavier supercharged four-cylinder diesel engine with the volume of 2 dm^3 with an automatic six-gear gearbox. Then, according to the found values of vehicles inertia [4–6], a virtual vehicle of its weight parameters are illustrated in Table 1 was chosen (Fig. 2).

Table 1 Values of the chosen virtual vehicle

	Virtual vehicle—choice of values for virtual bodywork with a driver (without aggregate)	Virtual vehicle with chosen petrol engine	Virtual vehicle with chosen diesel engine
m (kg)	1050	137	265
J_X (kg m^2)	388	404.6	422.6
J_Y (kg m^2)	1000	1232.6	1408.4
J_Z (kg m^2)	1275	1503.3	1680.6
D_{XY} (kg m^2)	0	−20.9	−36.4
D_{XZ} (kg m^2)	100	140.5	171.8
D_{YZ} (kg m^2)	−10	−12.8	−13.5

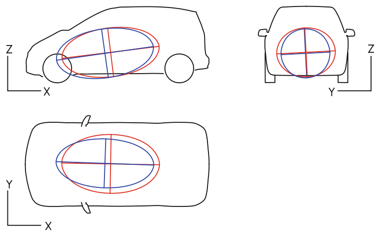

Fig. 2 Illustration of an ellipsoid of inertia for a virtual vehicle (*red*—a vehicle with spark ignition engine, *blue*—a vehicle with diesel engine) (color figure on line)

3 Influence of Inertia Matrix of a Vehicle on Dynamic Behaviour of a Vehicle

Mathematical description of the common vehicle movement represents a very complicated task. For simulations, models which are appropriately simplified are used. For the first approach, a simplified automobile model (often linear) is usually used. A simple model can relatively quickly offer quite good results which are important for a design and a construction of a control system. Then the next level of the model extension can follow. This extended model includes the possibility to assign more parameters of the vehicle. From the previous papers and simulations results it is obvious that for instance a simplified linear simulation model is considerably simpler for modelling. More simplifications were implemented; nevertheless, very good results were reached.

For a comparison, a linear simulation model of a vehicle [7, 8] was used. For comparing the vehicle behaviour with two different aggregates, an avoidance manoeuvre with the same value towards the side (around 2.2 m) was chosen. The vehicle with the diesel engine had bigger lateral force on the front axle and the wheels of the front axle had to turn at a greater angle. The rest of the outputs from the simulation model were about the same. In Fig. 3, the results of the simulation are illustrated. The vehicle with the diesel engine behaves like more understeering.

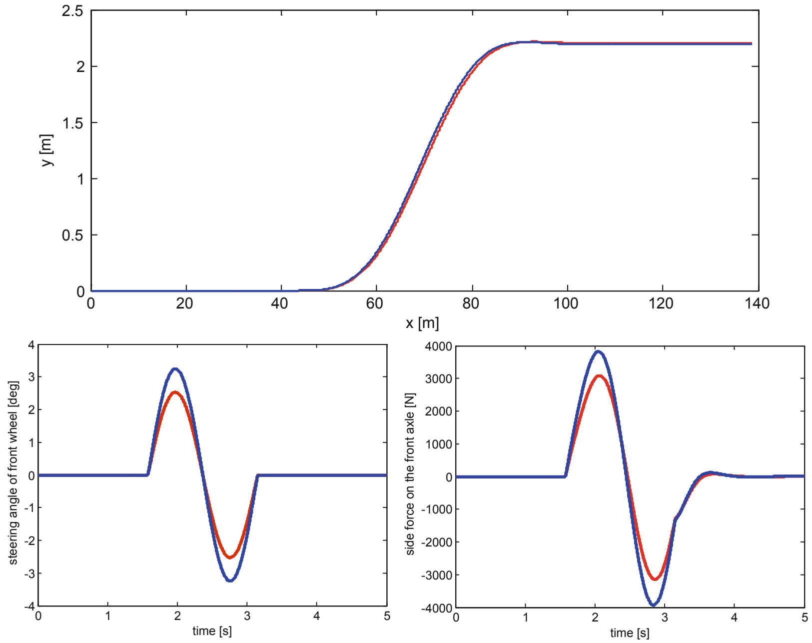

Fig. 3 Simulation results of an avoiding manoeuvre when using the simulation model of the vehicle: *bottom left-hand side*—angular displacement of the front wheels, *bottom right-hand side*—lateral force on the front axle (*red*—spark ignition engine vehicle, *blue*—diesel engine vehicle) (color figure on line)

4 Conclusions

The linear differential motion equations for a simple vehicle model have shown that the rotational vehicle motions will finish quickly if the vehicle weight is small, the moment of inertia around the vertical axis is small and the vehicle wheelbase is big. When dimensioning a motor vehicle, its reaction on both low and high frequencies of the steering-wheel rotation is important. The frequency characteristic is possible to discover both experimentally and by calculation. The evaluation of controllability according to the frequency characteristic process is, among others, performed according to the position of the amplitude maximum. The amplitude excess is the more distinct the lower the system attenuation is. For the given vehicle parameters, the amplitude maximum then depends on the driving speed [9].

It is further noted that on behalf of the fast driving manoeuvre, the amplitude maximum is supposed to lie at higher frequencies. If there is no amplitude excess (regarding the high damping), the decline of amplitude characteristics should appear as long as possible [10]. According to [11] there is no point for the vehicle resonance region in lying far beyond the value 5 s^{-1} by reason of the frequency response of the driver. On the basis of calculations, the assumption was confirmed that the resonance region position is dependent on design values and operating conditions: it moves towards the higher frequencies due to the small moment of inertia of a vehicle around its vertical axis, long wheelbase, use of tires with high lateral stiffness [9].

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