The Quantum Field Theory (QFT) Dual Paradigm in Fundamental Physics and the Semantic Information Content and Measure in Cognitive Sciences

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Abstract In this paper we explore the possibility of giving a justification of the "semantic information" content and measure, in the framework of the recent coalgebraic approach to quantum systems and quantum computation, extended to QFT systems. In QFT, indeed, any quantum system has to be considered as an "open" system, because it is always interacting with the background fluctuations of the quantum vacuum. Namely, the Hamiltonian in QFT always includes the quantum system and its inseparable thermal bath, formally "entangled" like an algebra with its coalgebra, according to the principle of the "doubling" of the degrees of freedom (DDF) between them. This is the core of the representation theory of cognitive neuroscience based on QFT. Moreover, in QFT, the probabilities of the quantum states follow a Wigner distribution, based on the notion and measure of quasiprobability, where regions integrated under given expectation values do not represent mutually exclusive states. This means that a computing agent, either natural or artificial, in QFT, against the quantum Turing machine paradigm, is able to change dynamically the representation space of its computations. This depends on the possibility of interpreting QFT system computations within the framework of category theory logic and its principle of duality between opposed categories, such as the algebra and coalgebra categories of QFT. This allows us to justify and not only to suppose, like in the "theory of strong semantic information" of L. Floridi, the definition of modal "local truth" and the notion of semantic information as a measure of it, despite both measures being defined on quasiprobability distributions.

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1 Introduction: A Paradigm Shift

Perhaps the best synthesis of the current paradigm shift in fundamental physics is the positive answer that it seems necessary to give to the following question: "Is physics legislated by cosmogony?". Such a question is the title of a visionary paper written in 1975 by J.A. Wheeler and C.M. Patton and published in the first volume of a successful series of the Oxford University about quantum gravity [1].

Such a revolution, suggesting a dynamic justification of the physical laws, fundamentally amounts to the so-called *information-theoretic approach* in quantum physics as the natural science counterpart of a *dual ontology* taking information and energy as two fundamental magnitudes in basic physics and cosmology. This approach started from Richard Feynman's influential speculation that a quantum computer could simulate any physical system [2]. This is the meaning of the famous "it from bit" principle posited by R. Feynman's teacher, Wheeler [3, p. 75]. The cornerstones of this reinterpretation are, moreover, D. Deutsch's demonstration of the universality of the quantum universal Turing Machine (QTM) [4], and C. Rovelli's overall development of a *relational* quantum mechanics (QM) [5]. An updated survey of such an informational approach to fundamental physics is provided in the recent collective book, edited by H. Zenil, and with contributions, among others, from R. Penrose, C. Hewitt, G. J. Chaitin, F. A. Doria, E. Fredkin, M. Hutter, S. Wolfram, S. Lloyd, besides D. Deutsch himself [6].

There are, however, several theoretical versions of the information-theoretic approach to quantum physics. It is not important to discuss all of them here (for an updated list in QM, see, for instance [7]), even though all can be reduced to essentially two:

1. The first one is the classical "infinitistic" approach to the *mathematical physics* of information in QM. Typical of this approach is the notion of the *unitary evolution* of the *wave function*, with the connected, supposed *infinite* amount of information it "contains" being "made available" in different spatiotemporal cells via the mechanism of the "decoherence" of the wave function. Finally, essential for this approach is the necessity of supposing *an external observer* ("information for whom?" [7]) for the foundation of the notion and of the measure of information. This is ultimately Shannon's purely syntactic measure and notion of information in QM [5]. Among the most prominent representatives of such an approach, we can quote the German physicist Zeh [8, 9] and the Swedish physicist Tegmark [10].

2. The second approach, the emergent one today, is related to a "finitary"[1] approach to the *physical mathematics* of information, taken as a fundamental physical magnitude together with energy. It is related to quantum field theory (QFT), because of the possibility it gives of spanning the microphysical, macrophysical, and even the cosmological realms, within only one quantum theoretical framework, differently from QM [17].

In this chapter we discuss the relevance of this second approach for the theory of *semantic information* in both biological and cognitive sciences.

2 From QM to QFT in Fundamental Physics

The notion of the quantum vacuum is fundamental in QFT. This notion is the only possible explanation, at the fundamental microscopic level, of the *third principle of thermodynamics* ("The entropy of a system approaches a constant value as the temperature approaches zero"). Indeed, the Nobel Laureate Walter Nernst first discovered that, for a given mole of matter (namely an ensemble of an Avogadro number of atoms or molecules) for temperatures close to absolute zero, T_0, the variation of the entropy ΔS would become infinite (through division by 0).

Nernst demonstrated that, to avoid this catastrophe, we have to suppose that the molar heat capacity C is not constant at all, but vanishes, in the limit $T \rightarrow 0$, to make ΔS finite, as it has to be. This means, however, that near absolute zero, there is a mismatch between the variation of the bodys content of energy and the supply of energy from the outside. We can only avoid such a paradox by supposing that such a mysterious inner supplier of energy is the vacuum. This implies that absolute zero is unreachable. In other terms, there is an unavoidable fluctuation of the elementary constituents of matter. The ontological conclusion for fundamental physics is that we can no longer conceive physical bodies as isolated.

> The vacuum becomes a bridge that connects all objects among them. No isolated body can exist, and the fundamental physical actor is no longer the atom, but the field, namely the atom space distributions variable with time. Atoms become the "quanta" of this matter field, in the same way as the photons are the quanta of the electromagnetic field [18, p. 1876].

[1]For the notion of "finitary" computation, as distinguished from "infinitistic" (second-order computation) and "finististic" (Turing-like computation), see [11]. This notion depends on the category theory (CT) interpretation of logic and computation [12], as far as based on Aczel's non-well founded (NWF) set theory [13], justifying a *coalgebraic semantics* in quantum computing [14], as far as based on the CT principle of the *dual equivalence* between a Boolean initial algebra and a final coalgebra [15, 16]. The key notion of the *doubling of the degrees of freedom* between a q-deformed Hopf algebra and a q-deformed Hopf coalgebra, as representing each quantum system in quantum field theory, perfectly satisfies such a logic, as we see below.

For this discovery, eliminating once and forever the notion of "inert isolated bodies" of Newtonian mechanics, Walter Nernst, a chemist, is one of the founders of modern quantum physics.

Therefore, the theoretical, core difference between QM and thermal QFT can be essentially reduced to the criticism of the classical interpretation of QFT as a "second quantization" of QM. In QFT, indeed, the classical Stone von Neumann theorem [19] does not hold. This theorem states that, for system with a *finite* number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations (CCRs)[2] are all *unitarily equivalent to each other*, so as.to justify the exclusive use of Shannon information in QM.

On the contrary, in QFT systems, the number of degrees of freedom is not finite, so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anticommutation (fermions) relations exist. Indeed, through the principle of *spontaneous symmetry breaking* (SSB) in the vacuum ground state, infinitely (not denumerable) many quantum vacuum conditions, compatible with the ground state, exist there. Moreover, this not holds only in the relativistic (microscopic) domain, but also applies to nonrelativistic many-body systems in condensed matter physics, i.e. in the macroscopic domain, and even on the cosmological scale [17, pp. 18. 53–96].

Indeed, starting from the discovery, during the 1960s, of dynamically generated long-range correlations mediated by *Nambu–Goldstone bosons* (NGBs) [20, 21], and hence their role in the local gauge theory through the Higgs field, the discovery of these collective modes deeply changed fundamental physics. Above all, it appears as an effective, alternative method to the classically Newtonian paradigm of perturbation theory, and hence to its postulate of the asymptotic condition.

In this sense, "QFT can be recognized as an *intrinsically thermal* quantum theory" [17, p. ix]. Of course, because of the intrinsic character of the thermal bath, the whole QFT system can recover the classical Hamiltonian character, because of the necessity of still satisfying the energy balance condition of each QFT (sub) system with its thermal bath ($\Delta E = 0$), mathematically formalized by the "algebra doubling" between a q-deformed Hopf algebra and its "dual" (see note 2) q-deformed Hopf coalgebra, where q is a thermal parameter [22].

Therefore, in QFT an uncertainty relation holds, similar to the one of Heisenberg, relating the uncertainty on the number of field quanta to that of the field phase, namely

$$\Delta n \Delta \varphi \geq \varphi(\hbar),$$

[2] It is useful to recall here that the *canonical variables* (e.g. position and momentum) of a quantum particle do not commute among themselves, like in classical mechanics, because of Heisenberg's uncertainty principle. The fundamental discovery of D. Hilbert consists in demonstrating that each canonical variable of a quantum particle commutes with the Fourier transform of the other (such a relationship constitutes a CCR), allowing a geometrical representation of all the states of a quantum system in terms of a commuting variety, i.e. the relative "Hilbert space".

where n is the number of quanta of the force field, and φ is the field phase. If $\Delta n = 0$, φ is undefined so that it makes sense to neglect the waveform aspect in favor of the individual, particle-like behavior. On the contrary, if $\Delta\varphi = 0$, n is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e. within a given phase coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particle behavior, since the collective modes of the force field prevail.

In QFT there is a duality between *two dynamic entities*: the fundamental force field and the associated quantum particles that are simply the quanta of the associated field that is different for different types of particles. In this a way, quantum entanglement does not imply any odd relationship between particles like in QM, but is simply an expression of the unitary character of a force field. To sum up, according to this more coherent view, the Schrödinger wave function of QM appears to be only a statistical coverage of the finest structure of the dynamic nature of reality.

3 QFT of Dissipative Structures in Biological Systems

3.1 Order and Vacuum Symmetry Breakdowns

It is well known that a domain of successful application of QFT is the study of the microphysics of condensed matter, that is in systems displaying at the macroscopic level a high degree of coherence related to an *order parameter.* The "order parameter", which is the macroscopic variable characterizing the new emerging level of matter organization, is related to the *matter density distribution*. In fact, in a crystal, the atoms (or molecules) are "ordered" in well-defined positions, according to a *periodicity law* individuating the crystal lattice.

Other examples of such ordered systems in the condensed matter realm include magnets, lasers, superconductors, etc. In all these systems, the emerging properties related to the respective order parameters are neither the properties of the elementary constituents, nor their "summation", but new properties depending on *the modes in which they are organized*, and hence on *the dynamics controlling their interactions.* In this way, for each new macroscopic structure, e.g. crystal, magnet or laser, there corresponds a new "function" the "crystal function", the "magnet function", etc.

Moreover, all these emerging structures and functions are controlled by *dynamic parameters,* that in engineering terminology, we can define as *control parameters.* Changing one of them, the elements can be subject to different dynamics with different collective properties, and hence exhibit different collective behaviors and functions. Generally, the temperature is the most important of them. For instance, crystals beyond a given critical temperature—that is different for different materials—lose their crystal-like ordering, and the elements acquire as a whole the macroscopic structure-functions of an amorphous solid or, for higher temperatures, they lose any static structure, acquiring the behavior-function of a gas.

So, any process of *dynamic ordering*, and of *information gain*, is related with a process of *symmetry breakdown.* In the magnet case, the "broken symmetry" is the rotational symmetry of the magnetic dipole of the electrons, and the "magnetization" consists in the correlation among all (most) electrons, so that they all "choose", among all the directions, that of the magnetization vector.

To sum up, any dynamic ordering among many objects implies an "order relation", i.e. a *correlation* among them. What, in QFT, at the *mesoscopic/macroscopic* level is denoted as *correlation waves* among molecular structures and their chemical interactions, at the *microscopic* level any correlation, and more generally any interaction, are as many coherent oscillation modes of force fields, mediated by *quantum correlation particles.* They are called "Goldstone bosons" or "Nambu–Goldstone bosons (NGBs)" [20, 21, 23], with mass—even though always very small (if the symmetry is not perfect in finite spaces)—or *without mass at all* (if symmetry is perfect, in the abstract infinite space). The lower the inertia (mass) of the correlation quantum, the greater the distance over which it can propagate, and hence the distance over which the correlation (and the ordering relation) constitutes itself.

However, an important caveat is necessary regarding the different role of Goldstone bosons as quantum correlation particles, and the bosons of the different energy fields of quantum physics (Quantum Electro-Dynamics (QED), and Quantum Chromo-Dynamics (QCD)). These latter are the so-called *gauge bosons*: the photons γ of the electromagnetic field, the gluons g of the strong field, the bosons $W^{\pm}$ and the boson Z of the electroweak field, and the scalar Higgs boson H^0 of the Higgs field, common to all these interactions.

The gauge bosons are properly mediators of *energy exchanges* among the interacting elements they correlate, because they are effectively quanta of the energy field they mediate (e.g. the photon is the quantum of the electromagnetic field). Therefore, the energy quanta are bosons that can change the *energy state* of the system. For instance, in QED of atomic structures, they are able to change the fundamental state (minimum energy) into one of the excited states of the electronic "cloud" around the nucleus.

On the contrary, NGB correlating quanta are not mediators of interactions among elements of the system. They determine only the *modes of interaction* among them. Hence, any symmetry breakdown in the QFT of condensed matter of chemical and biological systems has one only gauge boson mediator of the underlying energy exchanges, the photon, since they are all electromagnetic phenomena. Therefore, the phenomena involved here, from which the emergence of *macroscopic* coherent states derives, implies the generation, effectively the *condensation*, of correlation quanta with negligible mass, in principle null: the NGB, indeed. This is the basis of the fundamental "Goldstone theorem" [24, 25]. NGBs acquire different names for the different modes of interaction, and hence of the coherent states of matter they determine*:* *phonons* in crystals, *magnons* in magnets, *polarons* in biological matter, etc. Indeed, what characterizes the coherent domains in living matter is the phase coherence of the *electric dipoles* of the organic molecules and of the water, in which only the biomolecules are *active*. Therefore, although the correlation quanta are real particles, observable with the same techniques (diffusion, scattering, etc.), not only in QFT of condensed matter, but also in QED and in QCD like the other quantum particles, wherever we have to deal with broken symmetries [21], nevertheless they do not exist *outside* the system they are correlating. For instance, without a crystal structure (e.g. when heating a diamond over 3545 °C), we still have the component atoms, but no longer phonons. In this regard, the correlation quanta differ from energy quanta, like photons. Because the gauge bosons are *energy* quanta, they cannot be "created and annihilated" without residuals.

So, in any quantum process of particle "creation/annihilation" in quantum physics, what is conserved is the energy/matter, mediated by the energy quanta (gauge bosons), not their "form", mediated by the NGB correlation quanta. Also in this regard, a dual ontology (matter/form) is fundamental for avoiding confusion and misinterpretation in quantum physics.

Moreover, because the mass of the correlation quanta is in any case negligible (or even null), *their condensation does not imply a change of the energy state of the system.* This is the fundamental property for understanding how, not only the stability of a crystal structure, but also the relative stability of the structures/functions of living matter, at different levels of self-organization (cytoskeleton, cell, tissue, organ, etc.), can depend on such basic *dynamic* principles. In fact, all this means that, if the symmetric state is a fundamental state (a minimum of the energy function corresponding to a *quantum vacuum* in QFT of dissipative systems), also the ordered state, after symmetry breakdown and the instauration of the ordered state, remains *a state of minimum energy*, thus being *stable* in time. In kinematic terms, it is a *stable attractor* of the dynamics.

3.2 Doubling of Degrees of Freedom (DDF) in QFT and in Neuroscience

We said that the relevant quantum variables in biological systems are the *electrical dipole vibrational modes* in the water and organic molecules, constituting the oscillatory "dynamic matrix" in which also neurons, glia cells, and the other mesoscopic units of brain dynamics are immersed. The condensation of massless NGB (polarons)—controlling the electrical dipole coherent oscillation modes, and corresponding, at the mesoscopic level, to the long-range correlation waves observed in brain dynamics—depends on the triggering action of an external stimulus for symmetry breakdown of the quantum vacuum of the corresponding brain state. In such a case, the "memory state" corresponds to a coherent state for the basic quantum variables, whose mesoscopic order parameter displays itself as the amplitude and phase modulation of the carrier signal.

In the classical Umezawa model of brain dynamics [26], however, the system suffered from an "intrinsic limit of memory capacity". Namely, each new stimulus produces an associated polaron condensation, cancelling the preceding one, for a sort of "overprinting". *This limit does not occur in dissipative QFT where the many-body model predicts the coexistence of physically distinct patterns, amplitude modulated and phase modulated*. That is, by considering the brain as it is, namely an "open", "dissipative" system continuously interacting with its environment, there does not exist only one ground (quantum vacuum) state, like in the thermal field theory of Umezawa, where the system is studied at equilibrium; On the contrary, in principle, there exist infinitely many ground states (quantum vacuums), thus giving the system a potentially infinite capacity of memory. To sum up, the solution to the overprinting problem relies on three facts [27]:

1. In a dissipative (nonequilibrium) quantum system, there are (in principle) infinitely many quantum vacuum's (ground or zero-energy) states, on each of which a whole set of nonzero energy states (or "state space" or "representation states") can be built.
2. Each input triggers one possible irreversible time evolution of the system, by inducing a "symmetry breakdown" in one quantum vacuum, i.e. by inducing in it an ordered state, a coherent behavior, effectively "freezing" some possible degrees of freedom of the behaviors of the constituting elements (e.g. by "constraining" them to oscillate on a given frequency). At the same time, the input "labels" dynamically the induced coherent state, as an "unitary non-equivalent state" of the system dynamics. In fact, such a coherent state persists in time as a ground state (polarons are not energetic bosons, but Nambu-Goldstone bosons) thus constituting a specific "long-term" memory state for such a specific coupling between the brain dynamics and its environment. On the other hand, a brain that is no longer dynamically coupled with its environment is either in a pathological state (schizophrenia) or simply dead.
3. At this point, the DDF principle emerges as both a physical and mathematical necessity of such a brain model: physical, because a dissipative system, even though in nonequilibrium, must anyway satisfy the *energy balance*; mathematical, because the zero energy balance requires a "doubling of the system degrees of freedom". The *doubled* degrees of freedom, say $\tilde{A}$ (the tilde quanta, where the nontilde quanta A denote the brain degrees of freedom), thus represent the environment to which the brain state is coupled. The environment (state) is thus represented as the "time-reversed *double*" of the brain (state) on which it is impinging. The environment is hence "modeled on the brain", but according to the finite set of degrees of freedom *the environment itself elicited* in the brain.

What is relevant for our aims is that, each set of degrees of freedom A and for its "entangled doubled" $\tilde{A}$, there is a relater *unique number* $\mathcal{N}$, i.e. $\mathcal{N}_A, \mathcal{N}_{\tilde{A}}$, which in modul, $|\mathcal{N}|$, *univocally*, *identifies* i.e. *dynamically labels*, a given *phase coherence domain*, i.e. a quantum system state entangled with its thermal bath state, in our case, *a brain state matching its environment state.* This depends on the fact that, generally, in the QFT mathematical formalism, the number $\mathcal{N}$ is a numeric value expressing the NGB condensate value on which a phase coherence domain *directly depends*. In an appropriate *set-theoretic interpretation*, because for each "phase coherence domain" x, $|\mathcal{N}|$ effectively *identifies univocally* such a domain, it corresponds to an "identity function Id_x" that, in a "finitary" coalgebraic logical calculus, corresponds to the *predicate satisfied by such a domain because it identifies it univocally*. In other words, Vitiello's reference to the predicate "magnet function" or "crystal function" we quoted at the beginning of Sect. 3.1 are not metaphors, but are expressions of a fundamental formal tool—the "co-membership notion"—of the coalgebraic predicate calculus (see Sect. 5.2). Regarding the DDF applied to the quantum foundation of cognitive neuroscience, we have illustrated elsewhere its logical relevance, for an original solution of the reference problem (see [28, 29]).

There exists a huge amount of experimental evidence in brain dynamics of such phenomena, collected by W. Freeman and his collaborators. This evidence found, during the last ten years, its proper mathematical modeling in the dissipative QFT approach of Vitiello and his collaborators, justifying the publication during recent years of several joint papers on these topics (see, for a synthesis, [30, 31]).

To sum up [32], Freeman and his group used several advanced brain imaging techniques such as multielectrode electroencephalography (EEG), electrocorticograms (ECoG), and magnetoencephalography (MEG) study what neurophysiologist to generally consider the *background activity* of the brain, often filtering it out as "noise" with respect to the synaptic activity of neurons they are exclusively interested in. By studying these data with computational tools of signal analysis with which physicists, differently from neurophysiologists, are acquainted, they discovered the massive presence of patterns of AM/FM phase-locked oscillations. They are intermittently present in resting and/or awake subjects, as well as in the same subject actively engaged in cognitive tasks requiring interaction with the environment. In this way, we can describe them as features of the background activity of brains, modulated in amplitude and/or in frequency by the "active engagement" of a brain with its surroundings. These "wave packets" extend over coherence domains covering much of the hemisphere in rabbits and cats [33–36], and regions of linear size of about 19 cm in human cortex [37], with near-zero phasedispersion [38]. Synchronized oscillations of large-scale neuron arrays in the β and γ ranges are observed by MEG imaging in resting and motor-task-related states of the human brain [39].

4 Semantic Information in Living and Cognitive Systems

4.1 QFT Systems and the Notion of Negentropy

Generally, the notion of information in biological systems is a synonym of the *negentropy* notion, according to E. Schrödinger's early use of this term. Applied, however, to QFT foundations of dissipative structures in biological systems, the notion of negentropy is not only associated with the *free energy*, as Schrödinger himself suggested [40], but also with the notion of *organization*, as the use of this term by A. Szent-Györgi first suggested [41]. The notion of negentropy is thus related with the constitution of *coherent domains* at different space–time scales, as the application of QFT to the study of dissipative structures demonstrates, since the pioneering work by Frölich [42, 43].

In this regard, it is important to emphasize also the key role of the notion of *stored energy* that such a multilevel spatiotemporal *organization* in coherent domains and subdomains implies (i.e. the notion of quantum vacuum "foliation" in QFT), as distinct from the notion of *free energy* of classical thermodynamics [44]. Namely, as we know from the discussion above, the constitution of coherent domains allows chemical reactions to occur at *different timescales*, with a consequent energy release, thus becoming immediately available exactly *where/when it is necessary*. For instance, resonant energy transfer among molecules typically occurs in 10^{-14} s, whereas the molecular vibrations themselves die down, or thermalize, in a time between 10^{-9} and 10^{1} s. Hence, this is a 100% highly efficient and highly specific process, being determined by the frequency of the vibration itself, given that resonating molecules can attract one another. Hence, the notion of "stored energy" is meaningful at every level of the complex spatiotemporal structure of a living body, from a single molecule to the whole organism.

This completes the classical thermodynamic picture of Szilard [45] and Brillouin [46], according to which the "Maxwell demon" for getting information to compensate the entropic decay of the living body must consume free energy from the environment. This means an increase of the global entropy according to the Second Law. However, this has to be completed in QFT with the evidence coming from the Third Law discussed in this paper.

This occurs at the maximum level in the biological realm in human brain dynamics. To illustrate this point as DDF applied in neuroscience, Freeman and his collaborators spoke about "dark energy" for the extreme reservoir of energy hidden in human brain dynamics. The human brain indeed has 2% of the human body mass, but dissipates 20–25% of the body resting energy. This depends on the extreme density of cells in the cortices (10^5/mm^3), with an average of 10^4 connections [47].

To conclude this discussion, we showed that the "dual paradigm" related to the QFT interpretation of the "information-theoretic" approach to quantum physics does not depend on the distinction between "energy" and "information", like in the QM interpretation, where the "information" notion and measure—differently from the "energy" ones—are "observer-related", and therefore, logically, only "syntactic". In the QFT interpretation, where "information" is a physical magnitude, i.e. a thermodynamic *negentropy*, the duality concerns the two components of the negentropy notion and measure. These are, respectively, the *energetic* component (quantum "gauge bosons") and the *ordering* component (quantum "Nambu–Goldstone bosons") of a phase coherence domain, including the two entangled quantum states of the system and of its environment.

On the other hand, precisely because "ordering" is also a fundamental semantic notion in set-theoretic logic, the "semantic information" notion and measure strictly depend on the *logical* and *mathematical* notion of "duality". This duality in category theory logic concerns two opposed categories, specifically, in theoretical computer science (TCS), the notion of the "dual equivalence" between an algebra and its coalgebra, on which the notion of "local truth" and "finitary computation", on the one hand, as well as the notion and measure of *semantic information*, on the other, strictly depend.

These two notions of "duality", physical and logical, are however strictly interconnected in QFT, because both depend on the notion of *NGB condensates*, as constituting, respectively, the "ordering" component of the negentropy in information physics, and the sufficient condition for interpreting QFT systems as computing systems. A short introduction to all these notions will be the object of the rest of this paper, in the framework of the recent "coalgebraic approach" to quantum computing in TCS.

4.2 Syntactic Versus Semantic Information in Quantum Physics

4.2.1 Shannon's Syntactic Theory of Information in QM and in Mathematical Communication Theory

The Shannon nature of the notion and measurement of information that can be associated with decoherence in QM, overall in the relational and hence computational interpretations of QM illustrated above, has been emphasized [5]. In fact, in both cases, the "information" can be associated with the uncertainty H removal, in the sense that, the "more probable" or "less uncertain" an event/symbol is, the less informative (or, psychologically, less "surprising") its occurrence is. Mathematically, in the mathematical theory of communication (MTC), the information H associated with the ith symbol x among N (=alphabet), can be defined as

$$H = \sum_{i=1}^{N} p(x_i) I(x_i) = -\sum_{i=1}^{N} p(x_i) \log p(x_i),$$

where $p(x_i)$ is the relative probability of the ith symbol x with respect to the N possible ones, and I is the information content associated with the symbol occurrence, that is, the inverse of its relative probability (the less probable it is, the more informative its occurrence is). The information amount H thus has the dimensions of a statistical *entropy*, being very close to the thermodynamic entropy S of statistical mechanics:

$$S = -k_B \sum_i p(x_i) \log p(x_i),$$

where x_i are the possible microscopic configurations of the individual atoms and molecules of the system (microstates) which could give rise to the observed macroscopic state (macrostate) of the system, and k_B is the Boltzmann constant. Based on the correspondence principle, S is equivalent in the classical limit, i.e.

whenever the classical notion of probability applies, to the QM definition of entropy by John von Neumann:

$$S = -k_{\mathrm{B}}\mathrm{Tr}(\rho \log \rho),$$

where ρ is a density matrix and *Tr* is the trace operator of the matrix. Indeed, it was von Neumann himself who suggested to Claude Shannon to denote as "entropy" the statistical measure of information H he discovered. The informativeness associated with (the occurrence of) a symbol in the MTC (or with an event in statistical classical and quantum mechanics) is only "syntactic" and not "semantic" [48, p. 3]. Effectively, the symbol (event) occurs as *uninterpreted* (context independent) and *wellformed* (determined), according to the *rules* of a *fixed* alphabet or code (i.e. according to the *unchanged laws* of physics).

Anyway, starting from the pioneering works of Mackay [49] and of Carnap and Bar-Hillel [50], in almost any work dealing with the notion of information in biological and cognitive systems, the vindication of its *semantic/pragmatic* character is a leit motiv. Particularly, because information concerns here self-organizing and complex processes, in them the "evolution of coding", and the notion of "local (contingent) truth" (semantics), in the sense of *adequacy* for an optimal fitting with the environment (pragmatics), are essential [51–53]. More specifically, in QFT differently from QM, the pragmatic information content is significant, defined as the ratio of the rate of energy dissipation (power) to the rate of decrease in entropy (negentropy) [53] a measure generally considered in literature as the proper information measure of self-organizing systems. Evidently, in the DDF formalism of QFT, in the relationship between a quantum system and its thermal bath (environment), and specifically, in neuroscience, the relationship between the brain and its contextual environment, the notion and measure of pragmatic information, as described in [53], play an essential role [47].

What is to be emphasized here, above all, is that the Wigner function (WF) in QFT, from which the probabilities of the physical states are calculated, is deeply different from the Schrödinger wave function of QM, not only because the former, differently from the latter, is defined on the phase space of the system; What is much more fundamental is that the WF uses the notion of *quasiprobability* [54], and not the notion of probability of the classical Kolmogorov axiomatic theory of probability [55].

Indeed, the notion of quasiprobability allows regions integrated under given expectation values to not represent *mutually exclusive states*, thus violating one of the fundamental axioms of Kolmogorov's theory i.e. the separation of variables in such distributions is not fixed, but, as is the rule in the case of phase transitions, can evolve dynamically (see the QFT interpretation of the "quantum uncertainty principle" at the end of Sect. 2). From the computability theory standpoint, this means that a physical system in QFT, against the TM and QTM paradigms, is able to change dynamically the "basic symbols" of its computations, since new collective behaviors can emerge from individual ones, or vice versa. In this way, this justifies the definition of the information associated with a WF as a "semantic information content".

The semantic information in QFT computations hence satisfies, from the logical standpoint, the notion of *contingent*, or better, *local truth*, thus escaping from the Carnap and Bar-Hillel paradoxes (CBPs) [50]. To introduce this notion, it might be pedagogically useful to discuss briefly the "theory of strong semantic information" (TSSI) developed by L. Floridi, essentially because it shares with QFT the same notion of quasiprobability. In the QFT usage of the quasiprobability notion there is no necessity of violating also the other axiom of Kolmogorov's axiomatic theory of probability, i.e. the axiom excluding the "negative probabilities". On the contrary, Floridi uses the notion of negative probabilities [56], so that the reference to his theory has only a "pedagogical" value in the present context.

4.2.2 Floridi's Semantic Information Theory

Following the critical reconstruction of both theories (CSI and TSSI) by Sequoiah-Grayson [57], the CSI approach is based on Carnap's theory of intensional modal logic [58]. In this theory, given n individuals and m monadic predicates, we have 2^{nm} possible worlds and 2^m Q-predicators, intended as individuations of possible types of objects, given a conjunction of primitive predicates either unnegated or negated. A full sentence of a Q-predicator is a Q-sentence, hence a possible world is a conjunction of n Q-sentences, as each Q-sentence describes a possible existing individual. The *intension* of a given sentence is taken to be the set of possible worlds that make it true, i.e. are included by the sentence. This is in relation with the notion of *semantic information* in CSI, here referred to as the *content* of a declarative sentence s and denoted by "Cont(s)". In this way, the CBP consists in the evidence that, because an always true sentence is true for all possible worlds, i.e. it does not exclude any world, it is empty of any semantic content (effectively, it is a tautology) the maximum semantic content is for the always false (i.e. contradictory) sentence, because it excludes any possible world.

In Carnap & Bar Hillel terms, "a self-contradictory sentence asserts too much: it is too informative for being true" [50, p. 229]. Effectively, it is well known also to common sense that tautologies have no information content. What is paradoxical for common sense is that contradictions have the maximum information content. For logicians, however, who know the famous pseudo-Scotus law, according to which anything can be derived from contradictions (i.e., the so-called "explosion principle"), this conclusion is not surprising, once we have defined the information content of a sentence s, Cont(s), as the set of all sentences (possible worlds) belonging to the same universe W of the theory excluded by s.

Of course, the limit of CSI consists in its abstraction, namely in the *logical* notion of truth, and the a priori probability that it supposes. Surprisingly, but not contradictorily, it is just this supposition of *a logical notion of truth* (=true in all possible contexts, or "worlds" in modal logic terms) that makes it impossible to use truth as a necessary condition for meaningfulness in CSI.

What makes the TSSI of Floridi and followers interesting is that it offers a theory and measures of the semantic information for *contingent* and not *necessary* propositions namely for propositions that are not *logically* true, i.e. true for all possible worlds, in contrast to both tautologies (i.e. logical laws) and/or general ontology propositions which are true for whichever "being as being". Namely, both the propositions of all empirical sciences and the propositions of specific ontologies are *true* for objects *actually* existing (or that *existed*, or that *will exist*) only in *some* possible worlds—in the limit *one*: the actual, "present" world. In other terms, the scientific and ontological theories are "models" (i.e. theories true only for a limited domain of objects), precisely because both have semantic content, differently from tautologies. I developed elsewhere [59] a formal ontology of the QFT paradigm in natural sciences, in which this notion of truth is logically and ontologically justified, as an alternative to Carnap's logical atomism, i.e. alternative to the formal ontology of the Newtonian paradigm in natural sciences, on which both CSI and BCP depend.

Hence, it is highly significant to develop a *theory* and a *measure* of information content such as TSSI, compatible with what S. Sequoiah-Grayson defines as the *contingency requirement of informativeness* (CRI), supposed in TSSI. Unfortunately, a requirement such as CRI cannot be *supposed*, but only *justified*, as G. Dodig-Crnkovic indirectly emphasizes in her criticism of TSSI [60], and this is the limit of TSSI. In fact, the CRI states [57]: «A declarative sentence s is informative *iff* s individuates at least some but not all w_i from W (where $w_i \in W$)». Sequoiah-Grayson recognizes that CRI in TSSI is an idealization. However, he continues,

> Despite this idealization, CRI remains a convincing modal intuition. For a declarative sentence s to be informative, in some useful sense of the term, it must stake out a claim as to which world, out of the entire modal space, is in fact the actual world.

This requirement is explicitly and formally satisfied in the formal ontology of the "natural realism" as an alternative to the "logical atomism" of CSI [59, 61]. Effectively the main reason, Floridi states, leading him to defend the TSSI is that only such a theory having truthfulness as *necessary condition* for meaningfulness can be useful in an epistemic logic. In it, indeed, the entire problem consists in the justification of the passage from belief as "opinion" to belief as "knowledge", intended as a *true* belief.

That a CRI is operating in TSSI is evident from the "factual" character of the semantic information content in it, and of its probabilistic measure. Starting from the principle that semantic information σ has to be measured in terms of distance of σ from w, we have effectively four possibilities. Using the same example of Floridi [56, p. 55ff.], let us suppose that there are exactly three people in the room: this is the situation denoted in terms of the actual world w. The four possibilities for σ as to w are:

(T) There are *or* there are not people in the room;
(V) There are some people in the room;
(P) There are three people in the room;
(F) There are *and* there are not people in the room.

By defining θ as the distance between σ and w, we have: θ (T) = 1; θ (V) = 0.25 (for the sake of simplicity); θ (P) = 0; θ (F) = −1. From these relations it is possible to define the *degree of informativeness i* of σ, that is:

$$i(\sigma) = 1 - \theta(\sigma)^2.$$

The graph generated by the equation above (Fig. 1a) shows this as θ ranges from the necessary false (F) (=contradiction) to the necessary true (T) (=tautology), both showing the maximum distance from the contingent true (P).

To calculate the quantity of semantic information contained in σ relative to $\iota(\sigma)$, we need to calculate the area delimited by the equation above, that is, the definite integral of the function $\iota(\sigma)$ on the interval [0, 1]. On the contrary, the amount of vacuous information, which we denote as β, is also a function of θ. More precisely, it is a function of the distance of θ from w, i.e.

$$\int_0^\theta \iota(\sigma)\mathrm{d}x = \beta.$$

It is evident that, in the case of (P), $\beta = 0$. From α and β, it is possible to calculate the amount of semantic information carried by σ, i.e. γ, as the difference between the maximum information that can be carried in principle by σ and the vacuous information carried effectively by σ, that is, in bit:

$$\gamma(\sigma) = \log(\alpha - \beta).$$

Of course in the case of (P):

$$\gamma(\mathrm{P}) = \log(\alpha).$$

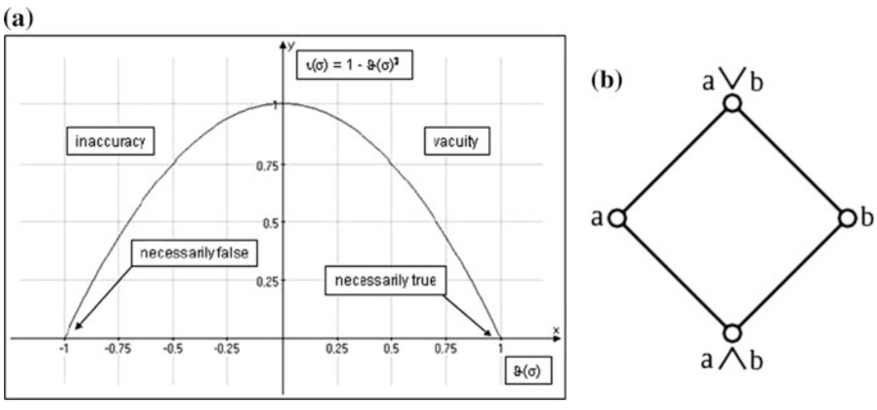

Fig. 1 **a** Degree of informativeness. From [56, p. 56], **b** Boolean lattice in equation logic

That confirms CRI in TSSI, that is, the contingently true proposition, namely denoting the actual situation *w* and/or expressing the true knowledge of *w*, carries the maximum semantic information about *w*.

5 Coalgebraic Semantics of Quantum Systems

5.1 Category Theory Logic and Coalgebraic Semantics

To satisfy Dodig-Crnkovic's criticism about the necessity of a *formal justification* of the notion of "local (contingent) truth" theory in logic and computability theory, let us start from the extension of the Boolean lattice (matrix) of Fig. 1b from the propositional calculus (Boolean equation logic) to the *monadic predicate calculus*, that is, where the proposition is $b = \neg a$. In such a case, the *meet* of the lattice ($a \wedge b$) would correspond to the *always false* proposition ($a \wedge \neg a$), and the *join* ($a \vee b$) would correspond to the *always true* proposition ($a \vee \neg a$) of the quasiprobability distribution of Fig. 1a, while the maximum of this distribution corresponds to the assertion of $|a|$ (and not of $a \mid b$, as in the lattice in figure) as "locally true". To make this representation computationally effective , it is necessary that we are allowed to associate this maximum to a measure of the *maximum of entropy* expressing the "matching" (convergence till equivalence) of the results of two "concurrent computations" of a system and of its environment, as the result of the "physical work" of the phase space dynamic reconfiguration (phase transition), consuming all the available "free energy", generated by the original "mismatch" between them.

What is highly significant for our aims is that in a way completely independent from quantum physicists—at least till the very last years (see Sect. 5.2 below)—logicians and computer scientists developed in the context of CT logic a coalgebraic approach to Boolean algebra semantics that only recently started to be applied also to quantum computing. Let us start from some basic notions of the CT logic (for a survey, see [12]).

The starting point of such a logic from set theory is that the fundamental objects of CT are not "elements" but "arrows", in the sense that also the set elements are always considered as domains-codomains of *arrows* or *morphisms*—in the case of sets, domains-codomains of *functions*.

In this sense, any object *A*, *B*, *C*, characterizing a category, can be substituted by the correspondent *reflexive morphism* $A \rightarrow A$ constituting *a relation identity* Id_A. Morover, for each triple of objects, *A, B, C,* there exists a *composition map* $AfBgC$, written as $g \circ f$ (or sometimes *f; g*), where *B* is the codomain of *f* and domain of *g*.[3]

[3]We recall that typical example of function composition is a recursive, iterated function: $x_{n+1} = f(x_n)$.

Therefore, a *category* is any structure in logic or mathematics with structure-preserving morphisms, e.g. in set-theoretic semantics, all the models of a given formal system, because sharing the same structure constitutes a category. In this way, some fundamental mathematical and logical structures are also categories: **Set** (sets and functions), **Grp** (groups and homomorphisms), **Top** (topological spaces and continuous functions), **Pos** (partially ordered sets and monotone functions), **Vect** (vector spaces defined on numerical fields and linear functions), etc.

Another fundamental notion in CT is the notion of *functor*, *F*, that is, an operation mapping objects and arrows of a category **C** into another **D**, *F:* **C** → **D,** so as to preserve compositions and identities. In this way, between the two categories, there exists a *homomorphism up to isomorphism*. Generally, a functor *F* is *covariant*, that is, it preserves arrows, directions and composition orders (e.g. in the QM attempt of interpreting thermodynamics within kinematics [62]); i.e. if $f: A \to B$, then $FA \to FB$; if $f \circ g$, then $F(f \circ g) = Ff \circ Fg$; if id_A, then $Fid_A = id_{FA}$. However, two categories can be equally homomorphic up to isomorphism if the functor *G* connecting them is *contravariant*, i.e. *reversing* all the arrows, directions and the composition orders, i.e. *G*: **C** → $\mathbf{D^{op}}$:

$$\text{if } f: A \to B, \text{ then } GB \to GA; \text{ if } f \circ g, \text{ then } G(g \circ f) = Gg \circ Gf; \text{ but if } id_A, \text{ then } Gid_A = id_{GA}.$$

Through the notion of contravariant functor, we can introduce the notion of *category duality*. Namely, given a category **C** and an *endofunctor E*: **C** → **C**, the contravariant application of *E* links a category to its opposite, i.e. E^{op}: **C** → $\mathbf{C^{op}}$. In this way it is possible to demonstrate the *dual equivalence* between them, in symbols: $\mathbf{C} \rightleftharpoons \mathbf{C^{op}}$. In CT semantics, this means that, given a statement α defined on **C**, α is true *iff* the statement α^{op} defined on $\mathbf{C^{op}}$ is also true. In other terms, truth is invariant for such an exchange operation over the statements, that is, they are *dually equivalent.* In symbols: $\alpha \rightleftarrows \alpha^{op}$, as distinguished from the ordinary equivalence of the logical tautology: $\alpha \leftrightarrow \beta$, defined within the very same category.

A particular category, indeed, that is interesting for our aims is the category of algebras, **Alg**. They constitute a category because any algebra $\mathcal{A}$ can be defined as *a structure on sets* characterized by an endofunctor projecting all the possible combinations (Cartesian *products*) of the subsets of the carrier set, on which the algebra is defined, onto the set itself, that is, $\mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$. The other category interesting for us is the category of coalgebras **Coalg**. Generally, a coalgebra can be defined as a structure on sets, whose endofunctor projects from the carrier set onto the *coproducts* of this same set, i.e. $\mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$. Despite appearances, an algebra and its coalgebra *are not dual.* This is the case, for instance, of a fundamental category of algebras in physics, that is, the category of *Hopf algebras*, **Halg**, generally used in dynamic system theory both in classical and in quantum mechanics, as we know. Each *HAlg* is essentially a *bi-algebra* because it includes two types of operations on/from the carrier set, where—because they are used to represent energetically closed systems—products (algebra, e.g., for calculating the energy of a single particle in a quantum state) and coproducts (coalgebra, e.g., for calculating the total energy of two particles in the same quantum state) can be defined on the same basis, and therefore *commute* among themselves. That is, there exists a complete *symmetry* between a *HAlg* and its *HCoalg,* so that they are *equivalent* and not *dually equivalent.* In this sense, any Hopf algebra is said to be *self-dual*, that is, isomorphic with itself. To make, on the contrary, a Hopf algebra dually equivalent with its coalgebra, as we know from thermal QFT, we have to introduce a q-deformation, where q is a thermal parameter. In this case, both coproducts and then products do not commute among themselves. In fact, in the case of coproducts used for calculating a quantum state total energy, they are associated to a system state energy and to its thermal-bath state energy, so that they cannot commute among themselves.

More generally, indeed, it is possible to define a dual equivalence between two categories of algebras and coalgebras by a contravariant application of the same functor. We might give two examples of this notion, the first in mathematics and computability theory concerning Boolean algebras, and the second in computational physics concerning QFT.

5.2 Coalgebraic Semantics of a Boolean Logic for a Contravariant Functor

The first example, concerning Boolean algebras, depends essentially on the fundamental representation theorem for Boolean algebras demonstrated in 1936 by the American mathematician M. Stone, five years after having demonstrated with John von Neumann the fundamental theorem of QM we quoted in Sect. 2. Indeed, the Stone theorem associates each Boolean algebra B to its Stone space $S(B)$ [63]. Therefore, the simplest version of the Stone representation theorem states that every Boolean algebra B is *isomorphic* to the algebra of partially *ordered by inclusion* closed-open (clopen) subsets of its Stone space $S(B)$, effectively an *ultrafilter*[4] of the power set of a *given set (interval) of real numbers* defined on $S(B)$.

Because each monotone function between a Boolean algebra A and a Boolean algebra B corresponds to a continuous function from $S(B)$ to $S(A)$ in the opposite direction so to make them *dual*, we can state that each endofunctor Ω in the category of the coalgebras on Stone spaces, **SCoalg,** *induces* a contravariant functor in the category of the Boolean algebras, **BAlg** [64, 65]. In CT terms, the theorem—effectively demonstrated by Abramsky in 1988—states the *dual equivalence* between them for the contravariant application of the "Vietoris functor" $\mathcal{V}$, i.e. **SCoalg(** $\mathcal{V}$) $\rightleftharpoons$ *BAlg*($\mathcal{V}$*). Let us deepen this fundamental point, by summarizing the essential steps leading to this result.

It is difficult to exaggerate the fundamental importance of the Stone theorem that, according to computer scientists, inaugurated the "Stone era" in computer science. In

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[4]We recall here that by an "ultrafilter" we mean the maximal partially ordered set defined on the power set of a given set ordered by inclusion, and excluding the empty set.

Particular, this theorem demonstrated definitively that Boolean logic semantics requires only *a first-order semantics* because it requires only *partially ordered* sets and not *totally ordered sets*. This result is particularly relevant for the foundations of computability theory. Indeed, the demonstration of the fundamental Lövenheim–Skolem theorem (1921) blocked the research program of E. Schröder of the so-called algebra of logic in the foundations of mathematics and of calculus [64], because it demonstrated that algebraic sets are not able to deal with *non-denumerable sets*, e.g. with the *totality* of real numbers. For this reason, and the subsequent fundamental demonstrations of Tarski's theory of truth as correspondence (1929) [65], and of Gödel's incompleteness theorems (1931) [66], the set-theoretic semantics migrated to higher-order logic, in order to grant the *total ordering* of sets, by some foundation axiom, e.g. the *axiom of regularity* in Zermelo-Fraenkel (ZF) set-theory. In this way, no *infinite chain of inclusions* among sets is allowed in *standard* set theory, so as to separate the semantic "set ordering" from the complete "set enumerability".[5]

Therefore, the further step for making the Stone theorem computationally effective for a Boolean first-order semantics, avoiding the limits of the Turing-like computation scheme, where a UTM is effectively a second-order TM as to an infinity of first-order TMs, and then strictly dependent on Gödel and Tarski theorems, is the definition of *nonstandard* set theories without foundation axioms. In this way, we allow infinite chains of set inclusions, according to the original intuition of the Italian mathematician E. De Giorgi [67, 68]. The most effective among the non-standard set theories is Aczel's set theories of *non-well-founded (NWF) sets* based on the *anti-foundation axiom* (AFA) [13]. The AFA, indeed, allowing set *self-inclusions* and therefore infinite chains of set inclusions, makes it also possible to define the powerful notion of set *co-induction* by *co-recursion, dual* to the algebraic notion of *induction* by *recursion*, both as formal methods of set definition and proof [15, 68, 69] (see Appendix A.1).

In this sense, the key role of the AFA is threefold:

1. Above all, it grants the *compositionality* of the set inclusion relations by prohibiting that the ordinary transitivity rule (TR), $\langle \forall u, v, w((uRv \wedge vRw) \rightarrow uRw) \rangle$, where R is the inclusion relation and u, v, w are sets—holds in set inclusions, because TR supposes the set total ordering. In this way, because only the "weaker" transitivity of the Euclidean rule (ER) $\langle \forall u, v, w((uRv \wedge uRw) \rightarrow vRw) \rangle$ between inclusions is allowed here, this means that the representation of sets ordered by inclusion as *oriented graphs*, in which the nodes are sets and the edges are inclusions with only one root (in our case the set u), *always* satisfies an "ascendant–descendant relationship" without "jumps" (each descendant always

[5]Two corollaries of the Lövenheim–Skolem theorem, demonstrated by Skolem himself in 1925, are significant for our aims, i.e. (1) that only *complete* theories are *categorical*, and (2) that the *cardinality* of an algebraic set depends intrinsically by the algebra defined on it. Think, for instance of the principle of *induction by recursion* for Boolean algebras, allowing a Boolean algebra to *construct* the sets on which its semantics is justified, blocking however Boolean computability on *finite* sets. It is evident that Zermelo's strategy of migrating to second-order set-theoretic semantics grants categoricity to mathematics on an *infinitistic* basis.

has its own ascendant i.e. they form a *tree*). This is the core of the "compositionality" of the *inclusion operator* of a coalgebra defined on NWF sets, i.e. the basis of the so-called tree-unfolding of NWF sets, starting from an "ultimate root" similar to the *universal set V*—which is allowed here, because of the possibility of set self-inclusion,[6] i.e. the disjunction of all sets forming the universe of the theory, like the "join" of a Boolean lattice. All this is the basis for extending the duality between the category **Stone** of Stone spaces, and the category **BAlg**, to the dual equivalence between the category of the coalgebras **Coalg** and the category of the algebras **Alg**, for an induced contravariant functor Ω^*, i.e. $\mathbf{Coalg(\Omega)} \rightleftharpoons \mathbf{Alg(\Omega^*)}$ [16, p. 417ff].[7]

2. Secondly, the AFA and the "final coalgebra theorem" justify *the coalgebraic interpretation of modal logic* in the framework of *first-order logic* (see the fundamental van Benthem theorem in this regard [70]) because the principle of set unfolding for partially ordered sets within an unbounded chain of set inclusions gives us an algebraically "natural" interpretation of the modal *possibility operator* "$\diamond$", in the sense that $< \diamond\alpha >$ means that "α is true in *some* possible worlds" [71–74], so as to give a computationally effective (first-order logic, where the predicate calculus is complete) justification to Thomason's early program of "reduction of the second-order logic to the modal logic" [75], made effective by another celebrated theorem, the Goldblatt–Thomason theorem. Because any set tree can be modeled as a *Kripke model*, this theorem defines rigorously which elementary classes of Kripke models are modally definable (for a deep discussion of this theorem, see [76, pp. 33–43]. For an intuitive treatment of these notions, see Sect. A.2 in the Appendix).
3. Thirdly, in the fundamental paper of 1988 [11], Abramsky first suggested that the endofunctor of modal coalgebras on Stone spaces, defined on NWF sets, is the so-called Vietoris functor $\mathcal{V}$.[8] In this way we can extend the duality between coalgebras and algebras for the induction of a contravariant functor Ω^*, to the *dual equivalence* between modal coalgebras on Stone spaces and modal Boolean algebras for the induction of a contravariant functor $\mathcal{V}^*$, i.e., $SCoalg(\mathcal{V}) \rightleftharpoons$

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[6]Recall that set self-inclusion is not allowed for standard sets because of Cantor's theorem. This impossibility is the root of all semantic antinomies in standard set theory, from which the necessity of a second-order set-theoretic semantics ultimately derives.

[7]This depends on the trivial observation that a coalgebra $C = \langle C,\gamma : C \rightarrow \Omega C \rangle$, where γ is a transition function characterizing C, over an endofunctor Ω: $\mathbf{C} \rightarrow \mathbf{C}$ can be seen also as an algebra in the opposite category $\mathbf{C^{op}}$, i.e. $\mathbf{Coalg(\Omega)} = (\mathbf{Alg(\Omega^{op})})^{\mathbf{op}}$ [16, p. 417].

[8]The fundamental property of $\mathcal{V}$ is that it is the counterpart of the power set functor $\wp$ in the category of the topological spaces (i.e. for continuous functions) such as the Stone space category, **Stone**. This functor maps a set S to its power set $\wp(S)$ and a function $f: S \rightarrow S'$ to the image map $\wp f$ given by $(\wp f)\ (X) := f\,[X]\ (= \{\ f(x) \mid x \in X\})$. Applied to Kripke's relational semantics in modal logic, this means that Kripke's *frames* and *models* are nothing but "coalgebras in disguise". Indeed, a *frame* is a set of "possible worlds" (subsets, s) of a given "universe" (set, S) and a binary "accessibility" relation R between worlds, $R \subseteq S \times S$. A Kripke's *model* is thus a frame with an *evaluation function* defined on it. Now R can be represented by the function $R[\bullet]: S \rightarrow \wp(S)$, mapping a point s to the collection $R[s]$ of its successors. In this way, frames in modal logic correspond to coalgebras over the *covariant* power set functor $\wp$. For such a reconstruction see [16, p. 391].

BAlg($\mathcal{V}^*$**)** [16, p. 393ff]. This depends on the fact that $\mathcal{V}$ is a functor defined on a particular category of topological spaces, the category of the vector spaces **Vect** we introduced in Sect. 5.1. Vector spaces are fundamental in physics; also the Hilbert spaces of the quantum physics mathematical formalism belong to this category, and, overall, the topologies of Stone spaces are the same of the C^*-algebras associated to Hilbert spaces in quantum physics. On the other hand, the morphisms characterizing the vector space category are, indeed, linear functions, so if we apply to modal coalgebras van Benthem's "correspondence theorem" [77] and the consequent "correspondence theory" [70] between the modal logic and the decidable fragments of the first order monadic predicate calculus, associating each axiom of modal calculus with a first-order formula (see Appendix A.2 for some examples), we obtain the following amazing result that Abramsky first suggested [11], and Kupke et al. developed [74]. Namely, we can formally justify the modal coinduction (tree unfolding) of predicate domains so as to justify *the modal operators* of "possible converse membership" or "possible co-membership", $\langle \ni \rangle$, and of "actual co-membership", i.e. $\neg\langle\neg\ni\rangle$, that is, $[\ni]$, where the angular and square parentheses are reminders of the possibility-necessity, "$\Diamond$-$\Box$" operators, respectively [16, p. 392ff].

What, intuitively, all this means for our aims is that, because modal coalgebras admit only a *stratified (indexed) usage* of the necessity operator $\Box$ and of the universal quantifier $\forall$, since a set *actually* exists as far as effectively *unfolded* by a co-inductive procedure, the semantic evaluations in the Boolean logic effectively consist in a *convergence* between an inductive "constructive" procedure, and a co-inductive "unfolding" procedure, namely they effectively consist in the superposition *limit/colimit* between two concurrent inductive/coinductive computations (see Appendix A.1). This is the core of Abramsky's notion of *finitary objects* as "limits of finite ones", definable only on NWF sets, finitary objects that according to him are the most proper objects for mathematical modeling of computations [11].

This is also the core of the related notion of *duality* between an *initial Boolean algebra*, starting from a *least* fixed point, $x = f(x)$, and its *final Stonean coalgebra*, starting from a *greatest* fixed-point (see Appendix A.1), at the basis of the notion of *universal coalgebra* as a "general theory of both computing and dynamic systems" [15]. This theory allows one to justify a *formal semantics of computer programming* as satisfaction of a given program onto the physical states of a computing system, outside the Turing paradigm. Indeed, this approach systematically avoids the necessity of referring to an UTM for formally justifying the *universality* in computations, because of the possibility of referring to algebraic and co-algebraic universality.[9] At the same time, this theory is able to give a *strong formal foundation* to the notion of *natural computation*, as far as we extend such a coalgebraic

[9]However, see the fundamental remarks about the limits of *decidability* and *computability* in this first-order modal logic semantic approach in [76], in which it is said, just in the conclusion, that one of the most promising research programs in this field is related to the coalgebraic approach to modal logic semantics.

semantics to quantum systems and quantum computation. This research program was inaugurated by S. Abramsky and his group at Oxford only a few years ago, both in fundamental physics [78], and in QM computing [14], even though it has its most natural implementation in a QFT foundation of both quantum physics and quantum computation [79].[10]

From the standpoint of the *natural ontology* of cognitive neuro dynamics in the framework of a QFT foundation of it (see Sect. 3.2 above), all this, roughly speaking, means that it is *logically* true that the *(sub)class* of horses is a *member* of the *(super)class* of mammalians *iff*, *dually*, it is *ontically* (dynamically) true that a *co-membership* of the *species* of horses to the *genus* of mammalians occurs, from some step *n* onward of the universe evolution (="natural unfolding" of a biological evolution tree), I.e.

$$\ominus_{\forall n(n>m)}\left(\underbrace{horse \in mammmalian}_{\text{Algebra}(\Omega^*)}\ \underbrace{\Omega^* \leftarrow \Omega \leftrightarrows}_{\text{Onto}-\text{logical}\,iff}\ \underbrace{horse \ni mammalian}_{\text{Co}-\text{Algebra}(\Omega)}\right)$$

In other terms, we are faced here with an example of a "functorially induced" homomorphism, from a coalgebraic *natural structure* of *natural kinds* (genera/species) into a *logical structure of predicate domains* (class/subclass), as an example of modal *local truth*, applied to a theory of the ontological natural realism, in the framework of an evolutionary cosmology [59, 82],[11] where it is nonsensical to use not indexed (absolute) modal operators and quantifiers, given that emergence of physical laws depends on the universe evolution. In parenthesis, this also gives a solution to the otherwise unsolved problem, in Kripke's relational semantics, of the denotation of *natural kinds* (the denoted objects of common names, such as "horses" or "mammalians" in our example) and of the connected Kripke's and Putnam's *causal theory of reference* (see on this point my previous discussion about these problems in [28]). Finally, this gives a logical interpretation as *predicate* (e.g. "being horse") of the "doubled number", i.e. $\mathcal{N}_A, \mathcal{N}_{\tilde{A}}$, as identity functions relative to two mirrored (doubled) sets of degrees of freedom, A and $\tilde{A}$, one relative to a logical realm (the Algebra(Ω*)), the other to its dual natural realm (the Coalgebra(Ω)), the latter satisfying (making true) *naturally*—i.e. *dynamically* in

[10]This depends on the fact that *contravariance* in QM algebraic representation theory can have only an *indirect justification*, as Abramsky elegantly explained in his just quoted paper. QM algebraic formalism is, indeed, intrinsically based on von Neumann's *covariant* algebra, so that only Hopf algebras' self-duality are "naturally" (in the algebraic sense of the allowed functorial transforms) justified in it [62, 80, 81].

[11]In this regard, the famous Aristotelian statement synthesizing his "intentional" approach to epistemology—"not the stone is in the mind, but the form of the stone"—has an operational counterpart in the homomorphism algebra coalgebra of QFT neuro dynamics.

this QFT implementation—the former (see above, Sect. 3.2). The co-membership relation in the coalgebraic half has its physical justification in QFT by the general principle of the "*foliation* of the QV" at the ground state, and of the relative Hilbert space into physically inequivalent subspaces", allowing "the building up" via SSB of ever more complex phase coherence domains in the QV, given their stability in time. They do not depend, indeed, on any energetic input (they depend on as many NGB condensates $|\mathcal{N}|$, each correspondent to a SSB of the QV at the ground state), but on as many "entanglements" with stable structures of the environment [83]. This justifies Freeman and Vitiello in suggesting that this is the fundamental mechanism of the formation of the so-called long-term memory traces in brain dynamics [47], i.e. the formation of the "deep beliefs" in our brains by which each of us interprets the world, based on her/his past experience, using the recently diffused AI jargon in artificial neural network computing [84].

Anyway, apart from this "ontological" exemplification, which is useful however to connect the present discussion with the rest of this chapter, all this means extending to a Boolean lattice L of the monadic predicate logic the modal semantics notions of co-induction and/or of "tree unfolding", so as to give a formal justification of the modal notion of "local truth" also in a computational environment.[12] Indeed, because such a co-inductive procedure of predicative domains justification is defined on NWF sets supporting set self-inclusion, i.e. $x \to \{x\}$, for each of these co-induced domains also the relative Id_x, i.e. the relative predicate φ, is defined, without any necessity to refer to Fregean second-order axioms, such as the comprehension axiom of ZF set theory, i.e. $< \forall x \exists y \; x \in y \; \equiv \; \varphi x >$. This justifies the general statement that, in CT coalgebraic semantics, there exists a Tarski-like model theory [12], without, however, the necessity to refer to higher-order languages for justifying the semantic meta language [86], according to Thomason's reduction program.

We can thus conclude this section by affirming that the previous discussion satisfies the first requirement of Dodig-Crnkovic's criticism to a theory of semantic information at the end of Sect. 4.2.2, i.e. the necessity of a *formal justification* of the theory of "local truth", essential for the notion and measure of Floridi's *semantic information* that can be naturally given in the context of a coalgebraic (modal) semantics of predicate logic. Quoting the first concluding remark of V. Goranko and M. Otto's contribution to the *Handbook of Modal Logic* devoted to the Kripke model theory of modal logic [87, p. 323], we can conclude too:

> Modal logic is local. Truth of a formula is evaluated at a current state (possible world); this localization is preserved (and carried) along the edges of the accessibility relations by the restricted, relativized quantification corresponding to the (indexed) modal operators.

[12]This result has been recently formally obtained [85]. For an intuitive explanation of this result, see below the two Appendices, Sects. 7.1–7.2.

5.3 Coalgebraic Semantics of Quantum Systems

We now have only one last step to perform: implementing the theory of local truth in a QFT system, that is, demonstrating that the curve of the quasi probability diagram of Fig. 1a as an information measure of the degree of semantic informativeness represents a measure of maximal entropy, expressing the fact that a given *dynamic cognitive system* (e.g. brain dynamics in the QFT interpretation depicted in Sect. 3.2) consumed all the *free-energy* deriving from the mismatch with its thermal bath, for the reorganization "work" (in the thermodynamic sense) of its inner state, so to match the outer state (thermal-bath), and then minimizing the free energy of the whole system (brain + thermal bath). In other terms, we have to interpret the maximal entropy physical measure as a logical measure of *maximal local truth* in the statistical sense. To sum up, we have to interpret consistently a QFT dynamic system as a *computing system.*

In the light of the discussion above it is necessary and sufficient for such an aim to demonstrate that the collections of the "*q*-deformed Hopf algebras" and the "*q*-deformed Hopf coalgebras" of the QFT mathematical formalism constitute two *dually equivalent categories* for the contravariant application of the same functor *T*, that is, the contravariant application of the so-called *Bogoliubov transform.* This is the classical QFT operator of "particle creation-annihilation", where the necessity of such a contravariance depends on the constraint of satisfying anyway the *energy balance principle,* i.e. $\boldsymbol{q}$**-HCoalg(*T*)** $\rightleftharpoons$ $\boldsymbol{q}$**-HAlg(*T**)**. It is useful to recall here that the *q-deformation* parameter characterizing each pair of *q*-deformed Hopf algebra coalgebra is physically a *thermal parameter*, so as to constitute the "evolution parameter" of the universe in a QFT interpretation of cosmology, via SSBs of the QV, according to Wheeler's suggestion with which we started our paper that in the new physics paradigm "cosmogony is the legislator of physics".

On the other hand, mathematically, this parameter is related to the "Bogoliubov angle", θ, characterizing each different application of the transform—where, as we know, the angle, with the frequency and the amplitude, are the three main parameters characterizing generally the phase of a given waveform. For a systematic presentation of the QFT mathematical formalism, see [17, pp. 185–235].

The complete justification of a coalgebraic interpretation of this mathematical formalism is given elsewhere [79], because we cannot develop it here. Nevertheless, at least two points of such a justification are important to emphasize, for justifying the interpretation of the maximal entropy in a QFT system as a semantic measure of information, i.e. as a statistical measure of *maximal local truth* in a CT coalgebraic logic for QFT systems.

Firstly, the necessary condition to be satisfied in order that a coalgebra category for some endofunctor Ω, i.e. **Coalg(Ω)**, can be interpreted as a *dynamic* and/or *computational* system, is that it satisfies the formal notion of *state transition system* (STS). Generally a STS is an abstract machine characterized as a pair $(S, \rightarrow)$, where S is a set of states, and $((\rightarrow) \subseteq S \times S)$ is a transition binary relation over S. If p, q belong to S, and (p, q) belongs to $(\rightarrow)$, then $(p \rightarrow q)$, i.e. there is a transition over S. To allow a dynamic/computational system to be represented as a STS on a functorial coalgebra for some functor Ω, it is necessary that the functor admits a *final coalgebra* [16, p. 389]; I.e.:

Definition 1: (Definition of final coalgebra for a functor). A functor $\Omega : \mathbf{C} \rightarrow \mathbf{C}$ is said to admit a final coalgebra iff the category **Coalg(Ω)** has a final object, that is, a coalgebra $\mathbb{Z}$ such that from every coalgebra $\mathbb{A}$ in **Coalg(Ω)**, there exists a unique homomorphism, $!\mathbb{A}: \mathbb{A} \rightarrow \mathbb{Z}$.

This property has a very intriguing realization—and this is the sufficient condition to satisfy for formalizing a QFT system as a computing system—into the final coalgebra associated with a particular abstract machine, the so-called infinite state black-box machine $\mathbb{M}\langle M, \mu\rangle$ [16, p. 395]. It is characterized by the fact that the machine internal states, x_0, x_1, …, cannot be directly observed, but only some values ("colors", c_n) associated with a state transition μ. I.e. $\mu\ (x_0) = (c_0,\ x_1)$, $\mu\ (x_1) = (c_1,\ x_2)$, … In this way, the only "observable" of this dynamics is the infinite sequence of behaviors or *stream beh* $(x_0) = (c_0,\ c_1,\ c_2,\ \ldots) \in C^{\omega}$ of value combinations or "words" over the dataset C. The collection C^{ω} forms a *labeled* STS for the functor $C \times \mathcal{I}$, where $\mathcal{I}$ is the set of all the identity functions (labels), as far as we endow C^{ω} with a transition structure γ splitting a stream $u = c_0c_1c_2, \ldots$ into its "head" $h(u) = c_0$, and its *tail* $t(u) = c_1c_2c_3\ldots$. If we pose $\gamma\ (u) = (h(u),\ t(u))$, it is possible to demonstrate that the behavior map $x \mapsto beh\ (x)$ is the unique homomorphism from $\mathbb{M}$ to this coalgebra $\langle C, \gamma\rangle$, that is, the final coalgebra $\mathbb{Z}$ in the category $\mathbf{Coalg}(C \times \mathcal{I})$.[13]

The abstract machine $\mathbb{M}$ is used in TCS for modeling the *coalgebraic semantics* of programming relative to infinite datasets, so-called *streams*: think, for instance, of the internet and more generally of all the ever-growing databases ("big data") [15]. The application of $\mathbb{M}$ for characterizing the QFT dynamics as a "computing dynamics" is evident in the light of the discussion above because we are allowed to interpret the thermodynamic functor T (Bogoliubov transform) characterizing the category ***q*-HCoalg(*T*)** as a functor able to associate the observable c of each "word" (phase coherence domain) of the QFT infinite dataset C, i.e. the infinite CCRs characterizing the QV, with the correspondent I_c, so that $T = (C \times \mathcal{I})$. Indeed, each I_c corresponds in the QFT formalism to the NGB condensate numerical value $|\mathcal{N}|$ univocally identifying each phase coherence domain, i.e. a "word" of the QV "language". In this way, the QV, because it is endowed with the SSB state transition —effectively a phase-transition—structure γ, selecting every time one CCR (*head*) from the others (*tile*), corresponds to the final coalgebra $\mathbb{Z}$ of the category ***q*-HCoalg(*T*)**. Moreover, the dynamics of $\mathbb{M}_{\mathrm{QFT}}$ is a *thermo*-dynamics; i.e. its state (phase) transition is "moved" by the II Principle (energy equipartition), in a way

[13]In parenthesis, in the machine $\mathbb{M}$ the general coalgebraic principle of the *observational* (or *behavioral*) *equivalence* among states holds in the following way. Indeed, for every two coalgebras (systems) $\mathbb{S}_1, \mathbb{S}_2 \in \mathbf{Coalg}(C \times \mathcal{I})$, $(!c_{\mathbb{S}_1} = !c_{\mathbb{S}_2}) \Rightarrow (!x_{\mathbb{S}_1} = !x_{\mathbb{S}_2})$. All scholars agree that this has an immediate meaning for quantum systems logic and mathematics, as a further justification for a coalgebraic interpretation of quantum systems.

that must satisfy, on the one hand, the "energy arrow contravariance" related to the I Principle, and, on the other one, without consuming all the QV energy "reservoir" as requested by the III Principle.[14] All this implies the necessity of doubling the behavior map, i.e. $x \mapsto beh(x, \tilde{x})$, and all the related objects and structures—i.e. the necessity of "echoing" each word of the QV language—so as to satisfy finally the "dual equivalence" characterizing the QFT categorical formalism, i.e. ***q*-HCoalg (*T*)** $\rightleftharpoons$ ***q*-HAlg(*T**)**. In logical terms, the functor induction $T \leftarrow T^*$ means that the semantics (coalgebra) induces its own syntax (algebra). This, on the one hand, justifies the computer scientist's interest toward a coalgebraic approach to quantum computation for managing streams, and on the other, demonstrates that the QFT interpretation of this approach is the more promising one. In fact, what we intended using the metaphor of "word echoing" within the model of the $\mathbb{M}_{QFT}$ is effectively the DDF principle determining the *dynamic choice*, observer independent, of the structure (syntax) of the "composed Hilbert space" of a QFT system as based on the *dual equivalence* (semantics) of one pair q-HCoalg(T) $\rightleftharpoons$ q-HAlg(T^*) representing the whole dissipative system [79].

All this is related to the second, final, observation, justifying the interpretation of the maximal entropy in a QFT "doubled" system as a semantic measure of information, i.e. as a statistical measure of *maximal local truth* in the CT coalgebraic logic. In the QFT mathematical formalism, this maximum of the entropy measure is formally attained when the above-illustrated DDF principle (far-from-equilibrium energy balance) between a system (algebra) and its thermal bath (coalgebra) is *dynamically* (=automatically) satisfied. This means that we are allowed to interpret the QFT *qubit* of such a natural computation as an "evaluation function" in the semantic sense. Indeed, in the QFT "composed Hilbert space" including also the thermal bath degrees of freedom, $\tilde{A}$, i.e. $\mathcal{H}_{A,\tilde{A}} = \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}}$, for calculating the static and dynamic entropy associated with the time evolution generated by the free energy, i.e. $|\phi(t)\rangle, |\psi(t)\rangle$, of the qubit mixed states $|\phi\rangle, |\psi\rangle$, one needs to double the states by introducing the tilde states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, relative to the thermal bath, i.e. $|0\rangle \rightarrow |0\rangle \otimes |\tilde{0}\rangle$ and $|1\rangle \rightarrow |1\rangle \otimes |\tilde{1}\rangle$. This means that such a QFT version of a qubit implements effectively the CNOT (controlled NOT) logical gate, which flips the state of the qubit, conditional on a *dynamic* control of an effective input matching [79].

6 Conclusion

I used many times in this paper, also in the title, the expression "new paradigm". Th. Kuhn, who coined the fortunate expression "paradigm shift", said that the "scientific community" has to decree this shift every time it happens in the history

[14] A condition elegantly satisfied in the QFT formalism by the *fractal structure* of the systems phase space and, therefore, by the *chaotic character* of the macroscopic trajectories (phase transitions) defined on it, generally [88], and specifically in dissipative brain dynamics [32].

of science. Therefore, if we agree in giving to the Stockholm Royal Academy the honor and the duty of representing the scientific community at its higher levels, it decreed just a few months ago that we are living one of these turns characterizing the history of modern science. In the official conference press release for announcing to the world that the 2015 Nobel Prize in Physics was awarded to the physicists T. Kajita and A. B. McDonald for their observational discovery of the neutrino mass, the Academy stated that "the new observations had clearly showed that the Standard Model cannot be the complete theory of the fundamental constituents of the universe" [89].

In this paper we defended the idea that QFT interpreted as a "thermal field theory" is a candidate for constituting, above all, the proper theory of the "physics beyond the Standard Model" because it is able to give physics a strong formal alternative to the "perturbative methods" and their "asymptotic states" that lie at the basis of the Standard Model "mechanistic" interpretation of the statistical distinction between "fermions" and "bosons", in terms of "particles" and of "force field quanta", respectively. The validity of perturbative methods relies indeed on the possibility of correctly defining asymptotic states for the system, namely states defined in infinitely distant space time regions, so as to make interactions negligible, in the presumption that this representation is not falsifying the nature of the physical system to be represented. In this light, the "paradigm turn" with which today we are faced is therefore not only with respect to the Standard Model physics (QED and QCD, i.e. the so-called standard QFT), but also to QM, the "many-body dynamics" extension of classical mechanics, from Laplace on. Therefore, the presumption of correctly representing a system as isolated lies at the bottom of the same origins of modern physics and modern calculus. This presumption cannot hold, however, in the case of QFT systems, as they are intrinsically "open" to background QV fluctuations, or, more generally, when we have to reckon with system phase transitions. In all these cases, the QFT alternative picture of representing both fermions and bosons as quanta of the relative force field is more suitable. On the other hand, we showed that the paradigm shift we are discussing, because it involves the foundations of modern science, also involves the foundations of mathematics and computability theory, as far as both are related to non standard set theories.

The alternative formalism offered by thermal QFT is, therefore, the doubled algebra representation of a quantum system and of its thermal bath, through the mathematical formalism of the DDF between a q-deformed Hopf algebra and its q-deformed Hopf coalgebra, as illustrated in this chapter. Such formalism has been successfully applied not only in fundamental physics, the physics of neutrino oscillations included [17, pp. 91–95], but also in condensed matter physics, biological matter, and brain dynamics. For this reason, in this chapter we deepened the possibility of justifying in such a formalism also the notion and measure of "semantic information", generally associated with biological and neural information processing. For this aim, we discussed an information-theoretic interpretation of the DDF in QFT systems, in the framework of the coalgebraic approach to quantum computation, recently introduced as an alternative to the information-theoretic interpretation of QM systems as quantum Turing machines. This allowed us to give a formal justification of the notion of “local truth”, associated with the measure of semantic information that is therefore interpreted as a measure of maximal entropy, because it minimizes the “free energy” associated with the mismatch with the system environment thermal bath. Practically, this measure expresses the “entanglement” that dynamically occurred, as signaled by the flipping of the associated qubit, between the degrees of freedom of the system and of its thermal bath, within the same representation space including both [79]. This result is ultimately based on the possibility of justifying the dual equivalence between the categories of the q-deformed Hopf algebras and the q-deformed Hopf coalgebras, allowing one to interpret quantum computations of QFT systems in the framework of category theory logic. This initial result opens the way to new promising scenarios in quantum natural and artificial computation to be explored in the near future.

Appendix A

A.1 Induction and Coinduction as Principles of Set Definition and Proof for Boolean Lattices

The collection of clopen subsets of a Stone space, to which a Boolean algebra is isomorphic, according to the Stone theorem is effectively an ultrafilter U (or the maximal filter F) on the power set, $\wp(S)$, of the set S. Namely, it is the *maximal partially ordered set* (maximal poset) within $\wp(S)$ ordered by inclusion, i.e. ($\wp(S)$, $\subseteq$), with the exclusion of the empty set. Any filter F is *dual* to an *ideal I*, simply obtained in set (order) theory by inverting all the relations in F, that is, $x \leq y$ with $y \leq x$, and by substituting *intersections* with *unions*. From this derives that each ultrafilter U is dual to a greatest ideal that, in Boolean algebra, is also a *prime* ideal, because of the so-called *prime ideal theorem*, effectively a corollary of the Stone theorem, demonstrated by himself. All this, applied to the Stone theorem, means that the collection of partially ordered clopen subsets of the Stone space, to which a Boolean algebra is isomorphic, corresponds to a Boolean logic complete lattice L for a *monadic first-order predicate logic.* From this, the definition of *induction* and *coinduction* as dual principles of set definition and proof is immediate, as soon as we recall that the fixed point of a computation F is given by the equality $x = F(x)$ [68, p. 46]:

Definition 2 (*sets inductively/coinductively defined by F*). For a complete Boolean lattice L whose points are sets, and for an endofunction F, the sets

$$F_{\text{ind}} := \bigcap\{x | F(x) \leq x\}$$
$$F_{\text{coind}} := \bigcup\{x | x \leq F(x)\}$$

are, respectively, the sets *inductively* defined by a *recursive* F, and *coinductively* defined by a *co-recursive* F. They correspond, respectively, to the *meet* of the pre-fixed point and the *join* of the post-fixed points in the lattice L, i.e. the least and greatest fixed points, if F is monotone, as required from the definition of the category **Pos** (see above, Sect. 5.1).

Definition 3 (*induction and coinduction proof principles*). In the hypothesis of Definition 2, we have:

$$\begin{array}{ll} \textit{if } F(x) \le x \textit{ then } F_{\text{ind}} \le x & \text{(induction as a method of proof)} \\ \textit{if } x \le F(x) \textit{ then } x \le F_{\text{coind}} & \text{(coinduction as a method of proof)} \end{array}$$

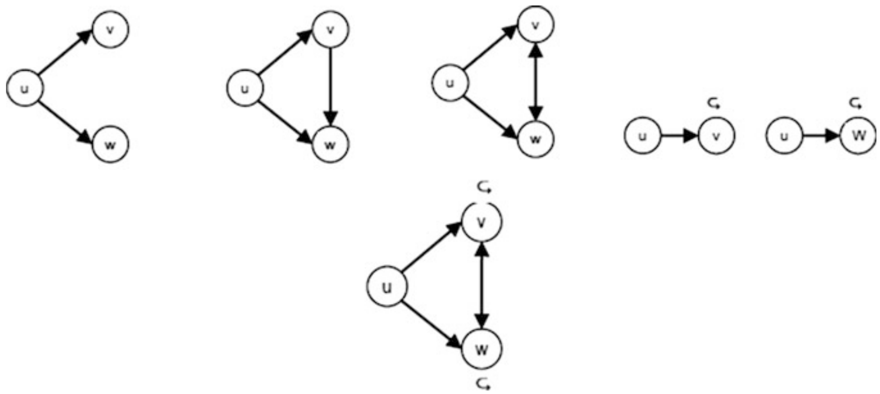
These two definitions are the basis for the *duality* between an *initial algebra* and its *final coalgebra*, as a new paradigm of computability, i.e. Abramsky's *finitary* one, and henceforth for the duality between the *universal algebra* and the *universal coalgebra* [15].

A.2 Extension of the Coinduction Method to the Definition of a Complete Boolean Lattice of Monadic Predicates

The fundamental result of the above-quoted Goldblatt–Thomason theorem and van Benthem theorem is that a set-tree of NWF sets—effectively a set represented as an oriented graph where nodes are sets, and edges are inclusion relations with subsets governed by Euclidean rule—corresponds to the structure of a Kripke *frame* of his relational semantics, characterized by a set of "worlds" and by a two-place accessibility relation R between worlds, e.g. the second graph from left below corresponds to the graph of the number 3, with $u = 3$, $v = 2$, $w = 1$. Therefore for understanding intuitively the extension of the coinduction method to the domains of monadic predicates of a Boolean lattice, let us start from (1) the "Euclidean rule (ER)" ‹$\forall u,v,w\ ((uRv \wedge uRw) \rightarrow vRw)$› (see the second from left graph below), driving all the NWF set inclusions and that is associated by van Benthem's correspondence theorem to the modal axiom **E** (or **5**): ‹$\Diamond\alpha \rightarrow \Box\Diamond\alpha$›, of the modal propositional calculus, and (2) from the "seriality rule (SR)" ‹$\forall u \exists v\ (uRv)$› (an example of this axiom is given by the fourth or fifth graph below)—that has an immediate physical sense, because it corresponds to whichever energy conservation principle in physics, e.g. the I Principle of Thermodynamics—and that is associated to the modal axiom **D:** ‹$\Box\alpha \rightarrow \Diamond\alpha$›. The straightforward first-order calculus, by which it is possible to formally justify the definition/justification by coinduction (tree unfolding) of an *equivalence class* as the domain of a given monadic predicate, through the application of the two above rules to whichever triple of objects ‹u, v, w›, is the following:

For ER, ‹$\forall u,v,w\ ((uRv \wedge uRw) \rightarrow vRw)$›; hence, for seriality, ‹$\forall u,v\ (uRv \rightarrow vRv)$›; finally: ‹$\forall u,v,w\ [((uRv \wedge uRw) \rightarrow (vRw \wedge wRv \wedge vRv \wedge wRw)) \leftrightarrow$

$((v \equiv w) \subset u)]\rangle$, I.e. $(v \equiv w)$ constitutes an equivalence subclass of u, say **Y**, because a "generated" transitive[15]-symmetric-reflexive relation holds among its elements, which are therefore also "descendants" of their common "ascendant", u. More intuitively, using Kripke's relational semantics graphs for modal logics, where $\langle u, v, w\rangle$ are also "possible worlds" (models) of a given universe W, and where R is the two-place "accessibility relation" between worlds, the above calculus reads:



The final graph constitutes a Kripke-like representation of the **KD45** modal system, also defined in literature as "secondary **S5**", since the equivalence relationship among all the possible worlds characterizing **S5** here holds only for a subset of them, in our example, the subset of worlds $\{w, v\}$.

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[15]Remember that the transitive rule in the NWF set theory does not hold only for the inclusion operation, i.e. for the superset/subset ordering relation.

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