

A Neural Hysteresis Model for Smart-Materials-Based Actuators

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Abstract. In this paper, a constraint factor (CF) is presented. The CF and an odd m -order polynomial form a new hysteretic operator (HO) together. And then, an expanded input space is constructed based on the proposed HO. In the expanded input and output spaces, the one-to-multiple mapping of hysteresis is transformed into a one-to-one mapping so that a neural network can be used to develop a neural hysteresis model. The model parameters are computed by using the least square method. Finally, the neural hysteresis model is employed to approximate a real data from a magnetostrictive actuator in an experiment. The experimental results demonstrate the proposed approach is effective.

Keywords: Hysteresis · Hysteretic operator (HO) · Constraint factor (CF) · Smart materials · Neural networks

1 Introduction

In the past decades, smart materials, such as piezoelectric materials, magnetostrictive materials, shape memory alloys, etc., have been widely used in many fields. The piezoelectric and magnetostrictive actuators have been specially used for micro-displacement systems [1, 2]. Since smart-materials-based actuators have advantage in the output force, position resolution, and response speed [3], they have been attached importance in ultra-precision positioning systems [4]. However, the inherent hysteresis nonlinearity in

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smart-materials-based actuators, which is non-differentiable and multi-valued mapping, frequently leads to undesired tracking errors, oscillations, and even instability [5]. The model-based scheme for hysteresis compensation is currently popular in control systems [6]. A large amount of hysteresis models have been proposed in the past decades, such as PI model [7], KP model [8], Preisach model [9], Maxwell slip model [10], Jiles-Atherton model [11], Duhem model [12], Bouc-Wen model [13, 14], and so on. However, the ultra-precision positioning systems need more accurate hysteresis models so as to meet the requirement of science and technology.

Three-layer feed-forward neural networks (NNs) have been regarded as one of the best ways to model nonlinear systems because they can implement all kinds of nonlinear mapping. However, the mapping of hysteresis consists of one-to-multiple and multiple-to-one mappings, while the NNs are incapable of identifying one-to-multiple mapping [15], so the one-to-multiple mapping has to be eliminated so that the neural approaches can be used to model hysteresis. Ma [16] proposed a hysteretic operator (HO), expanded the input space of NN from 1-dimension to 2-dimension based on the HO so that an NN-based hysteresis model was established, and named the method as expanded-space method. Zhao [17], Dong [18], Zhang [19] and Ma [20, 21] proposed respectively new HOs and constructed neural hysteresis models, thereby improving the expanded-space method.

In this paper, a new HO, which is made up of a constraint factor (CF) and an odd m -order polynomial, is proposed to expand the input space of NN. And then, based on the proposed HO, the one-to-multiple mapping of hysteresis is transformed into one-to-one mapping so that the neural approach can be used to identify the expanded mapping. Finally, a NN-based hysteresis model is developed and used to approximate a set of real data from a magnetostrictive actuator. The experimental results demonstrate that the proposed model is effective.

2 HO Construction

2.1 HO Definition

In this paper, the HO is consisted of a CF and an odd m -order polynomial with the constant term. The function, $c(x) = 1 - e^{-x}$, is used as the CF of HO. The role of CF is to constrain the amplitude of HO curve and ensure the curve passes through the origin in every minor coordinate system. Therefore, in the i th minor coordinate system, the HO is defined as follows:

$$f(x_i) = (1 - e^{-x_i})(a_0 + \sum_{j=1}^m a_j x_i^{2j-1}) \quad (1)$$

where x_i and f are respectively any input and the corresponding output of HO in the i th minor coordinate system.

In the main coordinate system, the HO is described as

$$h(x) = \begin{cases} h(x_{ei}) + f(x - x_{ei}) & x > x_{ei} \\ h(x_{ei}) - f(x_{ei} - x) & x < x_{ei} \end{cases} \quad (2)$$

where $[x_{ei}, h(x_{ei})]$ are the coordinates of the origin of the i th minor coordinate system in the main coordinate system, x is any input and h is the corresponding output of HO.

2.2 Parameter Computation

As known to all, the best method of determining polynomial coefficients is the least square method. Thus, the least square method is adopted to compute the HO parameters based on the samples used for training neural network.

In terms of the least square method, the residual δ_i is written as

$$\delta_i = y_i - f(x_i) \quad (3)$$

Consequently, the sum of square residuals is shown as follows:

$$S = \sum_{i=1}^n \delta_i^2 = \sum_{i=1}^n [y_i - f(x_i)]^2 = \sum_{i=1}^n [y_i - (1 - e^{-x_i})(a_0 + \sum_{j=1}^m a_j x_i^{2j-1})]^2 \quad (4)$$

To minimize S , the partial derivatives of S with regard to a_0, a_1, \dots, a_m should be set to zeros. Therefore, the $(m + 1)$ partial derivative equations are given as follows:

$$\begin{cases} \frac{\partial S}{\partial a_0} = -2 \sum_{i=1}^n [(1 - e^{-x_i}) \cdot y_i - (1 - e^{-x_i})^2 (a_0 + \sum_{j=1}^m a_j x_i^{2j-1})] = 0 \\ \frac{\partial S}{\partial a_k} = -2 \sum_{i=1}^n [(1 - e^{-x_i}) \cdot x_i^{2k-1} \cdot y_i - (1 - e^{-x_i})^2 (a_0 x_i^{2k-1} + \sum_{j=1}^m a_j x_i^{2(j+k-1)})] = 0, k = 1, 2, \dots, m \end{cases} \quad (5)$$

Rearranging the Eq. (5) gives

$$\begin{cases} a_0 \cdot \sum_{i=1}^n (1 - e^{-x_i})^2 + \sum_{j=1}^m a_j \cdot \sum_{i=1}^n (1 - e^{-x_i})^2 x_i^{2j-1} = \sum_{i=1}^n (1 - e^{-x_i}) \cdot y_i \\ a_0 \cdot \sum_{i=1}^n (1 - e^{-x_i})^2 x_i^{2k-1} + \sum_{j=1}^m a_j \cdot \sum_{i=1}^n (1 - e^{-x_i})^2 x_i^{2(j+k-1)} = \sum_{i=1}^n (1 - e^{-x_i}) \cdot x_i^{2k-1} \cdot y_i, k = 1, 2, \dots, m \end{cases} \quad (6)$$

3.1 The Proposed Model

In this section, the neural hysteresis model is employed to approximate the real data. The activation function of the hidden layer is the sigmoid function and that of the output layer is the linear function. The comparison of the different orders shows that the 9-order polynomial is most suitable to the HO in this experiment. The HO parameters are listed in the Table 1.

Table 1. HO Parameters

Parameter	Value
a_0	-13.1053
a_1	3.5024e02
a_2	-4.7518e03
a_3	3.8646e04
a_4	-1.9549e05
a_5	6.3845e05
a_6	-1.3467e06
a_7	1.7721e06
a_8	-1.3230e06
a_9	4.2793e05

To determine the optimal number of hidden nodes, the number from 1 to 100 is tried in this experiment. The best 3 performances are listed in the Table 2. It can be seen from the Table 2, that the neural hysteresis model has the best performance when the number of hidden nodes becomes 5. Therefore, a three-layer feed-forward neural network with two input nodes, 5 hidden nodes and one output node was employed to approximate the real data in this experiment. After 229 iterations, the training procedure finishes. The mean square error (MSE) of model prediction is 0.0611. The Figs. 1 and 2 illustrate the model prediction and absolute error respectively.

Table 2. The top 3 performances of NN with different number of hidden neurons

No. of hidden nodes	MSE
5	0.0611
6	0.0677
2	0.0690

3.2 PI Model

In addition, to compare with the proposed model, the PI model was also applied to approximate the measured data. The model thresholds were calculated via the following formula:

$$r_i = \frac{i - 1}{N} [\max(x(k)) - \min(x(k))] \tag{11}$$

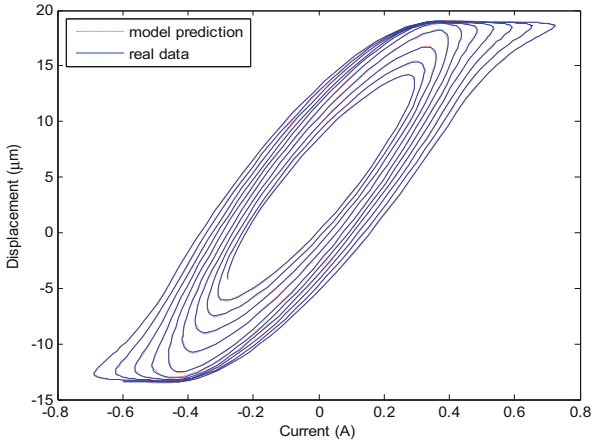


Fig. 1. Comparison of the proposed model prediction and the real data

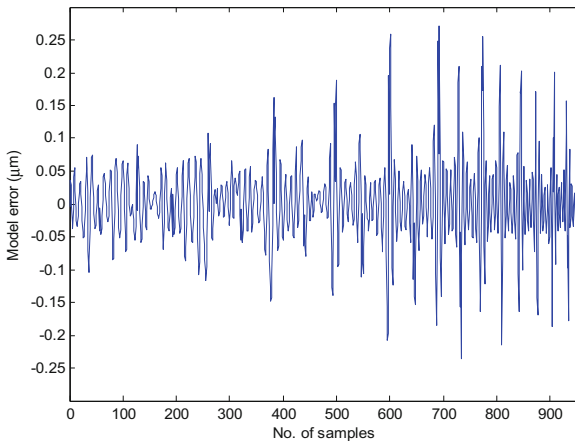


Fig. 2. The absolute error of the proposed model

where N is the number of backlash operators and $i = 1, 2, \dots, N$.

The Matlab nonlinear optimization tool was used to determine the weights of backlash operators. However, the calculation time of the PI model increases along with the increase of N , so only the PI models containing 1–5000 backlash operators were tried. The top three performances are listed in the Table 3. Therefore, $N = 4994$ is selected in this experiment, which leads to 20-hour calculation time. The MSE of model prediction is 0.4327. Figures 3 and 4 display the model prediction and absolute errors respectively.

3.3 Comparison

In the above experiments, the MSE of the proposed neural hysteresis model is 85.88 % smaller than that of the PI model, which demonstrates that the proposed neural model can better approximate the real data measured from the magnetostrictive actuator than the PI model.

Table 3. Performances of different no. of backlash operators

No. of backlash operators	Performance
4994	0.4327
4997	0.4330
5000	0.4348

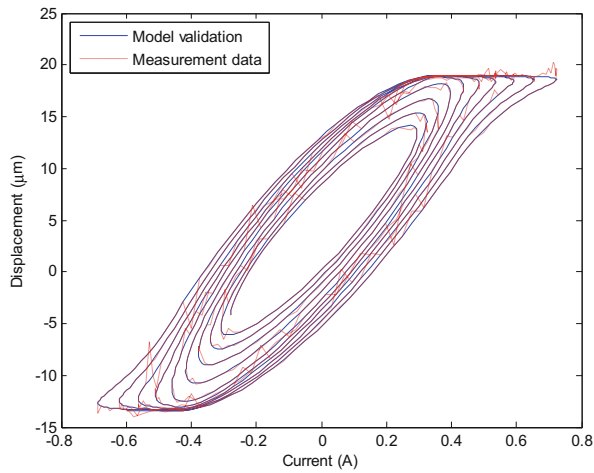


Fig. 3. Comparison of the PI model prediction and the real data

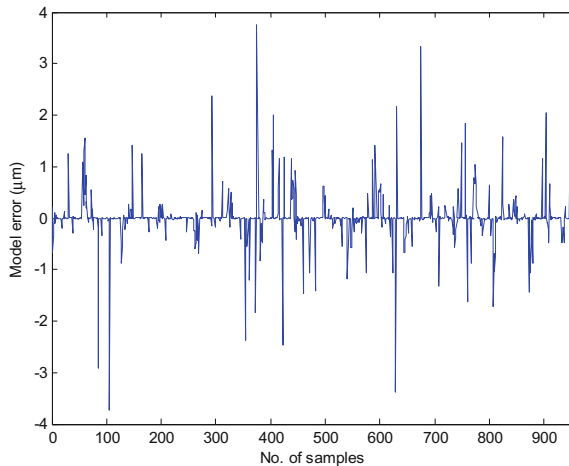


Fig. 4. The absolute error of the PI model

4 Conclusions

In this paper, a new HO is proposed. The HO consists of two components: a CF and an odd m -order polynomial. And then, based on the constructed HO, the one-to-multiple mapping of hysteresis is transformed into a continuous one-to-one mapping by expanding the input space of NN. In this way, the expanded mapping only contains one-to-one and multiple-to-one mappings, which can be identified using the neural approaches. Finally, an experiment is implemented to verify the proposed hysteresis model. The verification performance approves the proposed approach.

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