# Topological Structure Synthesis of 3T1R Parallel Mechanism Based on POC Equations

Tingli Yang[1], Anxin Liu[2(✉)], Huiping Shen[1], and Lubin Hang[3]

[1] Changzhou University, Changzhou, China
{yangtl,shp65}@126.com
[2] PLA University of Science and Technology, Nanjing, China
liuanxinn@163.com
[3] Shanghai University of Engineering Science, Shanghai, China
hanglb@126.com

**Abstract.** A systematic method for topological structure synthesis of PM based on POC equations is introduced. The complete synthesis process includes the following several steps: (a) synthesize all candidate SOC and HSOC branches based on the POC equation for serial mechanisms, (b) determine branch combination schemes and geometrical conditions for branches to be assembled on two platforms based on POC equation for PMs, (c) check the obtained PMs for design requirements and obtain all usable PMs. Topological structure synthesis of 3T1R PM which can be used in SCARA robot is discussed in detail to illustrate to procedure of this method. 18 types of 3T1R PMs containing no prismatic pair are obtained.

**Keywords:** Parallel mechanism (PM) · Topological structure synthesis · Position and orientation characteristic (POC) · Single open chain (SOC) · Hybrid single open chain (HSOC)

## 1 Introduction

The SCARA parallel robot can achieve 1 rotation and 3 translations and has been used widely in industries such as sorting, packaging and assembling. The SCARA serial robot first developed by Hiroshi [1] has 4 dofs. It has a cantilever design. So it features low rigidity, low effective load and high arm inertia. It is very hard for its end effector to move at high speed.

The SCARA parallel robot FlexPicker developed by ABB Corporation [2] has one output platform and one sub-platform (see Fig. 10). It is obtained by adding a RUPU branch to the original Delta parallel robot [3]. The SCARA parallel robot Par4 developed by Pierrot [4–6] has one output platform and 2 sub-platforms (see Fig. 9a). Relative motion between the two sub-platforms is converted to rotation of the output platform. This robot features high flexibility, high speed and high rigidity. Based on Par4 robot, Adept Corporation developed the SCARA parallel robot Quattro, which boasts the highest operation speed [7–9]. Some innovative work has also been done by Huang Tian on this type of SCARA parallel robot [10–13]. The SCARA parallel robot X4 has one output platform and 4 parallel-connected branches (see Fig. 7a). It features simple structure and can realize a rotation within the range of ±90° [14].


N. Kubota et al. (Eds.): ICIRA 2016, Part I, LNAI 9834, pp. 147–161, 2016.
DOI: 10.1007/978-3-319-43506-0_13

Obviously, the most important part of the above SCARA parallel robots is the 3T1R PM involved. In this paper, topological structure synthesis of 3T1R PM is discussed systematically using the method based on POC equations [15–23] in order to provide the readers with more comprehensive understanding about the topological structure of 3T1R PM and to obtain some more new 3T1R PMs with potential practical use.

The topological structure synthesis method based on POC equations introduced in this paper is the further development of our early work on topological structure synthesis of PMs and now becomes a systematic and easy-to-follow method for topological structure synthesis of PMs due to some new achievements of this paper, such as: (a) general method for topological structure synthesis of simple branches based on POC equation for serial mechanisms; (b) general method for topological structure synthesis of complex branches based on replacement of sub-SOCs in a SOC branch by topologically equivalent sub-PMs; (c) general method for determination of geometrical conditions for assembling branches between the fixed platform and the moving platform; and (d) general criteria for selection of driving pairs.

## 2 Theoretical Bases

### 2.1 POC Equation for Serial Mechanism

The POC equation for serial mechanism [15, 16, 21, 23] is

$$M_S = \bigcup_{i=1}^{m} M_{J_i} = \bigcup_{j=1}^{k} M_{sub-SOC_j} \tag{1}$$

where, $M_S$—POC set of the end link relative to the frame link, m—number of kinematic pairs, $M_{J_i}$—POC set of the i$^{th}$ kinematic pair (Table 1), $M_{sub-SOC_j}$—POC set of the j$^{th}$ sub-SOC (12 sub-SOCs containing R and P pairs only are listed in Table 2).

**Table 1.** POC set of kinematic pair

| P pair | R pair | H pair |
|---|---|---|
| $\begin{bmatrix} t^1(\parallel P) \\ r^0 \end{bmatrix}$ | $\begin{bmatrix} t^1(\perp(R,\rho)) \\ r^1(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^1(\parallel H) \cup t^1(\perp(H,\rho)) \\ r^1(\parallel H) \end{bmatrix}$ |
| $\mathrm{Dim}\{M_J\} = 1$ | | |

Note: "∥" means "parallel to", "⊥" means "perpendicular to".

"Union" operation rules for Eq. (1) include 8 linear operation rules and 2 nonlinear criteria [21, 23].

### 2.2 POC Equation for PM

The POC equation for PM [15, 16, 21, 23] is

**Table 2.** POC set of sub-SOC

| Sub-SOCs | $SOC\{-R \parallel R-\}$<br>$SOC\{-R \perp P-\}$ | $SOC\{-R \parallel R \parallel R-\}$<br>$SOC\{-R \parallel R \perp P-\}$<br>$SOC\{-P \perp R \perp P-\}$ | $SOC\{\diamondsuit(P, P, \cdots, P)\}$ | $SOC\{-R\vert P-\}$ | $SOC\{-RR\widehat{RR}-\}$ | $SOC\{-\widehat{RRR}-\}$ |
|---|---|---|---|---|---|---|
| POC set ($M_s$) | No.1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 |
| | $\begin{bmatrix} t^2(\perp R) \\ r^3(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^2(\perp R) \\ r^3(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^2 \\ r^0 \end{bmatrix}$ | $\begin{bmatrix} t^1(\parallel P) \cup t^1(\perp(R,\rho)) \\ r^1(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^2(\perp \rho) \\ r^2 \end{bmatrix}$ | $\begin{bmatrix} t^2(\perp \rho) \\ r^3 \end{bmatrix}$ |
| | $\mathrm{Dim}\{M_s\} = 2$ | $\mathrm{Dim}\{M_s\} = 3$ | $\mathrm{Dim}\{M_s\} = 2$ | $\mathrm{Dim}\{M_s\} = 2$ | | $\mathrm{Dim}\{M_s\} = 3$ |
| | No. 1* | No. 2* | No. 3* | No. 4* | No. 5* | No. 6* |
| | $\begin{bmatrix} t^1(\perp(R,\rho)) \\ r^1(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^2(\perp R) \\ r^1(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^2 \\ r^0 \end{bmatrix}$ | $\begin{bmatrix} t^1(\parallel P) \\ r^1(\parallel R) \end{bmatrix}$ | $\begin{bmatrix} t^0 \\ r^2 \end{bmatrix}$ | $\begin{bmatrix} t^0 \\ r^3 \end{bmatrix}$ |
| | $\mathrm{Dim}\{M_s\} = 2$ | $\mathrm{Dim}\{M_s\} = 3$ | $\mathrm{Dim}\{M_s\} = 2$ | $\mathrm{Dim}\{M_s\} = 2$ | | $\mathrm{Dim}\{M_s\} = 3$ |

Note: The base point o' of end link of the sub-SOC lies on the pair axis for No.1*–No.6*.

$$M_{Pa} = \cap_{j=1}^{(v+1)} M_{b_j} \tag{2}$$

where, $M_{Pa}$—POC set of the moving platform, $M_{b_j}$—POC set of the end link in the j[th] branch, $v$—number of independent loop.

"Intersection" operation rules for Eq. (2) include 12 linear operation rules and 2 nonlinear criteria [21, 23].

According to Eq. (2), there is

$$M_{b_j} \supseteq M_{Pa} \tag{3}$$

## 2.3 DOF Formula

(1) The DOF formula [22] is

$$F = \sum_{i=1}^{m} f_i - \sum_{j=1}^{v} \xi_{L_j} \tag{4a}$$

$$\xi_{L_j} = \dim.\left\{ \left( \bigcap_{j=1}^{j} M_{b_j} \right) \bigcup M_{b_{(j+1)}} \right\} \tag{4b}$$

where, $F$—DOF of mechanism, $f_i$—DOF of the ith kinematic pair, $\xi_{L_j}$—number of independent displacement equations for the jth independent loop, $M_{b_j}$—POC set of the end link in the jth branch.

(2) Criteria for driving pair selection [22]
For the mechanism with DOF = F, select and lock (make the two links connected by a kinematic pair into one integral link) F kinematic pairs. Only if DOF of the obtained new mechanism is 0, the selected F kinematic pairs can all be used as driving pairs simultaneously.

# 3 Method for Topological Structure Synthesis of Branch

## 3.1 Method for Topological Structure Synthesis of Simple Branch

The branch containing no loop is called simple branch. It is also referred to as SOC branch. Main steps of SOC branch topological structure synthesis include:

**Step 1.** Determine POC set and DOF of SOC branch

(1) Determine POC set of SOC branch $(M_S)$ according to Eq. (3).
(2) Determine DOF of SOC branch according to dimension of $M_S$.

**Step 2.** Determine kinematic pair combination scheme of SOC branch
According to Eq. (1), the number of kinematic pairs of SOC branch containing only R and P pairs shall meet the following requirements:

(1) DOF of SOC branch

$$F = m_R + m_P \tag{5}$$

where, $F$ – DOF of branch, $m_R$ – number of R pair, $m_P$ – number of P pair.

(2) Range of R pair number

$$m_R \geq \dim\{M_S(r)\} \tag{6}$$

where, $\dim\{M_S(r)\}$ – number of independent rotation elements in $M_S$.

(3) Range of P pair number

$$m_P \leq \dim\{M_S(t)\} \tag{7}$$

where, $\dim\{M_S(t)\}$ – number of independent translation elements in $M_S$.

**Step 3.** Determine sub-SOCs contained in the SOC branch
With $M_S$ and the kinematic pair combination scheme being known, determine the sub-SOCs contained in SOC branch according to Eq. (1), Tables 1 and 2. Connect these sub-SOCs in tandem and obtain the desired SOC branch.
**Step 4.** Check POC set of the obtained SOC branch
For each SOC branch obtained in step 3, check whether its POC set complies with the design requirement according to Eq. (1).

**Example 4.1.** Topological structure synthesis of SOC branch with 4 DOFs (3T1R)

Step 1. Basic functions of the SOC branch

(1) There is $M_S = \begin{bmatrix} t^3 \\ r^1 \end{bmatrix}$. Select an arbitrary point on the end link as base point o'.
(2) F = dim{$M_S$} = 4.

Step 2. Determine kinematic pair combination scheme of SOC branch

Since $\dim\{M_S(r)\} = 1$ and $\dim\{M_S(t)\} = 3$, there must be $m_R \geq 1$ and $m_P \leq 3$ according to Eqs. (5, 6 and 7). So, there are 3 pair combination schemes according to Eq. (1) and Table 2: 3R1P, 2R2P and 1R3P.

Step 3. Determine sub-SOCs contained in the SOC branch

Case 1: 3R1P

According to Eq. (1) and Table 2, this branch contains $sub-SOC\{-R \parallel R \parallel R-\}$ and a P pair. The SOC branch shall be $SOC\{-R \parallel R \parallel R - P-\}$.

Case 2: 2R2P

According to Eq. (1) and Table 2, this branch contains $sub-SOC\{-R \parallel R - P-\}$ and a P pair. The SOC branch shall be $SOC\{-R \parallel R - P - P-\}$.

Case 3: 1R3P

According to Eq. (1) and Table 2, this branch contains $sub-SOC\{-R - P - P-\}$ and a P pair. The SOC branch shall be $SOC\{-R - P - P - P-\}$.

Step 4. Check POC set of the obtained SOC branch

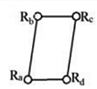
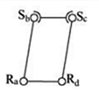
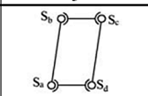
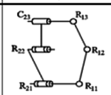
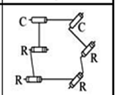
### 3.2 Method for Topological Structure Synthesis of Hybrid Branch

The branch containing one or more loop(s) is called hybrid branch. It is also referred to as HSOC branch.

If two kinematic chains have same POC set, they are considered as topologically equivalent. Similarly, if a sub-PM (PM) and a sub-SOC have same POC set, the sub-PM and the sub-SOC are topologically equivalent. If a SOC branch and a HSOC have the same POC set, the HSOC is topologically equivalent to the SOC branch.

Two-branch sub-PM, i.e. single-loop mechanism, is usually used in HSOC branch. These sub-PMs, together with their topologically equivalent sub-SOCs and there POC sets are listed in Table 3.

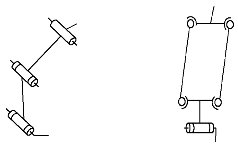
**Table 3.** Two-branch sub-PMs and their topologically equivalent sub-SOCs

| No. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Sub-PMs |  |  |  |  |  |
| POC set | $\begin{bmatrix} t^1(\parallel(ad)) \\ r^0 \end{bmatrix}$ | $\begin{bmatrix} t^1(\parallel bc) \\ r^1(\parallel \mathrm{bc}) \end{bmatrix}$ | $\begin{bmatrix} t^1(\parallel(ad)) \cup t^1(\perp(ad)) \\ r^1(\parallel(ad)) \end{bmatrix}$ | $\begin{bmatrix} t^2(\perp R_{11}) \\ r^0 \end{bmatrix}$ | $\begin{bmatrix} t^3 \\ r^0 \end{bmatrix}$ |
| Equivalent sub-SOC |  |  |  |  |  |

Generally, a HSOC branch can be obtained by replacing a sub-SOC in a SOC branch with a topologically equivalent sub-PM. According to Eq. (1), this HSOC branch and the original SOC branch have the same POC set.

For example, the SOC branch $\{-R \parallel R \parallel C-\}$ in Fig. 1a can be re-expressed as $\{-R \parallel R \parallel R \parallel P-\}$. POC set of its sub-SOC $\{-R \parallel R \parallel P-\}$ is $\begin{bmatrix} t^1(\parallel P) \cup t^1(\perp R) \\ r^1(\parallel R) \end{bmatrix}$. So this sub-SOC is topologically equivalent to No. 3 sub-PM in Table 3.

Replace sub-SOC $\{-R \parallel R \parallel P-\}$ with this No. 3 sub-PM and obtain the HSOC branch in Fig. 1b. The branch in Fig. 1a and the branch in Fig. 1b have the same POC set.



(a) (b)

**Fig. 1.** Two topologically equivalent (3T1R) branches

## 4 Geometrical Conditions for Assembling Branches

Since POC set of each branch contains more elements than (or at least the same number of elements as) POC set of the PM, these branches shall be so assembled between the two platforms that some elements of their POC sets are eliminated to obtain the desired POC set of the PM. The basic equations used to determine geometrical conditions for assembling branches will be discussed in the following two different cases.

**Case 1.** After the first k branches are assembled, a sub-PM will be obtained. If POC set of this sub-PM still contains any element that is not included in POC set of the PM, the (k + 1)th branch shall be so assembled between the two platforms that this unwanted element shall be eliminated during intersection operation between POC sets.

A. If a rotational element is to be eliminated, the following three intersection operation equations shall be used.

$$\left[r^1(\parallel R_i)\right]_{bi} \cap \left[r^1(\parallel R_j)\right]_{bj} = \left[r^0\right]_{Pa}, \quad \text{if } R_i \nparallel R_j \tag{8a}$$

$$\left[r^1(\parallel R_i)\right]_{bi} \cap \left[r^2(\parallel \diamondsuit(R_{j1}, R_{j2}))\right]_{bj} = \left[r^0\right]_{Pa}, \quad \text{if } R_i \nparallel (\diamondsuit(R_{j1}, R_{j2})) \tag{8b}$$

$$\begin{aligned} &\left[r^2(\parallel \diamondsuit(R_1, R_2))\right]_{bi} \cap \left[r^2(\parallel \diamondsuit(R_{j1}, R_{j2}))\right]_{bj} \\ &= \left[r^1(\parallel (\diamondsuit(R_{i1}, R_{i2}) \cap (\diamondsuit(R_{j1}, R_{j2})))\right]_{Pa}, \text{if } (\diamondsuit(R_{i1}, R_{i2})) \nparallel (\diamondsuit(R_{j1}, R_{j2})) \end{aligned} \tag{8c}$$

B. If a translational element is to be eliminated, the following three intersection operation equations shall be used.

$$\left[t^1(\| P_i^*)\right]_{bi} \cap \left[t^1(\| P_j^*)\right]_{bj} = \left[t^0\right]_{Pa}, if\ P_i^* \nparallel P_j^* \tag{9a}$$

$$\left[t^1(\| P_i^*)\right]_{bi} \cap \left[t^1(\| P_j^*)\right]_{bj} = \left[t^0\right]_{Pa}, if\ P_i^* \nparallel P_j^* \tag{9b}$$

$$\begin{aligned} &\left[t^2(\| \diamondsuit(P_1^*, P_2^*))\right]_{bi} \cap \left[t^2(\| \diamondsuit(P_{j1}^*, P_{j2}^*))\right]_{bj} \\ &= \left[t^1(\| (\diamondsuit(P_{i1}^*, P_{i2}^*) \cap (\diamondsuit(P_{j1}^*, P_{j2}^*)))\right]_{Pa}, if\ (\diamondsuit(P_{i1}^*, P_{i2}^*)) \nparallel (\diamondsuit(P_{j1}^*, P_{j2}^*)) \end{aligned} \tag{9c}$$

Note: P* – translation of P pair or derivative translation of R pair or H pair.

**Case 2.** After the first k branches are assembled, a sub-PM will be obtained. If POC set of this sub-PM is identical to POC set of the PM, the $(k + 1)^{th}$ branch shall be so assembled between the two platforms that no element shall be eliminated during intersection operation between POC sets.

C. The following intersection operation equations shall not eliminate any rotational element.

$$\left[r^1(\| R_i)\right]_{bi} \cap \left[r^1(\| R_j)\right]_{bj} = \left[r^1(\| R_i)\right]_{Pa}, if\ R_i \parallel R_j \tag{10a}$$

$$\left[r^1(\| R_i)\right]_{bi} \cap \left[r^2 \diamondsuit\left((R_{j1}, R_{j2})\right)\right] = \left[r^1(\| R_i)\right]_{Pa}, if\ R_i \parallel (\diamondsuit(R_{j1}, R_{j2})) \tag{10b}$$

$$\begin{aligned} &\left[r^2(\| \diamondsuit(R_1, R_2))\right]_{bi} \cap \left[r^2(\| \diamondsuit(R_{j1}, R_{j2}))\right] \\ &= \left[r^2(\| \diamondsuit R_{i1}, R_{i2})\right]_{Pa}, if\ (\diamondsuit(R_{i1}, R_{i2})) \parallel (\diamondsuit(R_{j1}, R_{j2})) \end{aligned} \tag{10c}$$

$$\left[r^1(\| R_i)\right]_{bi} \cap \left[r^3\right]_{bj} = \left[r^1(\| R_i)\right]_{Pa} \tag{10d}$$

$$\left[r^2(\| \diamondsuit(R_{i1}, R_{i2}))\right]_{bi} \cap \left[r^3\right]_{bj} = \left[r^2(\| \diamondsuit(R_{i1}, R_{i2}))\right]_{Pa} \tag{10e}$$

$$\left[r^3\right]_{bi} \cap \left[r^3\right]_{bj} = \left[r^3\right]_{Pa} \tag{10f}$$

D. The following intersection operation equations shall no eliminate any translational element.

$$\left[t^1(\| P_i^*)\right]_{bi} \cap \left[t^1(\| P_j^*)\right]_{bj} = \left[t^1(\| P_i^*)\right]_{Pa}, if\ P_i^* \parallel P_j^* \tag{11a}$$

$$\left[t^1(\| P_i^*)\right]_{bi} \cap \left[t^2(\| \diamondsuit(P_{j1}^*, P_{j2}^*))\right] = \left[t^1(\| P_i^*)\right]_{Pa}, if\ P_i^* \parallel (\diamondsuit(P_{j1}^*, P_{j2}^*)) \tag{11b}$$

$$\left[t^2(\| \diamondsuit(P_1^*, P_2^*))\right]_{bi} \cap \left[t^2(\| \diamondsuit(P_{j1}^*, P_{j2}^*))\right] \\ = \left[t^2(\| \diamondsuit P_{i1}^*, P_{i2}^*)\right]_{Pa}, if\ (\diamondsuit(P_{i1}^*, P_{i2}^*)) \parallel (\diamondsuit(P_{j1}^*, P_{j2}^*)) \tag{11c}$$

$$\left[t^1(\| P_i^*)\right]_{bi} \cap \left[t^3\right]_{bj} = \left[t^1(\| P_i^*)\right]_{Pa} \tag{11d}$$

$$\left[t^2(\| \diamondsuit(P_{i1}^*, P_{i2}^*))\right]_{bi} \cap \left[t^3\right]_{bj} = \left[t^2(\| \diamondsuit(P_{i1}^*, P_{i2}^*))\right]_{Pa} \tag{11e}$$

$$\left[t^3\right]_{bi} \cap \left[t^3\right]_{bj} = \left[t^3\right]_{Pa} \tag{11f}$$

By the way, if POC set of the sub-PM obtained after the first k branches are assembled is already identical to POC set of the PM, the (k + 1)$^{th}$ branch can be replaced by any simple branch whose POC set contains at least all elements of POC set of the PM, e.g. the $SOC\{-S-S-R-\}$ branch.

## 5 Topological Structure Synthesis of 3T1R PM

**Step 1 Design requirements.** $M_{Pa} = \begin{bmatrix} t^3 \\ r^1 \end{bmatrix}$; DOF = 4; No P pair or only driving P pair shall be contained; Each branch contains only one driving pair and all driving pairs must be allocated on the same platform.

**Step 2 Determine POC set of SOC branch.** According to Eq. (3), POC set of the SOC branch shall be $M_{b_j} = \begin{bmatrix} t^3 \\ r^1 \end{bmatrix}, \begin{bmatrix} t^3 \\ r^2 \end{bmatrix}$ or $\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}$

**Step 3 Topological structure synthesis of SOC branch.**

(1) Topological structure synthesis of SOC branch with POC set $\begin{bmatrix} t^3 \\ r^1 \end{bmatrix}$

As discussed in Example 4.1, three candidate SOC branches are obtained (see Table 4).

(2) Topological structure synthesis of SOC branch with POC set $\begin{bmatrix} t^3 \\ r^2 \end{bmatrix}$

Four candidate SOC branches are obtained based on a synthesis process similar to that described in Example 4.1 (see Table 4).

(3) Topological structure synthesis of (3T3R) SOC branch with POC set $\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}$

As we all know, the simplest such SOC branch is $SOC\{-S-S-R-\}$.

**Table 4.** Feasible SOC branches and corresponding HSOC branches

| $M_{b_j}$ | SOC branch | HSOC branch |
|---|---|---|
| $\begin{bmatrix} t^3 \\ r^1 \end{bmatrix}$ | (1) $SOC\{-R \parallel R \parallel R-P-\}$ | (1) $HSOC\{-R \parallel R \parallel R-P^{(4R)}-\}$ |
| | (2) $SOC\{-R \parallel R-P-P-\}$ | (2) $HSOC\{-R \parallel R^{(4S)}-P^{(4S)}-P^{(4S)}-\}$<br>(3) $HSOC\{-R(\parallel \Diamond(P^{(4R)}, P^{(4R)}) \parallel R-\}$<br>(4) $HSOC\{-R \parallel R-(\Diamond(P^{(5R1C)}, P^{(5R1C)})-\}$ |
| | (3) $SOC\{-R-P-P-P-\}$ | (5) $HSOC\{-R-2\{R \parallel R^{(4S)}-P^{(4S)}-P^{(4S)}-\}-\}$<br>(6) $HSOC\{-R-Delta\,PM-\}$ |
| $\begin{bmatrix} t^3 \\ r^2(\parallel \Diamond(R, R^*)) \end{bmatrix}$ | (4) $SOC\{-R \parallel R \parallel R-R \parallel R-\}$ | |
| | (5) $SOC\{-R \parallel R \parallel R-P-R-\}$ | (7) $HSOC\{-R \parallel R \parallel R-P^{(4R)}-R-\}$ |
| | (6) $SOC\{-R \parallel R-P-R \parallel R-\}$ | (8) $HSOC\{-R \perp R^{(2R2S)}-P^{(2R2S)}-R \parallel R-\}$ |
| | (7) $SOC\{-R-P-P-R \parallel R-\}$ | (9) $HSOC\{-R-P^{(4S)}-P^{(4S)}-R^{(4S)} \parallel R-\}$<br>(10) $HSOC\{-\Diamond(P^{(5R1C)}, P^{(5R1C)}) \parallel R \parallel R \perp R-\}$ |
| $\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}$ | (8) $SOC\{-S-S-R-\}$ | |

**Step 4 Topological structure synthesis of HSOC branch.** Sub-SOCs containing P pairs can be replaced by the following sub-PMs.

(1) $sub-SOC\{-P-\} = sub-PM : \{-P^{(4R)}-\}$.
(2) $sub-SOC\{-R-P-\} = sub-PM : \{-R^{(2R2S)}-P^{(2R2S)}-\}$.
(3) $sub-SOC\{-R-P-P-\} = sub-PM : \{-R^{(4S)}-P^{(4S)}-P^{(4S)}-\}$.
(4) $sub-SOC\{-P-P-P-\} = sub-PM : \{-2\,SOC\{-R \parallel R^{(4S)}-P^{(4S)}-P^{(4S)}-\}-\}$.
(5) $sub-SOC\{-P-P-\} = sub-PM : \{-\Diamond(P^{(5R1C)}, P^{(5R1C)})-\}$.
(6) $sub-SOC\{-P-P-P-\} = sub-PM : \{-Delta\,PM-\}$.

After topologically equivalent replacement, 6 types of 3T1R branches and 5 types of 3T2R branches which contain no P pairs are obtained (see Table 4), as shown respectively in Figs. 2 and 3.

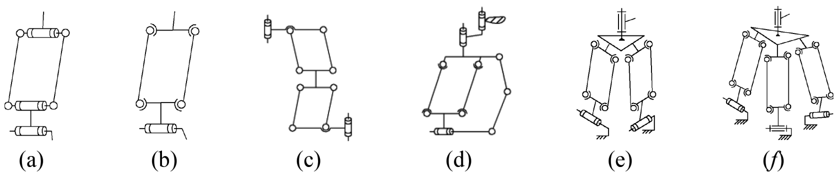



**Fig. 2.** Six types of 3T1R branches without P pairs

**Step 5 Branch combination scheme.** Based on the 2 SOC branches (No. 4 and No. 8 in Table 4) and 10 HSOC branches (refer to Table 4), different branch combination schemes which can generate the desired POC set and DOF of the PM can be obtained. During branch combination, we shall ensure that the branch shall have simple structure and certain symmetry. Each branch shall have only one driving pair and all driving pairs shall be allocated on the same platform. Table 5 shows some of these branch combination schemes.

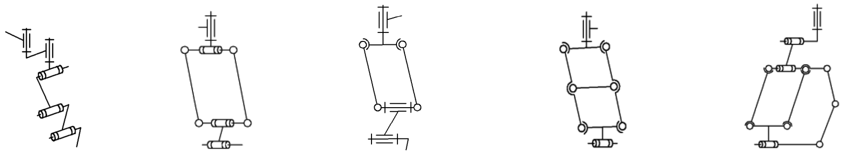



**Fig. 3.** Five types of 3T2R branches without P pairs

**Table 5.** Branch combination scheme of 3T1R PM

| Branch type | Branch combination scheme | |
|---|---|---|
| Table 4-SOC-(4) | I | $4-SOC\{-R\parallel R\parallel R-R\parallel R-\}$ |
| | 2 | $2-SOC\{-R\parallel R\parallel R-R\parallel R-\}\oplus 2-SOC\{-S-S-R\}$ |
| Table 4-HSOC-(7) | 3 | $4-HSOC\{-R\parallel R\parallel R-P^{(4R)}-R-\}$ |
| | 4 | $2-HSOC\{-R\parallel R\parallel R-P^{(4R)}-R-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(8) | 5 | $4-HSOC\{-R\perp R^{(2R2S)}\parallel P^{(2R2S)}-R\parallel R-\}$ |
| | 6 | $2-HSOC\{-R\perp R^{(2R2S)}\parallel P^{(2R2S)}-R\parallel R-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(9) | 7 | $4-HSOC\{-R-P^{(4S)}-P^{(4S)}-R^{(4S)}\parallel R-\}$ |
| | 8 | $2-HSOC\{-R-P^{(4S)}-P^{(4S)}-R^{(4S)}\parallel R-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(l) | 9 | $4-HSOC\{-R\parallel R\parallel R-P^{(4S)}-\}$ |
| | 10 | $2-HSOC\{-R\parallel R\parallel R-P^{(4S)}-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(5) | 11 | $2-HSOC\{-R\parallel R^{(4S)}-P^{(4S)}-P^{(4S)}-\}-\}$ |
| | 12 | $1-HSOC\{-R-2\{R\parallel R^{(4S)}-P^{(4S)}-P^{(4S)}-\}-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(6) | 13 | $1-HSOC\{-R-Delta\,PM-\}\oplus 1-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(3) | 14 | $2-HSOC\{-R(\parallel\diamondsuit(P^{(4R)},P^{(4R)})\parallel R-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-(4) | 15 | $1-HSOC\{-R\parallel R-(\diamondsuit(P^{(5R1C)},P^{(5R1C)})-\}\oplus 2-SOC\{-S-S-R-\}$ |
| Table 4-HSOC-1O) | 16 | $2-HSOC\{-\diamondsuit(P^{(5R1C)},P^{(5R1C)})\parallel R\parallel R\perp R-\}$ |

**Step 6 Determine geometrical conditions for assembling branches.** We will use scheme 1 in Table 5 as an example to explain the process to determine geometrical conditions for assembling branches between two platforms, as shown in Fig. 4a.

(1) Select 4 identical SOC branches

$$SOC\{-R_{j1}\parallel R_{j2}\parallel R_{j3}-R_{j4}\parallel R_{j5}-\},\mathrm{j}=1,2,3,4$$

(2) Select an arbitrary point on moving platform as the base point o’.
(3) Determine POC set of SOC branch

$$M_{bj}=\begin{bmatrix} t^3 \\ r^2(\parallel\diamondsuit(R_{j3},R_{j4}) \end{bmatrix},\mathrm{j}=1,2,3,4,$$

(4) Establish POC equation for PM

Substitute the desired POC set of the PM and POC set of each branch into Eq. (2) and obtain

$$\begin{bmatrix} t^3 \\ r^1 \end{bmatrix} \Leftarrow \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{13}, R_{14}) \end{bmatrix} \cap \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{23}, R_{24}) \end{bmatrix} \cap \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{33}, R_{34}) \end{bmatrix} \cap \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{43}, R_{44}) \end{bmatrix}$$

Where, "<=" means the POC set on the left side of the equation is to be obtained by intersection operation of all the POC sets on the right side of the equation

(5) Determine geometrical conditions for assembling the first two branches.
In order for the moving platform to obtain the desired POC set, intersection of POC sets of the first two branches must eliminate one rotational element. According to Eq. (8c), when $R_{13} \nparallel R_{23}$ and $R_{15} \parallel R_{25}$, the moving platform has only one rotation ($\|R_{15}$) and 3 translations. So the above POC equation can be rewritten as

$$\begin{bmatrix} t^3 \\ r^1 \end{bmatrix} \Leftarrow \begin{bmatrix} t^3 \\ r^1(\| R_{15}) \end{bmatrix} \cap \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{33}, R_{34}) \end{bmatrix} \cap \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{43}, R_{44}) \end{bmatrix}$$

Where, the first POC set on right side of the equation is POC set of the sub-PM form obtained after the first two branches are assembled.

(6) Determine geometrical conditions for assembling the other two branches
Since POC set of the sub-PM formed by the first two branches is already identical to the desired POC set of the PM, assembly of the other two branches shall not change POC set of the PM. According to Eq. (10b), geometrical conditions for assembling these two branches are: $R_{15} \parallel R_{25} \parallel R_{35} \parallel R_{45}$, $R_{11} \parallel R_{31}$ and $R_{21} \parallel R_{41}$.

(7) Determine allocation scheme of the 4 branches on platforms
According to geometrical conditions for assembling branches, there may be several different allocation schemes of the 4 branches on platforms. One feasible allocation scheme is:

$$SOC\{-R_{j1} \parallel R_{j2} \parallel \overbrace{R_{j3} \perp R_{j4}} \parallel R_{j5}-\}, R_{15} \parallel R_{25} \parallel R_{35} \parallel R_{45}, R_{11} \parallel R_{31}, R_{21} \parallel R_{41}, R_{21} \parallel R_{41},$$

(Axes of $R_{11}$, $R_{21}$, $R_{31}$ and $R_{41}$ lie in the same plane and form a square)
The PM assembled based on this allocation scheme is shown in Fig. 4a. The PMs shown in Fig. 4b and c also satisfy the geometrical assembling conditions.

(8) Discussion
Since POC set of the sub-PM formed by the first two branches and the two platforms is already identical to the desired POC set of the PM, the other two branches can be replaced by two simpler $SOC\{-S-S-R-\}$ branches, see Fig. 4d.
For each branch combination scheme in Table 5, corresponding PM(s) can be obtained similarly, as shown in Figs. 4, 5, 6, 7, 8, 9, 10, 11 and 12.

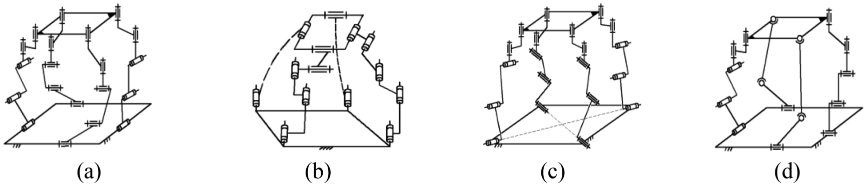



**Fig. 4.** Four PMs with No.6 SOC in Table 4

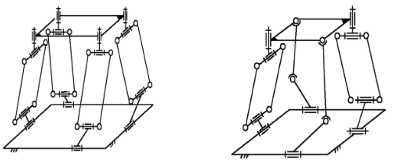


**Fig. 5.** Two PMs with No.7 HSOC in Table 4

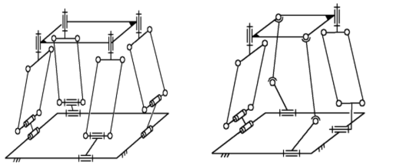


**Fig. 6.** Two PMs with No.8 HSOC in Table 4

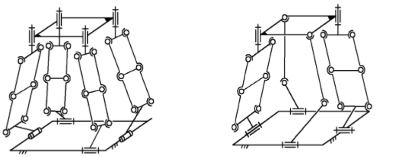


**Fig. 7.** Two PMs with No.9 HSOC in Table 4

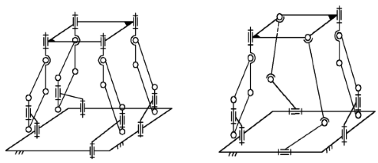


**Fig. 8.** Two PMs with No.1 HSOC in Table 4

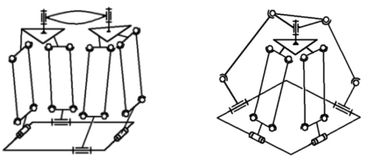


**Fig. 9.** Two PMs with No.5 HSOC in Table 4

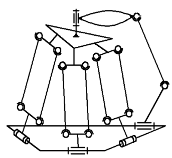


**Fig. 10.** PM with No.6 HSOC in Table 4

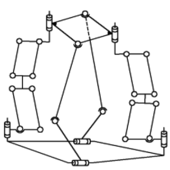


**Fig. 11.** PM with No.3 HSOC in Table 4

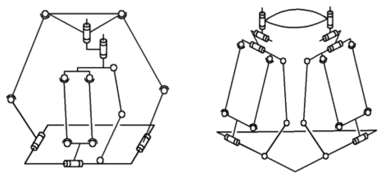


**Fig. 12.** Two PMs with No.4 HSOC in Table 4

**Step 7 Check DOF of the PM.** Check the above obtained 18 PMs for their DOFs. Take the PM in Fig. 4a as example, Eqs. (4a, b) are used to calculate its DOF.

(1) Check POC set of branch: POC set of each branch is

$$M_{bj} = \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{j3}, R_{j4}) \end{bmatrix}, \mathrm{j} = 1, 2, 3, 4.$$

(2) Calculate DOF of the PM: Substitute $M_{bj}$ into Eq. (4b) and obtain $\xi_{L1} = 6,\ \xi_{L2} = 5,\ \xi_{L3} = 5$. Then substitute $\xi_{L_j}$ into Eq. (4a) and obtain DOF of the PM

$$F = \sum_{i=1}^{m} f_i - \sum_{j=1}^{v} \xi_{L_j} = 20 - (6 + 5 + 5) = 4$$

All the other 17 PMs can be checked in the same way.

**Step 8 Select driving pairs.** According to criteria for driving pair selection (refer to Sect. 2.3), check whether the 4 R pairs on (Fig. 5) the fixed platform (Fig. 4a) can all be selected as driving pairs.

(1) Suppose the 4 R pairs $(R_{11}, R_{21}, R_{31}, R_{41})$ to be driving pairs and lock them, a new PM can be obtained. Topological structure of each branch is $SOC\{-R_{j2} \parallel R_{j3} - R_{j4} \parallel R_{j5}-\}, \mathrm{j} = 1, 2, 3, 4.$
(2) According to Eq. (1), POC set of each branch is

$$M_{bj} = \begin{bmatrix} t^3 \\ r^2(\| \diamondsuit(R_{j3}, R_{j4}) \end{bmatrix}, \mathrm{j} = 1, 2, 3, 4.$$

(3) Determine DOF of the new PM
Substitute $M_{bj}$ into Eq. (4b) and obtain $\xi_{L1} = 6,\ \xi_{L2} = 5,\ \xi_{L3} = 5$. Then there is

$$F^* = \sum_{i=1}^{m} f_i - \sum_{j=1}^{v} \xi_{L_j} = 16 - (6 + 5 + 5) = 0$$

(4) Since DOF of the obtained new PM is $F^* = 0$, the 4 R pairs (Fig. 6) on the fixed platform can be selected as (Fig. 8) driving pairs simultaneously according to Sect. 2.3.

All the other 18 PMs can be checked in the same (Fig. 11) way and find that the 4 R pairs on the fixed (Fig. 12) platform of each PM can be selected as driving pairs simultaneously.

## 6 Conclusions

(1) 18 types of 3T1R PMs with no P pairs are obtained using the topological structure synthesis of PMs based on POC equations, as shown in Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12. Among these 18 types of PMs, 3 types of 3T1R PMs shown in Figs. 7a, 9a and 10 respectively have been used in SCARA robot design. The other 15 types of 3T1R PMs also feature simple structure and may have the possibility to be used in new SCARA robot design.

(2) The topological structure synthesis method based on POC equations discussed in this paper is the further development of our early work on topological structure synthesis of PMs. New achievements of this paper include: (a) general method for topological structure synthesis of simple branches based on POC equation for serial mechanisms; (b) general method for topological structure synthesis of complex branches based on replacement a sub-SOC in a SOC branch with a topologically equivalent sub-PM; (c) general method for determination of geometrical conditions for assembling branches between platforms (for the same geometrical condition, there may be several different branch assembling schemes); (d) general criteria for selection of driving pairs.

(3) The three basic equations (Eqs. (1, 2 and 4a, b)) unveil the internal relations among topological structure, POC set and DOF. They are the theoretical basis for mechanism structure analysis and synthesis.

**Acknowledgment.** The authors acknowledge the financial support of the national science foundation of China (NSFC) with grant numbers of 51375062, 51365036, 51475050 and 51405039.

## References

1. Mitsubishi SCARA robots-RH Series [EB/OL], 16 May 2015. http://www.meau.com/eprise/main/sites/public/Products/Robots/Robot_Videos/default
2. ABB. IRB 360 FlexPicker [EB/OL], 16 May 2015. http://new.abb.com/products/robotics/industrial-robots/irb-360
3. Clavel, R.: Device for displacing and positioning an element in space. Switzerland, WO1987003528, 18 June 1987
4. Pierrot, F., Reynaud, C., Fournier, A.: Delta—a simple and efficient parallel robot. Robotica **8**(2), 105–109 (1990)
5. Fanuc. M-1iA series robots [EB/OL], 16 May 2015. http://robot.fanucamerica.com/products/robots/product.aspx
6. Pierrot, F., Nabat, V., Company, O., et al.: Optimal design of a 4-DOF parallel manipulator: from academia to industry. IEEE Trans. Rob. **25**(2), 213–224 (2009)
7. Pierrot, F.: High-speed parallel robot with four degrees of freedom: France, WO2006087399A1, 24 Aug 2006
8. An articulated traveling plate [EB/OL], 16 May 2015. http://www.adept.com/products/robots/parallel/quattro-s650h/downloads

9. Adept Quattro s650H [EB/OL], 16 May 2015. http://www.adept.com/products/robots/parallel/quattro-s650h/general
10. Huang, T., Zhao, X.,Wang, P.,et al.: Rod-wheel combined PM with three translations and a rotation: China, CN102152306A. Accessed 17 Aug 2011
11. Huang, T., Wang, M., Liu, S., et al.: A PM with three translations and one rotation: China, CN201907121U, 27 July 2011
12. Huang, T., Mei, J., Wang, H., et al.: A high speed PM with three translations and one rotation: China, CN101863024A, 20 Oct 2010
13. Yuhang, L., Jiangping, M., Songtao, L., et al.: Dynamic dimensional synthesis of a 4-DOF high-speed parallel manipulator (in Chinese). Chin. J. Mech. Eng. **50**(19), 32–40 (2014)
14. Xinjun, L., Fugui, X., Jinsong, W.: Current opportunities in the field of mechanisms in China (in Chinese). Chin. J. Mech. Eng. **51**(13), 2–12 (2015)
15. Yang, T.-L., Jin, Q., Liu, A.-X., et al.: Structural synthesis and classification of the 3-DOF translational parallel robot mechanisms based on the units of single-opened-chain (in Chinese). Chin. J. Mech. Eng. **38**(8), 31–36 (2002)
16. Jin, Q., Yang, T.-L.: Theory for topology synthesis of parallel manipulators and its application to three-dimension-translation parallel manipulators. ASME J. Mech. Des. **126**, 625–639 (2004)
17. Jin, Q., Yang, T.-L.: Synthesis and analysis of a group of 3-degree-of-freedom partially decoupled parallel manipulators. ASME J. Mech. Des. **126**, 301–306 (2004)
18. Yang, T.-L.: Theory and Application of Robot Mechanism Topology (in Chinese). China machine press, Beijing (2004)
19. Shen, H.-P., Yang, T.-L., et al.: Synthesis and analysis of kinematic structures of 6-DOF parallel robotic mechanisms. Mech. Mach. Theory **40**, 1164–1180 (2005)
20. Shen, H.-P., Yang, T.-L., et al.: Structure and displacement analysis of a novel three-translation PM. Mech. Mach. Theory **40**, 1181–1194 (2005)
21. Yang, T.-L., Liu, A.-X., Luo, Y.-F., et al.: Position and orientation characteristic equation for topological design of robot mechanisms. ASME J. Mech. Des. **131**, 021001-1-17 (2009)
22. Yang, T.-L., Sun, D.-J.: A general DOF formula for PMs and multi-loop spatial mechanisms. ASME J. Mech. Robot. **4**(1), 011001-1-17 (2012)
23. Yang, T.-L., Liu, A.-X., Shen, H.-P., et al.: On the correctness and strictness of the POC equation for topological structure design of robot mechanisms. ASME J. Mech. Robot. **5**(2), 021009-1-18 (2013)