Chapter 1
Introduction

The whole heliosphere is permeated by the solar wind, a supersonic and super-Alfvénic plasma flow of solar origin which continuously expands into the heliosphere. This medium offers the best opportunity to study directly collisionless plasma phenomena, mainly at low frequencies where large-amplitude fluctuations have been observed. During its expansion, the solar wind develops a strong turbulent character, which evolves towards a state that resembles the well known hydrodynamic turbulence described by Kolmogorov (1941, 1991). Because of the presence of a strong magnetic field carried by the wind, low-frequency fluctuations in the solar wind are usually described within a magnetohydrodynamic (MHD, hereafter) benchmark (Kraichnan 1965; Biskamp 1993; Tu and Marsch 1995; Biskamp 2003; Petrosyan et al. 2010). However, due to some peculiar characteristics, the solar wind turbulence contains some features hardly classified within a general theoretical framework.

Turbulence in the solar heliosphere plays a relevant role in several aspects of plasma behavior in space, such as solar wind generation, high-energy particles acceleration, plasma heating, and cosmic rays propagation. In the 1970s and 80s, impressive advances have been made in the knowledge of turbulent phenomena in the solar wind. However, at that time, spacecraft observations were limited by a small latitudinal excursion around the solar equator and, in practice, only a thin slice above and below the equatorial plane was accessible, i.e., a sort of 2D heliosphere.

In the 1990s, with the launch of the Ulysses spacecraft, investigations have been extended to the high-latitude regions of the heliosphere, allowing us to characterize and study how turbulence evolves in the polar regions. An overview of Ulysses results about polar turbulence can also be found in Horbury and Tsurutani (2001). With this new laboratory, relevant advances have been made. One of the main goals of the present work will be that of reviewing observations and theoretical efforts made to understand the near-equatorial and polar turbulence in order to provide the reader with a rather complete view of the low-frequency turbulence phenomenon in the 3D heliosphere.

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New interesting insights in the theory of turbulence derive from the point of view which considers a turbulent flow as a complex system, a sort of benchmark for the theory of dynamical systems. The theory of chaos received the fundamental impulse just through the theory of turbulence developed by Ruelle and Takens (1971) who, criticizing the old theory of Landau and Lifshitz (1971), were able to put the numerical investigation by Lorenz (1963) in a mathematical framework. Gollub and Swinney (1975) set up accurate experiments on rotating fluids confirming the point of view of Ruelle and Takens (1971) who showed that a *strange attractor* in the phase space of the system is the best model for the birth of turbulence. This gave a strong impulse to the investigation of the phenomenology of turbulence from the point of view of dynamical systems (Bohr et al. 1998). For example, the criticism by Landau leading to the investigation of intermittency in fully developed turbulence was worked out through some phenomenological models for the energy cascade (cf. Frisch 1995). Recently, turbulence in the solar wind has been used as a big wind tunnel to investigate scaling laws of turbulent fluctuations, multifractals models, etc. The review by Tu and Marsch (1995) contains a brief introduction to this important argument, which was being developed at that time relatively to the solar wind (Burlaga 1993; Carbone 1993; Biskamp 1993, 2003; Burlaga 1995). The reader can convince himself that, because of the wide range of scales excited, space plasma can be seen as a very big laboratory where fully developed turbulence can be investigated not only per se, rather as far as basic theoretical aspects are concerned.

Turbulence is perhaps the most beautiful unsolved problem of classical physics, the approaches used so far in understanding, describing, and modeling turbulence are very interesting even from a historic point of view, as it clearly appears when reading, for example, the book by Frisch (1995). History of turbulence in interplanetary space is, perhaps, even more interesting since its knowledge proceeds together with the human conquest of space. Thus, whenever appropriate, we will also introduce some historical references to show the way particular problems related to turbulence have been faced in time, both theoretically and technologically. Finally, since turbulence is a phenomenon visible everywhere in nature, it will be interesting to compare some experimental and theoretical aspects among different turbulent media in order to assess specific features which might be universal, not limited only to turbulence in space plasmas. In particular, we will compare results obtained in interplanetary space with results obtained from ordinary fluid flows on Earth, and from experiments on magnetic turbulence in laboratory plasmas designed for thermonuclear fusion.

1.1 The Solar Wind

"Since the gross dynamical properties of the outward streaming gas are hydrodynamic in character, we refer to the streaming as the solar wind." This sentence, contained in Parker (1958b) seminal paper, represents the first time the name "solar wind" appeared in literature, about 60 years ago.

The idea of the presence of an ionized gas continuously streaming radially from the sun was firstly hypothesized by Biermann (1951, 1957) based on observations of the displacements of the comet tails from the radial direction and on the ionization of cometary molecules. A similar suggestion seemed to come out from the occurrence of auroral phenomena and the continuous fluctuations observed in the geomagnetic lines of force. The same author estimated that this ionized flow would have a bulk speed ranging from 500 to 1500 km/s.

Parker (1958a) showed that the birth of the wind was a direct consequence of the high coronal temperature and the fact that it was not possible for the solar corona, given the estimated particle number density and plasma temperature, to be in hydrostatic equilibrium out to large distances with vanishing pressure. He found that a steady expansion of the solar corona with bulk speed of the order of the observed one would require reasonable coronal temperatures.

As the wind expands into the interplanetary space, due to its high electrical conductivity, it carries the photospheric magnetic field lines with it and creates a magnetized bubble of hot plasma around the Sun, namely the heliosphere.

For an observer confined in the ecliptic plane, the interplanetary medium appears highly structured into recurrent high velocity streams coming from coronal holes regions dominated by open magnetic field lines, and slow plasma originating from regions dominated by closed magnetic field lines. This phenomenon is much more evident especially during periods of time around minimum of solar activity cycle, when the meridional boundaries of polar coronal holes extend to much lower heliographic latitude reaching the equatorial regions.

This particular configuration combined with the solar rotation is at the basis of the strong dynamical interactions between slow and fast wind that develops during the wind expansion. This dynamics ends up to mix together plasma and magnetic field features which are characteristic separately of fast and slow wind at the source regions. As a matter of fact, in-situ observations in the inner heliosphere unraveled the different nature of these two types of wind not only limited to the large scale average values of their plasma and magnetic field parameters but also referred to the nature of the associated fluctuations.

It is clear that a description of the wind MHD turbulence will result more profitable if performed within the frame of reference of the solar wind macro structure, i.e. separately for fast and slow wind, without averaging the two.

Just to strengthen the validity of this approach, that we will follow throughout this review, we like to mention the following concept: “Asking for the average solar wind might appear as silly as asking for the taste af an average drink. What is the average between wine and beer? Obviously a mere mixing and averaging means mixing does not lead to a meaningful result. Better taste and judge separately and then compare, if you wish.” (Schwenn 1983)

However, before getting deeper into the study of turbulence, it is useful to have an idea of the values of the most common physical parameters characterizing fast and slow wind.

Table 1.1 Typical values of several solar wind parameters as measured by Helios 2 at 1 AU

Wind parameter	Slow wind	Fast wind
Number density	$\sim 15\,\mathrm{cm}^{-3}$	$\sim 4\,\mathrm{cm}^{-3}$
Bulk velocity	$\sim 350\,\mathrm{km\,s}^{-1}$	$\sim 600\,\mathrm{km\,s}^{-1}$
Proton temperature	$\sim 5 \times 10^4\,\mathrm{K}$	$\sim 2 \times 10^5\,\mathrm{K}$
Electron temperature	$\sim 2 \times 10^5\,\mathrm{K}$	$\sim 1 \times 10^5\,\mathrm{K}$
α-Particles temperature	$\sim 2 \times 10^5\,\mathrm{K}$	$\sim 8 \times 10^5\,\mathrm{K}$
Magnetic field	$\sim 6\,\mathrm{nT}$	$\sim 6\,\mathrm{nT}$

Table 1.2 Typical values of different speeds obtained at 1 AU

Speed	Slow wind	Fast wind
Alfvén	$\sim 30\,\mathrm{km\,s}^{-1}$	$\sim 60\,\mathrm{km\,s}^{-1}$
Ion sound	$\sim 60\,\mathrm{km\,s}^{-1}$	$\sim 60\,\mathrm{km\,s}^{-1}$
Proton thermal	$\sim 35\,\mathrm{km\,s}^{-1}$	$\sim 70\,\mathrm{km\,s}^{-1}$
Electron thermal	$\sim 3000\,\mathrm{km\,s}^{-1}$	$\sim 2000\,\mathrm{km\,s}^{-1}$

These values have been obtained from the parameters reported in Table 1.1

Table 1.3 Typical values of different frequencies at 1 AU

Frequency	Slow wind	Fast wind
Proton cyclotron	$\sim 0.1\,\mathrm{Hz}$	$\sim 0.1\,\mathrm{Hz}$
Electron cyclotron	$\sim 2 \times 10^2\,\mathrm{Hz}$	$\sim 2 \times 10^2\,\mathrm{Hz}$
Plasma	$\sim 2 \times 10^5\,\mathrm{Hz}$	$\sim 1 \times 10^5\,\mathrm{Hz}$
Proton-proton collision	$\sim 2 \times 10^{-6}\,\mathrm{Hz}$	$\sim 1 \times 10^{-7}\,\mathrm{Hz}$

These values have been obtained from the parameters reported in Table 1.1

Table 1.4 Typical values of different lengths at 1 AU plus the distance traveled by a proton before colliding with another proton

Length	Slow wind	Fast wind
Debye	$\sim 4\,\mathrm{m}$	$\sim 15\,\mathrm{m}$
Proton gyroradius	$\sim 130\,\mathrm{km}$	$\sim 260\,\mathrm{km}$
Electron gyroradius	$\sim 2\,\mathrm{km}$	$\sim 1.3\,\mathrm{km}$
Distance between 2 proton collisions	$\sim 1.2\,\mathrm{AU}$	$\sim 40\,\mathrm{AU}$

These values have been obtained from the parameters reported in Table 1.1

Since the wind is an expanding medium, we ought to choose one heliocentric distance to refer to and, usually, this distance is 1 AU. In the following, we will provide different tables referring to several solar wind parameters, velocities, characteristic times, and lengths.

Based on the Tables above, we can conclude that, the solar wind is a super-Alfvénic, supersonic and collisionless plasma, and MHD turbulence can be investigated for frequencies smaller than $\sim 10^{-1}$ Hz (Table 1.3).

1.2 Dynamics vs. Statistics

The word *turbulent* is used in the everyday experience to indicate something which is *not regular*. In Latin the word *turba* means something confusing or something which does not follow an ordered plan. A *turbulent boy*, in all Italian schools, is a young fellow who rebels against ordered schemes. Following the same line, the behavior of a flow which rebels against the deterministic rules of classical dynamics is called turbulent. Even the opposite, namely a *laminar* motion, derives from the Latin word *lámina*, which means stream or sheet, and gives the idea of a regular streaming motion. Anyhow, even without the aid of a laboratory experiment and a Latin dictionary, we experience turbulence every day. It is relatively easy to observe turbulence and, in some sense, we generally do not pay much attention to it (apart when, sitting in an airplane, a nice lady asks us to fasten our seat belts during the flight because we are approaching some turbulence!). Turbulence appears everywhere when the velocity of the flow is high enough,[1] for example, when a flow encounters an obstacle (cf., e.g., Fig. 1.1) in the atmospheric flow, or during the circulation of blood, etc. Even charged fluids (plasma) can become turbulent. For example, laboratory plasmas are often in a turbulent state, as well as natural plasmas like the outer regions of stars. Living near a star, we have a big chance to directly investigate the turbulent motion inside the flow which originates from the Sun, namely the solar wind. This will be the main topic of the present review.

Turbulence that we observe in fluid flows appears as a very complicated state of motion, and at a first sight it looks (apparently!) strongly irregular and chaotic, both in space and time. The only dynamical rule seems to be the impossibility to predict any future state of the motion. However, it is interesting to recognize the fact that, when we take a picture of a turbulent flow at a given time, we see the presence

Fig. 1.1 Turbulence as observed in a river. Here we can see different turbulent wakes due to different obstacles encountered by the water flow: simple stones and pillars of the old Roman Cestio bridge across the Tiber river

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[1] This concept will be explained better in the next sections.

of a lot of different *turbulent structures* of all sizes which are actively present during the motion. The presence of these structures was well recognized long time ago, as testified by the amazing pictures of *vortices* observed and reproduced by the Italian genius Leonardo da Vinci, as reported in the textbook by Frisch (1995). The left-hand-side panel of Fig. 1.2 shows, as an example, some drawings by Leonardo which can be compared with the right-hand-side panel taken from a typical experiment on a turbulent jet.

Turbulent features can be recognized even in natural turbulent systems like, for example, the atmosphere of Jupiter (see Fig. 1.3). A different example of turbulence in plasmas is reported in Fig. 1.4 where we show the result of a typical high resolution numerical simulations of 2D MHD turbulence. In this case the turbulent field shown is the current density. These basic features of mixing between order and chaos make the investigation of properties of turbulence terribly complicated, although extraordinarily fascinating.

When we look at a flow at two different times, we can observe that the general aspect of the flow has not changed appreciably, say vortices are present all the time but the flow in each single point of the fluid looks different. We recognize that the gross features of the flow are reproducible but details are not predictable. We have to use a statistical approach to turbulence, just as it is done to describe stochastic processes, even if the problem is born within the *strange* dynamics of a deterministic system!

Fig. 1.2 *Left panel*: three examples of vortices taken from the pictures by Leonardo da Vinci (cf. Frisch 1995). *Right panel*: turbulence as observed in a turbulent water jet (Van Dyke 1982) reported in the book by Frisch (1995) (photograph by P. Dimotakis, R. Lye, and D. Papantoniu)

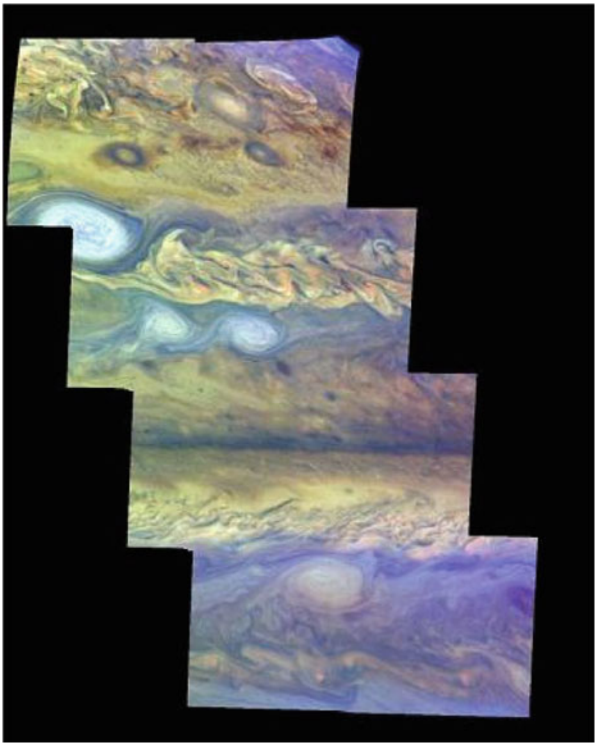

Fig. 1.3 Turbulence in the atmosphere of Jupiter as observed by Voyager

Turbulence increases the properties of transport in a flow. For example, the urban pollution, without atmospheric turbulence, would not be spread (or eliminated) in a relatively short time. Results from numerical simulations of the concentration of a passive scalar transported by a turbulent flow is shown in Fig. 1.5. On the other hand, in laboratory plasmas inside devices designed to achieve thermo-nuclear controlled fusion, anomalous transport driven by turbulent fluctuations is the main cause for the destruction of magnetic confinement. Actually, we are far from the achievement of controlled thermo-nuclear fusion. Turbulence, then, acquires the strange feature of something to be avoided in some cases, or to be invoked in some other cases.

Turbulence became an experimental science since Osborne Reynolds who, at the end of nineteenth century, observed and investigated experimentally the transition from laminar to turbulent flow. He noticed that the flow inside a pipe becomes turbulent every time a single parameter, a combination of the viscosity coefficient η, a characteristic velocity U, and length L, would increase. This parameter $Re = UL\rho/\eta$ (ρ is the mass density of the fluid) is now called the *Reynolds number*. At lower Re, say $Re \leq 2300$, the flow is regular (that is the motion is laminar), but when Re increases beyond a certain threshold of the order of $Re \simeq 4000$, the flow becomes turbulent. As Re increases, the transition from a laminar to a turbulent state

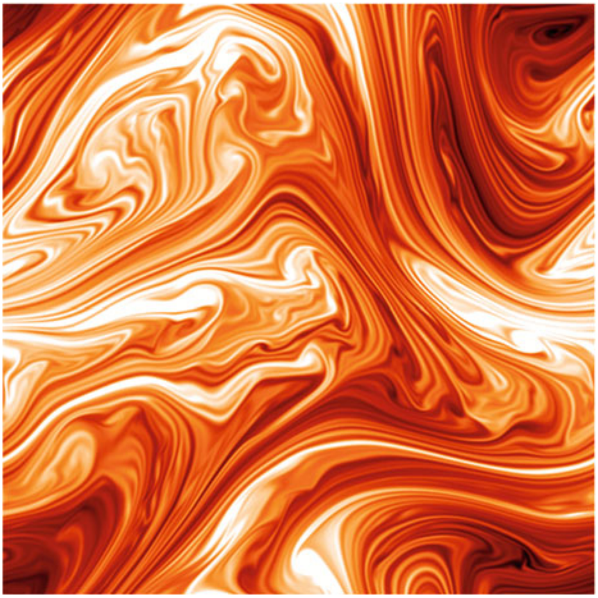

Fig. 1.4 High resolution numerical simulations of 2D MHD turbulence at resolution 2048 × 2048 (courtesy by H. Politano). Here, the authors show the current density $J(x, y)$, at a given time, on the plane (x, y)

occurs over a range of values of Re with different characteristics and depending on the details of the experiment. In the limit $Re \to \infty$ the turbulence is said to be in a fully developed turbulent state. The original pictures by Reynolds are shown in Fig. 1.6.

In Fig. 1.7 we report a typical sample of turbulence as observed in a fluid flow in the Earth's atmosphere. Time evolution of both the longitudinal velocity component and the temperature is shown. Measurements in the solar wind show the same typical behavior. A typical sample of turbulence as measured by Helios 2 spacecraft is shown in Fig. 1.8. A further sample of turbulence, namely the radial component of the magnetic field measured at the external wall of an experiment in a plasma device realized for thermonuclear fusion, is shown in Fig. 1.9.

As it is well documented in these figures, the main feature of fully developed turbulence is the chaotic character of the time behavior. Said differently, this means that the behavior of the flow is unpredictable. While the details of fully developed turbulent motions are extremely sensitive to triggering disturbances, average properties are not. If this was not the case, there would be little significance in the averaging process. Predictability in turbulence can be recast at a statistical level. In other words, when we look at two different samples of turbulence, even collected within the same medium, we can see that details look very different. What is actually common is a generic stochastic behavior. This means that the global statistical behavior does not change going from one sample to the other.

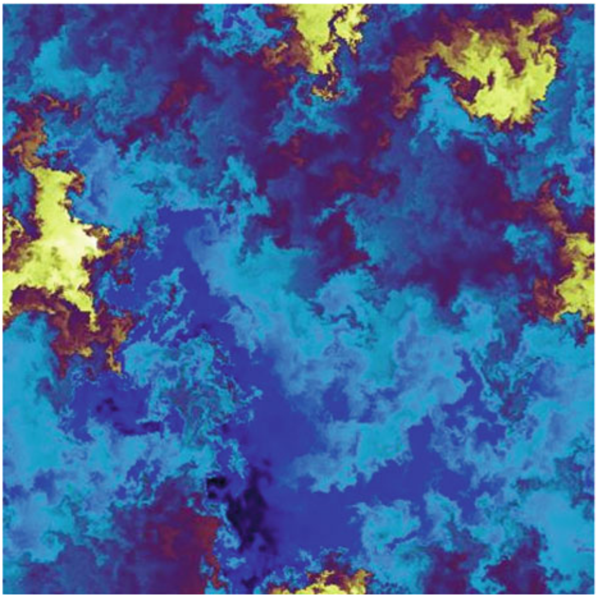

Fig. 1.5 Concentration field $c(x, y)$, at a given time, on the plane (x, y). The field has been obtained by a numerical simulation at resolution 2048×2048. The concentration is treated as a passive scalar, transported by a turbulent field. Low concentrations are reported in *blue* while high concentrations are reported in *yellow* (courtesy by A. Noullez)

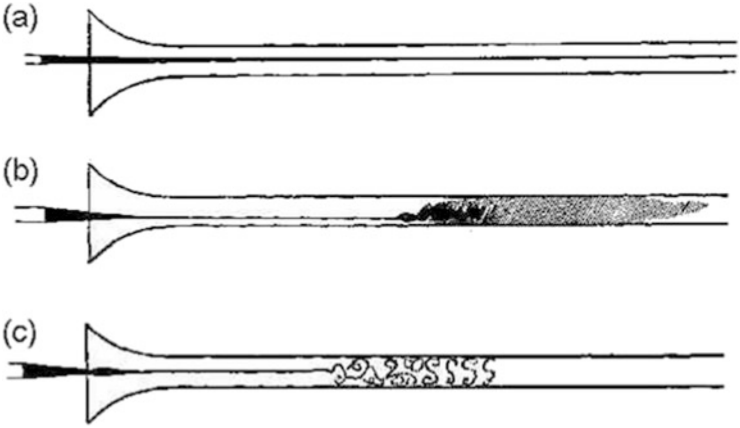

Fig. 1.6 The original pictures taken from Reynolds (1883) which show the transition to a turbulent state of a flow in a pipe as the Reynolds number increases [(**a**) and (**b**) panels]. Panel (**c**) shows eddies revealed through the light of an electric spark

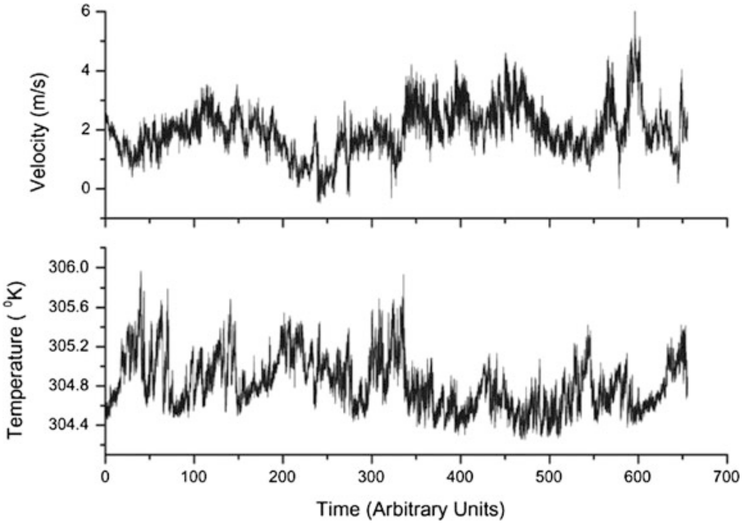

Fig. 1.7 Turbulence as measured in the atmospheric boundary layer. Time evolution of the longitudinal velocity and temperature are shown in the *upper and lower panels*, respectively. The turbulent samples have been collected above a grass-covered forest clearing at 5 m above the ground surface and at a sampling rate of 56 Hz (Katul et al. 1997)

The idea that fully developed turbulent flows are extremely sensitive to small perturbations but have statistical properties that are insensitive to perturbations is of central importance throughout this review. Fluctuations of a certain stochastic variable ψ are defined here as the difference from the average value $\delta\psi = \psi - \langle\psi\rangle$, where brackets mean some averaging process. Actually, the method of taking averages in a turbulent flow requires some care. We would like to recall that there are, at least, three different kinds of averaging procedures that may be used to obtain statistically-averaged properties of turbulence. The space averaging is limited to flows that are statistically homogeneous or, at least, approximately homogeneous over scales larger than those of fluctuations. The ensemble averages are the most versatile, where average is taken over an ensemble of turbulent flows prepared under nearly identical external conditions. Of course, these flows are not completely identical because of the large fluctuations present in turbulence. Each member of the ensemble is called a *realization*. The third kind of averaging procedure is the time average, which is useful only if the turbulence is statistically stationary over time scales much larger than the time scale of fluctuations. In practice, because of the convenience offered by locating a probe at a fixed point in space and integrating in time, experimental results are usually obtained as time averages. The ergodic theorem (Halmos 1956) assures that time averages coincide with ensemble averages under some standard conditions (see Sect. 3.2).

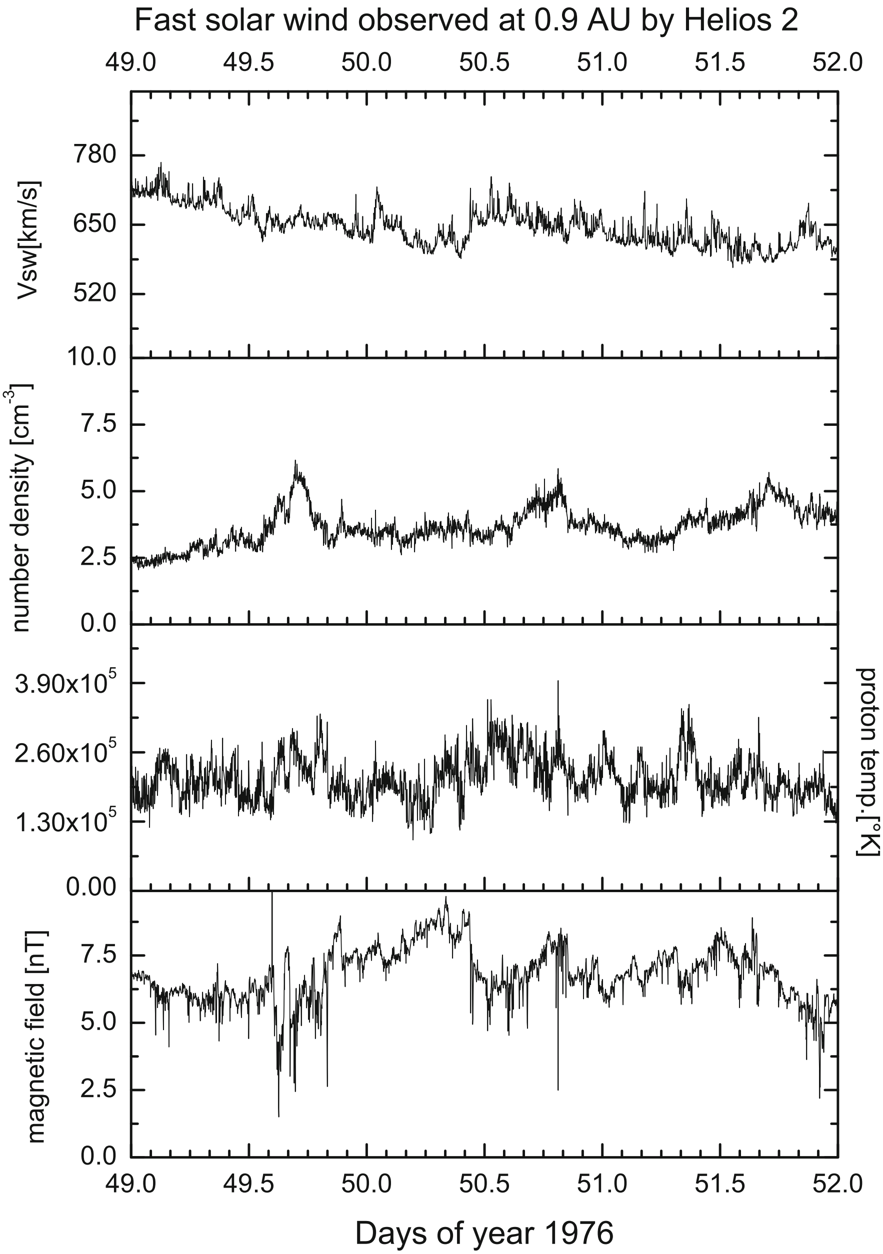

Fig. 1.8 A sample of fast solar wind at distance 0.9 AU measured by the Helios 2 spacecraft. *From top to bottom:* speed, number density, temperature, and magnetic field, as a function of time

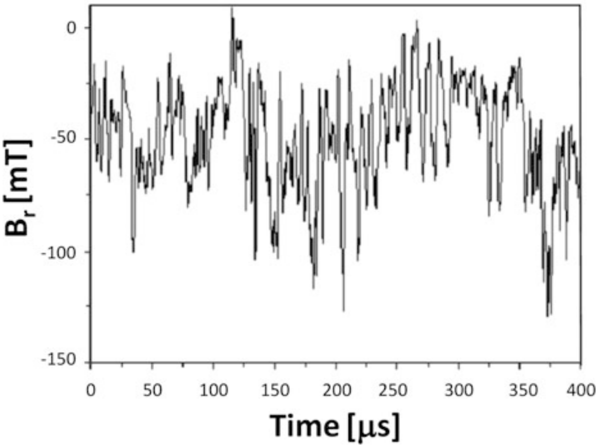

Fig. 1.9 Turbulence as measured at the external wall of a device designed for thermonuclear fusion, namely the RFX in Padua (Italy). The radial component of the magnetic field as a function of time is shown in the figure (courtesy by V. Antoni)

A different property of turbulence is that all dynamically interesting scales are excited, that is, energy is spread over all scales. This can be seen in Fig. 1.10 where we show the magnetic field intensity (see top panel) within a typical solar wind stream.

In the middle and bottom panels we show fluctuations at two different detailed scales. In particular, each panel contains an equal number of data points. From top to bottom, graphs have been produced using 1 h, 81 and 6 s averages, respectively. The different profiles appear statistically similar, in other words, we can say that interplanetary magnetic field fluctuations show similarity at all scales, i.e. they look self-similar.

Since fully developed turbulence involves a hierarchy of scales, a large number of interacting degrees of freedom are involved. Then, there should be an asymptotic statistical state of turbulence that is independent on the details of the flow. Hopefully, this asymptotic state depends, perhaps in a critical way, only on simple statistical properties like energy spectra, as much as in statistical mechanics equilibrium where the statistical state is determined by the energy spectrum (Huang 1987). Of course, we cannot expect that the statistical state would determine the details of individual realizations, because realizations need not to be given the same weight in different ensembles with the same low-order statistical properties.

It should be emphasized that there are no firm mathematical arguments for the existence of an asymptotic statistical state. As we have just seen, reproducible statistical results are obtained from observations, that is, it is suggested experimentally and from physical plausibility. Apart from physical plausibility, it is embarrassing that such an important feature of fully developed turbulence, as the existence of a statistical stability, should remain unsolved. However, such is the complex nature of turbulence.

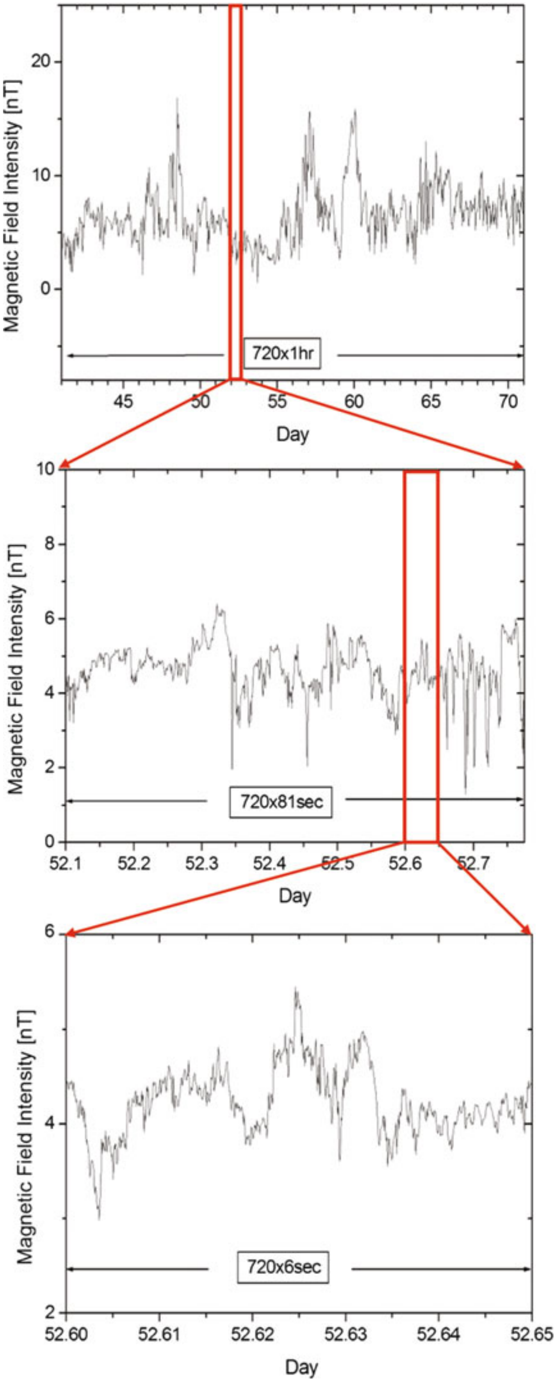

Fig. 1.10 Magnetic intensity fluctuations as observed by Helios 2 in the inner heliosphere at 0.9 AU, for different blow-ups. Each panel contains an equal number of data points. From *top to bottom*, graphs have been produced using 1 h, 81 and 6 s averages, respectively

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