Everybody Counts: Designing Tasks for *TouchCounts*

Nathalie Sinclair and Rina Zazkis

Abstract *TouchCounts* is an open-ended multi-touch App, which provides unconventional opportunities for engagement with the concept of a number, counting, and number operations. We describe a series of tasks designed for use in *TouchCounts*, which take advantage of the affordances of this environment. We elaborate on various aspects of the tasks as related to their pragmatic and epistemic values. We discuss the learning potential of the tasks, compare *TouchCount* tasks with similar tasks performed with physical manipulatives and provide a few illustrative examples of children's engagement with the tasks.

Keywords Counting · Number operations · Cardinal number · Ordinal number · Subitising

1 Introduction

For many open-ended, expressive digital environments for mathematics learning, the role of the task can be very important. In a Logo environment, for example, or a dynamic geometry environment (DGE), the learner starts with a blank screen and infinite possibility for engagement. In such environments, designing tasks that enable purposeful mathematical engagement, without becoming overly prescriptive, can be challenging. The challenge is increased by the impetus to design tasks that are not already doable, or even possible, in non-digital environments. In other words, good tasks in these environments should take advantage of the affordances of the given tools. Having students draw five different triangles can be done in a DGE, but having them drag one triangle into five different configurations is

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something that takes advantage of the continuous and direct manipulation affordance of most DGEs.

In this chapter, we elaborate on several tasks that we have designed for use in a particular open-ended multi-touch App called *TouchCounts* (Jackiw and Sinclair 2014), which provides unconventional engagement with the introductory concept of number and number operations. Learning to count in a contemporary world is as basic as learning to walk and talk. However, it is known that young children often experience difficulty in creating a one-to-one correspondence between the counted objects and assigning the number attributed to the last counted object as an enumerator of the total. Furthermore, initial experiences with arithmetic operations may present a challenge for learners, especially when the operations are approached by means of direct modelling (Coles 2014). How can technology assist with these challenges? This chapter describes a series of tasks that have been specifically designed to take advantage of the affordances of *TouchCounts*. We analyse these tasks in terms of their novel potential for supporting the development of number, as well as the different functions they draw on in terms of how children are invited to count, operate and attend to both ordinal and cardinal dimensions of number.

2 Theoretic Perspectives on Task Design in Expressive Environments

While there are many features of the App design that are of importance to shaping the kinds of tasks that are possible and productive, our focus in this paper is on task design. In our analysis of the tasks, we consider on two different aspects of the task. The first relates to its use of the digital technology. The second relates to the type of values offered by the task.

To begin, we adapt Laborde's (2001) typology of tasks developed by secondary teachers using the dynamic geometry software *Cabri*. She found that the teachers designed the following type of tasks:

- The technology is "used mainly as facilitating material aspects of the task while not changing it conceptually";
- The software is "supposed to facilitate the mathematical task that is considered as unchanged";
- The software "is supposed to modify the solving strategies of the task due to the use of some of its tools and to the possibility that the task might be rendered more difficult";
- The task itself "takes its meaning" from the software.

Tasks that do not change the mathematical activity conceptually can be said to make weak use of the technology. We are thus interested in identifying tasks that make strong use of technology, as well as in better understanding how these tasks change solution strategies or ways of thinking. Tasks that make strong use of technology will probably differ from tasks designed for non-digital environments. While we will highlight some of these differences, our main focus in this paper will be on task design.

In discussing the use of software in mathematics education, (Artigue 2002) distinguishes between their epistemic and pragmatic values:

> Epistemic (what you learn while you are doing this; as they contribute to the understanding of the objects they involve), pragmatic (what you achieve; I would like to stress that techniques are most often perceived and evaluated in terms of pragmatic value, that is to say, by focusing on their productive potential (efficiency, cost, field of validity).) (p. 248).

Similarly, tasks can have epistemic and/or pragmatic value. That is, they can change the techniques that are used, particularly in making a task easier to solve or more precise. They can also have epistemic value in terms of contributing to mathematical understanding in a certain way. We assume that there is some relation between the values of the software and those of the tasks, that is, that tasks that have epistemic value will draw on the epistemic values afforded by the software's design.

A final consideration relates to feedback. Mackrell et al. (2013) distinguish among different kinds of feedback: evaluation feedback is related to completion of a task or part of a task; strategy feedback aims to support or amend student approaches while she is engaged in a task; and, direct manipulation feedback, which "is the response of the environment to student action" (p. 83). One of the benefits of working with computers is that it provides a neutral form of feedback that the teacher cannot, often providing a sufficient indication of whether the task was completed successfully or what can be adjusted to achieve a successful completion. However, this is not always the case. Therefore, a guiding feature in our task design is to allow direct manipulation feedback to serve as evaluation feedback.

3 A Multi-touch Application for Counting and Operating

The multi-touch device is a novel technological affordance in mathematics education. Through its direct mediation, it offers opportunities for mathematical expressivity by enabling children to produce and transform screen objects with fingers and gestures, instead of engaging and operating through a keyboard or mouse. This makes it highly accessible, but also opens the way for new, tangible forms of mathematical communication (Jackiw 2013). In this section, we describe *TouchCounts*, whose design was motivated by multi-touch affordances.

Unlike many 'educational games' that can be found for the iPad, *TouchCounts* is open-ended and exploratory, rather than practice- and level-driven—it follows in the tradition of constructionist and expressive technologies in mathematics education (Papert 1980; Noss and Hoyles 1996) and supports the development of number by offering modes of interaction with objects that involve fingers and gestures. Specifically, it aims both (1) to engage one-to-one correspondence by allowing every finger touch to summon a new sequentially-numbered object into existence, one whose presence is both spoken aloud and symbolically labelled and (2) to enable gesture-based summing and partitioning, by means of pushing objects together and pulling them apart in ways that expose very young children to arithmetic operations. With these new affordances, however, come new questions related to design decisions (such as "What touch-based actions on the screen might better support and enable mathematical activity?"), as well as questions related to the development of number and how this particular technology may shape current curricular trajectories and, in the process, potentially disrupt them.

Currently, there are two sub-applications in *TouchCounts*, one for Enumerating and the other for Operating. After we describe each of the two worlds, we present and analyse a series of tasks, where the first set are to be used in the Enumerating world and the second set in the Operating world. In our analysis we refer to some comments or actions that we have observed children make. These are drawn from an ongoing study that involved iterative testing of the application with children (aged three to eight) in four different educational settings (one day-care and two primary school children either at school or in after-school care). Some of this research has been reported elsewhere (see Sinclair and Heyd-Metzuyanim 2014; Sinclair and Pimm 2014).

3.1 The Enumerating World

In this world, a user taps her fingers on the screen to summon numbered objects (yellow discs). The first tap produces a disc containing the numeral '1'. Subsequent taps produce successively numbered discs. As each tap summons a new numbered disc, *TouchCounts* audibly speaks the number word for its number ("one", "two", …, if the language is set to English). Fingers can be placed on the screen one at a time or simultaneously. With five successive taps, for instance, five discs (numbered '1' to '5') appear sequentially on the screen, which are counted aloud one by one (see Fig. 1a). However, if the user places two fingers on the screen simultaneously, two consecutively numbered discs appear at the same time (Fig. 1b), but only the higher-numbered one is named aloud ("two", if these are the first two taps). One small instance of opportunity lies in a new sense of the times-two table: the number of 'times' two fingers simultaneously touch the screen. The entire 'world' can be reset, to clear all numbered discs and return the 'count' of the next summoned disc to one. Note that the discs always arrive in order, with their symbolic names imprinted upon them.

From an adult perspective, the number of taps (whether made sequentially or simultaneously) is also the number of discs on the screen, a fact which can tacitly reinforce the cardinality principle, since the last number 'counted' (spoken aloud by *TouchCounts*) is exactly "how many" numbered discs there are to be seen. In traditional research in the area of early counting, it is a well-documented finding that even after children have counted a set of things (up to five, say), when they are asked "how many" objects are in that set, they will often count the objects again

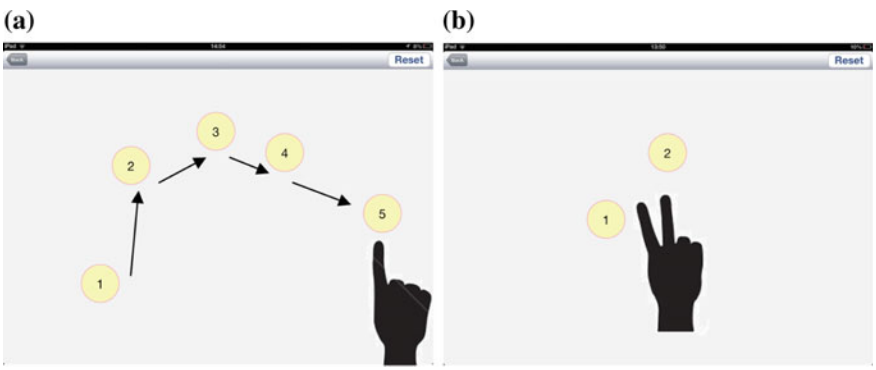

Fig. 1 **a** Five sequential taps—"one, two, three, four, five" is said (the *arrows* are only to indicate the sequence; they are not shown on the screen). **b** A simultaneous two-finger tap—only "two" is said (both discs appear simultaneously)

(Baroody and Wilkins 1999). The "how many?" question seems to provoke a routine of sequential counting.

In *TouchCounts'* Enumerating World, however, the child is engaged in a somewhat different practice—rather than counting a *given* set, she is actively *producing* a set with her finger(s) (perhaps aiming at a pre-given total) and the elements of that set seem to count themselves (both aurally and symbolically) as they are summoned into existence. One distinction that *TouchCounts* makes is that, orally, each number word in succession replaces (and eradicates) the previous one. At the end of the spoken count, no trace is left of what has been said. On the screen, however, each action leaves a visual trace, in the form of (one or more) numeral-bearing discs, of what has once been summoned into being.

If the 'gravity' option for this World is turned on in the App, then as long as the learner's finger remains pressed to the screen, the numbered object holds its position beneath her fingertip. But as soon as she 'lets go' (by lifting that finger), the numbered object falls toward and then disappears "off" the bottom of the screen, as if captured by some virtual gravity. With 'gravity' comes the option of a 'shelf', a horizontal line across the screen (in Fig. 2). If a user releases her numbered object above the shelf, it falls only to the shelf, and comes to rest there, visibly and permanently on screen, rather than vanishing out of sight 'below'. (Thus, Fig. 2 depicts a situation in which there have been four taps below the shelf—these numbered objects were falling—and then a disc labelled '5' was placed above the shelf by tapping above it.) Since each time a finger is placed on the screen a new numbered disc is created beneath it and, once released by lifting the finger, it begins to fall, one cannot "catch" or reposition an existing numbered object by re-tapping it. This is not a conventional "dragging" world.

Discs dropping away (under 'gravity') mirror the way spoken language fades rapidly over time, with no trace left—the impermanence of speech. Also, with discs disappearing, any sense of cardinality goes too: in the absence of the presence of

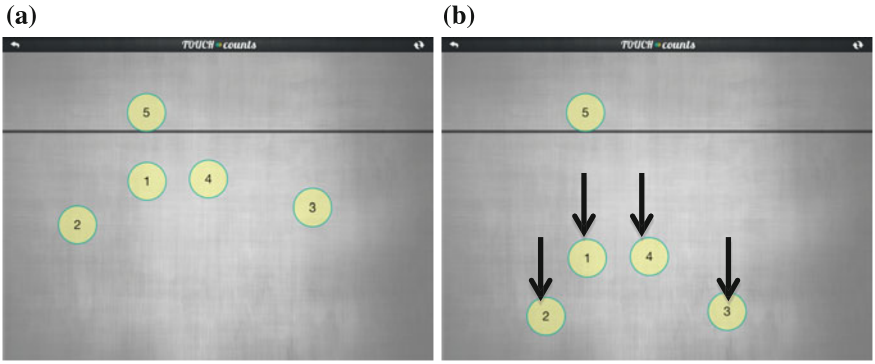

Fig. 2 **a** Five sequential taps—"one, two, three, four, five" is said (*the arrows are only to indicate the sequence; they are not shown on the screen*). **b** A simultaneous two-finger tap—only "two" is said (both discs appear simultaneously)

'1', the disc labelled '2' is simply the second one to have been summoned. So the Enumerating World with 'gravity' enabled (it is an option) is almost entirely an ordinal one, with the shelf acting as a form of visible memory.

One of the characteristics of *TouchCounts*, then, is that the computer handles the counting (the iPad is the one who announces and manages the arrival of various figures onto the ritual scene). The design intent was to help move young users towards transitive counting, even though the general setting provides a mix of cardinal and ordinal elements.

3.2 The Operating World

Whilst tapping on the screen in the Enumerating World creates sequentially numbered objects, tapping on the screen in the Operating World creates autonomous numbered sets, which we refer to as herds. The user's creation choreography starts by placing one or several fingers on the screen, which immediately creates a large disc that encompasses all the fingers and includes a numeral corresponding to the combined number of fingers touching the screen. At the same time, every one of the fingers in contact with the screen creates its own much smaller (and unnumbered) disc, centred on each fingertip. When the fingers are lifted off the screen, the numeral is spoken aloud and the smaller discs are then lassoed into a 'herd' and arranged regularly around the inner circumference of the big disc (Fig. 3a shows herds of 3 and 4). The small discs all move in either a clockwise or counter-clockwise direction to emphasise that they are to be seen as one unit. Herds of size one wander around the screen in order to make them more difficult to place one's finger on, in order to encourage children to operate with herds that are greater than one in number.

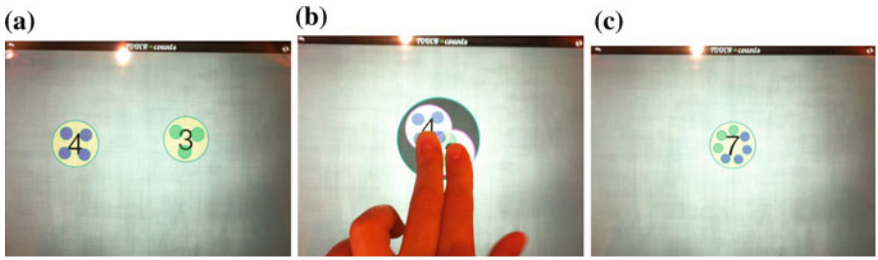

Fig. 3 **a** The herds. **b** Pinching two herds together. **c** The sum of two herds

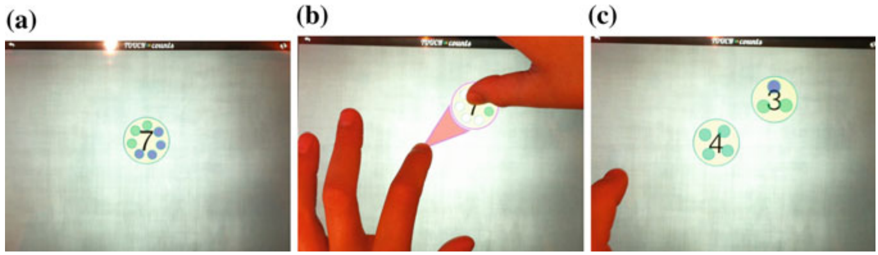

Fig. 4 **a** An initial herd of 7. **b** Left finger (in this instance) swiping outside the herd. **c** Resulting separation of 7 into herds of 4 and 3: *TouchCounts* announces “four”

Unlike in the Enumerating World, herds can be interactively dragged, either to move them around on the screen or to operate upon them. After two or more such arrangements have been produced (as in Fig. 3b) they can either be pinched together (addition) or ‘unpinched’ (subtraction or partition). In the case of pinching together, two fingers are required—one on each herd—to make the herds merge. Dynamically, they then become one herd that contains the ‘digital’ counters from each previous herd, thus adding them together. The new herd is labelled with the associated numeral of the sum (Fig. 3c), which *TouchCounts* announces aloud. Moreover, the new herd keeps a distinguished trace of the previous herds, which can be seen by means of the differentiated colours of the individual small discs. Multiple herds can be pinched together simultaneously. Note that the pinching gesture is entirely symmetric, both with respect to the pinching fingers and with respect to the herds, so that adding does not have the kind of order implied by the directionality of verbal or written expressions such as ‘two plus three’ or ‘2 + 3’.

An inverse pinch gesture (‘unpinching’) can be made in order to decompose a given herd into two herds. The gesture can be described either as ‘separating’, which supports the idea of partitioning, or as ‘taking out’ or ‘removing’, which supports the idea of subtracting. In both cases, two fingers are placed in the herd—

while one stays put, the second swipes out of the herd. This distinction of roles between the two fingers supports the needed directionality of subtraction. The further the swipe travels, the more will be taken out from the starting herd (and of course, at the extreme, everything can be taken out of the starting herd) (Fig. 4b). When the swiping finger is lifted, two new herds are formed and *TouchCounts* announces the number that has been taken out (Fig. 4c). In the extreme case (where everything is removed), a new herd is formed under the finger that has swiped, while in the location of the previous herd the numeral '0' appears briefly but then fades away.

The pinching gesture draws on one of the four grounding metaphors for addition, that of *object collection* (see Lakoff and Núñez 2000). Both adding and either subtracting or partitioning offer children the *action* of operating without necessarily requiring them to calculate the result. Unlike with the calculator, which can also perform addition and subtraction, *TouchCounts* first requires the production of herds that will be labeled by a numeral (indicating "how many" are in the herd) and then enacts the gathering/splitting mechanisms in which the two herds join or separate, both visually and temporally.

Children can pinch two herds together or split a herd apart relatively easily (though some children find it challenging, at least initially, to place their fingers right on the herds and often produce new herds of one). They can do this, obviously, without knowing what the sum or difference will be, without knowing that the transformation occurring reflects the operation of addition or subtraction and, most importantly, without thinking of those herds as cardinal numbers. In this sense, *TouchCounts* invites the children into a gesture-mediated form of operations. We note that while a teacher might introduce the word 'adding' to describe the process, neither that word nor its symbolic counterpart (e.g., '+') appears on the screen: the iPad only announces the result of the operation. As such, language such as "making" or "putting together" or "joining" can all be used to accompany the action of pinching discs together.

4 *TouchCounts* Tasks

In this section, we describe and analyse six tasks that have been created for use with *TouchCounts*. They have been developed over the course of on-going work with young children in a variety of settings and geographic locations. In developing the tasks we consider what engages the children and also what can focus attention on the mathematics of counting and operating. Some of the tasks were initiated by the children, which are of particular interest because of their adidactical nature.

4.1 Plucking Out Numbers for the Shelf

Put 36 above the shelf

This is an extension and variation on the previously explored task in which a child was asked to put 5 on the shelf. 36 has no particular attributes other than being a relatively big number. It is expected that a child will not count up by ones, but find a more efficient way.

Attending to the task requires a continuous control and comparison of the number that is on the screen and the target number. For example, if a child repeatedly increases the number of discs "counting by 5's", there is a point at which she will have to stop and evaluate a situation. The task is a good introduction to developing ideas related to addition and subtraction, considering a "relative distance" from the current number to the target number. If a screen has 30 on it, a child, maybe implicitly, starts evaluating whether to put another 5 or whether to "play it safe" and reach 36 increasing by ones.

This task provides a teacher with an opportunity to observe children's learning. If a child's preferable method is to increase the number by 1s, she can be encouraged to use a different way to "get there faster". Changing the task to a bigger number, 360 for example, may serve as a motivation for a child to seek a more efficient strategy. Also, it is interesting to observe if a child semi-randomly increasing the count or is using some kind of a strategy, like counting by threes.

The analog outside of the *TouchCounts* environment is making available to a child a large amount of counters in some container and asking her to put 36 counters on the table. One can definitely count by ones, or put some amount on the table, count up and adjust. Such a task is made both easier and harder within *TouchCounts*. It is easier because the child does not have to produce the number names or symbols. It is also harder because the target number in this case cannot be adjusted both up and down. The constraint in *TouchCounts* is that there is no option for adjusting down. Consider the case that by some oversight a child reached 37 or any other number bigger that 36. Working with counters, some can be returned to the container. Working with *TouchCounts*, the only way to complete the task is to start over. This requires a "calculated" approach, which is the epistemic value of the task.

If the task is repeated, one may focus on a minimal number of steps. I can complete the task in 4 steps: 10, 10, 10, 5–36! Less than 4 may require collaboration (or use of toes). The task is also appropriate for a group work. (Think of it as a simplified NIM game.) 2 or 3 children sharing a screen and taking turns. The "winner" is the one who puts 36 on the shelf. Each child, in turn, can touch the screen once or twice, and can use 1 or all his fingers.

Put numbers with 0 at the end on the shelf

We are purposefully avoiding the language of 'multiples of 10' in setting this task. However, it is expected that a child will develop to idea of counting by 10's. The epistemic value here is connecting the multitude and the sound to the written representation. That is, a child learns that the number that comes after 9 (or 19, or 39) is the one that has zero at the end. The same of course can be requested with ANY digit at the end. A challenge is to do this with eyes closed.

Put numbers that have digits 3 or 7 on the shelf
This is a more complex variation, especially when several children work on the task. This can be a game of concentration, when the iPad is shared between two children or more. A child has to predict whether the next number has in it one of the two given digits. Note that this task does not focus on the last digit, but any digit.

4.2 Continue the Pattern

A teacher repeats 3 times the following: taps twice below the shelf and then taps once above the shelf. The numbers 3, 6, 9 appear on the shelf.

Can you put those numbers on the shelf like I did?
The pragmatic value of this task is to place the same numbers on the shelf as the teacher, and to try to mimic the finger choreography of the teacher. The epistemic value is in the embodiment of the skip counting, that is, the opportunity to connect rhythmic taps to a sequence of multiples of 3. Another pragmatic value, particularly if the child has mainly been using one finger at a time to make numbers, is for the child to use two fingers simultaneously. This connects to the an added epistemic value, which is in becoming familiar with different strategies for making numbers more efficiently (a two-tap followed by a one two is quicker than a one tap followed by a one tap and then another one tap), especially for bigger numbers. If children have developed adequate tool fluency, the teacher might ask whether 30 will ever appear on the shelf, for example.

This task resembles hand clapping tasks that teachers use to help children develop a more rhythmic sense of the multiples. It changes somewhat the emphasis only on the multiples since the number preceding the multiple is also said aloud by *TouchCounts* ('two, three, five, six, eight, nine…'). It also puts the fingers in charge of the production of the multiples, so that the actual numbers do not have to be known in advance—it is enough for the rhythm of the fingers to remain constant. An extension to this task might involve placing the sequence 4, 8, 12 on the shelf by tapping a triplet below the shelf and a single above.

After putting the required numbers on the shelf, one first grade child decided to place 5, 10 and 15 on the shelf. However, instead of placing four fingers below the shelf simultaneously, he continued using two fingers at a time. This produced a different kind of pattern (two tap below, two tap below, one tap above, repeat) that was successful in terms of achieving his goal. Once the teacher suggested a different approach (a four tap below followed by a one tap above), the child tried making a ten tap below followed by a one tap above, and was delighted to produce the sequence 11, 22, 33.

4.3 Inverse Gestural Subitising

Making 4 all-at-once

This task, which is best performed in the Enumerating world, preferably with no-gravity, involves the children pressing four fingers simultaneously on the screen. It is a task that is related to subitising. In subitising tasks, students must determine quickly, without counting, the number of objects in an array, which they then either say or type using a keyboard. Instead of producing a spoken or alphanumeric action based on a visual prompt, inverse gestural subitising requires that the children produce an action (quickly lifting up their fingers and pressing them on the screen, instead of pressing fingers one by one) based on an oral prompt. Unlike conventional subitising tasks, which rarely extend beyond five, inverse gestural subitising with *TouchCounts* has no upper limit, in the sense that a child may use all her fingers to make ten/10, but she can also work collaboratively with other children to produce even higher targets.

The making of 4 all-at-once can also be used in the Operating World as a quick way of producing and operating on numbers. The pragmatic value is clear in that the children know that they have to hear *TouchCounts* say ‘four’ (and only ‘four’). The epistemic value is in the use of a finger gesture that expresses cardinality because ‘four’ is the number of fingers that are lifted and that will touch the screen at the same time. If more fingers touch the screen, *TouchCounts* will not say ‘four’, which provides immediate feedback to the student, who can then reset and try again, perhaps even guided by the fact that the number *TouchCounts* said was too high.

In one episode involving a four-year-old boy as well as several other children aged 4 or 5, the teacher asks “Make 7 all-at-once”. The boy looks at his fingers and counts to seven on them, unfurling them one at a time palms facing him. He then turns his hands around and places the unfurled fingers on the screen. *TouchCounts* says “eight”, indicating that he actually touched the screen at eight different places (perhaps accidentally touching with an unfurled finger or with his palm). The teacher asks if he wants to try again, which he does. He then immediately stretched out seven fingers (without counting them out) and places them on the screen. *TouchCounts* says “seven”. It is clear in this case, and in many others we have observed, that the temporal counting out of a number quickly turns into a gesture (both communicative and manipulative) for expressing cardinality.

4.4 Count by n *in Both Worlds*

Count by 3s (in the Enumerating world)
Count by 3s (in the Operating world)

This task is often done in early years school settings, and involves “skip-counting”. It is a primarily oral task in that the student must utter the correct sequence of numbers, a sequence, which is often learned by heart (like the counting sequence) at the younger ages. The only way for students to know that they are wrong is if the teacher tells them, and their options for self-correcting are limited to them trying to remember the correct answer.

In *TouchCounts*, the task involves taking actions that makes *TouchCounts* produce the oral sequence of numbers. The task itself thus changes, as do the solution strategies and the opportunity for feedback. Although the hand gesture for doing so can be quite similar in both worlds, the visual display on the screen looks very different. In the Enumerating World, one can count by threes, for example, by repeatedly tapping three fingers on the screen simultaneously. *TouchCounts* will say "three, six, nine, twelve, …". In the no-gravity setting, numbered yellow discs will appear where the fingers have tapped the screen so that the total number of discs will be equal to the value of the last multiple created. In the gravity setting, the yellow discs that have been created below the shelf will have fallen away, leaving only the yellow numbered discs that were created above the shelf. Therefore, there may very well be less numbered discs on the screen than the value of the last multiple created. A dexterous tapping of the fingers could also leave all the multiples of three on the shelf while the other numbers fall away.

In the Operating World, tapping three fingers simultaneously will produce a herd of three. In order to count by 6, a child might either create a three with three fingers, then a six with six fingers, then a nine with nine fingers and so on. This gets quite challenging once the count gets to twelve, though children could work together to produce 12, 15, 18, etc. Another method would be to create a second herd of three, join it to the first herd so as to obtain six, then create another herd of three, join it with the herd of six so as to obtain nine, and so on. Thus, *TouchCounts* would be heard saying "three, three, six, three, nine, three, twelve, …" Similarly, a child could produce many herds of three and then begin to join herds of three to a running count in order to get *TouchCounts* to say "three, six, nine, twelve, …"

The pragmatic function of the task is clear, in that students have to produce a certain sequence in the right order, without skipping any elements. But the epistemic value resides in the multiple ways in which the task can be solved, which children then are invited to compare. In each of the ways of solving the task (in both worlds), the three-fingers-lifted gesture functions as a pragmatic and epistemic ways. Pragmatically, it is the way to get *TouchCounts* to say just three, without having to count up to three. Epistemically though, it also expresses three as a single action, which relates it to the cardinality of 3, rather than the ordinality. While in the Enumerating world, counting by threes is a question of succession, in the Operating World, an additional action is required, which is the combining of two herds in order to produce a sum.

The importance of this additional action is evident in the interaction of a 5-year-old called Chloe, who had decided that she wanted to make a really big number. She began by making a 5 in the Operating World by tapping the screen with a five-fingers-lifted gesture. She then made another 5 and combined it with the first one. Then she made a third five and combined it with the 10 and, finally, combined a fourth five with the 15. Once she saw the new herd, which was labeled with a 20, she said "That's why they say five, ten [short pause] fifteen, twenty". Chloe had clearly engaged in skip-counting before but now, having produced the sequence of numbers 5, 10, 15, 20 by successive addition of herds of 5, she made a connection between skip counting and adding.

There are two important aspects of *TouchCounts* that differentiate it from other environments. First, it takes care of the computation so that Chloe can attend to the result of her successive adding. She may well have been able to perform the additions herself, but that would likely have shifted her focus of attention away from the pattern she was producing. It is important to note that in doing the calculation, *TouchCounts* offers both symbolic and aural results, and it is perhaps the latter that helped Chloe make a connection to skip-counting, which is most often a ritual, spoken aloud event in the classroom. This distinguishes the task from what Chloe might have done on a calculator, which would also take care of the computation, because the calculator does not announce the sums out loud. The second distinguishing feature is the gestural interface for performing the addition. This gesture, which has both pragmatic and epistemic functions, draws Chloe's attention to the adding operation, which is very different from the successive counting that might occur in the Enumerating World.

4.5 Make Them Equal

A teacher puts on the screen 2 herds, for example 4 and 6.
A student is asked to "make these two equal".

Imagine this task outside of the *TouchCounts* environment. You have in front of you two piles of marbles, 4 in one and 6 in another. A solution that appears obvious initially is to move one marble from the six-pile to the four-pile. In a way, you created the following equation: $4 + 1 = 6 - 1$.

Is there another way? Of course one can think of hiding 2 marbles in a pocket, an action that can be modelled as $4 = 6 - 2$. Adding more marbles to the table is another option of course, if those are available. However, the immediate action, and also a self-imposed restriction, is that of creating a balance using only what is available and all of what is available.

Now turn to the *TouchCounts* task. Obviously, the two options described above are available. In this sense, the task may be seen as facilitating the mathematics since the students do not need to actually perform the operations, but can instead focus on strategy. But the environment easily affords a variety of other solutions, at times unintentional or self-correcting. For example, Tiki wanted to pull out 1 from six and join it with 4. However, unintentionally, she pulled out 2. She immediately recognized that the desired outcome had been achieved. In this sense, the task also modifies the solving strategies, thereby offering a form of strategy feedback.

While availability of additional marbles is uncertain, creation of new numbers is a matter of a touch. As such, it opens many options for achieving a balance. For example, a child can "make 1" and "make 3" and then rejoin, which can be modelled by 4 + 3 = 6 + 1.

If this is a preference, a constraint can be introduced:

8 and 4 on the screen. Make those equal, without creating more numbers.

This constraint can be examined with an "impossible" task

4 and 7 on the screen. Make those equal, without creating more numbers.

This could be done in a more elaborate way by proposing the following sequence of tasks:

Make equal herds from:
Given: 8 and 4
Given: 8 and 12
Given: 8 and 5.

The pragmatic value of the tasks is clear: equal herds are created. Direct manipulation feedback of the symbols provides children with quick feedback as to whether or not they have achieved their goal, as does the aural feedback to some extent. However, the epistemic value is far reaching. The task(s) open the exploration of equalities and equations. In a way, what a child is concerned with can be modelled as $4 + x = 6 + y$. And, of course, the epistemic value is enhanced if a teacher is asking for alternative solutions. The last task invites initial considerations of parity.

A possible variation is to start with a larger number and ask a child to split it into two equal parts, or 3 equal parts. The epistemic value of such an exercise is that it can serve as initial informal introduction to the concepts of division, division with remainder or divisibility. Of course we do not intend to say that a child who successfully splits 9 into 3 equal groups has acquired the concept of divisibility. But we do claim that this is an experiential hands-on—in fact, fingers on—initiation into the multiplicative structure.

4.6 Families of Partitions

How many different ways can you make 7?

This task, which is used in the Operating World, invites children to use pinching (addition) to make 7 in different ways. This task is frequently undertaken at the primary school (sometimes with physical manipulatives), where students are asked to come up with different combinations such as 3 + 4 and 5 + 2. The task is important in drawing attention to partitioning and in preparing for work with subtraction. Strategies for solving this task involve looking for different combinations of number that sum to 7, and then perhaps writing the sums as equations. The task is used with children who have already been introduced to addition and perhaps know some addition facts.

In *TouchCounts*, there are three main differences. The first is that *TouchCounts* performs the addition, so that students are focused more on experimenting with different combinations of numbers. The second is the visual display of the sum, which looks both like a new number because it came from the addends, but also like any other number that can be used to perform operations with. The third is that the output of the addition is given as a new number that retains the trace of its composition (by colour). That means that the 7 that one makes from pinching 3 and 4 looks different than the 7 that one gets from pinching 2 and 5. This provides some visual feedback on how to produce more combinations.

The pragmatic value of this task might not be as straightforward in that *TouchCounts* does not tell you when you have made all the combinations, and nor does the task. Indeed, there can be some vagueness about the task since 7 made all-at-once, for example, could be considered as a way of making 7 that is equivalent to 7 + 0. The epistemic value of the task is in the seeking of strategies for producing different combinations as well as in the strategies used to adjust combinations that do not produce the target number of 7. So, for example, if a student puts 5 and 3 together to get 8, they might use the feedback to reduce one of the two addends.

5 Discussion

We have described a series of tasks that have been designed for use with *TouchCounts*, based in part on the functionalities of the App, but also on interactions with young children. In fact, there were several tasks that we tried with children and did not analyse above, and this occurred primarily for reasons related to feedback. For example, one of the tasks that we tried early on was to show children a repeated sequence of touching with two fingers below the shelf and then one finger above, and then to invite them to continue. The goal was to develop a rhythmic sense of the skip counting by three. However, the children sometimes touched the screen with an extra finger, which meant that the numbers on the shelf were no longer multiples of three. The feedback offered by *TouchCounts* was thus not well aligned with the pragmatics and epistemic goal of the task. This led to the design of the ‘Counting the pattern’ task we discussed above. In general, we have found that given the nature of the *TouchCounts* feedback, it is imperative for feedback to provide information that enables the learner to assess (either through seeing or hearing) her action *in relation to the goal*, a type of feedback that has been called direct manipulation feedback (see Mackrell et al. 2013). This is true of exploratory environments more generally. When a learner drags the vertex of an isosceles triangle, the feedback is not evaluative, but can provide visual information about whether the given triangle can indeed be dragged into an equilateral configuration. While the importance of goal-aligned feedback is especially true when working on a given task, we emphasize that children can also develop goals based on the feedback from *TouchCounts*. So, for example, a child who notices the numeral 44 on a disc (even if she was not trying to get to 44) might wonder how to make numbers that have two fives or two eights.

One of the important features of feedback is that it offers immediate information, which is directly related to one's actions, and which can guide further actions, especially if a goal has not been reached. In the case of the 'Put 5 above the shelf' task, when a child taps for the fifth time below the shelf, she can see that she has not succeeded in the task, but can also use the oral feedback provided to notice that "four" comes before "five", which may in turn result in a change in strategy. In the Enumerating world though, she cannot simply "undo". She must press Reset and start all over again. In a sense, the Enumerating World is unforgiving because it does not allow for much tinkering. This contrasts with most other software programs, from Word to DGEs, which almost always allow the last action to be undone. It also contrasts with the Operating world, where undoing is often possible. For example, if a child wanted to make a herd of 5, but mistakenly only made a herd 4, she can make another herd of 1 and pinch it together with the herd of 4. Or, she can simply push it off the visible screen. These differences are important to take into account when designing tasks and when assessing the learning potential of tasks. Having to try many times to put 5 on the shelf may serve an important purpose if a child is still struggling with the counting sequence from 1 to 5. However, in the extended task where a child is asked to put 5, 10, 15, 20 on the shelf, but mistakenly places 5, 10, 15 and 19 on the shelf, there may be less value in starting over again.

In the Operating world, it is the possibility to tinker that opens up a multiplicity of solutions to certain tasks (like Make two herds the same). If children were only allowed to use the existing herds, there would be a highly constrained focus to the task, but given that the child can make new herds, the number of solution strategies increases. While this might be challenging for teachers with a specific learning outcome in view, it likely offer the children a greater sense of agency.

6 Conclusions

In our analyses, we highlighted both the pragmatic/epistemic values of the tasks as well as the nature of the task in terms of its use of *TouchCounts*. As is evident in these analyses, the pragmatic value of a task is almost always equivalent to the completion of the task itself, which may in part be due to the nature of mathematics involved and the age of the children. Further, the epistemic value of each task usually depended strongly on the making of certain bodily actions that, in turn, entailed a particular aspect of number. For example, in the task "Making 4 all-at-once", a child can learn the physical action of lifting four fingers and touching them to the screen, and it is precisely this physical action that is intended to bring about the cardinality of four.

Finally, each task made use of *TouchCounts* in a way that either offered new ways of thinking about number, as compared to similar physical manipulatives-based tasks, or derived its meaning from *TouchCounts* altogether (so that equivalent tasks in a non-digital environment do not exist). As Laborde (2001) notes, it can be very difficult to teachers to design such tasks because they are accustomed to certain ways of working with number that involve different tasks, different questions and different challenges. In an exploratory environment, it is thus very important for teachers to have access to well-designed tasks. A further challenge for teachers and parents will be to achieve a fine balance between offering the kinds of tasks we have described in this chapter and enabling children to engage in self-directed mathematical exploration as well, so that they are not exclusively following a set of given tasks.

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