Task Design Potential of Using an Interactive Whiteboard for Implementing Inquiry-Based Learning in Mathematics

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Abstract This chapter explores the role and potential of using an Interactive Whiteboard (IWB) for inquiry-based learning. A case study on how a French school teacher uses an interactive whiteboard is presented, illustrating how an IWB expands the milieu (Brousseau in *Theory of didactical situations in mathematics*. Dordrecht: Kluwer, 1997) of the learning situation and the collective part of the class investigation and suggests a mesogenesis-topogenesis-chronogenesis heuristic for digital pedagogical task design.

Keywords Interactive whiteboard · Inquiry-based learning

1 Introduction

This chapter proposes a theoretical perspective on task design, by illustrating, a case-study where the Interactive WhiteBoard (IWB) is integrated into a French primary school (10 and 11 years old) and used to discuss theoretical underpinnings of inquiry-based learning. This research study was set up to investigate teachers' practices in proposing and solving a mathematical modelling problem. The purpose of this chapter is to analyse how teachers' use of an IWB encourages or not the implementation of inquiry-based learning in the classroom. More specifically, the chapter attempts to explore the following questions: What might the role and potential of an IWB for inquiry-based learning be? What new dimensions might an IWB bring about in class activities in the context of inquiry-based learning?

In the first part I review theoretical backgrounds for using an IWB and inquiry-based learning. In the second part I present a case study of a teacher who employed an IWB to carry out inquiry-based learning during a mathematical modelling class. In the last parts I will discuss aspects of IWB inquiry-base task design.

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2 Theoretical Background

2.1 The IWB and Teaching Practices

The IWB was massively introduced in the UK in order to change the teachers' practices towards greater interactivity between students and their teacher. Consequently many British studies have been conducted on teaching practices in connection with the IWB. Miller et al. (2005) identified three stages of development in the effective use of the IWB[1]:

- Supported didactic: The teacher makes some use of the IAW but only as a visual support to the lesson and not as an integral tool to conceptual development. There is little interactivity, student involvement or discussion.
- Interactive: The teacher makes some use of the potential of the IAW to stimulate students' responses from time to time in the lesson and to demonstrate some concepts. Elements of lessons challenge students to think, by the use of a variety of verbal, visual and aesthetic stimuli.
- Enhanced interactive: This approach is a progression from the previous stage, marked by a change of thinking on the part of teachers. They now seek to use the technology as an integral part of most lessons, and look to integrate concept and cognitive development in a way that exploits the interactive capacity of the technology. These teachers are aware of the techniques available, are fluent in their use and structure lessons so that there is considerable opportunity for students to respond to IAW stimuli—as individuals, pairs or groups—with enhanced interactive learning. The IAW is used as a means of prompting discussion, explaining processes and developing hypotheses or structures; these are then tested by varied application. A wide variety of materials are used including 'home-grown' and internet resources and IAW specific and commercial software (p. 4).

However, Smith et al. (2006) observed that the most common use of the IWB remains as a tool for projecting content and

> traditional patterns of whole class interaction persist despite the emphasis on interactive whole class teaching in the national strategies and the introduction of IWBs in the English primary school classroom (p. 455).

A requirement for taking advantage of the potential of the IWB is to change teacher behaviour: the teacher-instructor who shows the knowledge to be learned must become a teacher-go-between who organizes the meeting between students and knowledge to be learnt. Miller et al. (2008) suggested a new pedagogy for using the IWB which they termed *at the board, on the desk, in the head*:

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[1] Miller et al. used the abbreviation IAW instead of IWB.

> A typical lesson will have students interacting with the teacher, the IWB and with each other and would involve some of the similar features found in lessons that are typified by the approach of Swan (2005) (p. 3)

This pedagogy aims at leading teachers to question the differences in information processing when the students work on the desk, at the IWB or in “their head”. Thus teachers turn their attention to their own practices for improving their understanding on how IWBs can improve learning.

Chevallard (2002a), in the Anthropological Theory of the Didactic, postulates the existence of a close link between mathematical knowledge, mathematical organizations (used by the class and the conditions created by the teacher) and didactical organizations (for using this mathematical knowledge). Thus, the type of teaching activities based on this linkage proposed to students will affect the type of the IWB usages employed. In this chapter, the context for this linkage is a mathematical modelling activity which requires an inquiry-based learning approach.

2.2 Inquiry-Based Learning

The use of inquiry-based learning in science education is a long-standing constructivist pedagogic tradition after John Dewey, Jerome Bruner, Jean Piaget and Lev Vygotsky. Even if there are some differences between the pragmatic philosophy of Dewey and the rationalist philosophy of Bachelard (Artigue and Blomhoj 2013), inquiry-based learning is a pedagogic embodiment of a Bacherlard-like vision of sciences:

> First of all we must know how to state problems. Whatever one might say, in the scientific life, problems do not arise by themselves. It is precisely this sense of problem that gives the mark of the true scientific mind. For a scientific mind, all knowledge is an answer to a question. Without a question, there cannot be scientific knowledge. Nothing goes without saying. Nothing is given. Everything is built. (Bachelard, 1934; p. 17, my translation).

Minner et al. (2010), who synthesised research about the effect on students’ learning through inquiry-based science education, noted:

> The term inquiry has figured prominently in science education, yet it refers to at least three distinct categories of activities—what scientists do (e.g., conducting investigations using scientific methods), how students learn (e.g., actively inquiring through thinking and doing into a phenomenon or problem, often mirroring the processes used by scientists), and a pedagogical approach that teachers employ (e.g., designing or using curricula that allow for extended investigations) (p. 476).

In France, inquiry-based learning appears in curricula of science teaching as a teaching method based on “seven essential moments” (BOEN No 5 special issue of August 25, 2005):

(1) Choice of a problem by the teacher;
(2) Appropriation of the problem by the students;

(3) Formulation of conjectures, explanatory hypothesis, possible protocols;
(4) Investigation or the resolution of the problem by students;
(5) Argued exchange on students' proposals;
(6) Acquirement and the structuration of knowledge;
(7) Operationalization of knowledge.

Inquiry-based learning is not only looking for a solution to a problem, it is also a pedagogic approach where inquiry is at the basis of learning. Thus at the end of primary school, the personal skills booklet, introduced on June 14th 2010, attests students' capacity to "practice an inquiry-based method: observe, inquire; handle and experiment, formulate a hypothesis and test it, argue; try several pathways of solution" [my translation].

In this chapter I am interested in the role and potential of using an IWB for the implementation of inquiry-based learning in mathematics classroom. What features of the IWB can be used in the context of inquiry-based task design? First of all, criteria that can be used to determine whether inquiry-based learning has been well implemented in a teaching situation are needed.

2.3 Criteria for Implementation of Inquiry-Based Learning

The Anthropological Theory of the Didactic (Chevallard 2006), like the Theory of Didactical Situations (Brousseau 1997), is based on a postulate: *a student learns by adaptation in interaction with a milieu*. The solution of the studied problem is produced via a student's autonomous confrontation with this milieu. Chevallard (2011) suggested studying milieu through three processes: mesogenesis, topogenesis and chronogenesis. *Mesogenesis*, the genesis of the milieu, is the process by which the milieu of a situation is produced, developed and enriched. *Topogenesis*, the genesis of the positions, is the process by which the duties of the teacher and students in a teaching situation are allocated; that is, how the activity is divided between the teacher and the students. *Chronogenesis*, the genesis of the didactic time, is the process by which the temporality of knowledge acquisition is modified. These three processes form criteria for implementation of inquiry-based learning.

The aim of inquiry-based learning is to search for answers for a given question. Chevallard (2002b) described the process of studying a question through a 5-step cycle:

(1) Observation of already existing resources
(2) Experimental and theoretical analysis of these initial resources
(3) Assessment of these resources
(4) Development of the final answer
(5) Justification and illustration of the answer produced

The function of resources for studying the inquiry process is crucial and Chevallard (2006) differentiates media and milieu as two types of resource. A *media* is defined as “any social system pretending to inform some segment of the population or some group of people about the natural or social world” and a *milieu* is defined as “any system that, as far as the question that you address to it is concerned, is devoid of intentions and therefore behaves like a fragment of nature—a system that intends neither to please or to displease you nor to defeat you of your hopes” (ibid, p. 29). Media and milieu are distinguishable from one another by the didactical intent concerning the acquisition of information. Media is a resource produced with the intention of providing information about something for someone. For instance, the texts in scientific literature are media for a physicist who is searching for existing information about the phenomenon under study. Milieu is a system of objects that produces feedback without any didactic intention towards students. For instance, an experimental process may be used as a milieu for studying a physical phenomenon.

When initially approaching a problem, the first action is to look for different media to find whether the answer, or part of the answer, already exists. Documentary research, experiments, observations are then used to create a milieu to test and validate the information provided by these media. The *dialectic between media and milieu* (Chevallard 2006) is the didactical dynamics that puts to the test the resources to produce the materials from which the answer is developed. Thus, for instance, the outcome of a survey on the Internet (Ladage and Chevallard 2011; Chevallard and Wozniak 2013) is based on a dual assessment of the reliability level and reception quality of the information provided by a media: “Is it right?” and “Do I understand?” respectively. Accumulation and testing of resources contribute to the validation of the produced answer and the construction of a milieu is fed by the validation of information provided by the consulted media. Thus milieus providing feedback to media may combine and evolve into a larger milieu of the problem situation.

The mesogenesis criterion: the milieu of a problem situation is constructed out of the dialectic between media and milieus. In inquiry-based learning, the purpose is to learn how to produce an answer to a problem. Students must find their own ways to validate the information provided by the media. Therefore the teacher must remain in the background. This means that the role of the students in the learning process should grow individually and collectively whereas the teacher is in an assistant position.

The topogenesis criterion: the teacher remains in the background leaving the individual students and the whole class to develop key roles in constructing the milieu from which the answer is developed. Since the milieu is constructed and organized by the class, the issue of controlling the time for doing it becomes crucial. Because of time constraints due to the demand of the curriculum, the teacher sometimes reduces the role of the students in the topogenesis process in order to “move the course onwards.” In inquiry-based learning, it is important for the teacher to modify the didactical time allowing students to complete the inquiry process.

The chronogenesis criterion: the teacher gives enough time for students to complete the inquiry process.

Under these theoretical elements, a list of questions can be drawn to evaluate the implementation of inquiry-based learning:

- Mesogenesis: What is the milieu made of? Did the milieu evolve during the inquiry process? Has the dialectic of media and milieus been used? How has the answer to the problem been validated?
- Topogenesis: How and by whom is the milieu made? What are the roles of the teacher and students in building the answer to the problem?
- Chronogenesis: How has the teacher managed the time of the inquiry process? Does s/he shorten the time of the inquiry process?

3 A Case Study on Teachers' Use of an IWB

Wozniak (2012) examined teaching practices and their effects on students' learning during a sequence of problem-based lessons. In the study, teachers were given a larger degree of freedom to design their teaching sequence. In one case, the teacher used an IWB as a tool in the classroom to carry out a sequence of inquiry-based mathematics lessons.

3.1 The Problem

The problem to be solved was introduced by a photo. Figure 1 is a scaled-down version of the original 16.1 cm by 12 cm photo distributed to the students.

This kind of problem is called a *Fermi Problem*:

> Enrico Fermi (1901–1954), who in 1938 won the Nobel Prize for physics for his work on nuclear processes, was known by his students for posing open problems that could only be solved by giving a reasonable estimate. Fermi problems such as how many piano tuners are there in Chicago? share the characteristic that the initial answer of the problem solver is that the problem could not possibly be solved without recourse to further reference material. (Peter-Koop 2004, p. 457)

These problems, based on real world situations, are characterized by a problem statement that does not include numbers and whose solution relies on a modelling activity. Amongst the Fermi problems, the problem of the Giant is of the "Pictorial Problems or Picture Mathematics" type (Herget and Torres-Skoumal 2007) where information required to answer the question must be extracted from a photo or picture.

This photo was taken at an amusement park in England. The leg of a Giant is partly visible. What is roughly the height of the Giant?

Fig. 1 The problem of the Giant [my translation]

3.2 Description of the Case Study

The teacher designed five lessons for a total length of 4 h. During the first lesson (45 min), students were given freedom to conduct individual inquiry starting with three questions:

1. In your opinion, how tall is the Giant?
2. Explain how you obtained this result.
3. What elements are missing to calculate the Giant's height?

Some students asked the teacher about the accuracy level of the answer (in particular whether the answer is an integer or a decimal number). The teacher wrote on the IWB the numerical answers given by the students for the first question and on a paperboard the information that the students would like to know to solve the problem. The teacher ended the lesson with the remark *"I'll look at what you have written on your documents and we will try to see among all the elements that are there, what is useful and what is useless"* [my translation].

The teacher started the second lesson (1 h and 5 min) by showing a selection of four student productions from the previous lesson. The goal was to "*compare these solutions with what we lack and try to see how each of these four solutions attempted to circumvent what we needed*" [my translation]. One production was a mere opinion "*for me it is 12 m*", the other three solutions were of the same type as what the teacher had in mind. The height of the Giant is calculated by multiplying the foot-length (as measured in the photo) of the Giant by a coefficient k. Afterwards a collective discussion was initiated on identifying the essential elements needed to solve the problem by considering the list set out at the end of the first lesson. Homework was given for the next session: a survey on the heights of adults.

The third lesson (55 min) began with a discussion on the three necessary elements to produce the answer to the problem: the heights of humans, the foot-length of the boot, Giants are human-like (similar in proportion). Then the teacher collected the results of the investigation on the "*average height of a person*" and the class discussed how each student collected and produced this information. Students measured their foot-lengths and their heights. They then established the number of transfer of their foot-length to their bodies to determine their heights. The data thus obtained was collected in a spreadsheet projected on the IWB. Using the Spreadsheet, the students verified that the ratio (height/foot-length) was between 5 and 6. A final discussion started around how to determine the ratio k by transfer, its validation by calculation and the usefulness of knowing k to solve the problem. Afterwards, the teacher gave students homework for the next session: a survey to determine the ratio (height/foot-length) to verify whether the ratio they found was the same for an adult.

In the fourth lesson (1 h and 2 min), the teacher collected the results of the investigation about the relationship k = height/foot-length. A discussion was carried out on the differences between the measurements made on students and adults and the technique that could be used to calculate the height of the Giant. The teacher concluded "*A child is not a miniature adult*". The students made a final individual research structured by three questions:

1. In your opinion, how tall is the Giant?
2. Explain how you reached this result.
3. Explain what enabled you to find the height of the Giant.

Numerical answers were collected on the IWB followed by a discussion focussing on the comparison with the students' answers from the first session.

In the fifth and final lesson, students summarized their findings and wrote in their notebooks the solution obtained.

To assess the implementation of inquiry-based learning in the sequence of lessons described above, in the following section I apply the three criteria discussed in Sect. 2.3 on the students' construction of the answer to the Giant problem.

3.3 Mesogenesis, Topogenis, Chronogenesis

Regarding mesogenesis, the milieu is gradually enriched during the lessons through the survey after lesson 2 and the two data collections on the ratios foot-length to student heights and foot-length to adult height (lessons 3 and 4). To carry out the investigation on the average height of a person, students interviewed their parents and grandparents. Furthermore, some students consulted a website, used the heights of football players or consulted a book. The sharing of different information allowed the implementation of the dialectic of media and milieus. There was a document found on the website of the INSEE (National Institute for Statistics and Economical Studies) which allowed the teacher to validate students' answers:

> I went on the internet and I went to the INSEE. […] And there is such a study there… Here it is: the currently reported size of men of age 18 to 65 is 1.75 m and the average size of women is 1.63 m. It was you Lisa who found this. You have found the same thing, you have found it in a book. In my case, I am basing [my information] on a study, therefore, on data collected by the INSEE. So if you wish so, do you agree to take 1.75 m? [my translation]

Regarding topogenesis, the teacher guided the lessons by explicitly asking students to perform certain tasks leading gradually to the solution. During lesson 2, a student put forward the idea that a relationship could exist between the size of the shoe and the height of a man. The teacher then organized debates and allowed time in the classroom to make sure that the ideas were well discussed by the whole class. He did not express judgements on the ideas but instead he asked this student to continue the investigation:

> Can you look that up for next time? You'll tell us if there is a link between the foot-size of a person and his/her height. I let you take care of it. [my translation].

Similarly, after the first data collection (lesson 3), the teacher suggested to the students that the ratio may not be the same for an adult. After a class discussion, the teacher concluded:

> Is the same ratio found for an adult? It will be your job for the next lesson. You will do a survey at home. You will try to complete the table with one or two adults. Man or Woman? [my translation]

The milieu is constructed by the class under teacher's management and guidance. The teacher was well aware of the mathematics involved as he said during the post lesson interview:

> Some students let themselves get carried away by the class without understanding why we were looking for the average height of an adult or why the search for a potential invariance of the human body proportions could be important [my translation].

The teacher plays a key role in the study: he helps the students clarify the problem and identify relevant data, organize the comparison of students' answers, coordinate the collective reflection and commission the writing. He indeed used an inquiry-based method in the sense of Dewey.

Regarding chronogenesis, the teacher organized the lessons to be implemented over a longer than usual, but acceptable, duration. In particular he delayed conclusive sharing to allow students to make judgements on their productions first. This drove the lesson in a certain direction. Furthermore, he assigned outside classroom data collection and documentation research activities, and used skilful questioning techniques in collective discussions to guide the students' thoughts.

The milieu for this lesson sequence was made out of ample resources and was being constructed constantly during the lessons. The teacher organized discussions among students until agreeable answers were reached. In this way, the dialectic of media and milieus was pedagogically and fruitfully realized.

4 Discussion on Inquiry-Based IWB Task Design

4.1 Roles and Functions of the IWB

The teacher used the IWB in every lesson and it was a tool that students were comfortable using to conduct surveys or to collect data. The IWB was placed near the teacher's desk and the black board of the classroom to facilitate its use as a classroom tool and students had no difficulty getting their numerical data in the IWB spreadsheet suggesting that they were familiar with using it in the classroom. Rather than just enhancing presentation, the IWB enriched the milieu with respect to sustaining communication in the class and fostering interactivity. Thus, the IWB digitally enhanced collective communication and interactivity.

The IWB was obviously used as a display tool. The Fermi Problem photo was presented using the IWB in lesson 1. Students' procedures were presented and discussed collectively at the IWB before individual work was started (lesson 2). Furthermore the teacher used the IWB to validate the students' research and to gather numerical data on a spreadsheet for computational checking, and to justify the choice of the average size of a man via the Internet (lesson 3).

The IWB was used as a "guardian of collective memory" (lesson 4). Throughout the collaborative lesson sequence, the IWB was a digital environment conducive to producing a collective answer by facilitating the analysis of students' proposals, validating data, keeping track of intermediate results and creating a space to synthesize. The teacher played an important mediating role between the IWB and the students, guiding their investigation with skill and flexibility. It was the teacher who proposed to study the morphology of adults since students did not seem to be aware that there may be physical differences between a child and an adult.

Thus with respect to a mathematical modelling classroom, an IWB can be used as a modelling tool conducive to the struction and formation of solution discourses. Mediation using the IWB can facilitate a constructive implementation of the inquiry-based learning.

4.2 Inquiry-Based IWB Task Design Considerations

Studies have been conducted on using IWBs to support the paradigm shift from a transmissive pedagogy to a pedagogy that focuses on interaction between students and the teacher. Classroom integration of IWBs requires understanding of how the IWB can afford pedagogy. Thus instead of focusing attention on the technology itself, the focus should be on the teachers' pedagogical practices. Wood and Ashfield (2008) made the following observation:

> While initially this study intended to focus upon the way in which the IWB could support whole-class interactive teaching, it became increasingly apparent that the teacher's interpretation of whole-class interactive teaching itself was the primary factor in developing materials and opportunities for children to engage with their own method. In terms of creative teaching, it is essentially the teacher who determines what resource to use and how it will be utilised (ibid, p. 94)

From the above, it seems essential that the teacher uses the IWB as an environment enabling an answer built collectively.

From the point of view of mesogenesis, the IWB clearly enriched the milieu and could be used as a "window on the world", for example when the teacher showed a document from the INSEE website through it. It is uncommon in France that a teacher valorises a student's idea and conducts an Internet query. French teachers usually claim strong reservations about this practice. The IWB provides teachers with a dynamic writing window with access to knowledge beyond the classroom. Teachers should design activities making use of the IWB's features to create a milieu made up of multi-digital resources like photos, maps, software (e.g. spreadsheets, dynamic geometry software), and documental research. An IWB is an effective platform (milieu) to design activities based on sharing that could facilitate the implementation of the dialectic of media and milieus.

From the point of view of topogenesis, the IWB "memorizes" the displays and processes numerical data. A "collective space of student work" can thus be formed where the dialectic between media and milieus takes place by comparing the different student contributions. Teachers should design autonomous activities for students to search and to share under the IWB platform. Teachers may take the role of mediators who guide and organize student-student discussions within the collective space produced by the IWB.

From the point of view of chronogenesis, the IWB "escalates the efficiency" to collect and compare student work and thus speeds up the formation of solution discourse. The teacher should design activities allowing students how to access previous IWB captured data. The task design should make use of the IWB also as a storage space of the class to foster a progressive built up collective answer space to the problem. An IWB task should also be designed to manage the class memory.

4.3 Inquiry-Based Digital Task Design Activity Sequence

I have considered how a teacher made use of an IWB for the implementation of inquiry-based learning for solving a modelling problem. What is observed in an IWB classroom is *a decreased individual topogenesis for the benefit of increased* *mesogenesis*. By "memorizing" the class activities, the IWB naturally becomes a digital environment for collaborative study. The IWB creates conditions for the establishment of a collective pedagogy and an enrichment of the milieu. For instance, the use of the Internet on the IWB offers simultaneous multiple windows to facilitate inquiry based learning: search engines, translation services, dictionaries, encyclopedias, institutional sites, calculators, dynamic geometry software, spreadsheets, etc. The IWB case study described above confirms an observation of Wood and Ashfield (2008):

> This research seems to indicate that it is the skill and the professional knowledge of the teacher who mediates the interaction, and facilitates the development of students' creative answers at the interface of technology, which is critical to the enhancement of the whole-class teaching and method processes (p. 84).

The IWB is an example of a digital environment that can store and compute students' data to form a collective sample space for students' autonomous learning. Metagenesis, topogenesis and chronogenesis can be regarded as a heuristic triplet for pedagogical digital task design. These features combined suggest that for digital inquiry-based modelling task design, collective interactivity and simultaneous multiplicity should play a dominant role. The case study discussed in this chapter is an example to support such a task design approach. Task design in digital environment like IWB could make use of these two features to organize the classroom dynamic among mesogenesis, topogenesis and chronogenesis.

Base on the theoretical considerations discussed in Sect. 2, I would like to end the chapter by proposing criteria about the inquiry-based digital tasks design.

1. *Activity criterion*: The activity is based on solving a problem and a digital environment (IWB in the case of this chapter) is used to promote a collective study by data sharing.
2. *Mesogenesis criterion*: Students solve the problem by accessing to (digital) ressources and the digital environment is used to implement the dialectic of media and milieus in order to (1) produce an initial milieu of the problem situation by exploring existing (digital) resources; (2) develop the milieu by going deeper into the problem situation via experimental or theoretical analysis of these initial resources; and from this assessment result form a milieu that can provide possible explanations for the problem situation; (3) produce and justify the final answer to the problem situation via formulation of conjectures, explanatory hypothesis and possibly protocols and in this way the solution is validated by students' argumentation and discussion. For example, the IWB features facilitate literature survey and add real value by using resources like photos, maps or suitable digital tools (spreadsheet, dynamic geometry software, for example) for controlling the gradual shaping of the mathematical solution to the problem.
3. *Topogenesis criterion*: The students are autonomous and the teacher encourages students to contact directly with the digital environment (for example, the IWB environment) for gathering and incorporating data and ressources. The teacher is

a study director who organizes discussions between students and does not impose himself as a mediator between the IWB and the students.

4. C*hronogenesis criterion*: The didactic time governs the temporality of knowledge acquisition using the IWB preseres the students' previous works. Thus, the IWB is the memory container of the class that progressively builds a collective solution to the problem.

These activity criteria serve as a guideline to design inquiry-based digital tasks where collective study is essential and investigation is the motor of learning. The genesis of a digital-based (collective) milieu for a problem situation and the ease to shift attention between different resources are key factors enabling students to explore and formulate different possible solutions for a real-life problem situation.

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