Designing Interactive Dynamic Technology Activities to Support the Development of Conceptual Understanding

Gail Burrill

Abstract Technology can make a difference in teaching and learning mathematics when it serves as a vehicle for learning and not just as a tool to crunch numbers and to draw graphs. This paper discusses a technology leveraged program to develop student understanding of core mathematical concepts. A sequence of applet-like dynamically linked documents allows students to take a meaningful mathematical action, immediately see the consequences, and then reflect on those consequences in content areas associated with the middle grades U.S. Common Core State Standards. The materials are based on the research literature about student learning, in particular enabling students to confront typical misconceptions, and designed to support carefully thought out mathematical progressions within and across the grades.

Keywords Conceptual understanding · Learning progressions · Interactive dynamic technology · Action consequence principle

1 Introduction

Researchers have investigated challenges in teaching and learning certain mathematical concepts such as fractions, ratios and proportions for years. Burrill and Dick (2008), in investigating student achievement on high stakes state assessments, identified core mathematical concepts in which students consistently underperformed. On international assessments, scores in the United States are usually below international averages. In addition, studies of U.S. texts reported that mathematics concepts addressed in mathematics textbooks are not well constructed, with presentations more mechanical than conceptual (Ginsburg and Leinwand 2009).

These concerns led to the development of the Common Core State Standards for Mathematics (CCSSM) (2010), which aims to improve mathematics education in

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the US by providing a focused and coherent set of standards to guide the teaching and learning of mathematics. The CCSSM emphasize the development of both conceptual understanding and procedural fluency. As mentioned above, prior emphasis in typical curricular materials was to a large degree on procedural fluency. Building Concepts was developed as a technology-based approach to developing mathematical understanding of core concepts that lead to computational proficiency in the mathematical strands outlined in the CCSSM.

2 Building Concepts

2.1 Learning Progressions

Almost all content strands in the CCSSM are supported by progressions documents (http://ime.math.arizona.edu/progressions/ 2011), narratives describing the learning progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents outline the important mathematical concepts in each content strand. These documents provide the framework for Building Concepts activities. The underlying premise is that static pictures or examples in the progression documents are made interactive in the Building Concepts activities.

Interactive dynamic technology is not new in mathematics education. Early work with SimCalc (Roschelle et al. 2000) used such technology to link real contexts with graphical representations of those contexts and provided opportunities and experiences for students to develop understanding of the mathematics of change and variation. Dynamic geometry software (Laborde 2001) allowed students to interact directly with objects, their shapes and measurements related to those shapes, looking for consequences that are invariant with respect to a certain shape. Computer algebra systems allowed students to make changes in variable values and parameters of functions and see immediate consequences (Heid 1995). Each of these projects involved "active learning" experiences, which laid the foundation for the "action consequence" principle that guides the development of the technology platform for the Building Concepts activities.

2.2 Action Consequence Principle

Many studies have pointed to the effectiveness of active learning where students are engaged in the process of learning by actively processing, applying, and discussing information in a variety of ways (Kilpatrick et al. 2001; National Research Council (NRC) 2012, 1999; Michael and Model 2003). The theories of Mezirow (1997), Kolb's learning cycle model (1984), and the work of Zull (2002) on brain theory all suggest that people learn through the mechanism of participating in an immersive mathematics experience, reflecting on these experiences, and attempting similar strategies on their own. Mezirow introduced the notion of transformative learning as a change process that transforms frames of reference for the learner. Key elements in this process are an "activating event" (Cranton 2002) that contributes to a readiness to change (Taylor 2007). This is followed by critical reflection where the learner works through his understanding in light of the new experiences, considering the sources and underlying premises (Cranton 2002). The third element of this process is reflective discourse or dialogue in an environment that is accepting of diverse perspectives (Mezirow 2000). The final step is acting on the new perspective, central for the transformation to occur (Baumgartner 2001). Kolb's model of experiential learning (1984) is a cycle containing four parts: concrete experience, reflective observation, abstract conceptualization, and active experimentation; experimentation leads once again to concrete experience.

Dynamic interactive technology provides a virtual environment in which these kinds of learning opportunities can take place. Interactive dynamic technology goes beyond linking students to multiple representations—visual, symbolic, numeric and verbal—by providing them with visual representations they can directly manipulate and control (Roschelle et al. 2000; Sacristan et al. 2010). Interactive dynamic technology allows the learner to use technological tools to "explore and deepen understanding of concepts" (CCSSM). Too often mathematics learning technologies are used as a "servant", where the user employs the technology to create a graph, perform calculations or generate a table. Building Concepts represents a shift in the use of technology from "carrying out mathematical processes" to "learning mathematics" (Dick and Burrill 2009).

This perspective is supported by a number of studies that suggest the strategic use of technological tools can enhance the development of proficiencies such as problem solving and mathematical reasoning (Kastberg and Leatham 2005; Roschelle et al. 2010; Suh and Moyer 2007). Such technologies can help students transfer mental images of concepts to visual interactive representations that lead to a better and more robust understanding of the concept. Building Concepts activities were designed to embody this notion of active learning, employing an "action/consequence" principle, where the learner is to "deliberately take a mathematical action, observing the consequences, and reflecting on the mathematical implications of the consequences (Mathematics Education of Teachers II 2012, p. 34)". The software supports tasks that provide opportunities for the student to make mathematical choices and reflect on what happens because of those choices. The next section addresses the approach to content in Building Concepts and how it embodies the action consequence principle.

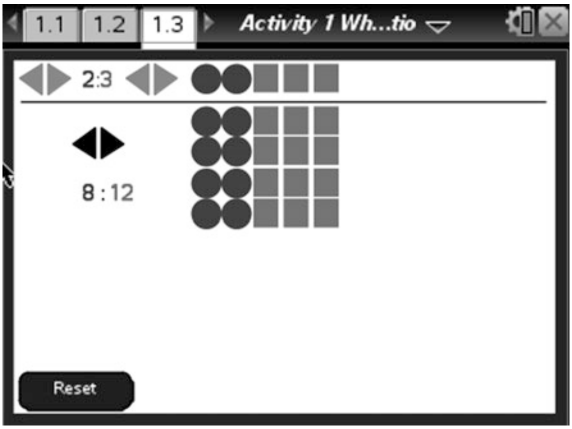
3 Building Conceptual Understanding

3.1 A Coherent Development of Concepts

The content in the K-8 CCSSM is designed to be focused and coherent within and across grades with an emphasis on conceptual understanding that lays the foundation for procedural fluency. Many traditional current materials in the U.S. covered a plethora of ideas in two textbook pages, giving students little opportunity to develop any one idea fully let alone make connections among ideas. Building Concepts is designed to thoughtfully consider the key ideas in building conceptual understanding of important mathematical concepts. Thus, the activities focus on fundamental concepts, typically one per activity, in a carefully developed sequence of explorations aligned with the progressions documents. For example, the progression for ratio and proportional relationships defines a ratio as a pair of non-negative quantities both of which are not 0, emphasizes equivalent ratios and suggests that pairs of quantities in equivalent ratios be recorded in a table. Figure 1 displays the screen from *What is a Ratio*?, the first activity in *Building Concepts*: *Ratios and Proportional Reasoning*, where the concept of ratio is introduced. Students see a physical representation of this pairing, two circles to three rectangles, and generate representations of equivalent ratios (action/consequence). They observe patterns in the rectangular array, noticing the "pairing" as each row is added to the representation and think about the numbers involved, initially additively—adding two blue circles and three green squares each time, but the multiplicative interpretation is also visible in the total number of circles and squares. The arrow keys at the top allow students to change the original ratio to verify their observations and conjectures with different numbers.

Figure 2, from *Building a Table of Ratios*, displays an original ratio, its physical representation and a table of the numbers that compose equivalent ratios. Now students are expected to reason about relationships among the numbers they see in

Fig. 1 Equivalent ratios



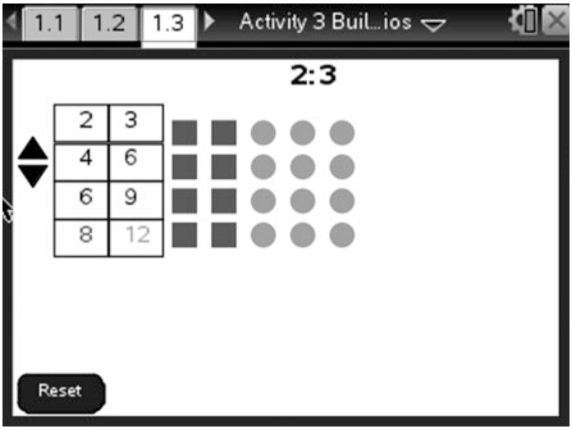


Fig. 2 Associating numbers and ratios

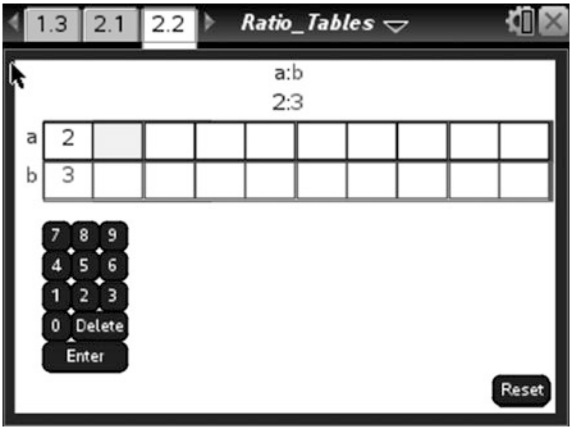


Fig. 3 Ratio table

the table, as well as the physical pattern, and observe the multiplicative relationship between equivalent ratios. In *Ratio Tables* (Fig. 3), students move to the abstract, where the physical representation of squares and circles is absent but can be recalled as a basis for thinking about the ratios in the table. The next page in the file moves one step farther, allowing students to generate equivalent ratios in any order, building multiplicative understanding of equivalent ratios.

Another activity in *Ratios and Proportional Relationships* focuses on developing understanding of how to compare ratios and strategies for doing so. Figure 4 displays a problem posed in the progressions document about mixing cans of yellow and red paint. The activity, *Comparing Ratios*, allows students to associate visual representations of cans of paint with equivalent ratios displayed numerically in a table (Fig. 5). Students refer to earlier work with equivalent ratios to answer questions such as: Is a mixture of 2 red to 6 yellow a different shade than a mixture of 5 red and 15 yellow? In considering different approaches for comparing the two

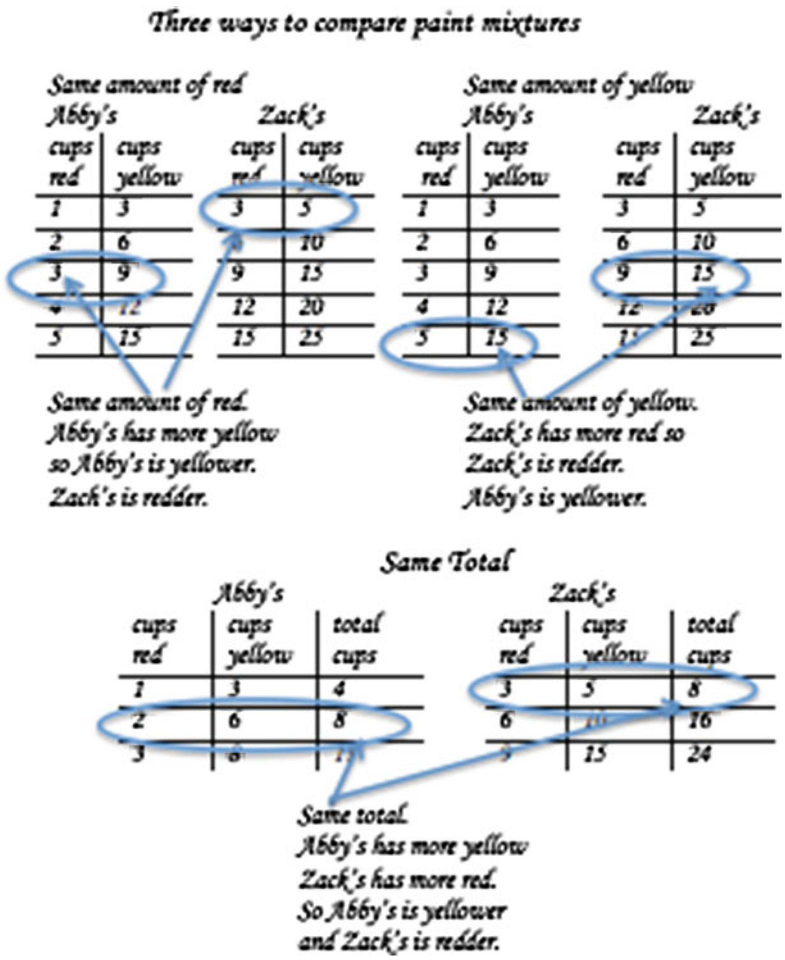


Fig. 4 Mixture problem

mixtures, students investigate whether any of the equivalent ratios can be useful in deciding which mixture would be more yellow.

When students select pairs of ratios with a common unit, such as 6 cans of red paint (Fig. 6), the "action/consequence" move produces a visual representation of the ratios that helps them make the comparison with respect to cans of yellow paint. They confront misconceptions such as pairing cans of red and yellow and counting the number of left over cans of yellow paint to determine the mixture that will have the most yellow as well as consider what other pairs of ratios might also be used to make the comparison.

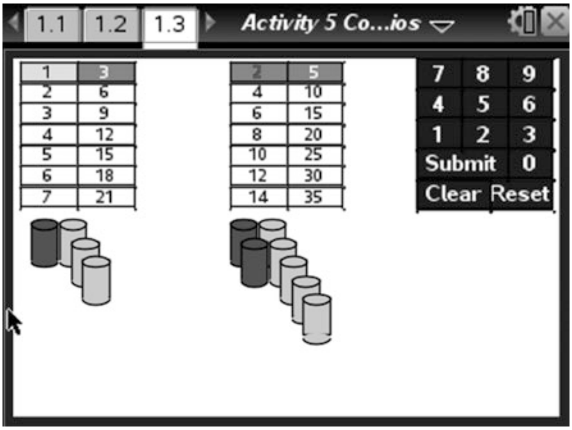


Fig. 5 Visualizing ratios

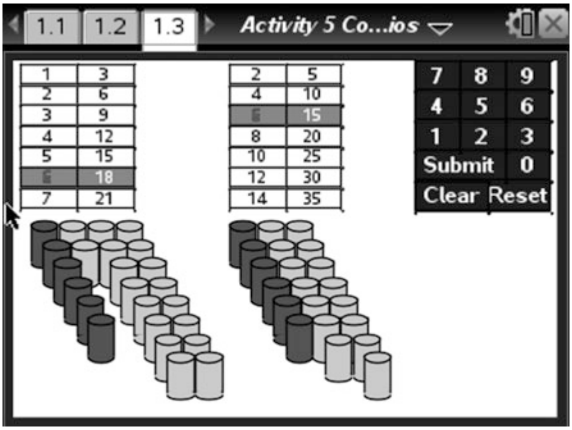


Fig. 6 Equivalent ratios

A third example from *Ratios and Proportional Relationships* illustrates how static diagrams in the progressions documents (Fig. 7) were made interactive. In Fig. 8, students repeatedly generate a horizontal and then vertical change associated with a collection of equivalent ratios, observe the corresponding table and consider the slope triangles and their relationship to each other and the line from the perspective of repeated addition. In Fig. 9, students enter values in the table to generate equivalent ratios by multiplication, and the resulting pair is graphed leading to the notion of scaling and similarity.

Because the activities within a content strand are based on the progressions for that content area, they are sequenced in a developmental order, beginning in middle grades and extending into high school. Figure 10 shows the progression of ratio concepts from the initial concept of ratio as pairings of quantities through a trajectory that leads to proportions to slope to functions and a parallel trajectory that leads to geometric ideas of scaling, similarity and trigonometric ratios. Similar

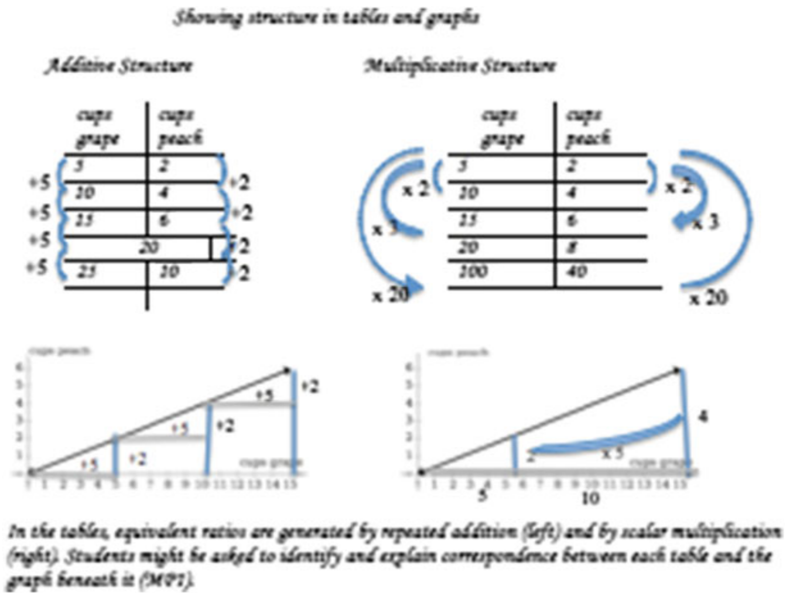
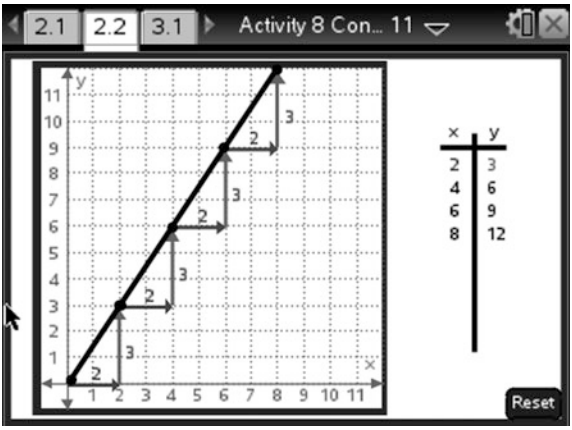


Fig. 7 Graphing ratios

Fig. 8 Additive structure



progression maps describe other content strands. While some of the activities can be used "out of sequence", the cumulative learning built into the complete set for a strand will not happen if the activities are used without regard to the progression. The activities can be associated with a grade level in the CCSSM but could be used at any grade level as long as the sequence of ideas is maintained.

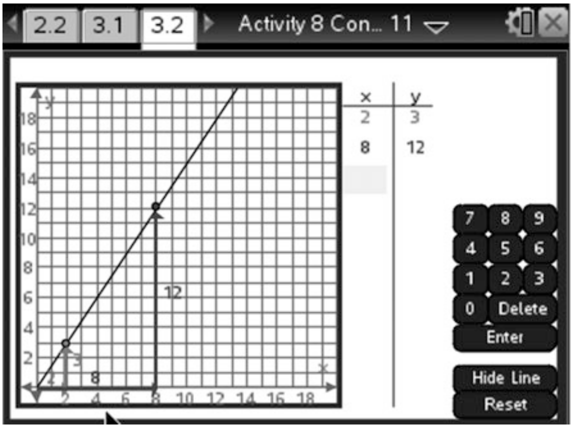


Fig. 9 Multiplicative structure

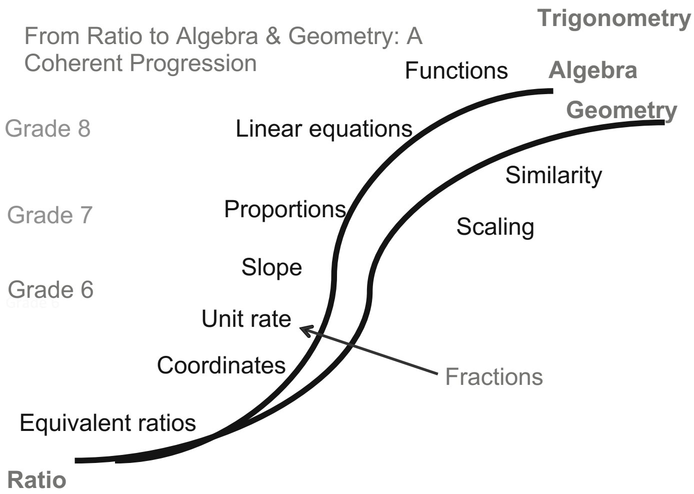


Fig. 10 Ratio learning progression

3.2 The Tasks

The tasks in each activity were constructed following the advice of Black and William (1998) with respect to formative assessment: "Tasks have to be justified in terms of the learning aims that they serve, and they can work well only if opportunities for pupils to communicate their evolving understanding are built into the planning (p. 143)." Thompson (2002) argued that the goal of a task is to have students participating in conversations that foster reflection on some mathematical "thing". Thus, the majority of tasks in the activities create opportunities to discuss

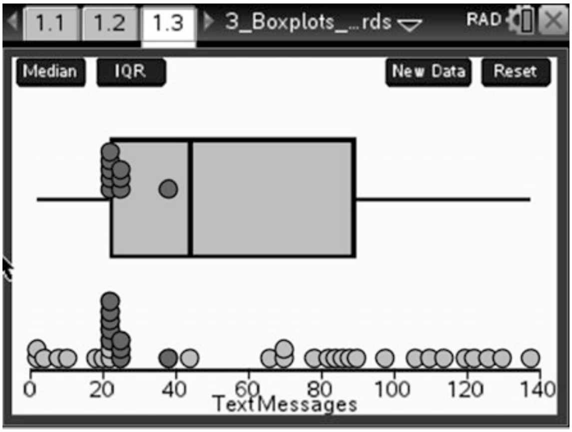


Fig. 11 Highlighting values

particular mathematical objects or ideas that need to be understood and to ensure that specific conceptual issues and misconceptions will arise for students as they engage in discussions.

Misconceptions: The tasks in the activities have been designed in light of the research related to student learning, challenges and misconceptions (e.g., Zehavi and Mann 2003). The well documented misconception that $(a+b)^2 = a^2 + b^2$ is deliberately addressed in the first activity, *What is an Exponent*?, in the Expressions and Equations strand, where students experiment with "distributing" exponents over all four operations, using the definition of exponents to examine expressions and, as is done in much of this CCSSM strand, making connections back to arithmetic to help their thinking. The misconception is confronted again in later activities in *Building Concepts*: *Expressions and Equations*.

A common misconception in the statistics and probability strand relates to box plots: the longer the section, the more data in that section. To build understanding of the connection between the data and a box plot, a dot plot "morphs" into the box plot, and students compare the number of data values in each section of the box plot (Fig. 11). Moving points in the dot plot immediately displays the effect on the corresponding box plot (Fig. 12), reinforcing the fact that medians and quartiles are summary measures based on counting.

Tough to teach/tough to learn concepts: Many students struggle with adding fractions, where they typically follow an algorithm they do not understand. The CCSS stress the number line as a representation for fractions and the unit fraction as a building block for developing operations with fractions. Figure 13 displays a screen from the activity on adding fractions with a common denominator, where students visibly see how addition is the concatenation of two fractions both multiples of the same unit fraction. They explicitly change the denominator of the fractions and observe the results, giving them a physical model for the algebraic formula, $a/b + c/b = (a + c)/b$. Students consider the number of unit fractions in each of the two fractions and justify why the sum of the two fractions is the total

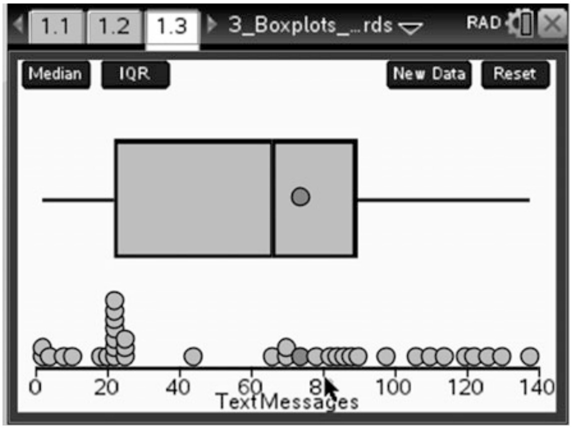


Fig. 12 Moving points

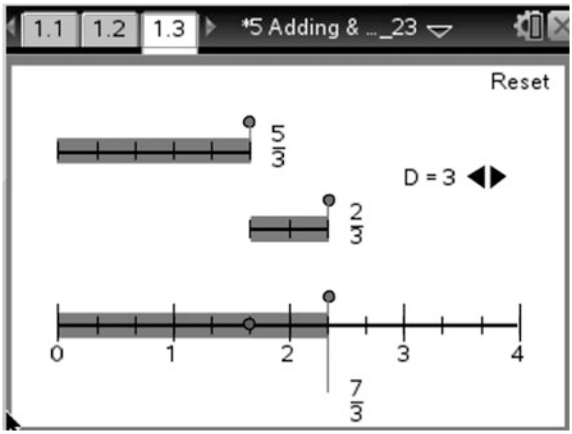


Fig. 13 Adding fractions

number of unit fractions. Reflecting on the process of adding fractions with unlike denominators in light of the visual representation of the sum of unit fractions, students recognize that to add they must find equivalent fractions based on a common unit fraction.

A second example of fragile conceptual understanding with respect to fractions is the fact that a fraction has meaning only when the unit related to the fraction is known. Figure 14 illustrates an activity in which students use geometric models to create equivalent fractions and compare their work with others to make sense of a "unit" using a visual model of equivalence.

In statistics, the conceptual transition from data represented in bar graphs to plotting data on a number line has long been problematic. One consequence is the fragile understanding of histograms, and another is the "make everything into a bar graph" approach to graphical representations. In the activity *Mean as Fair Share* students explore giving dogs "fair shares" of bags of dog food, first using a "take

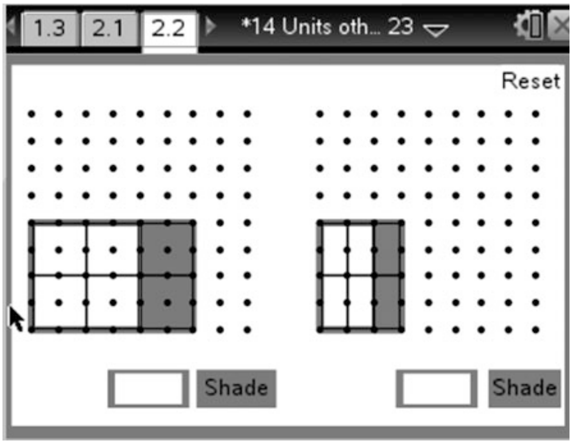


Fig. 14 Different size units

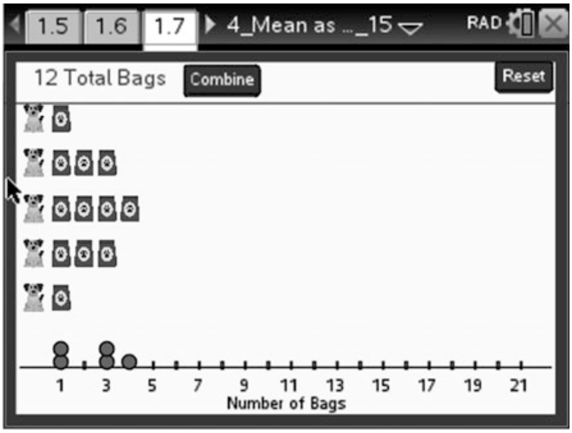


Fig. 15 Fair shares

from the most and give to the least" strategy and then using a pooling strategy. At the end of the activity students return to the first strategy, but this time the display includes a number line (Fig. 15) where each dot represents a dog and its position on the number line indicates the number of bags of dog food assigned to that dog. Selecting a dot highlights the dog in the pictograph and vice versa. As students move the bags from dog to dog, the corresponding points move. When each dog has its "fair share", the points are stacked at the mean number of bags of dog food per dog. This lays the foundation for the next activity, which extends the definition of mean as fair share to mean as the balance point of a distribution of data.

Note that the nature of the activities indicates they are not intended to be used for "doing" mathematical procedures but rather provide a foundation for reasoning about the mathematics that can support the transition to procedural fluency. When students have a solid conceptual foundation, they can reason about the mathematics, are less susceptible to common errors, less prone to forgetting and are able to see connections and build relationships among ideas (NRC 1999).

3.3 Posing Questions

In addition to making sure that the tasks surface misconceptions and develop conceptual understanding of "tough to teach/tough to learn" concepts, the questions for each of the activities are created using some general guidelines below:

1. Activate prerequisite knowledge before it is used: e.g., Remember what the solution to an equation represents. How is the solution to the equation reflected in the picture on the screen? How do you know? (*Equations and Operations*)
2. Point out things to notice so students focus on what is important to observe; e.g., When you increase the value of the denominator of a unit fraction, how does the number of equal parts in the interval from 0 to 1 change? What happens to the length of those parts? (*What is a Fraction*?)
3. Ask for justifications and explanations; e.g., Make a conjecture about which data set will have the largest mean. Explain why you think your conjecture might be correct. Use the file to check your thinking (*Mean as Balance Point*).
4. Make connections to earlier tasks or to an immediately previous action taken by the student. (Questions should not come out of the blue.); e.g., Look at your answers for question 2 and see if you want to change them now that you have looked at the values when they are ordered (*Median and Interquartile Range*).
5. Include both positive and negative examples in developing understanding of definitions, theorems and rules; e.g., Which of the following seems like the best definition of an exponent? Explain your reasoning. An exponent a) is a multiplier; b) is a factor, c) tells how many times a number is used as a factor; d) tells you to multiply a number by another number (*What is an Exponent*?).
6. Have students consider the advantages/disadvantages of each approach when it is possible to carry out a task using multiple strategies; e.g., Petra claims you should always use a unit fraction or a unit rate for solving missing value ratio problems. Do you agree? Give an example to support your thinking? (*Double Number Line*)
7. Be explicit about possible misconceptions: e.g., Decide whether the following statements are true or false. Give an example to support your thinking.
   (a) Some equations have more than one solution. (b) Some equations do not have any solutions. (c) Some equations have an infinite number of solutions. (d) Some expressions always have even numbers as outputs (*What is an Equation*?).

Choosing good tasks, paying attention to cognitive demand and to student misconceptions, and asking the right questions are not the whole story. When a dynamic interactive platform is integral to the development of ideas, the platform must support both the mathematics and the user as a learner, i.e., careful attention must be paid to the design of the activities. The next section describes the principles used in designing Building Concepts interactive activities.

4 Design Principles for Building Concepts Activities

4.1 Mathematical Fidelity

To have mathematical fidelity, the software should be mathematically correct; for example, the boundary line for the graph of $y < 2x + 3$ should not be solid; a side of a triangle in the Euclidean plane should not be associated with a negative slope without reference to a coordinate system. To maintain mathematical fidelity, what students view onscreen should always be mathematically acceptable, i.e., two box plots on the same screen should refer to the same scale. Some technologies have serious flaws in their ability to be mathematical faithful (for example, round off error and limited precision can result in bizarre graphs in given situations). This suggests that the design of activities using the software should consider the context and the mathematical behaviours of objects on screen.

4.2 User Experience

To support the action/consequence principle, the user interface should eliminate obstacles that get in the way or distract the user from easily and immediately being able to attach meaning to both the action and the consequence. The design of the tool should pay attention to cognitive processes appropriate for students' reasoning and knowledge base. A decimal scale on the horizontal axis of a dot plot of data such as that in Fig. 16 (Statkey 2012), which might be appropriate for upper level students, would be conceptually difficult for younger students to interpret. They would typically struggle with why three of the dots are colored and what 0.025 on the right side of the screen represents. The aim should be to support mathematical thinking as opposed to finding results efficiently (the shortest way to a solution) or a "showy" illustration with little mathematical substance. Students should not be asked to spend time sorting out the actions on the screen but rather on making sense of the mathematics they can observe. (For example, in Fig. 17, dragging the point L changes measures on the screen, but the display is cluttered and does not help students see the connections between the sides, angles and proportions.)

Even the location of plots within a panel can make a difference in student understanding. Budgett and colleagues (2013) hypothesized that the vertical arrangement of the graphics panel within the dynamic visualization tool

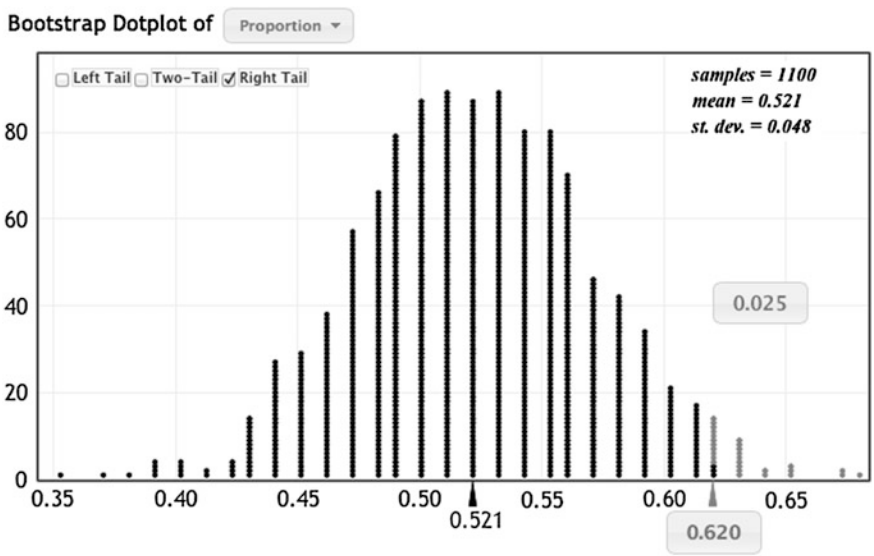


Fig. 16 Sampling proportions

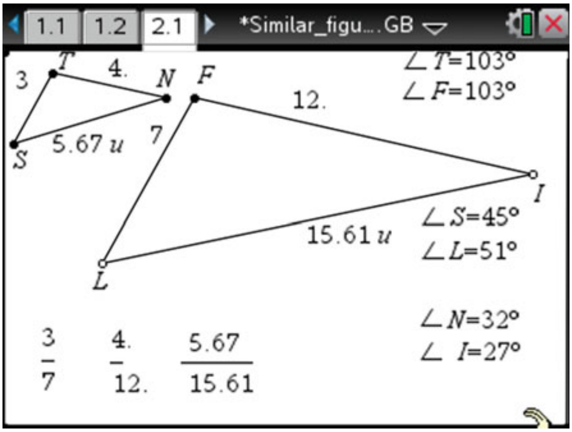


Fig. 17 Similarity

(http://www.stat.auckland.ac.nz/~wild/VIT/) used in their study of randomization in statistics made a difference in what students took from the activities.

Design principles used for creating websites can be useful in thinking about the user interface when designing interactive applets. Visual design guidelines advocated by the US Department of Health and Human Services (http://www.usability.gov/what-and-why/visual-design.html) include:

- Unity: everything on a page visually and conceptually belongs together; e.g., an image must relate to the text it is next to, for the overall message to make sense.
- Gestalt: users perceive the overall design as opposed to individual elements.
- Space: placement of objects reduces noise and increases readability. Simple designs are best.
- Hierarchy: difference in importance of items is conveyed using font sizes, colors, and placement on the page. Usually, items at the top are perceived as most important.
- Contrast: some items stand out because of differences in size, color, direction, and other characteristics.
- Consistency: continuity is maintained throughout a design where pieces work together over an interface. This simplifies learning the interface for the user.

Implicit in these principles is the notion of clarity, (Schwier and Misanchuk 1993) where the meaning of an image is readily apparent to the viewer and the message is reduced to the absolute essentials.

In Building Concepts, these guidelines from web design were adapted to ensure that the experience would maximize learning opportunities for students by creating interactive files that:

- Use simple but mathematically meaningful actions (examples: entering a value, changing a parameter by clicking on a directional arrow, dragging a point on a number line) (gestalt);
- Have visible cues for actions students can take and for the consequences students should be noticing or thinking about (space);
- Minimize use of text on screen (clarity);
- Use color only to make connections and enhance understanding (contrast);
- Locate changing quantities as close together as possible (proximity/unity);
- Display information in order of importance, in terms of position, font size (contrast);
- Use same core design features within and across the files (reset, representation of moveable dots, behaviour of arrow keys, etc.) (consistency).

Color is often misused in creating visual representations. If objects and text are colored gratuitously, the color can introduce unnecessary distractions rather than suggesting important connections. Figure 18 shows the color wheel (invented by Newton in 1666 when he transformed the bar of colors created by light passing through a prism into a segmented circle, where the size of segment differed according to wavelength and width in the spectrum) can help designers choose effective color combinations. To find a harmonious color scheme, use any two colors straight across from each other on the wheel, any three colors that are the vertices of an equilateral triangle or of an isosceles triangle or any four colors that are the vertices of a rectangle. Thus, blue and orange or purple, red-orange and yellow-green could be used to enhance visual images.

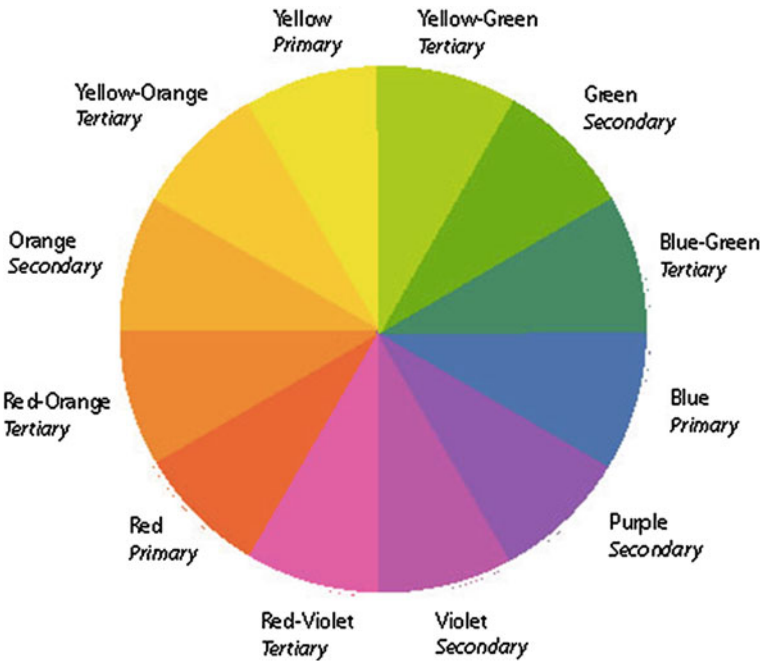


Fig. 18 Color wheel (sustland.umn.edu)

One facet of hierarchy or emphasis is sometimes not obvious; displaying information in order of importance has relevance for the location of buttons and elements that change. Eye tracking studies suggest that people scan computer screens in an "F" pattern, starting from the top and left of the screen. The right side of the screen is rarely seen. This suggests the design of interactive files should position important interactions or information at the top or left of the screen. http://shortiedesigns.com/2014/03/10-top-principles-effective-web-design/.

Figures 19 and 20 from *Building Concepts*: *Equations and Operations* illustrate the use of both color and emphasis. The additive change is colored green in both files (+10 and +4), a purplish pink color is used for the variable, and blue is used for the constant on the right. A violet color, complementary to the pinkish color, represents the multiplicative factor on Sect. 2.2. The arrow buttons and the important things to observe, the changing equation, are at the top. The cues signaling which objects are moveable are given by the handles on the line segments.

Designing technology interfaces should be attentive to user interface issues to fully exploit the action consequence principle, making both the action and consequences transparent and immediate. The next section describes TI Nspire software as a choice for the interactive dynamic documents.

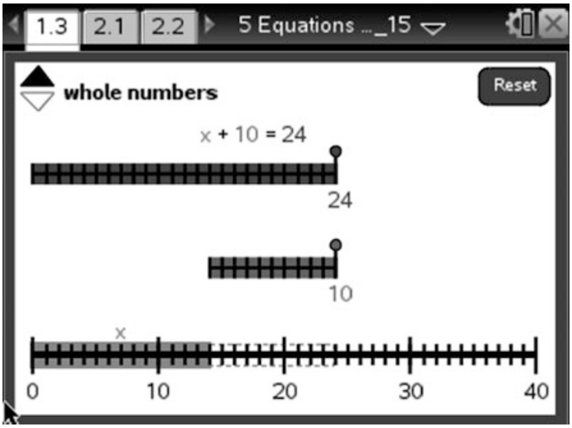


Fig. 19 Equation $x + a = b$

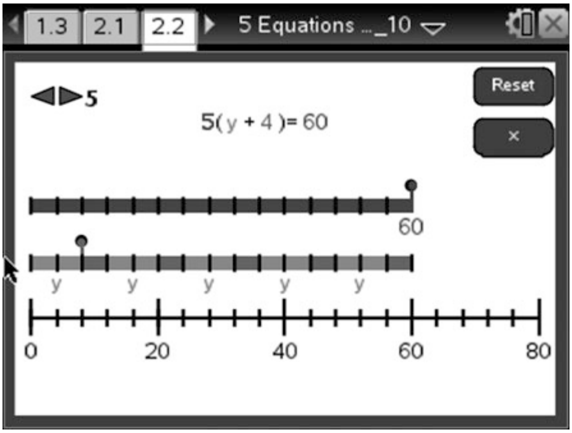


Fig. 20 Equation $cx = b$

5 Use of TI Nspire Platform

The TI Nspire platform easily lends itself to the construction of the interactive documents, which are related to the applet-family concept (Dick and Burrill 2009). The documents are written in Lua and can be used on handhelds, computers or iPads. The developer has devised several "meta" programs, such as an interactive number line, buttons, and clickers that are used frequently in creating the files. Lua allows the creator to program behaviors using the infrastructure of the Nspire, but the end product restricts the user from interacting with any of the Nspire applications (spreadsheet, graphs, geometry, data collection). This has two advantages. First, the documents provide "safe" environments in which students can play with a mathematical idea in a variety of ways but where the opportunity to go astray is limited (Dick 2008). Second, the technology learning curve is short. The user needs only to know how to get the files on their device, find and open them, and turn pages. The interactivity is restricted to dragging, clicking or selecting an object, and entering numerical values.

Challenges to using the documents vary with the hardware platform. A user might experience some frustration due to lack of familiarity with the touchpad on the handheld device. To address this, when possible, the movements are enabled by using the arrows and tab keys on the handheld keyboard. In some instances, such as changing the factor in a multiplication problem, the user can select the handheld Menu and use the document specific options given there instead of moving the cursor over the number and making the change directly. Entering numerical values poses a problem on the iPad with its touchscreen interface. This was addressed by building a keypad into the document when it was necessary (See Fig. 3). Screen size, especially on the handheld, limits the use of multiple displays.

One challenge related to the computer software and displaying the documents on LCD projectors is the diffusion of color; projectors have various interfaces that change, mute or enhance certain colors in ways that vary from machine to machine. This makes testing the colors a time consuming task, and one that still may not produce effective results for some projectors.

Building Concepts is intended to serve each of the content strands in the CCSSM. The next section describes the activities in three of those strands.

6 Building Concepts Content Strands

6.1 Building Concepts: Fractions

Building Concepts: *Fractions* (2014) consists of a series of 15 interactive dynamic files designed to develop concepts related to fractions and operations with fractions. The development, based on the approach advocated by Wu (2011) and aligned with the CCSSM, uses number lines and area models to help students visualize a fraction as a number (Mack 1995) and develop the arithmetic operations and concepts related to the meaning of fraction, with a particular emphasis on equivalent fractions and their role in fraction operations. A strong emphasis is placed on the notion of a unit fraction and the role of unit in interpreting fraction relationships (Clark and Roche 2009; Lamon 1999). The file and questions are intended to support the transition from words to symbols (Sowder 1992), and enable students to recognize that fractions can be larger than 1, understand that whole numbers can be represented as fractions (Siegler and Pyke 2012), and recognize a larger denominator does not determine the larger fraction (Fazio et al. 2012).

6.2 Building Concepts: Ratios and Proportions

The 15 activities in *Building Concepts*: *Ratios and Proportions* (2015) develop ratio as pairings of quantities that vary together. Consistent with the CCSSM and countries such as Japan (Ministry of Education 2008), while a fraction such as a unit rate can be associated with a ratio, ratio as a concept is broader than fraction. Students engage in visual and interactive strategies (i.e., double number lines, ratio tables) for solving problems involving ratios and proportion to overcome the difficulty they typically have using algorithms (Lamon 2007; Singh 2000), where they often think any problem with three values and one unknown is a proportional relationship. The activities provide a deliberate and careful investigation into the difference between multiplicative and additive situations (Lamon 1999). A major focus is on equivalent ratios. Research suggests students can create equivalent ratios using simple numbers such as doubling and halving (Empson and Knudsen 2003) but have trouble with more complicated situations. The activities relate collections of equivalent ratios to ordered pairs in a coordinate grid, develop the notion of a slope triangle and the relation of slope to unit rates, introduce proportional relationships and connect them to graphs, and relate proportional reasoning to scale factors.

6.3 Building Concepts: Statistics and Probability

The activities in *Building Concepts*: *Statistics and Probability* (2016) are closely aligned with the CCSSM and also with the *Guidelines for Assessment and Instruction in Statistics Education* (Franklin et al. 2007), which describe the statistical process as consisting of four parts: formulating a question, collecting data, analyzing data and interpreting results. All of these are enacted in the presence of variability, a recurring theme in the statistics and probability progression. The activities begin with a focus on asking a statistical question and looking at distributions of life spans and maximum speed of animals. Measures of center and spread are introduced together, recognizing that either measure alone tells a very incomplete story about the distribution of the data. This helps students take both center and spread into account when reasoning about variation in a variety of situations (Shaughnessy et al. 1999). When appropriate, data points are moveable to call attention to features of the data and of the different plots. Probability is introduced through games, and a choice of simulation models such as coins and spinners allows students to simulate probability distributions and sampling distributions. The activities enable students to experience variability by comparing random samples, generating simulated distributions of sample statistics, and observing the effect of sample size on sampling distributions (Hodgson 1996; del Mas et al. 1999). The middle grade activities end with an investigation fitting models to data sets and examining the error in various models.

Each file is accompanied by supporting materials that include (1) a description of the mathematics that underlies the file; (2) a description of the file and how to use it; (3) possible mathematical objectives for student learning; (4) sample questions for student investigation; and (5) a set of typical assessment tasks.

7 Issues and Potential Pitfalls

7.1 Potential Pitfalls in Designing Activities

The platform affords a vast number of opportunities to enact the action/consequence principle by exploiting the dynamic linkages that can be created between virtually any two objects (where an object is a number, or an expression, or a graph, or a point, or a geometric figure, or a spreadsheet cell, or …). Using this feature allows for the creation of mathematical scenarios or "microworlds" where a student can take an action on at least one of the objects and immediately see a change in the linked object(s). But it is also very easy to create a microworld that leaves the student as a passive observer where the "consequence" may be simply an animation triggered by pushing a button. Design decisions must be made as to what consequences are supplied by the device and which must be supplied by the student. For example, in a probability simulation, should the student or the device record the outcomes of tossing a coin until you have four heads? When should the process be automated with the number of successes being plotted for 500 repetitions of the simulation? The challenge is to design the interaction in ways that engage students in thinking about the mathematics and not just observing an outcome. Without careful guidance and questioning, this can too easily happen when students use a computer algebra system (CAS), where they turn over the mechanics to the device but are not engaged in reasoning about the process.

Another potential pitfall is to ensure that the focus of the activities is on developing conceptual understanding and not just "doing" the mathematics. Essential to the action/consequence principle is the notion that the action taken by the student is a purposeful choice that has mathematical meaning for the student. When the action is pushing a button and the consequence is the graph of a large data set, the action itself may not be perceived by the student as mathematically meaningful; the interaction between mathematical objects is missing, and the same results could have been achieved with simple graphing software. Such results, while obviously very useful, do not push students to reflect on connections to underlying concepts. In contrast, moving a point and observing no change in the interquartile range for a distribution of data provokes the opportunity to reflect on what an interquartile range is and how one is constructed. The challenge is to focus some of these output driven procedures in a more conceptual direction.

7.2 The Role of the Teacher

Technology alone will not make a difference in student learning. The teacher is the mediator of the interaction between the students, the technology and the learning (Drijvers 2012; Laborde 2001; Roschelle et al. 2010; Suh 2010). Research about effective use of interactive applets in learning statistical concepts suggests teachers should engage students in activities that not only help them confront their misconceptions but also provide them with feedback (del Mas et al. 1999). Allowing students to engage in unfettered "play" with interactive technology is appealing but by itself will not maximize learning opportunities; even a well-designed interactive activity is unlikely to be effective unless students' interaction with it is carefully structured by the teacher (Lane and Peres 2006). Managing classrooms to effectively use technology means involving students in discussing observations after an activity to focus on important observations, helping them become aware of missed observations, and engaging them in reflecting on how important observations are connected (Chance et al. 2007).

The teacher notes offer potential questions for the activities, suggestions for structuring lessons and for managing discussions in ways that support learning. But implementing the activities requires that teachers have confidence in their content knowledge and an understanding of what it means to teach, something not all school teachers in the U.S. are prepared to do.

7.3 Changes in Content

Perhaps the biggest challenge to the CCSSM and to Building Concepts is that "teachers prefer to teach as they were taught" (Cheek and Castle 1981; Kennedy 1991). The CCSSM advocates for coherent and consistent mathematical stories across the grades and not only organizes but presents mathematics from a different perspective. Unfortunately, many teachers work new ideas into their old curriculum and traditional practice rather than accepting an entirely new approach.

8 Conclusion

Very preliminary results from piloting seem promising. Initial results of a study using *Building Concepts*: *Fractions* (2014) with teachers in a preservice methods course for elementary teachers suggest that the activities made a difference for teachers' understanding of and ability to use fractions. Pre and posttest scores were compared for those who used the technology based activities with those who received instruction in use of concrete manipulatives, such as Cuisenaire rods and fraction strips. Evidence from a school test site suggests that students struggling with mathematical concepts outperformed other students on the state assessment after using *Building Concepts*: *Ratios and Proportional Reasoning* (2015). Other pilot sites are currently being established for the materials as they are being developed.

In Ben-Zvi and Friedland (1997) noted that technology for teaching and learning has evolved over the years, progressively allowing the work to shift to a higher cognitive level enabling a focus on planning and anticipating results rather than on carrying out procedures. Since then technology has provided powerful new ways to assist students in exploring and thinking about ideas, allowing them to focus on interpretation of results and understanding concepts rather than on computational mechanics. And technology continues to change and offers opportunity to rethink what and how we operate in our classrooms and how that is related to the world outside of the classrooms (Gould 2011). If we use technology to do what we have been doing, we will get the same results (Ehrmann 1995). This paper proposes a program based on an action consequence principle to add new thinking to new technology to enhance mathematics teaching and learning for all students.

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