Designing for Mathematical Applications and Modelling Tasks in Technology Rich Environments

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Abstract Mathematical modelling and applications is a well-established field within mathematics education. Research in mathematical modelling and applications has maintained a focus on how to enhance students' capabilities in using mathematics learnt in school to solve problems identified in, or derived from, the real world. While significant progress has been made in understanding the processes that underpin the successful applications of mathematics in real world contexts, there has been limited research into how to design tasks that are authentic reflections of the role of digital technologies in solving problems situated in the work place or daily life. This chapter draws on data sourced from a research and development project that investigated the use of digital technologies in teaching and learning mathematical modelling and applications to identify principles of effective task design. The instantiation of these principles within classroom practice is illustrated through a classroom vignette. This chapter concludes with a reflection on the research needed to further develop understanding of the role of technology as an enabler of principles of design for mathematical modelling tasks.

Keywords Modelling · Mathematics · Technology · Design · Applications

1 Introduction

While significant progress has been made in understanding the processes that underpin the successful applications of mathematics in real world contexts, there has been limited research into how to design tasks that are authentic reflections of the role of digital technologies in solving problems situated in the work place or daily life (Geiger et al. 2010). This is despite the noteworthy progress of research that explores both the themes of mathematical modelling and applications and the use of digital tools to enhance mathematics learning.

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Mathematical modelling and applications is a well-established field within mathematics education. Research in mathematical modelling and applications has maintained a focus on how to enhance students' capabilities in using mathematics learnt in school to solve problems identified in, or de-rived from, the real world as well as how the modelling process itself is played out while attempting to solve real world problems. A broadly accepted description of the act of modelling outlines a cyclic process that involves: the formulation of a mathematical representation of a real world situation (model); using mathematics in conjunction with the model to derive initial results; interpreting the resulting outcome in terms of the given situation to determine the validity of the model; and, if necessary, revising the model until it is determined to be effective (e.g., Blomhoj and Hojgaard Jensen 2003; Blum and Niss 1991). Simply put, the purpose of models is to interpret real world situations and/or make predictions about the future or past states of modelled systems (English et al. 2005). The need to build models, however, is motivated by a requirement to: measure some property of a system; decide between alternatives; allow one to replicate a system; predict the outcome of a system; explain the outcome of a system; and understand how to manipulate a system (Thompson and Yoon 2007). While there is now a large and still developing corpus of research related to mathematical modelling within educational contexts, to this point, studies have tended to coalesce around mathematical, cognitive, curricular, instructional, and teacher education perspectives (e.g., Cai et al. 2014).

Similarly, the body of knowledge related to the use of digital tools in mathematics classrooms has increased rapidly over the past two decades. Studies in this area, however, have tended to report on advantages to instruction in mathematical thinking and learning within content specific domains such as number (e.g., Kieran and Guzma'n 2005), geometry (e.g., Laborde et al. 2006), algebra and calculus (e.g., Ferrara et al. 2006) or social aspects of classroom practice such as collaborative investigative practice (e.g., Beatty and Geiger 2010).

The potential for digital tools to enhance the teaching and learning of modelling has been recognised, as is evident in this statement from Niss et al. (2007).

> Many technological devices are highly relevant for applications and modelling. They include calculators, computers, the Internet, and computational or graphical software as well as all kinds of instruments for measuring, for performing experiments etc. These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment (p. 24).

While there is a developing body of research that lends weight to this potential (e.g., Geiger et al. 2010; Villarreal et al. 2010), there is still much work to be done on how technology can be used in tandem with mathematical knowledge to work on problems that exist in the real world, as Zevenbergen (2004) observes:

> While such innovations [ICTs] have been useful in enhancing understandings of school mathematics, less is known about the transfer of such knowledge, skills and dispositions to the world beyond schools. Given the high tech world that students will enter once they leave schools, there needs to be recognition of the new demands of these changed workplaces (p. 99).

Zevenbergen's (2004) statement identifies a shortcoming of school mathematics instruction and implies there is a need to develop tasks and learning experiences where expectations of how real world problems are tackled and solved include the integrated use of digital tools. Others, such as Hoyles et al. (2010), have noted the need for the development of techno-mathematical literacies—new mathematics based competencies required by societies in which digital technologies are becoming ubiquitous.

The aim of this chapter is to explore one approach to the design and implementation of mathematical modelling tasks that integrate digital technologies. In doing so, the chapter will address the following research question—What are the principles of design for technology rich modelling and applications tasks that result in effective learning experiences for students?

The first section of this chapter will outline the theoretical framework that provides the background for this study, comprising of a review of the role of digital tools as mediators of mathematical learning and a discussion of general principles of task design. In the second section the research design and methods employed through the study are described. The third section will present one teacher's principles of task design and provide an example of how these were implemented via a classroom vignette. The final section will reflect on the effectiveness of the teacher's principles and compare these to the general principles of tasks design outlined in the theoretical framework.

2 Digital Tools as Mediators of Mathematical Learning

In developing principles of task design for technology integrated modelling and applications tasks, consideration must be given to the role of artifacts and instruments in mathematics teaching and learning. Verillon and Rabardel's (1995) iconic work on the distinction between an artifact and an instrument provides insight into the role of artifacts in mediating learning by distinguishing between an artifact, which includes both physical and sign tools that have no intrinsic meaning of their own, and an instrument in which an artifact is used in a meaningful way to work on a specific task. Different tasks make different demands on the user and their relationship with the artifact. The development of this relationship, and thus how the artifact is used, is known as instrumental genesis. Instrumental genesis is complex and involves, firstly, a process where the potentialities of the artifact for performing a specific task are recognised and the artifact is transformed into an instrument (instrumentalisation), and, secondly, a process that takes place within the user in order to use the instrument for a particular task (instrumentation) (Artigue 2002). Instrumentation generates schemas of instrumented action that are either original creations by individuals or pre-existing entities that are appropriated from others. An instrument, therefore, consists of the artifact and the user's associated schemas of instrumented action. Instrumental genesis is also a dynamic process between the instrument and the user, as the constraints and affordances of the artifact shape the user's conceptual development while at the same time the user's perception of the possibilities of the artifact during instrumentation can lead to the use of the artifact in ways that were not originally intended by the designers of a tool (Drijvers and Gravemeijer 2005).

Instrumental genesis has been used to explain how digital tools are transformed into instruments for learning through interaction with teachers and students (e.g., Artigue 2002). A teacher's activity in promoting a student's instrumental genesis is known as instrumental orchestration (Trouche 2005). This process recognises the social aspects of learning as it allows for the sharing of schemas as instrumented action that individuals have developed within a small group or whole class. A teacher can facilitate the appropriation of these schemas by other students by making the nature of these schemas explicit through orchestration of classroom interaction around the schemas through careful and selective questioning.

More recently, others have attempted to extend our understanding of an instrumental approach to the role of artifacts in mediating learning by recognising that the genesis of an artifact into an instrument takes place within highly interactive environments, such as school staff rooms or mathematics classrooms, where a number of artifacts are used simultaneously. Gueudet and Trouche (2009) extend the definition of artifact by introducing the term *resources* to encompass any artifact with the potential to promote semiotic mediation in the process of learning. Resources include entities such as computer applications, student worksheets or discussions with a colleague. A resource is appropriated and reshaped by a teacher, in a way that reflects their professional experience in relation to the use of resources, to form a schema of utilisation—a process parallel to the creation of a schema of instrumented action within instrumental genesis. The combination of the resource and the schema of utilisation is called a document. Documental genesis is an ongoing process in which utilisation schemas are reshaped as a teacher gains more experience through the use of a resource.

The idea that learning and problem solving are processes that require strategic deployment of a range of resources in an integrated fashion with the potential to transform tasks and learning environments is echoed in Kaput et al.'s (2007) perspective on the role of digital technologies in mathematics education. From their perspective "technologies and tools co-constitute both the material upon which they operate and the conditions, particularly social conditions, within which such operations occur" (p. 172). This means that digital technologies should be considered as essential infrastructure for mathematical problem solving in current and future societies.

3 General Principles for Task Design in Mathematics

As tasks are integral to many dimensions of mathematics learning, including mathematical content, processes, and modes of working, Burkhart and Swan (2013) argue for the importance of task design to improve mathematics instruction. For teachers, task selection, adaptation, and creation are intertwined with choices of pedagogies for realising opportunities that lie within specific tasks (Sullivan and Yang 2013). Evidence that coherent research and development approaches to task design are effective in improving teaching practice is provided by the long term success of programs such as Connected Mathematics (Lappan and Phillips 2009). At the same time, Schoenfeld (2009) argues for greater communication between designers and researchers as many designers do not make their design principles explicit, and so it is difficult for others, including teachers, to adopt effective approaches to task creation and adaptation. Thus, partnerships between teachers and researchers, where understandings of principles of task design and the effective integration of tasks with pedagogical approaches are explored, refined and documented, holds potential for improving teaching and learning practices in mathematics.

As most tasks are developed for implementation within specific curriculum and school contexts, the *fit to circumstance* of tasks with local conditions and constraints is a vital consideration for effective implementation (Kieran et al. 2013). Such circumstances include local curriculum specifications as well as other affordances, requirements or restrictions, for example, resources available within a particular school.

An appropriate level of *challenge* is important for students when engaging with tasks if real learning is to take place (Hiebert and Grouws 2007). Most guidelines for systemic improvement in learning outcomes stress the need for teachers to extend students' thinking, and to pose extended, realistic, and open-ended problems that challenge students (e.g., City et al. 2009). By posing challenging tasks, and adopting associated pedagogies, teachers provide opportunity for students to take risks, to justify their thinking, to make decisions, and to work with other students (Sullivan 2011). At the same time, students often resist engaging with challenging tasks and attempt to influence teachers to reduce the demands of an activity (Sullivan et al. 2013). Thus, for students to engage with the type of tasks that require the use of unfamiliar or developing capabilities, the completion of tasks must appear to be achievable, that is, tasks must be *challenging yet accessible*. In order for students to engage fully with tasks, however, activities must not only be accessible but also *transparent* in relation to their expected outcomes: that is, it is clear what is required of a student to achieve success with a task (Burkhart and Swan 2013).

As students need to take risks in order to extend their thinking, they must be provided with *opportunity to make decisions* (Geiger et al. 2014). Such opportunities also provide instances where students can exercise and develop their capacities to use mathematics critically. While closely linked to the notion of *challenge*, the opportunity to make decisions does not necessarily mean that highly complex or sophisticated mathematics is required to make judgments.

The articulation of carefully constructed principles for the design of a task does not guarantee the effectiveness of an activity as learning is also influenced by the choice of pedagogy. I have previously argued, in collaborations with other colleagues, that teachers must also adopt *investigative pedagogies* to fully realise the opportunities that such tasks afford (e.g., Goos et al. 2013). Such pedagogies must provide students with the opportunity to speculate, test ideas, and argue for or defend conjectures (Diezmann et al. 2001).

In order to be assured of the quality of a task, activities must also be developed, appraised, trialled, evaluated, and re-trialled in *iterative cycles of design and improvement* (Maass et al. 2013). Thus effective activities will take time to develop and require a commitment to reflective practice by teachers who aspire to be effective designers of instructional tasks.

4 A Technology-Rich, Modelling Task Oriented Research Project

This chapter reports on an aspect of a larger study that investigated the role of digital technologies in enhancing mathematical modelling teaching practice through a design research approach. The study was conducted with individual teachers working in secondary schools across two states within Australia. Six teachers were recruited from six schools; three from each of two different Australian states. Schools were drawn from across different educational systems (government and non-government) and were representative of a range of socio-economic characteristics. Teachers were invited into the project because of their reputations as effective teachers of mathematics, with particular skills in the use of digital tools in promoting students' learning. The project was managed by two university based researchers—one in each state. These researchers were primarily responsible for the conceptual development of the project and classroom data collection including lesson observations, teacher and student interviews, and collection of student samples. Teachers were primarily responsible for the development and implementation of technology demanding mathematical modelling tasks. Researchers played a vital role in providing feedback about the effectiveness of tasks trialled in teachers' classrooms. Together teachers and researchers developed principles of design for effective tasks based on their shared experiences while trialling tasks in individual mathematics classrooms.

The specific aspect of the study reported in this chapter is the work of one teacher and his students in a Year 11 (15–16 years of age) mathematics class. His curriculum context mandated the teaching, learning and assessment of mathematical modelling as a key objective of a state-wide syllabus (educational authorities are state based in Australia). Technology as a tool for teaching and learning mathematics was also prescribed in the Mathematics B program (incorporating the study of functions, calculus and statistics) in which his students were enrolled. Students had almost unrestricted access to digital technologies including: handheld digital devices with mathematical facilities such as data and function plotters and Computer Algebra Systems; computers with mathematically enabled applications; the internet; and electronic white boards.

Table 1 Research design and schedule

Time	Activity
Sept–Dec Year 1	Teacher workshops in each state: research team outline the aims of the project; offer prototype tasks; discussion of principles which underlie prototype tasks
Jan–April Year 2	Lesson observations; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialled tasks in relation to modelling and the use of digital tools
April–June Year 2	Lesson observations; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialled tasks in relation to modelling and the use of digital tools
July Year 2	Teacher workshops in each state: teachers share exemplars of digital tool and modelling tasks; discussion on principles which underlie teacher developed tasks; research team offer accounts of practice from classroom observations
Aug–Sept Year 2	Lesson observations; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialled tasks in relation to modelling and the use of digital tools
Oct–Dec Year 2	Final project meeting and focus group interview in each state; teachers share exemplars of modelling and digital tool tasks; further discussion on principles that underlie teacher-developed tasks

The research design consisted of three components: (1) two whole day teacher professional learning meetings which took place at the beginning and middle of the project; (2) three classroom observations for each teacher; and (3) a focus group interview near the end of the project that involved all teachers. The scheduling and purpose of each of these activities is out-lined in Table 1. Further detail on the research methodology can be found in Geiger et al. (2010).

5 Principles of Task Design in Technology Demanding Modelling Tasks

The teacher whose work is the focus of this chapter demonstrated keen insight into his own design processes and how these developed through the duration of the project. A major feature of this teacher's approach to task design was the integration of digital technologies. Towards the end of the project, he identified what he believed to be the characteristics of technology integrated modelling tasks. These principles and associated descriptions are presented in Table 2.

The teacher also provided insight into the role of digital tools in relation to each principle of design. An outline of these in-sights along with supportive statements drawn from the inter-view data follows.

The use of digital tools is a mandatory element of the state-wide senior secondary mathematics syllabuses, and so the use of technology was a matter of *compliance*. Genuinely authentic problems are mathematically complex. The representational capabilities of digital tools allow students to accommodate this

Table 2 Characteristics of effective modelling tasks

Principles	Description
Syllabus compliance	The task must meet the requirements of the syllabus for content knowledge and the dimensions related to applications and technology
Authenticity and relevance	Tasks must be set in an authentic or life-related context. The task must be of interest to the teacher and be of potential interest to the student
Open-endedness	The mathematics necessary to solve the problem set up in the task should not be immediately apparent. The task must be open-ended in nature providing for opportunity for multiple solution pathways
Connectivity	Ideally the task must make links to different content areas within the syllabus
Accessibility	The task must provide opportunity for students to link to their previous learning. There should be provision for multiple entry and exit points. The task should allow for the introduction of scaffolding prompts or hints
Development	The task must provide challenge and so encourage students to go beyond what they presently know and can do through the modelling process. Students' engagement with the task should provide feedback to the teacher about the development of their understanding

complexity and thus provide access to *authentic* problems that otherwise might be considered beyond the scope of their capabilities.

> If we didn't have the CAS calculators we couldn't do half the stuff that we do. From my perspective it is the integration of the whole lot together. We have a set of data and we try and build a model from that. We do a scatter plot and we make decisions about the model. We build a model and make some sorts of predictions.

Digital tools also provide the means for students with gaps in their content knowledge to *access* challenging problem scenarios.

> Lower achievers may be struggling with differentiation or integration at that particular point in time…but they can still have access to the problem. My lower achieving kids can still engage in the problem and still make some meaningful contributions. If they don't get caught up in all that manipulation they can still be thoughtful about it.

The nature of *authentic open-ended* problems means there is no clear solution pathway and students need to evaluate options as they progress toward a solution. The teacher argued that digital tools offer facilities that are essential for exploring possible solution pathways. Technology also provides the means for connecting different types of mathematical knowledge, for example, data representations and functional relationships that modelled patterns in the data.

> Selecting authentic, open tasks to model generally implies the students will need to make use of technology. Even if the teacher has scaffolded the task to facilitate access to the context, there is a requirement that the task be sufficiently open for there to be multi-representations of the solution and perhaps different solutions.

The *authenticity* and *open-endedness* of a problem is enhanced if students are required to collect data relevant to a problem from an original source; a capacity provided by digital tools in his classroom.

> There is often a need to collect data and then to determine whether a relationship exists within that data. Students may need to collect primary data, through the use of probes, or from a video that is then analysed using the technology or use secondary data collected from a newspaper, magazine, web site or some other source.

Used effectively, digital tools provide immediate feedback to students about their initial attempts to build models and solve problems thus progressing students' understanding of the underlying mathematics at the core of the task and hence their mathematical *development*.

> Technology has a significant role to play in the provision of feedback to the student in the first instance, about the models they have built and how well they fit the context being investigated. In mathematical modelling it is important to look for consensus between the mathematics and the context, hence, it is necessary to consider the validity of the conclusions in terms of the context.

While a number of these principles are consistent with the principles of task design presented earlier in this chapter, there are also points of departure. The commonalities and differences between the teachers "home grown" principles and those developed from research literature in the field will be outlined in the commentary that accompanies the following illustrative example.

6 Exemplar Task and Commentary

Principles for the design of technology demanding modelling tasks are evident in the following description of a task developed and then implemented by the teacher in his Year 11 mathematics classroom—the Algal Bloom Problem outlined in Fig. 1. In developing this task, the teacher expected his students to build a mathematical model for these data by first creating a scatterplot using their CAS active calculator. The calculators were equipped with a computer algebra system, as well as data plotting and regression function capabilities among other facilities.

In previous lessons, students had gained experience with developing models by finding single functions that fit data from different situations drawn from real world contexts. Students had also been introduced to piece-wise functions but had not yet been asked to fit these to real-life data. For the Algal Bloom Problem, the data plot suggests a piecewise function (one part linear and one part power function) would be appropriate. The teacher had hoped that students would then use the plotting functions on their calculators to determine the general form of suitable functions and, in due course, develop an equation that best fit the data. In doing so, the teacher expected students to make use of a piece-wise function, which was covered in earlier work but had not been used to model real-life data via previous examples. Students were then asked to use the model they had created to respond to the question at the end of the task. Further, they were asked to list any assumptions they made in developing their model and to comment on any limitations they believed were inherent in the response they provided.

The CSIRO has been monitoring the rate at which Carbon Dioxide is produced in a section of the Darling River. Over a 20 day period they recorded the rate of CO2 production in the river. The averages of these measurements appear in the table below.

The CO2 concentration [CO2] of the water is of concern because an excessive difference between the [CO2] at night and the [CO2] used during the day through photosynthesis can result in algal blooms which then results in oxygen deprivation and death of the resulting animal population and sunlight deprivation leading to death of the plant life and the subsequent death of that section of the river.

From experience it is known that a difference of greater than 5% between the [CO2] of a water sample at night and the [CO2] during the day can signal an algal bloom is imminent.

Rate of CO2 Production versus time

Time in Hours	0	1	2	3	4	5	6	7	8	9
Rate of CO2 Production	0	-0.042	-0.044	-0.041	-0.039	-0.038	-0.035	-0.03	-0.026	-0.023

Time in Hours	10	11	12	13	14	15	16	17	18	19
Rate of CO2 Production	-0.02	-0.008	0	0.054	0.045	0.04	0.035	0.03	0.027	0.023

Time in Hours	20	21	22	23	24					
Rate of CO2 Production	0.02	0.015	0.012	0.005	0					

Is there cause for concern by the CSIRO researchers?
Identify any assumptions and the limitations of your mathematical model.

Fig. 1 Algal bloom problem

When observing the lesson in which this task was used, the researcher noticed that while every student was able to produce a plot of the data using their handhelds, few had drawn the conclusion that a piecewise function was necessary to model the data. Most students tried using a single function, generally by trying to generate a model for the data using the digital handhelds regression model facility—a facility that did not allow for the fitting of piece-wise functions. When their single functions were plotted on their screens with the original data points it was obvious that their various functions were a poor fit. In response to students surprise at their results, the teacher encouraged students to have a closer look at the nature of their data and explore a wider range of possibilities for fitting a model.

Sometime later, two students, working together near the researcher, attempted to fit a piecewise function to the data, and after performing fine adjustments to each part of their function were happy with the result. Their success prompted a subdued celebration by the two students which attracted the teacher's attention. After discussing their conjectured model with the teacher, students went on to complete the task. A short period of time after his discussion with these students, the teacher called for the attention of the whole class and asked them about their progress. The two students near the researcher volunteered and outlined their attempt. When they announced they had decided to make use of a piecewise function, sections of the class responded in different ways. A small number of students indicated agreement with the approach the pair of students were proposing even though the details of the functions other students had used were different. Most students, however, expressed exasperation that they had not noticed an obvious feature of the plotted data. These students then returned to the task and were able to develop a piecewise function that fitted the data for themselves. A small minority of students needed more direct assistance from the teacher and were then able to develop a model based on a piece-wise function by the end of the lesson. The lesson concluded when the teacher asked the students to do further work on their assumptions and limitations for homework.

7 Comparing Views on Task Design

Two views of task design have been presented in this chapter. The first, as a set of general principles drawn from the literature and the second as a set of principles specific to modelling tasks devised by a teacher of mathematics. The exemplar task discussed in the previous section satisfied the teacher's "home grown" principles of modelling task design as well as the general principles developed from the literature. The purpose for and use of digital tools in this task were also consistent with relevant elements of both sets of task design principles.

7.1 Parallels Between Two Perspectives on Task Design

The use of modelling tasks and digital tools is consistent with mandatory requirements of the relevant state curriculum authority and so observes both *syllabus compliance* and a *fit to circumstance* for the specific curriculum context. Further, it was a mandatory requirement of the relevant syllabus for technology to be incorporated into the teaching, learning, and assessment of mathematics.

Consistent with both sets of principles, the task is *open-ended* in that a variety of mathematical models are plausible and the use of different models will lead to different, but still valid, responses to the problem. The available digital tools are a crucial facilitating resource that provided the facility to trial a range of functions to fit a complex underlying pattern and offered immediate feedback on the appropriateness of a conjectured function allowing students to develop specific solutions from a wide range of possibilities.

Students found the task to be *accessible*, an aspect common to both sets of principles, as it linked to mathematical knowledge they had studied in previous classes and the teacher made use of progress made by other students to provide a prompt when many were experiencing difficulty. Digital tools were also important for this aspect of design as they provided the means for students to trial different functions against the data and receive immediate feedback providing an entry point for most students and so enhanced the accessibility of the problem.

As the task required students to make use of mathematical knowledge they had already studied in previous lessons within an unfamiliar context it provided opportunity for students with the *challenge* needed for the development of their mathematical knowledge and their capacity to apply this knowledge in real world contexts—parallel aspects of the two sets of principles. There is clear evidence in the example that the students were challenged, as their attempt to directly apply mathematics they had learned in a previous lesson, without considering the specific circumstances of the real life situation, proved to be unsuccessful. Digital tools acted as a catalyst for progressing their attempts to solve the problem by providing feedback which indicated students' first single function conjectures were not consistent with the data. Further, because the technology included the capacity to plot multiple functions and so explore possible solutions, students were more easily able to employ their knowledge of different functions in finding a fit that involved a piece-wise approach. Thus, while digital tools were not integral to the challenge aspect of the task, technology was a vital resource deployed by the students in order to meet the challenge inherent in the task.

An essential part of this teacher's practice was the *continual improvement* of tasks over successive teaching cycles (typically revisiting tasks on a yearly basis). This was the first time the teacher had trialled this task but noted the tendency of his students to apply mathematical knowledge learned in the most recent lessons without considering the specific features of the plotted data. He saw that the task had provoked the need for students to consider additional functions and change their approach (for further detail see Geiger et al. 2010). The teacher indicated he intended to explore the possibility of designing similar features into other tasks (for other examples of such tasks see the materials developed as part of the MAACAS project http://www.qamt.org/maacas-project).

As outlined above, there is an inseparable interplay between the task and digital tools for some aspects of design in a manner consistent with documental genesis (Gueudet and Trouche 2009). This study, however, extends the work of Gueudet and Trouche (2009) from the general work of teachers to the specific activity of designing technology enhanced modelling tasks.

7.2 Divergence Between Two Perspectives on Task Design

The teacher created the task by drawing on "home grown" principles for developing effective technology active modelling tasks. These were mainly consistent with general principles of task design derived from research literature but addressed additional features in order to accommodate the demands of modelling tasks.

In the exemplar task, a national scientific body monitored the blue-green algae in the various river systems because of the related consequences for aquatic wildlife. Thus, this task represents a situation set in a life-related context consistent with the aim of achieving *authenticity and relevance*. This is an extension of the general principles of task design and accommodates an aspect that is the essence of mathematical modelling—its connection to real world situations and circumstances.

Different types of mathematics were necessary to explore the data (data representation, different forms of function) and so, students were expected to make *connections* to different types of mathematical knowledge. This is another aspect of task design that is important to modelling because the act of applying mathematics to the real world often requires the deployment of a range of mathematical knowledge. This is not necessarily the case in other types of mathematical tasks as these can have a focus on developing specific mathematical knowledge. The aspect of connectivity is also more fully realised through the use of digital tools. In this case, the available technology provided the option of viewing different types of mathematical representations (e.g., scatterplots and function graphs) on the screen at the same time, so enhancing the connection between these types of mathematical knowledge.

8 The Role of Digital Tools in Modelling Task Design

In the exemplar described earlier, the successful deployment of both sets of principles in designing tasks was dependant on the intersection of the potentialities of the task and available digital tools for a number of aspects of design. In implementing the task, the teacher anticipated how students would interpret the potentials of the task for learning and of the digital tool to act as a resource.

8.1 Transformation of the Task, Learning and Teaching

The relationship between student, teacher, task and digital tool represents a documental genesis (Gueudet and Trouche 2009) as each element within this genesis transforms the other in some way. The task is transformed, from the perspective of the students when they realise the need to make use of a piece-wise rather than a single function in order to model the data presented in the problem. This transformation occurs as a result of an attempt by the students to use a single function and receiving feedback via the digital device that this was an inappropriate model. The use of the digital tool changes from that of a device that provided a specific solution for students once they had made a decision on the general form of the function to model the data into a tool used to explore the data and eventually find a model that fitted the data to their level of satisfaction. Students' learning is also transformed during this same process as they realise the purpose of the task and the digital tool was not to implement prior learning in an automated fashion but to apply their knowledge and understanding in an original way by taking into account unfamiliar features of the data. The teacher had to transform his approach to the lesson when students took a path he had not anticipated—attempting to fit a single function to the data. He changed his approach by revising his orchestration of the lesson by utilising new resources at his disposal, in this case deploying the insight of the two students who had solved the problem. When the two students informed their classmates that an approach based on a single function regression was not appropriate and that the data was best represented by a piece-wise function led to class members revising their attempts at a solution and allowed for the expansion of their repertoire of function fitting skills.

8.2 Digital Tools as Enablers of Task Design Principles

From the perspective of instrumental genesis, nearly all of the teacher's principles of design required the use of digital tools as enablers of the task. The principle of *authenticity* and *relevance* required students to recognise the potential of the available digital tools to assist them in exploring and solving the problem described in the task from both purely mathematical and real world contexts. There was a necessary duality about the schemas of instrumented action required to accommodate the purely mathematical and contextual demands of the task as students needed to recognise that the real world context demanded the development of a piecewise rather than single function to model the production of CO2. Having decided that two functions were needed to model the data, a specific instrumentation of the digital tool was needed to find the most appropriate functions for each section of the piecewise function using a purely mathematical approach.

The *open-endedness* of the task placed students in a position where they were challenged to make choices among multiple potential solution pathways. Thus, students were required to make choices among existing schemas of instrumented action or to generate new schemas after recognising the potential of the digital tool for meeting the challenge defined by the task.

The principle of *connectivity* designed into this task required students to generate schemas of instrumented action that were inclusive of different types of mathematical content. The CAS active calculator students used while working with the task included the capacity to link statistical plots with the graphs of specific functions, and these functions could be developed using the regression facility of the calculator. Students needed to find ways of taking advantage of the capabilities when engaging with the demands of the task and pursuing a solution. This is a type of instrumental genesis in which the potential of an artifact is only realised through its instrumented action.

The task was designed to link the demands of the activity to students' previous learning as the separate functions required to build an appropriate piecewise function had been studied and applied to real world contexts in earlier classes although the use of multiple functions to model data had not been previously covered. Thus, the task was created to be *accessible* to students but, at the same time, required students to apply this previous learning in a more complex context one in which multiple functions were needed to model a phenomena rather than a single function—a genuine *challenge*. This meant that students' existing schemas of instrumented action required adaptation in order to accommodate a more complex scenario. By improvising and revising his approach to orchestrating students' learning the teacher promoted changes in students' schemas of instrumented action related to both the digital tool and also the task.

9 Conclusion

The episode included in this chapter demonstrates it is possible to design effective technology demanding modelling tasks, and so the approach offers direction for curriculum designers, teachers and teacher educators. Designing the modelling task itself appears to be largely consistent with general principles of mathematical task design although the teacher in this classroom vignette employed a number of additional principles specific to modelling. Further research is required into those elements of design for modelling tasks that differ from general principles of designing other mathematics tasks. The inclusion of digital tools did not emerge as a stand-alone element of the teacher's set of design principles; rather, technology acted as a vital enabler of a number of design principles. How digital tools can best enable the implementation of these aspects of design is another issue which requires further research. While the teacher had designed an engaging task based on his own principles, students took an approach that was not anticipated by their teacher. The teacher, however, was able to take advantage of students' original but inappropriate approaches, generating a dynamic learning environment where students' knowledge of using mathematics within real world contexts was transformed. This raises a challenge for teachers in how such triggers can be deliberately embedded in designed experiences in a way that provides space for the type of documental genesis described in this paper. This also indicates that further research is necessary to investigate how to take advantage of unanticipated events in a well-planned lesson.

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