Geodesy

36. Geodesy

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Continuous geodetic observations are fundamental to characterize changes in space and time that affect the Earth system. The advent of global navigation satellite systems (GNSSs), starting with the Global Positioning System (GPS) in the early 1980s, has significantly increased the range of geodetic applications and their precision. Significant improvements have progressively been made in the GNSS software packages developed by research institutes, leading to the determination of high-precision geodetic parameters and their temporal variations. The proliferation of dense GNSS networks (local, national, continental and global), composed of continuously observing stations, allows for a variety of geodetic and Earth science applications. Most areas of science, Earth observation, georeferencing applications, and society at large, today depend on being able to determine positions to millimeter-level precision. Point positions, to be meaningful and fully exploitable, have to be determined and expressed in a well-defined reference frame. All current global and regional reference frames rely on the availability of the international terrestrial reference frame (ITRF), which is the most accurate realization of the international terrestrial reference system (ITRS). One of the major modern achievements in geodesy today is the ability to determine highly precise global and regional terrestrial reference frames based on GNSS observations, fully connected to the ITRF. This chapter describes the use and applications of GNSS in geodesy, focusing on its role in the International Association of Geodesy's (IAG's) global geodetic observing system (GGOS) for monitoring our planet in space and time, GNSS-based reference frame implementation, Earth rotation and sea level monitoring.

36.1 GNSS and IAG's Global Geodetic Observing System

Geodesy is the science of the Earth's rotation, gravity and shape, including their evolution in time [36.1, 2]. These properties of the Earth change in time because the Earth is a dynamic system – it has a fluid, mobile atmosphere and oceans, a continually changing global distribution of ice, snow, and water, a fluid core that is undergoing some type of hydromagnetic motion, a mantle both thermally convecting and rebounding from the glacial loading of the last ice age, and mobile tectonic plates (Chap. 37). In addition, external forces due to the gravitational attraction of the Sun, Moon, and planets also act upon the Earth. These internal dynamical processes and external gravitational forces exert torques on the solid Earth, or displace its mass, thereby causing the Earth's rotation, gravity, and shape to change. Geodetic observing systems, including the space-geodetic techniques of very long baseline interferometry (VLBI), satellite laser ranging (SLR), global navigation satellite systems (GNSSs) like the US Global Positioning System (GPS), and the French Doppler orbitography and radio-positioning by integrated satellite (DORIS) system, provide the measurements of the Earth's rotation, gravity, and shape that are used to study the response of the Earth to these dynamical forces.

Observations of the Earth's variable rotation, gravity and shape also provide the basis for the realization of the reference systems that are required in order to assign coordinates to points and objects and thereby determine how those points and objects move in space and time (Fig. 36.1). The terrestrial reference frame (TRF) determined by geodetic measurements is the indispensable foundation for all sustainable Earth observations, in situ as well as airborne and spaceborne, and underpins all georeferenced data used by society. The TRF is therefore of fundamental importance to geodesy in particular, science in general, and society as a whole.

The global network of GNSS receivers is essential to determining the TRF. Of the different space-geodetic techniques, the GNSS network of observing stations is the densest. By colocating GNSS receivers with the stations of the other techniques it helps to integrate the separate technique-specific networks into one, integrated global observing system. GNSS also provides the means to access the TRF, allowing the absolute positions of GNSS receiver-equipped objects to be precisely given. Providing this ability to precisely position and navigate objects is one of the most important benefits of GNSS to science and society.

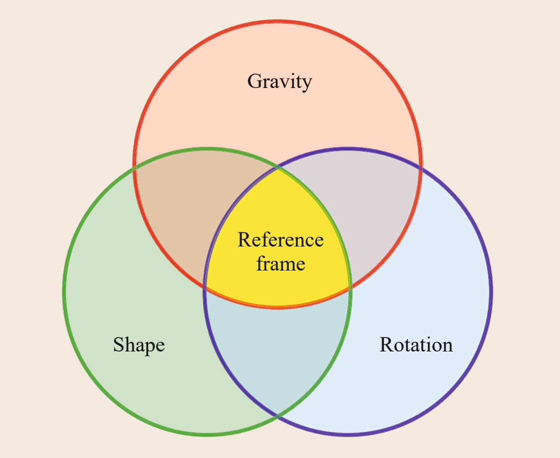

Fig. 36.1 Reference frames are determined from observations of the Earth's rotation, gravity, and shape. Reference frames also provide the means to integrate the three pillars of geodesy (rotation, gravity, and shape). These pillars are not independent of each other but are connected to each other by the common geophysical processes causing them to change. But to relate changes in the individual pillars to each other the changes must be given in the same reference frame (after [36.3, 4])

36.1.1 The International Association of Geodesy

The International Association of Geodesy (IAG), a founding association of the International Union of Geodesy and Geophysics (IUGG), is the international scientific organization devoted to the advancement of geodesy [36.5]. Its origin dates to 1862 when the Prussian General Johann Jacob Baeyer formed the central European arc measurement project with the ultimate goal of precisely determining the size and shape of the Earth. Today, more than 150 y later, the IAG continues to pursue this goal by advancing geodetic theory through research and teaching, by collecting, analyzing, modeling and interpreting observational data, by stimulating technological development, and by providing a consistent representation of the shape, rotation, and gravity of the Earth and planets including their temporal variations.

The IAG accomplishes its mission through the activities of its operating components, including its commissions, intercommission committees, services, and the global geodetic observing system (GGOS). Commissions represent the major fields of activity in geodesy and represent the IAG in all relevant scientific matters, promoting the advancement of science, technology, and international cooperation in these fields. The four IAG commissions are:

1. Reference frames
2. Gravity field
3. Earth rotation and geodynamics
4. Positioning and applications.

Intercommission committees address scientific matters that involve all of the commissions. There is currently one intercommission committee, the intercommission committee on theory.

Services organize the collection and reduction of geodetic observations and generate the geodetic products needed for scientific research and societal applications. The 14 IAG services span the relevant geometric, gravimetric, oceanographic, and related properties of the Earth. The geometric services of the IAG are the:

- International GNSS Service (IGS) (Chap. 33)
- International VLBI Service for Geodesy and Astrometry (IVS)
- International Laser Ranging Service (ILRS)
- International DORIS Service (IDS)
- International Earth Rotation and Reference Systems Service (IERS).

The gravimetric services of the IAG are the:

- International Gravity Field Service (IGFS)
- International Geoid Service (IGeS)
- International Gravimetric Bureau (BGI)
- International Center for Earth Tides (ICET)
- International Center for Global Earth Models (ICGEM)
- International Digital Elevation Model Service (IDEMS, to be confirmed).

The oceanographic services of the IAG are the:

- Permanent Service for Mean Sea Level (PSMSL)
- International Altimetry Service (IAS, to be confirmed).

The final service of the IAG, concerned with providing reference timescales, is the:

- Time Department of the International Bureau of Weights and Measures (BIPM).

36.1.2 The Global Geodetic Observing System

Recognizing the increasingly important role that geodesy plays in scientific research and societal applications, IAG established the global geodetic observing system (GGOS) in 2003, first as a project and then, in 2007, as a full component of the IAG. GGOS is meant to be *the* observing system of the IAG, organizing its technique-specific services under one unifying umbrella, thereby forming a comprehensive geodetic observing instrument integrating the hitherto separate pillars of geodesy (shape, rotation, and gravity) into one consistent observing system [36.6]. GGOS works with the other IAG components to provide unique, mutually consistent, and easily accessible geodetic constants, data and products for science and society. In addition, GGOS represents the IAG in the Group on Earth Observations (GEO) [36.7] and is IAG's contribution to the Global Earth Observation System of Systems (GEOSS) that is being constructed by GEO.

GGOS provides the basis on which future advances in geosciences can be built. By considering the Earth system as a whole (including the geosphere, hydrosphere, cryosphere, atmosphere and biosphere), monitoring Earth system components and their interactions by geodetic techniques and studying them from the geodetic point of view, the geodetic community provides the global geosciences community with a powerful tool consisting mainly of high-quality services, standards and references, and theoretical and observational innovations. The mission of GGOS is [36.5]:

1. *To provide the observations needed to monitor, map and understand changes in the Earth's shape, rotation and mass distribution*
2. *To provide the global frame of reference that is the fundamental backbone for measuring and consistently interpreting key global change processes and for many other scientific and societal applications*
3. *To benefit science and society by providing the foundation upon which advances in Earth and planetary system science and applications are built.*

The goals of GGOS are [36.5]:

1. *To be the primary source for all global geodetic information and expertise serving society and Earth system science*
2. *To actively promote, sustain, improve, and evolve the global geodetic infrastructure needed to meet Earth science and societal requirements*
3. *To coordinate the international geodetic services that are the main source of key parameters needed to realize a stable global frame of reference and to observe and study changes in the dynamic Earth system*
4. *To communicate and advocate the benefits of GGOS to user communities, policy makers, funding organizations, and society.*

In order to accomplish its mission and goals, GGOS depends upon the services, commissions, and intercommission committees of the IAG. The services provide the infrastructure, data and products on which all contributions of GGOS are based. The commissions and intercommission committees provide expertise and support for scientific development within GGOS. In summary, GGOS is IAG's central interface to the scientific community and to society in general.

Organizational Structure

The components of GGOS are shown in Fig. 36.2. The governing components of GGOS are its consortium, coordinating board, and executive committee. These components serve as its steering committee, setting the strategic direction for GGOS. The coordinating office, like central bureaus of IAG services, oversees and coordinates the day-to-day activities of GGOS. It serves as the secretariat of GGOS and manages GGOS web services and outreach activities. The science panel is an independent, multidisciplinary advisory board. It provides scientific advice and support to GGOS to ensure that GGOS remains focused on relevant scientific and societal needs. The GGOS interagency committee (GIAC) is a forum for coordinating and supporting the development, implementation, and operation of the geodetic infrastructure that is owned by governmental institutions. Membership in GIAC is open to any governmental organization that contributes resources to the operation and development of space-geodetic observing systems.

The Bureaus of GGOS

Along with the science panel and GIAC, the operating arms of GGOS are its bureaus and focus areas. GGOS currently has two bureaus: (1) The Bureau of Networks and Operations (2) The Bureau of Products and Standards.

Bureau of Networks and Observations. The goal of the Bureau of Networks and Operations (BNO) is to pursue the implementation of a network of space-geodetic observing systems of sufficient global distribution and capability that it will meet the needs of science and society as identified by GGOS [36.6]. To achieve this goal the Bureau works closely with the IAG services. In fact, the bureau is a consortium of service representatives supported by working groups, of which there are three:

1. *Working group on satellite missions* keeps GGOS informed about relevant satellite missions and supports GGOS in advocating for new missions that are needed to meet its goals.
2. *Working group on data and information systems* promotes the use of metadata standards and conventions for geodetic data and advocates for the interoperability of geodetic data centers.
3. *Working group on performance simulations and architectural trade-offs* uses simulations to assess the

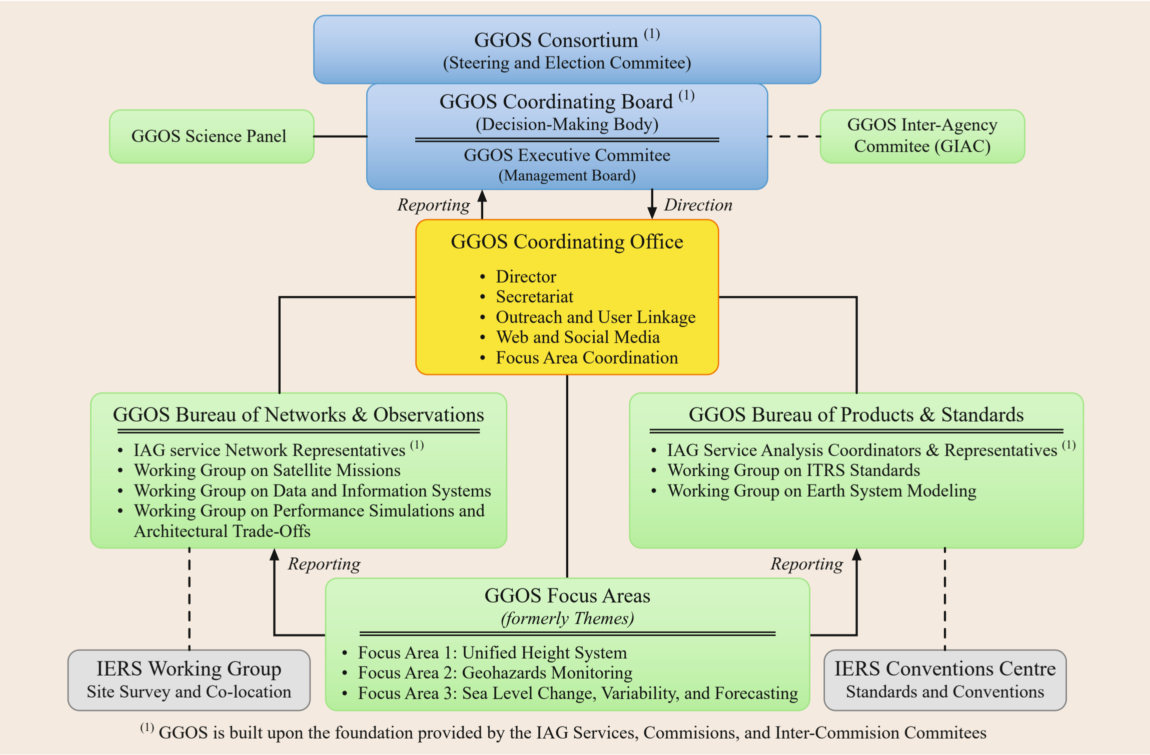

Fig. 36.2 The organizational structure of GGOS showing its governing components in *blue*, coordinating component in *yellow*, operating components in *green*, and affiliated components in *gray* (Courtesy of IAG-GGOS, reproduced under the CC BY-ND 4.0 license)

impact on geodetic data and products of different ground station architectures and their evolution, different space-based architectures and their evolution, and trade-offs between the ground- and space-based architectures including requirements on ground and space ties.

In pursuit of its goal, the BNO also works with the IERS working group on site survey and colocation. This working group is striving to improve the accuracy of the measurements of the relative positions of the reference points of colocated space-geodetic stations.

Bureau of Products and Standards. The goal of the Bureau of Products and Standards (BPS) is to make sure that the same standards and conventions are used by all components of the IAG. When combining data and products from different analysis centers or from different observing systems it is critical that they be determined using the same standards and conventions. Otherwise, inconsistencies can be introduced that can limit the accuracy of the combined data and products. As a first step towards meeting its goal, the BPS is compiling an inventory of the standards and conventions used by the organizations that generate IAG data and products. To help achieve its goal, the BPS has two working groups:

1. *Working group on ITRS standards* is pursuing the establishment of a new ISO (International Organization for Standardization) standard on global geodetic reference systems like the ITRS.
2. *Working group on Earth system modeling* is developing an integrated Earth system model that will apply to all observation techniques and all pillars of geodesy (rotation, gravity, and shape).

In pursuit of its goal, the BPS works closely with the IERS conventions center, that component of the IERS that is responsible for maintaining the constants, standards, and conventional models used by the IERS.

The Focus Areas of GGOS

GGOS focus areas are interdisciplinary in nature and address broad and critical issues that are important to science and society and that geodesy can contribute to but that need further development by or coordination within the geodetic community. GGOS currently has three focus areas:

1. Unified height system
2. Geohazards monitoring
3. Sea level change, variability and forecasting.

Unified Height System. The goal of a number of IAG working groups during the last few decades has been the unification of the more than 100 existing vertical reference systems. This involves defining and realizing a global reference level and determining the transformations between local height datums and the global, unified one. When this is achieved, all physical heights will be referred to the same global reference level. To aid in the realization of this goal, GGOS created a focus area, focus area 1, on the unified height system. To date, the activities of focus area 1 have been focused on determining a reliable value for the reference geopotential W_0 that can be used for the conventional reference level when realizing a global height system.

Geohazards Monitoring. Helping to mitigate the impact on human life and property of natural hazards such as earthquakes, volcanic eruptions, debris flows, landslides, land subsidence, tsunamis, floods, storm surges, hurricanes and extreme weather is one of the most important services that geodesy can provide to science and society. Since natural hazards often cause objects to be displaced and the Earth's surface to be deformed, GNSS plays a crucial role in this. For example, GNSS can be used to monitor the pre-eruptive deformation of volcanoes and the preseismic deformation of earthquake fault zones, aiding in the issuance of volcanic eruption and earthquake warnings. GNSS can also be used to rapidly estimate earthquake fault motion, aiding in the modeling of tsunami genesis and the issuance of tsunami warnings. GNSS observations are essential for understanding the processes causing the hazard, for assessing the risks of the hazard, for monitoring the development of the hazard, for deciding whether or not to issue an early warning, and to support rescue and damage assessment activities.

Recognizing the important role that geodetic observations play in disaster prevention and mitigation, GGOS created a focus area, focus area 2, on geohazards monitoring. The objective of focus area 2 is to improve the effectiveness of the geodetic community in supporting natural hazard identification, assessment, prioritization, prediction, and early warning. As an international organization, GGOS can be a very effective advocate for the role of geodesy in understanding and mitigating natural hazards. GGOS can also be an effective advocate for improving the geodetic data needed for natural hazards research including better spatial coverage, higher sampling rate, lower latency, and wider data availability, particularly of synthetic aperture radar (SAR) and GNSS data.

Sea Level Change, Variability and Forecasting. In 1990, 23% of the world's population lived both less than 100 km from the coast and less than 100 m above sea level. Nearly a quarter of the world's population is therefore vulnerable to the effects of a rising sea level. Since the long-term average rate of sea level rise is only a few mm/y, mitigation efforts can be planned well in advance. But great demands are placed on geodetic observing systems because the sea level rise signal is so small. For example, the terrestrial reference frame, which should be at least an order of magnitude more accurate than the amplitude of the signal being measured, needs to be accurate and stable to within about 0.1 mm/y to support studies of sea level change. This makes sea level change studies one of the most demanding applications of geodetic observing systems.

Recognizing the important role that geodetic observations play in sea level change studies, GGOS created a focus area, focus area 3, on sea level change, variability, and forecasting. The objective of focus area 3 is to improve our understanding of the causes and consequences of sea level change through the application of geodetic measurements.

GGOS and Reference Frames

As discussed above, GGOS is built upon the foundation provided by the IAG services, commissions, and intercommission committees. The IAG services coordinate the acquisition and analysis of geodetic observations of the Earth's time varying gravity, rotation, and shape (Fig. 36.1). An important goal of GGOS is to advocate for the improvement of the global geodetic infrastructure, including the GNSS infrastructure, that provides the geodetic observations. One of the most scientifically and societally important applications of geodetic observations is their use for determining reference frames. This chapter of the GNSS Handbook discusses the use of geodetic observations, especially GNSS observations, to determine reference frames and to study changes in the rotation of the Earth. The use of geodetic observations to study changes in the shape of the Earth are discussed in the chapters in the Handbook concerned with precise positioning (Chap. 25), generation of orbit products (Chap. 34) and geodynamics (Chap. 37).

36.2 Global and Regional Reference Frames

This section is divided in four parts. The first part introduces the types of reference frame representations for a deformable Earth, taking into account all sorts of linear and nonlinear motions. The second part deals with global reference frames, focusing on the International Terrestrial Reference Frame (ITRF), its derivatives formed by the International GNSS Service (IGS) and the IGS contribution to the ITRF construction, describing the fundamental role of the IGS network in connecting the three other techniques: very long baseline interferometry (VLBI), satellite laser ranging (SLR) and Doppler orbitography radiopositioning integrated by satellite (DORIS). The third part details the GNSS-based global and regional reference frames and how these frames are linked to the global ITRF, through the usage of IGS products. The fourth part gives general guidelines on how to realize GNSS-based local, regional and global reference frames, fully consistent with and optimally aligned to the ITRF.

36.2.1 Reference Frame Representations for the Deformable Earth

The Earth is a complex dynamic system that undergoes deformations caused by various geophysical processes that should be taken into account when constructing a reference frame. The frame is implemented through a geodetic network anchored to the Earth's crust and therefore can be called crust-based frame. The expression of the instantaneous station position $\boldsymbol{X}(t)$, at epoch t, can be written as the sum of its regularized position $\boldsymbol{X}_{\mathrm{R}}(t)$ and high frequency geophysical variations $\Delta\boldsymbol{X}_i(t)$ (see IERS conventions [36.8], Chap. 4)

$$\boldsymbol{X}(t) = \boldsymbol{X}_{\mathrm{R}}(t) + \sum_i \Delta\boldsymbol{X}_i(t) , \qquad (36.1)$$

and

$$\boldsymbol{X}_{\mathrm{R}}(t) = \boldsymbol{X}_{\mathrm{R}}(t_0) + \dot{\boldsymbol{X}}_{\mathrm{R}}(t - t_0) , \qquad (36.2)$$

where t_0 is the reference epoch of the station position and $\dot{\boldsymbol{X}}_{\mathrm{R}}$ its linear velocity. $\boldsymbol{X}_{\mathrm{R}}(t)$ is introduced here in order to obtain a position with more regular time variation, after removing high-frequency time variations caused by geophysical processes using conventional corrections $\Delta\boldsymbol{X}_i(t)$.

Chapter 7 of the IERS conventions [36.8] provides a full description of the currently agreed-upon conventional models that go into $\Delta\boldsymbol{X}_i(t)$, such as Earth tide, ocean loading, atmospheric pressure, and so on, and used by the analysis centers (ACs) dealing with space geodesy data. In addition to these conventional recommended models, other geophysical phenomena have a large impact on space geodesy observations and therefore need to be taken into account in reference frame implementation. We can categorize the resulting deformations of such phenomena at the surface of the deformable Earth into the following two types:

- Nearly linear motions that can be expressed in the mathematical geodesy formulation as constant with time. They are caused by two main types of processes: plate tectonics and glacial isostatic adjustment (GIA). Plate tectonics induce mainly horizontal motion, which is traditionally modeled via a rotation pole for each plate involving horizontal velocity components [36.9], while GIA implies horizontal and vertical deformations.
- Nonlinear motions that include periodic signals (e.g., annual, semi-annual or interannual) that are caused by nontidal loading effects due to the atmosphere, ocean circulation, terrestrial hydrology and ice melting [36.9]; ruptures provoked by earthquakes and volcanic eruptions; and slow transients or postseismic deformations.

Taking into account the above two types of motions, two categories of reference frame representations can be introduced: quasi-instantaneous reference frame and long-term or secular reference frame.

Quasi-Instantaneous Reference Frames

A quasi-instantaneous frame gives access to average station positions, using short timespan of space geodesy observations: commonly one day, and up to one week. In this case station positions are only valid at the central epoch of the observations used. More than one week of observations could of course mathematically be used, but the resulting averaged station positions would then be biased by tectonic motion effects. Long time series of quasi-instantaneous frame solutions naturally contain all types of linear and nonlinear station motions. The analysis and accumulation (rigorous stacking) of long time series of quasi-instantaneous frame solutions permit not only the study of all types of linear and nonlinear station motions they naturally contain, but also the construction of a long-term secular frame, such as the ITRF.

Examples of quasi-instantaneous reference frames are daily or weekly solutions provided by the analysis and combination centers of the IAG services of the four space geodesy techniques. In the particular case of GNSS, in addition to IGS global solutions, local, national and regional solutions are also produced by research groups for scientific studies and by institutions in charge of the maintenance of national reference frames based on GNSS permanent networks. In general, IGS analysis center daily or weekly solutions are generated by estimating not only station positions and EOPs, but also orbits, clocks and eventually other parameters such as troposphere gradients. Local, national and regional solutions are generally computed by fixing IGS products (orbits, clocks and EOPs) where the main target is the estimation of station positions, using either a network approach (all stations are adjusted together) or precise point positioning approach, on a station-by-station basis. General guidelines are provided in Sect. 36.2.4 on how to express or align a GNSS solution into the ITRF.

Long-Term Secular Reference Frames

A long-term or secular frame gives access to station positions at a given epoch t_0 and station linear velocities. Examples of long-term reference frames are the ITRF (Sect. 36.2.2) and a cumulative solution obtained by stacking time series of quasi-instantaneous reference frames. The choice of t_0 does not mathematically matter, but should be selected to be close to the central epoch of the stacked time series. The users can actually propagate station positions and their associated variances from the reference epoch t_0 to any other epoch t. For a given station with position vector $\boldsymbol{X}(t_0)$ at epoch t_0 and velocity vector $\dot{\boldsymbol{X}}$, its position $\boldsymbol{X}(t)$, at epoch t is given by

$$\boldsymbol{X}(t) = \boldsymbol{X}(t_0) + \dot{\boldsymbol{X}}(t - t_0)\,, \qquad (36.3)$$

and the variance propagation law gives its variance at epoch t as

$$\begin{aligned}\operatorname{var}(\boldsymbol{X}(t)) = {} & \operatorname{var}(\boldsymbol{X}(t_0)) + 2(t - t_0)\operatorname{cov}(\boldsymbol{X}, \dot{\boldsymbol{X}}) \\ & + (t - t_0)^2 \operatorname{var}(\dot{\boldsymbol{X}})\,.\end{aligned} \qquad (36.4)$$

The stacking of time series of quasi-instantaneous reference frame solutions is usually operated using (36.3) – a type of equation where the unknowns are station positions $\boldsymbol{X}(t_0)$, and station velocities $\dot{\boldsymbol{X}}$. Equation (36.3) could also be generalized to include transformation parameters in order to account for possible reference frame differences between individual quasi-instantaneous reference frames themselves (which might not have the same origin, scale and/or orientation) and with respect to the stacked/combined long-term solution. Minimum constraints equations as described in Sect. 36.2.4 could also be added to the stacking model ((36.3)-type) in order to express a cumulative GNSS long-term solution in the ITRF.

Geocenter Motion and Periodic Signals

A GNSS satellite, as any satellite, is theoretically orbiting around the center of gravity, or the center of mass (CM) of the total Earth system. Therefore in theory, the instantaneous CM position reflects the natural origin of the inertial frame in which the satellite orbit is expressed. Analysis of satellite geodesy data have clearly indicated for about two decades that the network of stations attached to the Earth's crust and materializing the reference frame has detectable translational motion with respect to CM, known as the geocenter motion [36.10]. This motion is often defined as the motion of the CM with respect to the center of figure (CF) of the solid Earth surface [36.11] and is believed to be the crust response to various geophysical fluid displacements within the Earth system, such as the atmosphere, oceans, terrestrial hydrology and ice sheets. It is assumed to include tidal, nontidal and secular components. The tidal parts of the geocenter motion induced by the atmosphere and oceans, with an amplitude that may reach up to 1 cm, are included in the models recommended by the IERS conventions [36.8] to be taken into account a priori in the station displacement of space geodesy techniques. The nontidal part of the geocenter motion, with an amplitude of a few mm, manifests itself in the form of periodic signals: annual, semi- and interannual, and is quantified through data analysis of time series of station positions determined by space geodesy techniques, or through external geophysical models. The secular part, often called the geocenter velocity, is believed to be less than 1 mm/y [36.12]. The detailed review by [36.12] is the most extensive article describing the theory of the geocenter motion and its geophysical implications, as well as its quantification over different timescales.

There are basically three main methods for estimating the nonlinear geocenter motion components:

1. The translational
2. The degree-1 load-induced deformation
3. The inverse approaches.

The translational approach consists in estimating the three equatorial components of the CF, which is in fact approximated by the barycenter of the implied geodetic network, often called the center of network (CN), with respect to CM. The translational approach is called a kinematic approach when the degree-1 coefficients of the gravity field are estimated, which are proportional to the geocenter motion components [36.13]. It is also called a network shift approach when the seven- or six-parameter similarity (Helmert) transformation formula (see for instance (36.7)) is used to infer the three translation components between a time series of quasi-instantaneous frames and a secular long-term frame such as the ITRF. Indeed, the ITRF origin is defined by the long-term average of the SLR CM realization. Fitting a sine and/or a cosine function yields in fact the amplitude and phase of annual and/or semi-annual signals of the geocenter motion.

Although the SLR technique suffers from its poor spatiotemporal network, leading to the so-called *network effect* when using the translational approach [36.14], it is the most precise space geodetic technique for the geocenter nonlinear motion estimation.

The estimability of the geocenter motion by GNSS via the kinematic or network shift approaches faces intrinsic complications due to an inherent coupling of the GNSS orbit dynamic parameters; [36.15, 16] showed that the GNSS geocenter Z-component is strongly correlated to a particular parameter of the solar radiation pressure. In [36.17] it was demonstrated, via a collinearity diagnosis formalism, that the inability of GNSS, as opposed to SLR, to properly sense the location of the geocenter CM is mostly explained by the estimation, in the GNSS case, of epoch-wise station and satellite clock offsets simultaneously with tropospheric parameters.

The degree-1 approach was first introduced in [36.18], using the spherical harmonics formalism, to infer not only the translational geocenter motion, but also the accompanying load-induced crust deformation using GNSS (GPS) time series of quasi-instantaneous frames of a globally distributed network of stations. They in fact demonstrated that the translational components of the geocenter motion are functions of degree-1 coefficients of the associated load deformation. It was then shown in [36.19] that the truncated higher-degree terms of the harmonic expansion of the load-induced deformation alias significantly into the degree-1 terms, and therefore higher-degree terms, up to 50, must be included in the estimation, leading so to the so-called inverse approach. Many authors (as referenced in [36.11]) have expanded and improved the inverse approach and its application, using not only GNSS(GPS) data that suffer from the network sparseness in ocean areas, but also data-assimilated ocean bottom pressure (OBP) models as well as GRACE gravity data.

In addition to geophysical (load-induced) periodic signals that explain about half of the observed GNSS seasonal power, other signals are also frequent and detected in the GNSS residuals of station position time series, such as the GPS draconitic errors [36.20]. The GPS draconitic year (of 351.2 d), is the period for the GPS orbit constellation to repeat its orientation with respect to the Sun. Harmonic signals of this period have actually been observed in the power spectra of nearly all IGS products [36.21].

36.2.2 Global Terrestrial Reference Frames

Following the terminology adopted by the geodetic community since the advent of space geodesy, we distinguish between a terrestrial reference system (TRS) and a terrestrial reference frame (TRF). While the former has a mathematical and physical foundations for its definition and properties, the latter represents its numerical realization constructed upon space geodesy observations (hence with uncertainties) and is accessible to the users through numerical values (e.g., positions as a function of time of a network of Earth crust-based points). The main physical and mathematical properties of a TRS (at the theoretical level) or of a TRF (at the realization level) are the origin, the scale, the orientation and their time evolution. The latter is usually expressed through rates (time variations) of translations (origin components), scale and rotations (orientation parameters).

While the origin and the scale (having physical properties) are the most critical parameters of interest to Earth science applications, the orientation and its time variation are of least consequence because they are arbitrarily and conventionally defined. In fact adopting a given orientation of the three axes of the reference system is a matter of conventions and convenience and would not change the relative shape of the implied geodetic network used to create the reference system. Continuous and long-term space geodesy observations are crucial for realizing a TRS that is able to precisely characterize and model Earth surface movements, such as tectonic plate motion. In the absence of technique-specific systematic errors, and if all geophysical processes are accurately accounted for in geodetic analysis, TRF origin and scale should be stable over time, i. e., should not exhibit any drift or discontinuities over the entire timespan of the implied geodetic observations.

None of the space geodesy techniques is able to provide all the necessary parameters for the TRF definition (origin, scale and orientation). While satellite techniques are sensitive to the Earth center of mass (a natural TRF origin; the point around which a satellite orbits), VLBI (whose TRF origin is arbitrarily defined through some mathematical constraints) is not. The scale is dependent on the modeling of some physical parameters, and the absolute TRF orientation (unobservable by any technique) is arbitrarily or conventionally defined through specific constraints. The utility of multitechnique combinations is therefore recognized for reference frame determination, and in particular for accurate reference definition. In principle, the particular strengths of one observing method can compensate for weaknesses in others if the combination is properly constructed, suitable weights are found, and accurate local ties in colocation sites are available.

The key element of a multitechnique combined frame, as used for the ITRF, is the availability of a sufficient number of globally distributed colocation sites. A colocation site is defined by the requirement that two or more space geodetic distinct instruments are operating at the same location or at locations very close to one another, which are very precisely surveyed in three dimensions, using geodetic classical surveys or the GPS technique. Classical surveys are usually direction angles, distances, and spirit leveling measurements between instrument reference points or geodetic markers. Adjustments by least squares of local surveys are generally performed by national geodetic agencies operating space geodesy instruments, yielding differential coordinates (local ties) connecting the colocated instrument reference points.

Figure 36.3 shows the four-technique colocation site at Yarragadee (western Australia), with the modern 12 m VLBI radio-telescope that started its operation in 2011, the National Aeronautics and Space Administration (NASA) SLR MOBLAS 5 system, the DORIS beacon, the GNSS pillars (called YARR, YAR2, YAR3) and the Gravimeter hut for campaign gravimetry measurements.

Intermarker distance and accuracy of the local tie are the two main criteria that must be considered for the definition of a colocation site [36.22]. Given the need for local tie vectors to be precise at the 1 mm level, and considering the increase in atmospheric refraction as a function of increased station separation, the distances between geodetic markers at colocation sites should not exceed 1 km. In addition, repeated surveys of the marker footprint are necessary for long-term local tie stability. The typical uncertainty of the local ties used for the ITRF is 2–5 mm (sometimes larger than 5 mm for the less precise ties). From the ITRF experience,

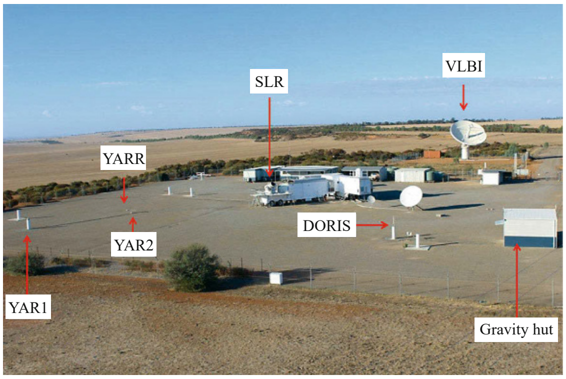

Fig. 36.3 Yarragadee (western Australia) four-technique colocation site (courtesy of Geoscience Australia)

discrepancies between local ties and space geodesy estimates are frequent as discussed in the following section. However, discrepancies mean that either local ties or space geodesy estimates (or both) are imprecise or in error. One of the major local tie limitations is in fact to precisely determine the eccentricity between the external physical reference point used by the surveyors and the point referenced by space geodesy data analysts, for example the intersection of axes of VLBI or SLR telescopes, the DORIS beacon or the electrical GNSS antenna phase center [36.23]. The estimated uncertainty for each internal instrument offset is probably not better than 2 mm, and consequently, the overall local tie error would be at best 3 mm per component.

The International Terrestrial Reference System (ITRS) and Frame (ITRF)

The international terrestrial reference system (ITRS) was developed by the geodetic community for the most demanding scientific applications under the auspices of the IERS. Following the IERS conventions and its updates [36.8, Ch. 4], the ITRS definition fulfills the following conditions:

1. It is geocentric, its origin being the center of mass for the whole Earth, including oceans and atmosphere.
2. The unit of length is the meter (SI). The scale is consistent with the geocentric coordinate time (TCG) for a geocentric local frame, in agreement with International Astronomical Union (IAU) and IUGG (1991) resolutions. This is obtained by appropriate relativistic modeling.
3. Its orientation was initially given by the Bureau International de l'Heure (BIH) orientation at 1984.0.
4. The time evolution of the orientation is ensured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth.

The most accurate realizations of the ITRS is called the international terrestrial reference frame (ITRF). The implementation of the ITRF is fundamentally based on the rigorous combination of geodetic products of the main space geodetic techniques (GNSS, VLBI, SLR, and DORIS), through their colocated measuring instruments at a certain number of core sites. The ITRF combination model is based on the linearized form of the general similarity transformation formula, as it will be detailed below.

There is no single ITRF, but rather a series of updated and improved versions of ITRF. The versions are identified by the year associated with the date of last data used in the analysis, and should not be confused with the date of applicability. The most recent versions are ITRF97, ITRF2000, ITRF2005 and the ITRF2008 [36.24–26]. Generally, as time progresses, there is less need for frequent updates, because more time may be needed to make significant improvements through the addition of new data and improved models. However, to satisfy increasing accuracy requirements, the ITRF will continue to be updated to incorporate more advanced models for the time-dependent reference coordinates. Since the tracking network equipped with the instruments of those techniques is evolving and the period of data available increases with time, the ITRF is constantly being updated.

For more than ten years, initiated first by the IGS, analysis centers of the three other space geodesy techniques (VLBI, SLR, DORIS) have made available time series of station positions and Earth orientation parameters (EOPs) in SINEX (software independent exchange) format [36.27]. The power of times series of station positions, allowing the control not only of the station behavior and in particular to monitor nonlinear motion, but also the frame physical parameters (origin and scale), led the ITRF center to consider them as input for the ITRF generation, starting with the ITRF2005 [36.25]. In addition to station positions and velocities, ITRF2005 and ITRF2008 [36.26] integrate also consistent daily EOPs. The latter was already used by the IERS EOP center in order to improve the consistency of the IERS operational series of EOPs with the ITRF [36.28]. Up to the ITRF2008, the ITRF input time series solutions are provided on a weekly basis by the IAG international services of satellite techniques: the IGS [36.29], the ILRS [36.30] and the IDS [36.31], and on a daily (VLBI session-wise) basis by the IVS [36.32]. Each per-technique time series is already a combination of the individual analysis center solutions of that technique. As an example, the GNSS (mainly GPS) submitted solution to the ITRF2008 is a combination of the first reprocessed solutions by the IGS analysis centers and covers the time period 1997.0-2009.5 [36.33]. Note that a very small portion of Global'naya Navigatsionnaya Sputnikova Sistema (GLONASS) observations were used by some IGS ACs that contributed to the reprocessing effort. Starting on 19 August 2012, the IGS switched to daily integration and therefore daily IGS SINEX files will be used in the future ITRF solutions.

The procedure adopted for the ITRF formation involves two steps [36.25, 26, 34]:

1. Stacking the individual time series to estimate a long-term solution per technique comprising station positions at a reference epoch, station velocities and daily EOPs

2. Combining the resulting long-term solutions of the four techniques together with the local ties in colocation sites.

The main two equations of the combination model are given below. They involve a 14-parameter similarity transformation, station positions and velocities, and EOPs and are written as

$$\begin{cases} \boldsymbol{X}_s^i & = \boldsymbol{X}_c^i + (t_s^i - t_0)\dot{\boldsymbol{X}}_c^i \\ & +\boldsymbol{T}_k + D_k\boldsymbol{X}_c^i + \mathbf{R}_k\boldsymbol{X}_c^i \\ & +(t_s^i - t_k)\left[\dot{\boldsymbol{T}}_k + \dot{D}_k\boldsymbol{X}_c^i + \dot{\mathbf{R}}_k\boldsymbol{X}_c^i\right] \\ \dot{\boldsymbol{X}}_s^i & = \dot{\boldsymbol{X}}_c^i + \dot{\boldsymbol{T}}_k + \dot{D}_k\boldsymbol{X}_c^i + \dot{\mathbf{R}}_k\boldsymbol{X}_c^i \,, \end{cases} \tag{36.5}$$

$$\begin{cases} x_s^p & = x_c^p + R_{yk} \\ y_s^p & = y_c^p + R_{xk} \\ UT_s & = UT_c - \frac{1}{f}R_{zk} \\ \dot{x}_s^p & = \dot{x}_c^p \\ \dot{y}_s^p & = \dot{y}_c^p \\ \mathrm{LOD}_s & = \mathrm{LOD}_c \,, \end{cases} \tag{36.6}$$

where for each point i, $\boldsymbol{X}_s^i$ (at epoch t_s^i) and $\dot{\boldsymbol{X}}_s^i$ are positions and velocities of technique solution s and $\boldsymbol{X}_c^i$ (at epoch t_0) and $\dot{\boldsymbol{X}}_c^i$ are those of the combined solution c. For each individual frame k, as implicitly defined by solution s, D_k is the scale factor, $\boldsymbol{T}_k$ the translation vector and $\mathbf{R}_k$ the rotation matrix. The dotted parameters designate their derivatives with respect to time. The translation vector $\boldsymbol{T}_k$ is composed of three origin components, namely T_x, T_y, T_z, and the rotation matrix of three small rotation parameters: R_x, R_y, R_z, following the three axes, respectively x, y, z. t_k is a conventionally selected epoch of the seven transformation parameters.

In addition to (36.5) involving station positions (and velocities), the EOPs are added by (36.6), making use of pole coordinates x_s^p, y_s^p and universal time UT_s as well as their daily rates $\dot{x}_s^p$, $\dot{y}_s^p$ and length-of-day LOD_s, where $f = 1.002737909350795$ is the conversion factor from universal time (UT) into sidereal time. The link between the combined frame and the EOPs is ensured via the three rotation parameters appearing in the first three lines of (36.6).

Note that (36.5) uses the linearized form of the general similarity transformation formula, neglecting second- and higher-order terms [36.8, 35].

In the first step of the ITRF construction, the first two lines of (36.5) and the entire (36.6) are used to estimate long-term solutions for each technique, by accumulating (rigorously stacking) the individual technique time series of station positions and EOPs. In the second step, the entire two equations are used to combine the long-term solutions obtained in step 1, together with local ties in colocation sites.

The number of colocation sites has evolved with time since the start of the ITRF combination activities in 1984, due to the decommission of certain historical sites for multiple reasons, and the appearance of a few new colocation sites. Figure 36.4 illustrates the distribution of the total number of VLBI, SLR and DORIS operating sites in 2015, as well as the IGS/GNSS colocated sites: IVS is managing a network of 49 radio telescopes located in 46 sites, ILRS 38 laser telescopes in 37 sites, IDS 53 beacons in 53 sites, and IGS more than 400 GNSS permanent, continuously operating receivers/antennas. All in all there are 90 colocated sites: 30 GNSS-SLR, 38 GNSS-VLBI and 43 GNSS-DORIS colocations. There are only 11 sites where VLBI and SLR are colocated, nine in the northern and only two in

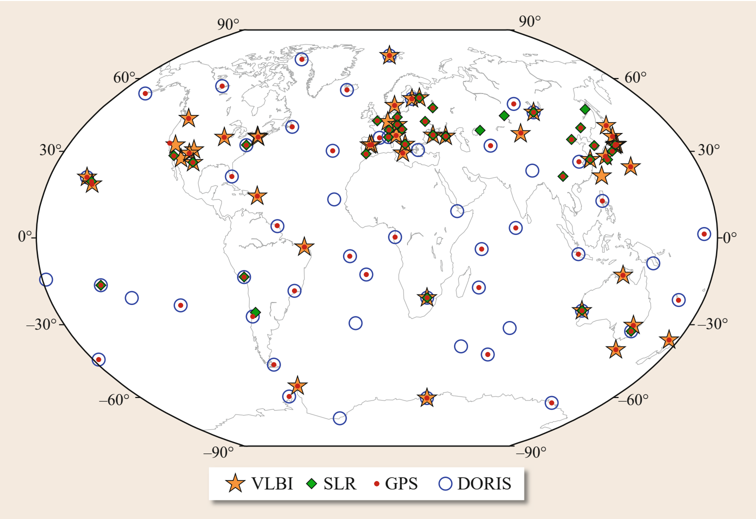

Fig. 36.4 VLBI, SLR and DORIS sites and their colocations with GPS

the southern hemisphere. Unfortunately more than half of the VLBI and SLR instruments are old generation systems. As a consequence, the improvement of the underlying geodetic infrastructure of ITRF is an important goal of GGOS [36.6], discussed in the previous section. The low number, the nonoptimal coverage, and the low performance of some of the 11 VLBI-SLR colocation sites are significant limiting factors to ensuring a precise connection between these two techniques in the ITRF implementation. In fact, GNSS is playing a major role in connecting the three other techniques, given the fact that almost all SLR and VLBI sites, as well as 43 DORIS sites are colocated with permanent IGS stations. The drawback of this situation is that if there is any GNSS-related bias, this will contaminate the parameters that define the ITRF, primarily the origin and the scale that are determined by SLR and VLBI. There most probably are other technique-specific errors related to the mismodeling of the instrumental measurement reference points, not only for GPS [36.23], but also for the other techniques. Indeed, based on ITRF2008 results [36.26], tie discrepancies for 47, 43 and 34% of the total local tie vectors between GPS-VLBI, GPS-SLR and GPS-DORIS respectively are larger than 6 mm, corresponding to the level of scale agreement between VLBI and SLR solutions included in the ITRF2008 adjustment.

IGS Reference Frames and Their Relationship with the ITRF

The IGS products were integrated in the IERS combined products in 1992 and have contributed since then to the ITRF starting with ITRF91 [36.36]. All the IGS products are expressed in and are consistent with the ITRF frames. At the inception of its activities, the IGS used directly the ITRF frames to be the underlying frame of its products [36.37–39]. Following the methodology of [36.38, 40], the IGS started in 2000 to form its own, internally more consistent GPS-only frame, but still inheriting the ITRF definition in terms of origin, scale and orientation [36.41]. A more detailed history of IGS reference frame realizations can be found in [36.42, 43]. Starting with GPS week 1400 (5 November 2006), the IGS switched from relative to absolute model corrections to account for antenna phase center variations (PCV) [36.44]. At the same time, the IGS adopted directly the ITRF2005 [36.25] to form its specific frame called IGS05, composed of about 100 sites whose coordinates were corrected to account for relative to absolute PCV differences. In order to preserve the ITRF2005 origin, scale and orientation, the IGS05 was aligned to the ITRF2005 using 14-parameter similarity transformation [36.33]. In reality, among the 14 parameters, only the scale factor was significant, representing the mean of the height relative to absolute differences over the IGS05 stations. On 17 April 2011, the IGS generated and adopted the IGS08 frame [36.45], derived from ITRF2008. The IGS08 is composed of positions and velocities of a reference set of 232 stable GNSS stations extracted from ITRF2008, where corrections were applied to 65 stations to ITRF2008 positions in order to comply with the antenna calibration models used in present-day GNSS data analysis [36.23] (igs08.atx, in use since GPS week 1632). On 17 October 2012, the IGS updated the IGS08, called IGb08 [36.45], by adding about 36 stations in replacement of some decommissioned or dormant IGS08 stations. It should be noted that ITRF2008, IGS08 and IGb08 are however equivalent at the global level (sharing the same underlying origin, scale and orientation), although station-dependent position differences can exist.

36.2.3 GNSS-Based Reference Frames and Their Relationship with the ITRF

This section deals with reference frames that are built using GNSS data only, but are nominally aligned with the ITRF in origin, scale and orientation. The first subsection presents the GNSS-specific frames that are implemented by the different GNSS providers and in which the broadcast orbits are expressed. The second subsection discusses the regional reference frames that are also based on GNSS data only, while aligned to the ITRF via common processed stations. The third subsection develops general and mathematical guidelines on how to optimally align global or regional frames to the ITRF using IGS products.

GNSS-Specific Reference Frames

In order to ensure the integrity of any GNSS system and to precisely determine satellite orbits of its constellation, a specific reference frame has to be defined and maintained over time. The computed orbits are then transmitted to the users via the GNSS navigation message that allow determination of the user location, which will be expressed in the reference frame of that of the used orbits. The GNSS systems and frames in existence with publicly available information and publications are WGS84 for GPS, whose newest realization is designated as G1674 [36.46], PZ-90 for GLONASS whose latest realization is PS-90.11 [36.47] introduced in early 2014, CGCS2000 for COMPASS [36.48], the Galileo terrestrial reference frame (GTRF) for Galileo where the first series of its realization is described in [36.49], the newest one being designated as GTRF14v01, and the Japanese geodetic system (JGS) for quasi-zenith satellite system (QZSS), which is believed to be consistent with or close to the newest Japanese geodetic datum 2011 (JGD2011) that was revised after the 2011 Tohoku Earthquake [36.50].

In an effort to ensure the interoperability of timing and geodetic references among the different GNSSs, working group D of the International Committee on GNSS (ICG) is actively interacting with the GNSS providers toward a more rigorous and accurate alignment of the GNSSs to a common time reference and to the ITRF. To our best knowledge, all recent and up-to-date realizations of all GNSS-specific geodetic reference systems are believed to be aligned to ITRF2008. However, almost all these realizations, except the GTRF series, are based on GNSS data with short timespans, most often a few days or one week of observations. While the GTRF series are aligned to the current ITRF version at the few millimeter level in both positions and velocities, the other GNSS-specific frames are obtained via the adjustment of station positions at the central epoch of the observations used, with no velocity estimates to account for time variations. Depending on the selected reference epoch of the adjusted positions of the control stations, the impact of, for example, tectonic motion will be at the few centimeter per year level. If we consider a scenario of 10 y before a new update of the control station positions is made, with no plate motion model applied, a 20–70 cm position error will be accumulated and mapped into the computed orbits, depending on the station locations. Consequently, we believe that the current realizations of GNSS-specific frames agree to each other and with the ITRF2008 at the few decimeter level. However, this level of agreement is certainly well below the inherent and typical uncertainty of the broadcast orbits. Over a ten-month period, [36.51] analyzed signal-in-space ranging errors (SISREs) for all current GNSS systems and showed that the global average SISRE values amount to 0.7 m (GPS), 1.5 m (BeiDou), 1.6 m (Galileo), 1.9 m (GLONASS), and 0.6 m (QZSS). As a consequence, the position of a real-time user, with single or multi-GNSS capabilities, is at the level of 1–2 m accuracy.

A way forward to improving the consistency of GNSS-specific frames at the few millimeter level is to follow the GTRF example, or to disclose data to the IGS of a subset of stations used in the ground segment, as it was the case of 11 stations of the National Geospatial Intelligence Agency (NGA) for the US Department of Defense, which were included in the ITRF2008 [36.26, 46]. The remaining challenge rests, however, in improving the intrinsic accuracy of the broadcast orbits of all GNSS constellations.

Regional and National Terrestrial Reference Frames

Since the start of the ITRF development, together with the advent of GNSS positioning performance, significant effort was and is still undertaken by national mapping agencies to redefine and modernize continental and national geodetic systems, so that they are compatible with the global ITRF.

The structure of IAG commission 1 (reference frames) includes a subcommission 1.3 dealing with the definitions and realizations of regional reference frames and their connection to the global ITRF. The commission offers a home for service-like activities addressing theoretical and technical key common issues of interest to regional organizations. Six regional organizations are part of IAG subcommission 1.3, distributed to cover all continents (AFREF for Africa, NAREF and SIRGAS for North and South Americas, EUREF for Europe, APREF for Asia and Pacific, and SCAR for Antarctica).

Regional reference systems and frames are defined with respect to the ITRS/ITRF, realized and maintained by the IAG regional entities; the best known and advanced ones are ETRS89 for Europe, NAD83 for North America, and SIRGAS for South America. These regional entities usually play a major role in redefining regional and national geodetic systems and their relationship to the ITRF. In addition, many countries have already redefined or are in the process of redefining their geodetic systems, directly connected to the ITRF, using their national permanent GNSS networks. The main purpose of regional and national reference frames is for georeferencing applications with centimeter precision and accuracy. There are three main categories of implementation of these reference frames:

1. Station positions at a given epoch, eventually updated more or less frequently. This is the case, for example, for NAD83 [36.52, 53] and SIRGAS [36.54] for North and South America respectively, and GDA94 for Australia [36.55]
2. Station positions and minimized velocities. This is the case of ETRS89 for Europe where velocities are minimized by removing the angular velocity of the Eurasian plate when transforming from ITRF to ETRS89 realization [36.56]
3. Station positions and deformation model corrections. A case example is the New Zealand geodetic datum 2000 (NZGD2000) [36.57] where a deformation model is elaborated to correct coordinates for the effect of regional-scale tectonic movements for all geodetic reference points. The accumulated displacements estimated by the deformation model allow the computation of station coordinates as if

they were observed at the fixed reference epoch of 2000.0.

36.2.4 General Guidelines for GNSS-Based Reference Frame Implementation

By design, the ITRF is to be regarded as a common global standard that provides the most accurate frame definition: long-term averages of the origin, scale and orientation, necessary for the consistency and interoperability of Earth science and societal applications. In the meantime, with the proliferation of dense GNSS networks at the local, national, continental and global levels, it is obviously impossible to include all worldwide permanent GNSS stations in the IGS (and consequently) in the ITRF networks. It becomes however desirable to express all local, national, continental and global GNSS network solutions in the ITRF.

In order to access the ITRF, it can be used directly, via its products (station positions and velocities), but also indirectly, using IGS products. In the following we describe general guidelines that allow the efficient expression of a GNSS-based solution of station positions in the ITRF, using IGS products (orbits, clocks and EOPs). This method, based on the equations of minimum constraints (MC) (see for instance [36.34, 58]), is described below for the case of an epoch solution (as a materialization of a quasi-instantaneous reference frame), involving one day or one week of GNSS observations. It can of course be applied to any kind of network (being global or regional) not only for positions, but also for velocities. It comprises the following steps:

1. Selection of a reference set of known ITRF/IGS stations and collecting their GNSS observation data provided in Receiver INdependent EXchange (RINEX) format from IGS data centers, covering the timespan (one day or one week) of the implied observations. It is highly advised to select a set of ITRF/IGS stations that are as homogeneously and globally distributed as possible, in order to achieve the best and an accurate expression in the ITRF.
2. Processing user station data together with the selected ITRF/IGS ones, using the preferred GNSS software. In this step, IGS orbits, clocks and EOPs should be fixed to the values consistent with the associated ITRF/IGS frame (ITRFyy, IGSyy). Fixing or tightly constraining ITRF/IGS reference station coordinates should by all means be avoided. Doing so would potentially introduce distortion in the solution due to possible outdated ITRF/IGS station coordinates after some events, such as earthquakes or equipment changes. Moreover, as the ITRF is a secular linear frame, fixing or tightly constraining ITRF/IGS reference station coordinates would also inhibit the geophysical signal embedded in the transformed solution one may want to preserve.
3. Propagation of the selected ITRF/IGS station positions ($\boldsymbol{X}_\mathrm{I}$) at the central epoch ($t_\mathrm{c}$) of the employed GNSS observations, using
$$\boldsymbol{X}_\mathrm{I}(t_\mathrm{c}) = \boldsymbol{X}_\mathrm{I}(t_0) + \dot{\boldsymbol{X}}_\mathrm{I} \cdot (t_\mathrm{c} - t_0) \ ,$$
where $\boldsymbol{X}_\mathrm{I}(t_0)$ are the ITRF/IGS station positions at epoch t_0 and $\dot{\boldsymbol{X}}_\mathrm{I}$ are their linear velocities.
4. Application of minimum constraints approach [36.34, 35], detailed below, which is believed to be implemented in all major scientific software packages. The derived solution will be expressed in the ITRF/IGS frame that is consistent with the used orbits.
5. Comparison of the estimated ITRF/IGS reference station positions to the official published values, propagated at epoch t_c in step 3, by fitting a similarity transformation of three, four or seven parameters selected in the MC application and checking for consistency. The estimated transformation parameters should all be zero. In addition, if large discrepancies (postfit residuals of the similarity transformation) are found for some stations (exceeding a certain threshold, say 1–2 cm, but depending on the targeted accuracy), these stations should be rejected from the ITRF/IGS reference set and the processing chain should be iterated. Care should also be taken in the time interval of the validity of the used IGS/ITRF coordinates, taking into account station position discontinuities.

The starting point of the MC concept is based on the seven-parameter similarity transformation between any two reference systems or frames. Therefore, the linearized relationship between any space geodesy TRF solution, for example GNSS-based solution ($\boldsymbol{X}_\mathrm{G}$) and the ITRF ($\boldsymbol{X}_\mathrm{I}$), over selected reference set of common stations, can be written as

$$\boldsymbol{X}_\mathrm{I} = \boldsymbol{X}_\mathrm{G} + \mathbf{A}\boldsymbol{\theta} \ , \qquad (36.7)$$

where the design matrix $\mathbf{A}$ is a stacked matrix made from elementary 3-row matrices

$$\mathbf{A}^i = \begin{pmatrix} 1 & 0 & 0 & x_\mathrm{a}^i & 0 & z_\mathrm{a}^i & -y_\mathrm{a}^i \\ 0 & 1 & 0 & y_\mathrm{a}^i & -z_\mathrm{a}^i & 0 & x_\mathrm{a}^i \\ 0 & 0 & 1 & z_\mathrm{a}^i & y_\mathrm{a}^i & -x_\mathrm{a}^i & 0 \end{pmatrix} \qquad (36.8)$$

with $i = 1 \dots n$ for a total of n sites, and where $\boldsymbol{\theta} = (T_x, T_y, T_z, D, R_x, R_y, R_z)^\top$ is the vector of seven transformation parameters. T_x, T_y, T_z are the three translation components, D is the scale factor, and R_x, R_y, R_z are the three rotation parameters. The approximate coordinates x_a^i, y_a^i, z_a^i of point i, appearing in the design matrix $\mathbf{A}$ can be taken from the ITRF/IGS reference solution. Note that (36.7) is only valid at the same and common epoch of the two station position sets ($\boldsymbol{X}_\mathrm{I}$ and $\boldsymbol{X}_\mathrm{G}$). It can also be generalized to 14 parameters when station velocities are involved in the process; see [36.24] for more details. Note also that the design matrix $\mathbf{A}$ can be reduced to the columns corresponding to the frame parameters of interest, e.g., columns 1, 2 and 3 for the origin components; 5, 6 and 7 for the orientation parameters. In case of a regional network, applying the MC approach on the three translation components (i. e., $\mathbf{A}$ is reduced to the three first columns) can be sufficient. It is however advisable to evaluate at least the following three options: translation, translation and scale, all seven parameters.

The unweighted least squares expression of (36.7) yields for $\boldsymbol{\theta}$

$$\boldsymbol{\theta} = \overbrace{(\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top}^{\mathbf{B}}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{G}) \,. \tag{36.9}$$

The approach of MC consists in using the matrix $\mathbf{B} = (\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top$ in such a way that $\boldsymbol{X}_\mathrm{G}$ will be expressed in the same frame as the ITRF solution $\boldsymbol{X}_\mathrm{I}$. Therefore to have $\boldsymbol{X}_\mathrm{G}$ expressed in the ITRF at a certain $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ level, an MC equation can be written as

$$\mathbf{B}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{G}) = \mathbf{0}\,(\boldsymbol{\Sigma}_{\boldsymbol{\theta}}) \,, \tag{36.10}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ is the variance matrix at which (36.10) is satisfied. It is a diagonal matrix containing small variances (to be selected at the user level) for each one of the seven transformation parameters. It is suggested to use 0.1 mm for translation parameters and equivalent amounts (i. e., 0.1 mm divided by the Earth radius) for the scale and orientation parameters.

In terms of normal equations, we can then write

$$\mathbf{B}^\top\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}\mathbf{B}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{G}) = \mathbf{0} \,. \tag{36.11}$$

The initial normal equation system of a GNSS-based solution before adding any kind of constraints can be written as

$$\mathbf{N}(\Delta\boldsymbol{X}) = \boldsymbol{K} \,, \tag{36.12}$$

where $\Delta\boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}_\mathrm{apr}$, with $\boldsymbol{X}$ being the unknown vector, $\boldsymbol{X}_\mathrm{apr}$ is the vector of a priori values, $\mathbf{N}$ is the unconstrained normal matrix and $\boldsymbol{K}$ is the right-hand side vector.

By fixing the IGS products (orbits, clocks and EOPs), the normal equation system (36.12) becomes invertible, but the underlying TRF could be far from that of the ITRF, i. e., defined at the level of the orbit precision (a few cm). The same normal equation system can also be obtained after removing classical constraints applied to a given GNSS-based solution.

Selecting a subset of ITRF stations ($\boldsymbol{X}_\mathrm{I}$), the MC equation becomes

$$\mathbf{B}^\top\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B}(\Delta\boldsymbol{X}) = \mathbf{B}^\top\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{apr}) \,. \tag{36.13}$$

Note that the right-hand side of (36.13) vanishes if the a priori values are taken from the ITRF/IGS selected solution.

Cumulating (36.12) and (36.13) yields

$$\begin{aligned}&(\mathbf{N} + \mathbf{B}^\top\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B})(\Delta\boldsymbol{X})\\ &\quad = \boldsymbol{K} + \mathbf{B}^\top\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{apr}) \,.\end{aligned} \tag{36.14}$$

The minimally constrained solution, expressed in the ITRF upon the selected stations is then

$$\begin{aligned}\boldsymbol{X} = {}& \boldsymbol{X}_\mathrm{apr} + (\mathbf{N} + \mathbf{B}^\top\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B})^{-1}\\ &\times (\boldsymbol{K} + \mathbf{B}^\mathrm{T}\boldsymbol{\Sigma}_\theta^{-1}\mathbf{B}(\boldsymbol{X}_\mathrm{I} - \boldsymbol{X}_\mathrm{apr})) \,.\end{aligned} \tag{36.15}$$

36.2.5 GNSS, Reference Frame and Sea Level Monitoring

Because of its ramifications around climate change and global warming, sea level monitoring requires the most stringent continuous geodetic observations that can only be addressed within the context of a global and stable reference frame. Two main data streams are used to infer sea level rise and its spatial and temporal variability: tide gage records and satellite altimetry data. The former dataset requires precise quantification of land vertical motion where tide gages are located, and the latter dataset imposes the precise knowledge of satellite orbits in a well-defined global reference frame. Both methods greatly benefit from the availability of GNSS observations. GNSS is the technique of choice to infer vertical crustal motion due to its ease of use and its connection to the ITRF through IGS products. Data collected by GPS receivers on board altimetry satellites and by ground-based receivers, together with DORIS and SLR data are used to precisely determine satellite orbits [36.59, 60].

To fully exploit tide gage records and accurately determine land vertical motion, it has been demonstrated that the GNSS processing strategy, together with the availability of an accurate reference frame are the main two limiting factors for improving our understanding of regional sea level variability in space and time. The processing strategy includes precise orbit determination of the GNSS satellites, an optimal treatment of GNSS terrestrial observations and an advanced method of reference frame determination. Using an improved processing strategy and GPS data spanning 10 y at tide gages, [36.61] determined vertical velocities, based on ITRF2005, with uncertainties several times smaller than the 1–3 mm/y associated with global sea level change. The same authors have also shown that GPS-based land motion corrections at tide gages and expressed in ITRF2005 perform much better than glacial isostatic adjustment (GIA) model predictions, both on the global and the regional scale. These results suggest that GNSS measurements are more appropriate than GIA models to capture localized vertical motions associated with, for example, plate tectonics, volcanism, sediment compaction, or underground fluid extraction.

Precise orbit determination (POD) is one of the main critical issues for an accurate determination of global sea level variability using altimetry data of ocean surface satellite topography missions (OSTM), such as TOPEX-Poseidon, and Jason-1 and 2. These missions carry onboard three tracking systems (DORIS, GPS and SLR) to meet the requirement of better than 1.5 cm radial accuracy for the operational orbit included in the geophysical data record products [36.60]. One of the main long-period error sources of POD is the stability of the origin of the reference frame, and in particular its z-component. In [36.62] it was shown that the difference between using an old reference frame called CSR95 (compatible with ITRF2000) of the Center for Space Research of the University of Texas at Austin, and ITRF2005 in orbit computation caused a change in the estimated mean sea level trend of -0.26 mm/y for the period from 1993 to 2002. The primary cause was shown to be the drift in the z-component of the origin between the two frames of 1.8 mm/y that also affected the regional sea level rates at the high latitudes by ± 1.5 mm/y. The review article [36.60] gives a summary of the different levels of performance of POD estimates as determined by different groups, using data of the three satellite techniques. They reported in particular that the 1 cm goal is met by both Jason-1 and 2 GPS-based reduced-dynamic orbits.

36.3 Earth Rotation, Polar Motion, and Nutation

Observations of the Earth's rotation show that while the Earth rotates about its axis once a day, it does not do so uniformly. Instead, the rate of rotation fluctuates by as much as a millisecond a day. The Earth wobbles as it rotates because its mass is not balanced about its rotation axis, and the Earth precesses and nutates in space. These variations in the Earth's rotation are caused by processes acting within the interior of the Earth such as glacial isostatic adjustment and core-mantle interaction torques, by processes acting at the surface of the Earth such as fluctuations in the transport of mass within the atmosphere and oceans, and by processes acting external to the Earth such as torques due to the gravitational attraction of the Sun, Moon, and planets [36.63–67].

In principle, only three time-dependent parameters, the Euler angles, are needed to fully characterize the varying orientation of the Earth in space. However, by convention, five parameters are actually used: two precession and nutation parameters that give the location of the reference pole in the space-fixed celestial frame, two polar motion parameters that give the location of the reference pole in the body-fixed terrestrial frame, and a spin parameter that gives the angular rotation of the Earth about the reference axis. The advantage of using five parameters instead of three is that with five parameters the externally forced precession/nutation motion of the Earth is largely separated from its internally excited wobbling motion, also known as polar motion.

Routine measurements of the Earth's time-varying rotation are currently provided by the space-geodetic techniques of satellite and lunar laser ranging (SLR and LLR), very long baseline interferometry (VLBI), global navigation satellite systems (GNSSs) like the global positioning system (GPS), and Doppler orbitography and radio positioning integrated by satellite (DORIS). Each of these techniques has its own unique strengths and weaknesses in its ability to determine the five Earth orientation parameters (EOPs). Not only is each technique sensitive to a different subset and/or linear combination of the Earth orientation parameters, but also the averaging time for their determination is different, as is the interval between observations and the precision with which they can be determined.

Because of the large number of Earth orbiting satellites that transmit GNSS signals and the large number of ground stations that receive them, continuous, uninterrupted measurements of the Earth's rotation are provided by GNSS. In addition, because the raw observables can be rapidly analyzed, GNSS can provide measurements of the Earth's rotation in near-real time. In this section, the contribution of GNSS to monitoring the rotational behavior of the Earth and to understanding the causes of the observed variations is discussed.

36.3.1 Theory of the Earth's Rotation

Changes in the rotation of the solid Earth are usually studied by applying the principle of conservation of angular momentum, which requires that changes in the rotation vector of the solid Earth are manifestations of either torques acting on the solid Earth or of changes in the mass distribution within the solid Earth that alter its inertia tensor. Angular momentum is transferred between the solid Earth and the fluid regions (the underlying liquid metallic core and the overlying hydrosphere and atmosphere) with which it is in contact; concomitant torques are due to hydrodynamic or magnetohydrodynamic stresses acting at the fluid/solid Earth interfaces. Using the principle of the conservation of angular momentum the equations governing small variations in both the rate of rotation and in the position of the rotation vector with respect to the Earth's crust can be derived [36.67–70].

Within a rotating, body-fixed reference frame, the equation that relates changes in the angular momentum $\boldsymbol{L}(t)$ of a rotating body to the external torques $\boldsymbol{\tau}(t)$ acting on the body is [36.71]

$$\frac{\partial}{\partial t}[\boldsymbol{h}(t)+\mathbb{I}(t)\cdot\boldsymbol{\omega}(t)] + \boldsymbol{\omega}(t)\times[\boldsymbol{h}(t)+\mathbb{I}(t)\cdot\boldsymbol{\omega}(t)] = \boldsymbol{\tau}(t)\,, \quad (36.16)$$

where $\boldsymbol{\omega}(t)$ is the angular velocity of the body with respect to inertial space and where $\boldsymbol{L}(t)$ has been written as the sum of two terms: (1) that part $\boldsymbol{h}(t)$ due to motion relative to the rotating reference frame, and (2) that part due to changes in the inertia tensor $\mathbb{I}(t)$ of the body caused by changes in the distribution of mass.

The Earth's rotation deviates only slightly from a state of uniform rotation, the deviation being a few parts in 10^8 in speed, corresponding to changes of a few milliseconds (ms) in the length of the day, and about a part in 10^6 in the orientation of the rotation axis relative to the crust of the Earth, corresponding to a variation of several hundred milliarcseconds (mas) in polar motion. Such small deviations in rotation are studied by linearizing (36.16).

Let the Earth be initially uniformly rotating about its figure axis and orient the body-fixed reference frame so that its z-axis is aligned with the figure axis. Under a small perturbation to this initial state, the initial relative angular momentum $\boldsymbol{h}_\mathrm{o}$ (which is zero because there is initially no relative angular momentum) will be perturbed to $\boldsymbol{h}_\mathrm{o}+\Delta\boldsymbol{h}$, the initial inertia tensor $\mathbb{I}_\mathrm{o}$ will be perturbed to $\mathbb{I}_\mathrm{o}+\Delta\mathbb{I}$, and the initial angular velocity vector $\boldsymbol{\omega}_\mathrm{o}$ will be perturbed to $\boldsymbol{\omega}_\mathrm{o}+\Delta\boldsymbol{\omega}$. Keeping terms to first order in small quantities and making a number of other assumptions including assuming that the Earth is axisymmetric, that the oceans remain in equilibrium as the rotation of the solid Earth changes, that the core is not coupled to the mantle, and that the rotational variations occur on timescales much longer than a day, then the Cartesian components of the linearized version of (36.16) can be written as [36.67]

$$\frac{1}{\sigma_\mathrm{o}}\frac{\partial m_x(t)}{\partial t}+m_y(t)=\chi_y(t)-\frac{1}{\Omega}\frac{\partial \chi_x(t)}{\partial t} \quad (36.17)$$

$$\frac{1}{\sigma_\mathrm{o}}\frac{\partial m_y(t)}{\partial t}-m_x(t)=-\chi_x(t)-\frac{1}{\Omega}\frac{\partial \chi_y(t)}{\partial t} \quad (36.18)$$

$$m_z(t)=-\chi_z(t)\,, \quad (36.19)$$

where Ω is the mean angular velocity of the Earth (rad/s), σ_o is the observed complex-valued frequency of the Chandler wobble (rad/s), and the dimensionless m_i are related to the elements of the perturbed rotation vector

$$\begin{aligned}\boldsymbol{\omega}(t)&=\boldsymbol{\omega}_\mathrm{o}(t)+\Delta\boldsymbol{\omega}(t)\\ &=\Omega\,\hat{\boldsymbol{z}}+\Omega\,[m_x(t)\,\hat{\boldsymbol{x}}+m_y(t)\,\hat{\boldsymbol{y}}+m_z(t)\,\hat{\boldsymbol{z}}]\,,\end{aligned} \quad (36.20)$$

with the hat denoting a vector of unit length.

The dimensionless $\chi_i(t)$ in (36.17)–(36.19) are known as excitation functions and are functions of the perturbed inertia tensor (kg-m^2) and relative angular momentum (kg-m^2/s) that are exciting the changes in the Earth's rotation [36.67]

$$\chi_x(t)=\frac{1.608[\Delta h_x(t)+0.684\,\Omega\,\Delta I_{xz}(t)]}{(C-A')\Omega}\,, \quad (36.21)$$

$$\chi_y(t)=\frac{1.608[\Delta h_y(t)+0.684\,\Omega\,\Delta I_{yz}(t)]}{(C-A')\Omega}\,, \quad (36.22)$$

$$\chi_z(t)=\frac{0.997}{C_\mathrm{m}\Omega}[\Delta h_z(t)+0.750\,\Omega\,\Delta I_{zz}(t)]\,, \quad (36.23)$$

where C is the axial principal moment of inertia of the entire Earth, C_m is that of just the crust and mantle, and A' is the average $(A+B)/2$ of the equatorial principal moments of inertia of the entire Earth.

The numerical coefficients of the inertia tensor terms in (36.21)–(36.23) are functions of load Love numbers [36.67], so (36.21)–(36.23) are valid for processes like atmospheric surface pressure variations that load the solid Earth causing it to deform. For processes that do not load the solid Earth, like earthquakes, the coefficients 0.684 in (36.21)–(36.22) and 0.750 in (36.23) should be set to 1.0.

36.3.2 Length-of-Day

Equations (36.19) and (36.23) relate changes in the axial component of the Earth's angular velocity to changes in both the axial component of relative angular momentum and in the zz-element of the inertia tensor. But GNSS observations do not give changes in the axial component of the Earth's angular velocity. Instead, they give changes in the length of the day. The length of the day is the rotational period of the Earth. Changes $\Delta\Lambda(t)$ in the length of the day are related to the time rate-of-change of the difference (UT1 − TAI) between Universal Time UT1 and atomic time TAI and to changes $\Delta\omega_z(t) = \Omega\, m_z(t)$ in the axial component of the Earth's angular velocity [36.67]

$$\frac{\Delta\Lambda(t)}{\Lambda_o} = -\frac{d(\mathrm{UT1}-\mathrm{TAI})}{dt} = -\frac{\Delta\omega_z(t)}{\Omega} = -m_z(t)\,, \tag{36.24}$$

where Λ_o is the nominal length-of-day (LOD) of 86 400 s. GNSS-observed changes in the length of the day are therefore related to the processes causing the length-of-day to change by

$$\frac{\Delta\Lambda(t)}{\Lambda_o} = \frac{0.997}{C_m\Omega}[\Delta h_z(t) + 0.750\,\Omega\,\Delta I_{zz}(t)]\,. \tag{36.25}$$

Figure 36.5 shows the changes in the length of the day $\Delta\Lambda(t)$ measured by GNSS during March 1997 to June 2014. Like a spinning ice skater whose speed of rotation increases as the skater's arms are brought closer to the body, the speed of the Earth's rotation increases and the length of the day decreases if its mass is brought closer to its axis of rotation. Conversely, the speed of the Earth's rotation decreases and the length of the day increases if its mass is moved away from the rotation axis. Observations of the length of the day like those shown in Fig. 36.5 show that it consists mainly of:

1. A linear trend of rate +1.8 ms/cy (not evident in Fig. 36.5 because of the shortness of the record)
2. Decadal variations having an amplitude of a few milliseconds
3. Tidal variations having an amplitude of about 1 ms
4. Seasonal variations having an amplitude of about 0.5 ms
5. Smaller amplitude variations occurring on all measurable timescales.

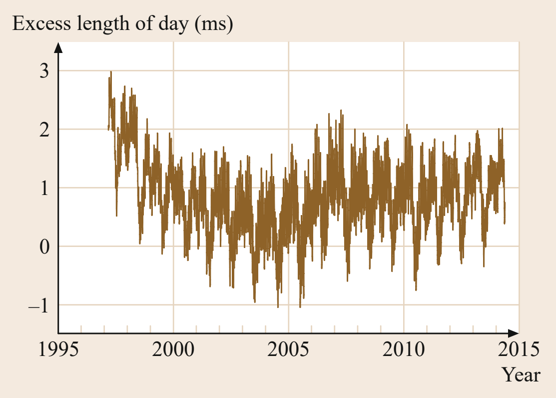

Fig. 36.5 Observed excess length-of-day values in milliseconds (ms) spanning March 1997 to June 2014 from the IGS final combined series. The excess length-of-day is the amount by which the length-of-day is longer (positive values) or shorter (negative values) than the nominal length-of-day of 86 400 s

A number of different dynamical Earth processes are responsible for the changes in the length of the day shown in Fig. 36.5. Tidal forces due to the changing gravitational attraction of the Sun, Moon, and planets deform the solid and fluid regions of the Earth, causing the Earth's rotation to change by causing its inertia tensor to change. In fact, solid-body tides, caused by the tidal forces acting on the solid Earth, are the dominant cause of length-of-day variations on intraseasonal to interannual timescales. Ocean tides, caused by the tidal forces acting on the oceans, are the dominant cause of subdaily length-of-day variations and contribute to length-of-day variations at longer periods.

Figure 36.6 shows a spectrum of the GNSS-observed length-of-day variations that are shown in Fig. 36.5. Peaks at the tidal frequencies are clearly evident. Nontidal variations in the length of the day occurring on timescales of a few days to a few years are predominantly caused by variations in the zonal atmospheric winds, with variations in atmospheric surface pressure, oceanic currents and bottom pressure, and water stored on land contributing much less. On longer timescales, decadal variations as large as a few milliseconds in the length of the day are caused by interactions between the fluid outer core and solid mantle of the Earth, and a secular trend of +1.8 ms/cy in the length of the day is caused by a combination of tidal dissipation in the Earth-Moon system (+2.3 ms/cy) and by glacial isostatic adjustment (−0.5 ms/cy). See [36.67] for a review of these and other causes of length-of-day changes.

36.3.3 Polar Motion

Equations (36.17)–(36.18) and (36.21)–(36.22) relate changes in the equatorial components of the Earth's angular velocity to changes in both the equatorial components of relative angular momentum and in the xz- and yz-elements of the inertia tensor. But GNSS observations do not give changes in the equatorial com-

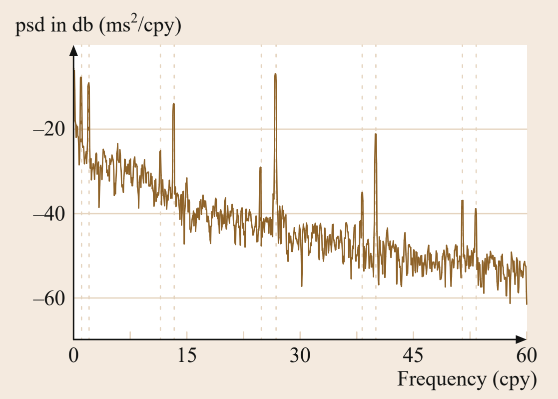

Fig. 36.6 Power spectral density (psd) estimates in decibels (db) computed by the multitaper method of the IGS Final combined length-of-day measurements spanning March 1997 to June 2014. *Vertical dashed lines* indicate the frequencies of the annual (1 cycle/y (cpy)) and semiannual (2 cpy) LOD variations and of the largest tidal variations in the monthly (13 cpy), fortnightly (27 cpy), termensual (40 cpy), and 7 d (51 cpy) tidal bands

ponents of the Earth's angular velocity. Instead, they give changes in the terrestrial position of the celestial intermediate pole (CIP). The CIP is the intermediate reference pole whose use allows the separation of nutation from wobble. The equatorial components of the Earth's angular velocity are related to the GNSS-observed position of the CIP in the terrestrial reference frame $\boldsymbol{p}(t) = p_x(t) - \mathrm{j}\, p_y(t)$, where the negative sign accounts for $p_y(t)$ being conventionally positive toward 90° West longitude, by [36.67]

$$\boldsymbol{m}(t) = \boldsymbol{p}(t) - \frac{\mathrm{j}}{\Omega}\frac{\mathrm{d}\boldsymbol{p}(t)}{\mathrm{d}t}\,, \quad (36.26)$$

where $\boldsymbol{m}(t) = m_x(t) + \mathrm{j}\, m_y(t)$ and j is the imaginary unit $\sqrt{-1}$. By combining (36.26) with (36.17)–(36.18) it can be shown that the GNSS-observed polar motion parameters $p_x(t)$ and $p_y(t)$ are related to the polar motion excitation functions $\boldsymbol{\chi}(t) = \chi_x(t) + \mathrm{j}\, \chi_y(t)$ by

$$\boldsymbol{p}(t) + \frac{\mathrm{j}}{\sigma_\mathrm{o}}\frac{\mathrm{d}\boldsymbol{p}(t)}{\mathrm{d}t} = \boldsymbol{\chi}(t) = \frac{1.608[\Delta\boldsymbol{h}(t) + 0.684\,\Omega\,\Delta\mathbb{I}(t)]}{(C - A')\Omega}\,, \quad (36.27)$$

where in this equation $\Delta\boldsymbol{h}(t) = \Delta h_x(t) + \mathrm{j}\,\Delta h_y(t)$ and $\Delta\mathbb{I}(t) = \Delta I_{xz}(t) + \mathrm{j}\,\Delta I_{yz}(t)$.

Figures 36.7a and 36.7b show the x- and y-components of polar motion, $p_x(t)$ and $p_y(t)$ respectively, measured by GNSS during July 1996 to June 2014. Much like the wobble of an unbalanced automobile tire, the Earth wobbles because the mass of the Earth is not balanced about its rotation axis. In the absence of excitation, the Earth would eventually stop wobbling because of dissipation processes in the oceans and solid, but not rigid, crust and mantle. But as long as mass continues to be horizontally transported towards or away from the poles the Earth will continue to wobble. Observations like those shown in Figs. 36.7a and 36.7b show that polar motion consists mainly of:

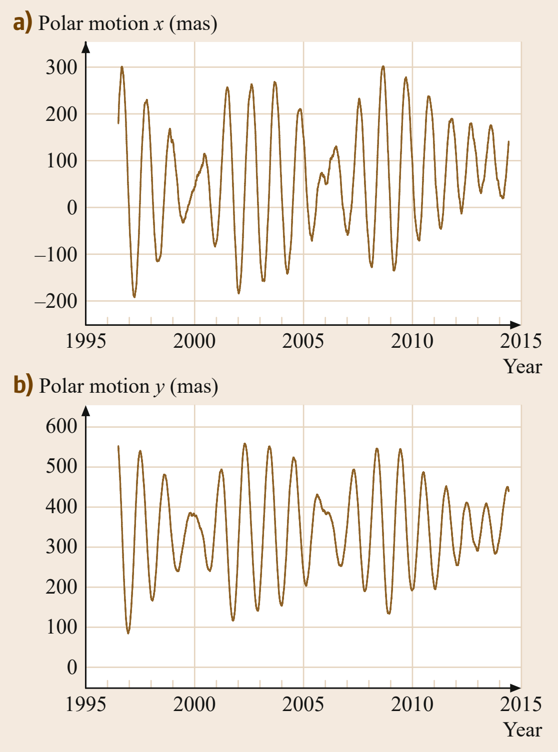

Fig. 36.7 The x-component **(a)** and y-component **(b)** of observed polar motion values in milliarcseconds (mas) spanning July 1996 to June 2014 from the IGS final combined series. The readily apparent beat pattern is caused by the 12-month annual and 14-month Chandler wobbles, which have similar amplitudes, constructively and destructively interfering with each other

1. A forced annual wobble having a nearly constant amplitude of about 100 mas
2. The free Chandler wobble having a period of about 433 d and a variable amplitude ranging from about 100–200 mas
3. Quasiperiodic variations on decadal timescales having amplitudes of about 30 mas known as the Markowitz wobble
4. A linear trend having a rate of about 3.5 mas/y and a direction towards 79° West longitude

5. Smaller amplitude variations occurring on all measurable timescales.

One of the great strengths of GNSS in measuring changes in the Earth's rotation is that it can measure not only the polar motion parameters themselves but also their time rate-of-change. By combining polar motion and polar motion-rate measurements according to the left-hand side of (36.27), GNSS allows direct measurements of the polar motion excitation functions to be made. These directly measured excitation functions can then be compared to models of, say, atmospheric and oceanic excitation to study the causes of the observed polar motion. Figure 36.8 shows a spectrum of the polar motion excitation functions determined from the GNSS-observed polar motion and polar motion-rate measurements. Like the changes in the length of the day, a number of different dynamical Earth processes are responsible for exciting polar motion. Because the tidal potential is symmetric about the polar axis, tidal deformations of the solid Earth do not cause it to wobble. But because ocean basins are asymmetrically distributed about the Earth, ocean tides do cause the Earth to wobble and small peaks at the fortnightly tidal frequencies are clearly evident in Fig. 36.8.

The annual wobble is a forced motion of the Earth that has been shown to be largely caused by the annual appearance of a high atmospheric pressure system over Siberia every winter [36.63]. This Siberian high-pressure system annually loads the Siberian crust, causing the Earth to wobble with an annual period. The Chandler wobble on the other hand is not a forced motion of the Earth, but is instead a free resonant motion of the Earth that occurs because the Earth is not rotating about its figure axis, the axis about which the Earth's mass is balanced. The Chandler wobble would freely decay with an exponential time constant of about 68 y if no mechanism or mechanisms were acting to excite it. Using atmospheric and oceanic general circulation models, it has been shown that the sum of atmospheric surface pressure and ocean-bottom pressure variations are the primary source of excitation of the Chandler wobble, with ocean-bottom pressure variations being about twice as effective as atmospheric pressure variations over land. On the longest timescales, the trend in the pole path has been shown to be caused by a combination of the viscoelastic response of the Earth to past changes in ice sheet mass and the elastic response of the Earth to present-day changes [36.72]; see [36.67] for a review of these and other causes of polar motion.

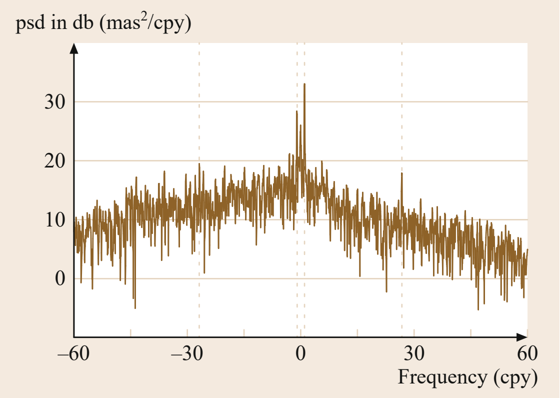

Fig. 36.8 Power spectral density (psd) estimates in decibels (db) computed by the multitaper method of the polar motion excitation functions $\boldsymbol{\chi}(t)$ spanning July 1996 to June 2014 formed by using (36.27) to combine the IGS final combined polar motion and polar-motion-rate measurements. *Vertical dashed lines* indicate the prograde and retrograde frequencies of the annual excitation (±1 cycle/y (cpy)) and of the fortnightly tidal term (±27 cpy). The retrograde component of polar motion excitation is represented by negative frequencies, the prograde component by positive frequencies

36.3.4 Nutation

Because of their great distance, the radio reference sources observed by VLBI exhibit negligible motion in the sky and can therefore be used to realize an inertial, celestial reference frame. This allows VLBI to determine all five of the Earth orientation parameters that are conventionally used to fully characterize the orientation of the Earth in space, including the two nutation parameters. But because the large nongravitational forces acting on artificial Earth-orbiting satellites cannot be accurately modeled [36.54], the orbits of satellites cannot be used to realize an inertial reference frame. Thus satellite techniques like GNSS can determine only a subset of the five EOPs. In particular, because of correlations between satellite orbital elements and both UT1 and the two nutation parameters, these Earth orientation parameters cannot be determined by satellite techniques like GNSS. However, their rate-of-change can be determined.

The rate-of-change of UT1, or length-of-day (36.24), was first routinely estimated from GPS data in June 1992 by the Center for Orbit Determination in Europe (CODE) analysis center. In [36.73] it was subsequently argued that there was no fundamental difference between estimating rates in UT1 and rates in nutation and that consequently GNSS should also be able to measure the rate-of-change of nutation.

They showed that the uncertainty in GNSS-measured nutation rates should grow linearly with the period of the nutation term and that GNSS should therefore be able to measure the rates of nutation terms having short periods. Using 3.5 y of GNSS data they were able to estimate the rates of 34 nutation terms having periods between four and 16 d.

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