Computing Probabilistic Assumption-Based Argumentation

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Abstract. We develop inference procedures for a recently proposed model of probabilistic argumentation called PABA, taking advantages of well-established dialectical proof procedures for Assumption-based Argumentation and Bayesian Network algorithms. We establish the soundness and termination of our inference procedures for a general class of PABA frameworks. We also discuss how to translate other models of probabilistic argumentation into this class of PABA frameworks so that our inference procedures can be used for these models as well.

Keywords: Probabilistic argumentation · Inference procedures · Bayesian networks

1 Introduction

Standard Abstract Argumentation (AA [3]) is inadequate in capturing argumentation processes involved probabilities such as the following.

Example 1 (Borrowed from [5]). John sued Henry for the damage caused to him when he drove off the road to avoid hitting Henry's cow.

– John: Henry should pay damage because Henry is the owner of the cow and the cow caused the accident (J_1).
– Henry: John was negligent as evidences at the accident location show that John was driving fast. Hence the cow was not the cause of the accident (H_1).

Let's try to construct an AA framework $\mathcal{F} = (AR, Att)$ to represent the judge's beliefs. The judge may consider J_1 as an argument proper, but not H_1 because according to him, the evidences at the accident location gives only some probability (p_0) that John was driving fast; and even if John was driving fast, the accident is caused by his fast-driving with some other probability p_1. Hence while the representation of J_1 is quite simple: $J_1 \in AR$, there is no perfect representation for H_1. $H_1 \notin AR$ (resp. $H_1 \in AR$) would mean that the judge would undoubtedly find for John (resp. Henry). However, in fact the chance that a party wins depends on the values that the judge assigns to p_0 and p_1.

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To remedy the above situation, several authors extend AA with probability theory, resulting in different models of Probabilistic Argumentation. Of our interest is the Probabilistic Assumption-based Argumentation framework of [5] ($PABA$) extending an instance of AA called Assumption-based Argumentation (ABA [2,4]). To anchor our contributions, let's loosely recall some technicalities. An ABA framework comprises inference rules in the form $c \leftarrow b_1, \ldots b_n$, representing that proposition c holds whenever propositions $b_1, \ldots b_n$ hold (b_i can be an assumption but not c). An $PABA$ framework is a triple $(\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ where $\mathcal{A}_p$ is a set of (positive) probabilistic assumptions, $\mathcal{R}_p$ is a set of probabilistic rules and $\mathcal{F}$ is an ABA framework. A probabilistic rule in $PABA$ also has the same form as an inference rule, except that its head is a proposition of the form $[\alpha : x]$ representing that the probability of probabilistic assumption α is x.

Example 2 (Cont. Example 1). The judge's beliefs is representable [5] by $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ where $\mathcal{A}_p = \{p_0, p_1\}$; $\mathcal{F}$ consists of assumptions $\sim forceMajeure$, $\sim johnNegligent$ (with contraries $forceMajeure$, $johnNegligent$) and inference rules $r_1, \ldots, r_5$; while $\mathcal{R}_p$ consists of probabilistic rules $r_6, \ldots, r_9$ where[1]

$r_1 : henryPay \leftarrow henryOwnerOfCow, cowCauseAccident, \sim forceMajeure$

$r_2 : cowCauseAccident \leftarrow \sim johnNegligent$ $\quad r_3 : henryOwnerOfCow \leftarrow$

$r_4 : johnNegligent \leftarrow drivingFast, p_1$ $\quad r_5 : drivingFast \leftarrow p_0$

$r_6 : [p_0 : 0.8] \leftarrow$ $\quad r_7 : [\neg p_0 : 0.2] \leftarrow$ $\quad r_8 : [p_1 : 0.75] \leftarrow$ $\quad r_9 : [\neg p_1 : 0.25] \leftarrow$

A possible world of $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ is a complete truth assignment over $\mathcal{A}_p$. For a possible world ω, $P(\omega)$ refers to the probability of ω generated by $\mathcal{P}$ (see Definition 6); and $\mathcal{F}_\omega$ denotes the revised version of $\mathcal{F}$ assuming that ω is the actual world. Each ABA semantics *sem* induces a $PABA$ semantics $Prob_{sem}$ stating the probability of the acceptability of a given proposition as follows.

$$Prob_{sem}(\pi) = \sum_{\omega \in \mathcal{W} : ABA\,\mathcal{F}_\omega \vdash_{sem} \pi} P(\omega)$$

where $\mathcal{W}$ is the set of all possible worlds; $ABA\,\mathcal{F}_\omega \vdash_{sem} \pi$ states that π is acceptable in $ABA\,\mathcal{F}_\omega$ under semantics *sem*.

Example 3 (Cont. Example 2). There are four possible worlds, in which the acceptability of proposition *henryPay* under any ABA semantics *sem* is shown below. Clearly $Prob_{sem}(henryPay) = 1 - P(\{p_0, p_1\}) = 1 - 0.8 \times 0.75 = 0.4$.

Possible world	$\{p_0, p_1\}$	$\{\neg p_0, p_1\}$	$\{p_0, \neg p_1\}$	$\{\neg p_0, \neg p_1\}$
henryPay is acceptable?	no	yes	yes	yes

[1] Probabilistic values are made up for demonstration.

Inference procedures for $PABA$ are procedures computing $Prob_{sem}(.)$ which have been unexplored. Note that since an $ABA\mathcal{F}$ can be represented by an $PABA$ framework with empty sets $\mathcal{A}_p$ and $\mathcal{R}_p$[2], inference procedures for $PABA$ subsume proof procedures for ABA, which have been developed in [2,4]. On the other hand, given a proof procedure for ABA semantics sem, one can computing $Prob_{sem}(\pi)$ by checking if $ABA\mathcal{F}_\omega \vdash_{sem} \pi$ for each possible world ω. Unfortunately this naive approach *always* results in an exponential blowup since there are as many as $2^{|\mathcal{A}_p|}$ possible worlds. It turns out that in the worst case we can not avoid this exponential blowup since $PABA$ subsumes Bayesian networks known to be exponentially complex in the worst case. However, as there are many inference algorithms Bayesian networks working efficiently in the average case, they may exist inference procedures for $PABA$ working efficiently in the average case. In this paper, we aim at developing such inference procedures. We establish the soundness and termination of our procedures for a general class of PABA frameworks. We also implement them to obtain an PABA inference engine capable of computing the credulous semantics and the ideal semantics of PABA[3]. Empirical evaluations of the engine, however, remains a future work.

The paper is organized as follows: Sect. 2 is a review of abstract argumentation and probabilistic assumption-based argumentation; Sect. 3 presents the theoretical basis of our inference procedures; Sect. 4 presents our inference procedures (due to space limitation, we present only the computation of PABA's credulous semantics and skip proofs of lemmas and theorems); Sect. 5 discusses translations of other models of probabilistic argumentation [7,8] into PABA in order to widen the applicability of our contributions and concludes.

2 Background on Argumentation

2.1 Abstract Argumentation

An AA framework [3] is a pair (AR, Att) where AR is a set of arguments, $Att \subseteq AR \times AR$ and $(A, B) \in Att$ means that A attacks B. $S \subseteq AR$ attacks $A \in AR$ iff $(B, A) \in Att$ for some $B \in S$. $A \in AR$ is acceptable wrt to S iff S attacks every argument attacking A. S is *conflict-free* iff S does not attack itself; *admissible* iff S is conflict-free and each argument in S is acceptable wrt S; *complete* iff S is admissible and contains every arguments acceptable wrt S; a *preferred* extension iff S is a maximal (wrt set inclusion) complete set; the *grounded* extension iff S is the least complete set; the *ideal* extension iff it is the maximal admissible set contained in every preferred extensions. An argument A is accepted under semantics $sem \in \{cr, gr, id\}$[4], denoted $AA\mathcal{F} \vdash_{sem} A$, iff A is in a sem extension.

[2] $ABA\mathcal{F} \vdash_{sem} \pi$ iff wrt this $PABA$ framework, $Prob_{sem}(\pi) = 1$.
[3] See https://pengine.herokuapp.com.
[4] Preferred/grounded/ideal semantics.

2.2 Assumption-Based Argumentation

As AA ignores the internal structure of argument, an instance of AA called Assumption-Based Argumentation (ABA [2,4]) defines arguments by deductive proofs based on assumptions and inference rules. Assuming a language $\mathcal{L}$ consisting of countably many sentences, an ABA framework is a triple $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ where $\mathcal{R}$ is a set of inference rules of the form $r : l_0 \leftarrow l_1, \ldots, l_n$ $(n \geq 0)$[5], $\mathcal{A} \subseteq \mathcal{L}$ is a set of assumptions, and $\overline{}$ is a (total) one-to-one mapping from $\mathcal{A}$ into $\mathcal{L}$, where $\overline{x}$ is referred to as the *contrary* of x. Assumptions do not appear in the heads of inference rules and contraries of assumptions are not assumptions.

A *(backward) deduction* of a conclusion π supported by a set of premises Q is a sequence of sets $S_1, S_2, \ldots, S_n$ where $S_i \subseteq \mathcal{L}$, $S_1 = \{\pi\}$, $S_n = Q$, and for every i, where σ is the selected proposition in S_i: $\sigma \notin Q$ and $S_{i+1} = S_i \setminus \{\sigma\} \cup \mathit{body}(r)$ for some inference rule $r \in \mathcal{R}$ with $\mathit{head}(r) = \sigma$.

An argument for $\pi \in \mathcal{L}$ supported by a set of assumptions Q is a deduction d from π to Q and denoted by (Q, d, π). An argument (Q, d, π) attacks an argument (Q', d', π') if π is the contrary of some assumption in Q'. For simplicity, we often refer to an argument (Q, d, π) by (Q, π) if there is no possibility for mistake.

A proposition π is said to be credulously/groundedly/ideally accepted in ABA $\mathcal{F}$, denoted ABA $\mathcal{F} \vdash_{cr} \pi$ (resp. ABA $\mathcal{F} \vdash_{gr} \pi$ and ABA $\mathcal{F} \vdash_{id} \pi$) if in the AA framework consisting of above defined arguments and attacks, there is an argument for π accepted under the credulous/grounded/ideal semantics.

2.3 Probabilistic Assumption-Based Argumentation

For clarity and modification, we break down the original definition of PABA (Definition 2.1 of [5] into two Definitions 1 and 2 below, where Definition 2 in fact slightly relaxes Definition 2.1 of [5], and as a result, our class of PABA frameworks subsumes the class of PABA frameworks in [5][6]. We also extend the definition of PABA's grounded semantics of [5] to define other semantics of PABA.

Definition 1 [5]. *A probabilistic assumption-based argumentation (PABA) framework $\mathcal{P}$ is a triple $(\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ satisfying the following properties*

1. $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ *is an ABA framework.*
2. $\mathcal{A}_p$ *is a finite set of* ***positive probabilistic assumptions****. Elements of* $\neg\mathcal{A}_p = \{\neg p \mid p \in \mathcal{A}_p\}$ *are called* ***negative probabilistic assumptions***[7]**.**
3. $\mathcal{R}_p$ *is a set of probabilistic rules of the form*

$$[\alpha : x] \leftarrow \beta_1, \ldots, \beta_n \; n \geq 0, x \in [0, 1], \alpha \in \mathcal{A}_p \cup \neg\mathcal{A}_p.$$

where $[\alpha : x]$*, called a* ***probabilistic proposition****, represents that the probability of probabilistic assumption* α *is* x*.*

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[5] For convenience, define $\mathit{head}(r) = l_0$ and $\mathit{body}(r) = \{l_1, \ldots l_n\}$.

[6] Any PABA framework in [5] is also an PABA framework in our extended definition, but the reverse may not hold.

[7] $\neg$ is the classical negation operator.

Definition 2 [5]. *PABA $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ is said to be **well-formed** if the following syntactic constraints are satisfied.*

1. *For each probabilistic assumption $\alpha \in \mathcal{A}_p \cup \neg\mathcal{A}_p$*
   (a) α does not occur in $\mathcal{A}$ as well as in the head of any rule in $\mathcal{R}$, and
   (b) $[\alpha : x]$ does not occurs in the body of any rule in $\mathcal{R}$ or $\mathcal{R}_p$.
2. *If a rule of the form $[\alpha : x] \leftarrow \beta_1, \ldots, \beta_n$ appears in $\mathcal{R}_p$, then $\mathcal{R}_p$ also contains a complementary rule $[\neg\alpha : 1 - x] \leftarrow \beta_1, \ldots, \beta_n$[8].*
3. *For each probabilistic assumption α, there exists a set of probabilistic assumptions $Pa_\alpha \subseteq \mathcal{A}_p$ such that for each maximally consistent subset $\{\beta_1, \ldots, \beta_m\}$ of $Pa_\alpha \cup \neg Pa_\alpha$, $\mathcal{R}_p$ contains a rule $[\alpha : x] \leftarrow \beta_1, \ldots, \beta_m$ (and complementary rule $[\neg\alpha : 1 - x] \leftarrow \beta_1, \ldots, \beta_m$).*
4. *If two rules of the form $r_1 : [\alpha : x] \leftarrow \ldots$ and $r_2 : [\alpha : y] \leftarrow \ldots$ appear in $\mathcal{R}_p$ and $x \neq y$, then either conditions below holds*
   (a) $body(r_1) \subset body(r_2)$ or $body(r_2) \subset body(r_1)$.
   (b) There is a probabilistic assumption $\alpha \in body(r_1)$ such that $\neg\alpha \in body(r_2)$

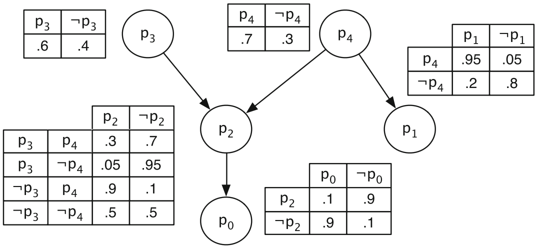
Note that in [5], the well-formedness condition consists of constraints 1, 2, 4(a); and a more rigid version of constraint 3 with $Pa_\alpha = \emptyset$, which implies that for each probabilistic assumption α, $\mathcal{R}_p$ must contain two rules of the forms $[\alpha : x] \leftarrow$ and $[\neg\alpha : 1 - x] \leftarrow$ (which, according to [5], encode the default/unconditional probability of α). So the $PABA$ given in Example 4 below[9] is not well-formed according to [5]. We do not require $Pa_\alpha = \emptyset$ because we want to have *Bayesian* PABA frameworks, defined as follows, to be well-formed.

Definition 3. *An $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ is said to **include** a Bayesian network $\mathcal{N} = (\mathcal{G}, CPTs), \mathcal{G} = (V, E)$ where V consists of only binary variables, if $\mathcal{A}_p, \mathcal{R}_p$ represents the same probabilistic information as $\mathcal{N}$[10]. A **Bayesian PABA** framework is an PABA framework that includes a Bayesian network.*

Example 4. Below are a Bayesian $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ and its network.

- $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ where $\mathcal{A} = \{\alpha, \beta, \gamma, \eta\}$ and $\overline{\alpha} = \neg\alpha$, $\overline{\beta} = \neg\beta$, $\overline{\gamma} = \neg\gamma$ and $\overline{\delta} = \neg\delta$ and $\mathcal{R}$ consists of $r_0 : \neg\alpha \leftarrow \alpha, p_0 \quad r_1 : \neg\alpha \leftarrow \beta, p_1 \quad r_2 : \neg\beta \leftarrow \alpha, p_2 \quad r_3 : \neg\gamma \leftarrow \delta, p_3$ and $r_4 : \neg\delta \leftarrow \gamma, p_4$
- $\mathcal{A}_p = \{p_0, p_1, p_2, p_3, p_4\}$ and $\mathcal{R}_p$ consists of the following probabilistic rules
  $[p_0 : .1] \leftarrow p_2 \quad [p_0 : .9] \leftarrow \neg p_2 \quad [p_1 : .95] \leftarrow p_4 \quad [p_1 : .2] \leftarrow \neg p_4$
  $[p_2 : .3] \leftarrow p_3, p_4 \quad [p_2 : .05] \leftarrow p_3, \neg p_4 \quad [p_2 : .9] \leftarrow \neg p_3, p_4$
  $[p_2 : .5] \leftarrow \neg p_3, \neg p_4 \quad [p_3 : .6] \leftarrow \quad [p_4 : .7] \leftarrow$

[8] In examples, we will not list complementary rules to save space.
[9] We will use this framework in running examples from now on.
[10] That is, each pair $\alpha, \neg\alpha$ of probabilistic assumptions of $\mathcal{P}$ corresponds to truth assignments of variable $\alpha \in V$ and vice versa; and each probabilistic rule in $\mathcal{R}_p$ corresponds to one entry of an CPT in $\mathcal{N}$ and vice versa.



For convenience, let's adopt some notations wrt an $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$.

– A **possible world** is a maximal (wrt set inclusion) consistent subset of $\mathcal{A}_p \cup \neg\mathcal{A}_p$. A **partial world** is a subset (not necessarily proper) of a possible world. $\mathcal{W}$ denotes the set of all possible worlds. For each $\omega \in \mathcal{W}$,
  - $ABA\,\mathcal{F}_\omega \triangleq (\mathcal{R}_\omega, \mathcal{A}, \overline{})$ where $\mathcal{R}_\omega \triangleq \mathcal{R} \cup \{p \leftarrow \mid p \in \omega\}$
  - $ABA\,\mathcal{P}_\omega \triangleq (\mathcal{R}_\omega \cup \mathcal{R}_p, \mathcal{A}, \overline{})$.
  - $AA\,\mathcal{P}_\omega$ denotes AA framework $(AR\,\mathcal{P}_\omega, Att\,\mathcal{P}_\omega)$ where $AR\,\mathcal{P}_\omega$ is the set of arguments of $ABA\ \mathcal{P}_\omega$, and $Att\,\mathcal{P}_\omega$ consists of three types of attacks as defined by Definition 4.
– An argument with conclusion being a probabilistic proposition (resp., non-probabilistic proposition) is referred to as a **probabilistic argument** (resp. **non-probabilistic argument**).

Definition 4 [5]. *Let $A = (Q, \alpha), A' = (Q', \alpha')$ be arguments in $AR\ \mathcal{P}_\omega$ for some possible world ω. A* ***attacks*** *A' if one of three conditions below holds:*

1. *(**type-1 attack**) A is a non-probabilistic argument and α is the contrary of some assumption in Q'.*
2. *(**type-2 attack**) A, A' are probabilistic arguments and A attacks A' by specificity as defined by Definition 5.*
3. *(**type-3 attack**) α is a probabilistic assumption, $A = (\emptyset, \alpha)$ and A' is a probabilistic argument with conclusion of the form $[\neg\alpha : x]$.*

Definition 5 [5]. *Let $A = (Q, \delta, [\alpha : x])$ and $A' = (Q', \delta', [\beta : y])$ be probabilistic arguments in $AR\ \mathcal{P}_\omega$ for some possible world ω. Further let $\delta = S_1, S_2, \ldots, S_m$, $\delta' = S'_1, S'_2, \ldots, S'_n$, and the rules used to derive S_2 from S_1 and S'_2 from S'_1 are r_1 and r'_1 respectively. A* ***attacks*** *A' by* ***specificity*** *if $body(r'_1) \subset body(r_1)$.*

The following definition extends the definition of $Prob_{gr}(.)$ in [5]. Intuitively, it tells how the probabilities of probabilistic assumptions, which are decided by the grounded semantics, propagate to influence the probabilities of accepting other propositions under an arbitrary semantics of argumentation.

Definition 6. *The probability that a proposition π is acceptable wrt semantics sem is* $Prob_{sem}(\pi) \triangleq \sum_{\omega \in \mathcal{W} : ABA\,\mathcal{F}_\omega \vdash_{sem} \pi} P(\omega)$ *where* $P(\omega) \triangleq \prod_{\alpha \in \omega : AA\,\mathcal{P}_\omega \vdash_{gr} (\_, [\alpha : x])} x$

For convenience, for a set $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ of partial worlds, we use $P(s_1 \vee s_2 \cdots \vee s_n)$ to refer to $\sum_{\omega \in \mathcal{W}, s \in \mathcal{S}: \omega \supseteq s} P(\omega)$.

Example 5 (Cont. Example 4). It is easy to verify that for any $\omega \in \mathcal{W}$:

– $ABA\mathcal{F}_\omega \vdash_{cr} \neg\alpha$ iff $\omega \supseteq \{p_1\}$. So $Prob_{cr}(\neg\alpha) = \sum_{\omega \in \mathcal{W}: \omega \supseteq \{p_1\}} P(\omega) = P(\{p_1\})$.
– $ABA\mathcal{F}_\omega \vdash_{id} \neg\alpha$ iff $\omega \supseteq s_1$ or $\omega \supseteq s_2$ where $s_1 = \{p_1, \neg p_2\}$ and $s_2 = \{p_0, p_1, p_2\}$. Hence $Prob_{id}(\neg\alpha) = \sum_{\omega \in \mathcal{W}: \omega \supseteq s_1 \text{ or } \omega \supseteq s_2} P(\omega) = P(s_1 \vee s_2)$.

In [5] Dung and Thang show that an PABA framework is *probabilistic coherent* ($\sum_{\omega \in \mathcal{W}} P(\omega) = 1$) if it is *probabilistically acyclic.*

Definition 7. *1. The dependency graph of ABA $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ is a directed graph of which nodes are sentences occurring in $\mathcal{F}$ and there is an edge from node p to node q if and only if*
(a) $\mathcal{R}$ contains a rule of the form $p \leftarrow \ldots, q, \ldots,$ or
(b) p is an assumption in $\mathcal{A}$ and q is the contrary of p.
2. The dependency graph of PABA $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ is a directed graph obtained from that of $\mathcal{F}$ by
(a) first, adding an edge from node p to a node q if $\mathcal{R}_p$ contains a rule of the form $[p : \_] \leftarrow \ldots, q, \ldots$
(b) then, for each $p \in \mathcal{A}_p$, merging node $\neg p$ with node p.

An PABA $\mathcal{P}$ is said to be ***probabilistically acyclic*** *if there is no infinite path starting from a probabilistic assumption in the dependency graph of $\mathcal{P}$.*

It turns out that probabilistic acyclicity is also sufficient for probabilistic coherence in our class of PABA frameworks.

Lemma 1. *Let $\mathcal{P}$ be an PABA framework as defined by Definitions 1 and 2. If $\mathcal{P}$ is probabilistically acyclic, then*

1. (Generalizing Lemma 2.1 of [5]) $\sum_{\omega \in \mathcal{W}} P(\omega) = 1$.
2. $0 \leq Prob_{gr}(\pi) \leq Prob_{id}(\pi) \leq Prob_{cr}(\pi) \leq 1$ for any proposition π[11]*.*

From now on, we restrict ourselves to probabilistically acyclic PABA frameworks that satisfy Definitions 1 and 2.

3 Computing PABA Semantics: Theoretical Basis

In this section, we present the theoretical basis for our inference procedures[12].

[11] If π does not occur in $\mathcal{P}$, then $Prob_{sem}(\pi) = 0$ for any semantics *sem*.
[12] From now on we assume an arbitrary but fixed $PABA\,\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ with $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ if not explicitly stated otherwise.

Definition 8. *Let* ***sem*** *be an argumentation semantics and* π *be a proposition. A partial world* s *is said to be* ***sem-sufficient*** *for* π *if* $ABA\,\mathcal{F}_\omega \vdash_{sem} \pi$ *for any partial world* $\omega \supseteq s$.

Note that if s is sem-sufficient for π then so is any super set of s.

Example 6 (Cont. Example 5). $\{p_1\}$ is cre-sufficient for $\neg\alpha$; while both $\{p_1, \neg p_2\}$ and $\{p_0, p_1, p_2\}$ are ideal-sufficient for $\neg\alpha$.

Definition 9. *Let* ***sem*** *be an argumentation semantics and* π *be a proposition.*

1. *A set* $\mathcal{S}$ *of partial worlds is said to be a* ***sem-frame*** *for* π *if each partial world in* $\mathcal{S}$ *is sem-sufficient for* π.
2. *A sem-frame* $\mathcal{S}$ *for* π *is said to be* ***complete*** *if for each possible world* $\omega \in \mathcal{W}$ *where* $ABA\,\mathcal{F}_\omega \vdash_{sem} \pi$, $\omega \supseteq s$ *for some* $s \in \mathcal{S}$.

Example 7 (Cont. Example 5). For $\neg\alpha$, $\mathcal{S}_1 = \{\{p_1\}\}$ is a complete cre-frame while $\mathcal{S}_2 = \{\{p_1, \neg p_2\}, \{p_0, p_1, p_2\}\}$ is a complete ideal-frame.

Note that there are multiple complete sem-frames for the same proposition. For example, cre-frame $\{\{p_1\}, \{p_1, \neg p_2\}\}$ is also complete for $\neg\alpha$.

Theorem 1 below is at the heart of our inference procedures.

Theorem 1. *If* $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ *is a complete sem-frame for a proposition* π*, then* $Prob_{sem}(\pi) = P(s_1 \vee s_2 \cdots \vee s_n)$.

So, continue Example 7, $Prob_{cr}(\neg\alpha) = P(\{p_1\})$; $Prob_{id}(\neg\alpha) = P(\{p_1, \neg p_2\} \vee \{p_0, p_1, p_2\})$.

4 Computing PABA Semantics: Inference Procedures

In this section we present our inference procedure computing PABA's credulous semantics. As Theorem 1 suggests, computing $Prob_{cr}(\pi)$ could be done via two steps: (1) generating a complete cre-frame $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ for π; and (2) computing $P(s_1 \vee s_2 \cdots \vee s_n)$. To reduce the load in step 2, we would like, in step 1, to arrive at a cre-frame "as small as possible". To this end, we develop the notion of *cre-frame derivation* (Subsect. 4.2), adapting on the notion of AB-dispute derivation of [2,4] for computing ABA's credulous semantics (recalled in Subsect. 4.1), and the notion of base derivation in [9] used to organize search spaces for dispute derivations in AA. In Subsect. 4.3, we shall show that if the given $PABA$ framework is Bayesian (see Definition 3), then existing Bayesian network inference algorithms can be used to compute $P(s_1 \vee s_2 \cdots \vee s_n)$.

4.1 AB-Dispute Derivations

AB-dispute derivations [2,4] simulate a dispute between two fictitious players: proponent and opponent. Formally, a AB-dispute derivation is a sequence of tuples $\langle \mathcal{P}_0, \mathcal{O}_0, A_0, C_0 \rangle \ldots \langle \mathcal{P}_i, \mathcal{O}_i, A_i, C_i \rangle \ldots$, where A_i is the set of defense assumptions (consisting of all assumptions occurring in the proponent's arguments) and C_i is the set of culprits (consisting of all opponent's assumptions that the proponent attacks). Multi-set $\mathcal{P}_i$ consists of propositions belonging to any of the proponent's potential arguments. Multi-set $\mathcal{O}_i$ consists of multi-sets of propositions representing the state of all of the opponent's potential arguments.

Definition 10 *(Modified from [2,4]). An AB-dispute derivation in ABA $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ using a selection strategy sl is a (possibly infinite) sequence of tuples $\langle \mathcal{P}_0, \mathcal{O}_0, A_0, C_0 \rangle, \ldots, \langle \mathcal{P}_i, \mathcal{O}_i, A_i, C_i \rangle, \langle \mathcal{P}_{i+1}, \mathcal{O}_{i+1}, A_{i+1}, C_{i+1} \rangle \ldots$ where*

1. *$\mathcal{P}_i$ is a multi-set of propositions, $\mathcal{O}_i$ is a set of finite multi-set of propositions, and A_i, C_i are set of assumptions.*
2. *For each step $i \geq 0$, selection strategy sl selects a proposition $\sigma \in \mathcal{P}_i$ or $\sigma \in S \in \mathcal{O}_i$, and*
   (a) If $\sigma \in \mathcal{P}_i$ is selected then
      i. if σ is an assumption then $\mathcal{P}_{i+1} = \mathcal{P}_i \setminus \{\sigma\}$ and $\mathcal{O}_{i+1} = \mathcal{O}_i \cup \{\{\overline{\sigma}\}\}$[13]
      ii. If σ is not an assumption, then there exists some rule $\sigma \leftarrow Bd \in \mathcal{R}$ such that $C_i \cap Bd = \emptyset$ and $\mathcal{P}_{i+1} = \mathcal{P}_i \setminus \{\sigma\} \cup (Bd \setminus A_i)$ and $A_{i+1} = A_i \cup (\mathcal{A} \cap Bd)$
   (b) If S is selected in $\mathcal{O}_i$ and σ is selected in S then
      i. If σ is an assumption, then
         A. either σ is ignored, i.e. $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\} \cup \{S \setminus \{\sigma\}\}$
         B. or $\sigma \notin A_i$ and $\sigma \in C_i$ and $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\}$
         C. or $\sigma \notin A_i$ and $\sigma \notin C_i$ and
            (C.1) if $\overline{\sigma}$ is an assumption, then $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\}$ and $A_{i+1} = A_i \cup \{\overline{\sigma}\}$ and $C_{i+1} = C_i \cup \{\sigma\}$
            (C.2) otherwise $\mathcal{P}_{i+1} = \mathcal{P}_i \cup \{\overline{\sigma}\}$ and $\mathcal{O}_{i+1} = \mathcal{O} \setminus \{S\}$ and $C_{i+1} = C_i \cup \{\sigma\}$
      ii. If σ is not an assumption, then $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\} \cup \{S \setminus \{\sigma\} \cup Bd \mid \sigma \leftarrow Bd \in \mathcal{R}$ and $Bd \cap C_i = \emptyset\}$

Definition 11. *1. An AB-dispute derivation **for a proposition** π is such that the first tuple $\langle \mathcal{P}_0, \mathcal{O}_0, A_0, C_0 \rangle = \langle \{\pi\}, \emptyset, \mathcal{A} \cap \{\pi\}, \emptyset \rangle$.*
*2. An AB-dispute derivation is said to be **successful** if it is ended by a tuple $\langle \emptyset, \emptyset, \_, \_ \rangle$.*

Example 8 (Cont. Example 4). Consider $ABA\,\mathcal{F}'$ obtained from $ABA\,\mathcal{F}$ by removing all probabilistic assumptions, i.e. $\mathcal{F}'$ contains $r'_0 : \neg\alpha \leftarrow \alpha \quad r'_1 : \neg\alpha \leftarrow \beta$ $r'_2 : \neg\beta \leftarrow \alpha \quad r'_3 : \neg\gamma \leftarrow \delta$ and $r'_4 : \neg\delta \leftarrow \gamma$. The following table shows a successful AB-dispute derivation for proposition $\neg\alpha$ in $\mathcal{F}'$. Note that in step 5 the proponent reuses $r'_1 : \neg\alpha \leftarrow \beta$ but β is not added into $\mathcal{P}_5$.

i	$\mathcal{P}_i$	$\mathcal{O}_i$	A_i	C_i	By rule (of Definition 10)	Remarks
0	$\{\neg\alpha\}$	$\{\}$	$\{\}$	$\{\}$		Proponent claims
1	$\{\beta\}$	$\{\}$	$\{\beta\}$	$\{\}$	2.a.ii	Proponent uses $r'_1 : \neg\alpha \leftarrow \beta$
2	$\{\}$	$\{\{\neg\beta\}\}$	$\{\beta\}$	$\{\}$	2.a.i	Opponent tries to attack β
3	$\{\}$	$\{\{\alpha\}\}$	$\{\beta\}$	$\{\}$	2.b.ii	Opponent uses $r'_2 : \neg\beta \leftarrow \alpha$
4	$\{\neg\alpha\}$	$\{\}$	$\{\beta\}$	$\{\alpha\}$	2.b.i.C1	Proponent selects α as a culprit
5	$\{\}$	$\{\}$	$\{\beta\}$	$\{\alpha\}$	2.a.ii	Proponent reuses $r'_1 : \neg\alpha \leftarrow \beta$

The following theorem states that AB-dispute derivations are sound for credulous acceptance in any *ABA* framework.

Theorem 2 *(Theorem 4.3 in [2]). If $\langle \mathcal{P}_0, \mathcal{O}_0, A_0, C_0\rangle, \dots, \langle \mathcal{P}_n, \mathcal{O}_n, A_n, C_n\rangle$ is a successful AB-dispute derivation for a proposition π, then A_n is an admissible set of assumptions and supports π.*

In their Theorem 4.4, the authors of [2] show that AB-dispute derivations are not complete in general, but complete for the class of *positively acyclic ABA* frameworks over finite languages. However AB-dispute derivation are indeed complete for a larger class of positively acyclic and *finitary ABA* frameworks.

Definition 12. *Let $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ be an ABA framework.*

1. *$\mathcal{F}$ is said to be **finitary** if for each node in the dependency graph of $\mathcal{F}$, there is a finite number of nodes reachable from it.*
2. *$\mathcal{F}$ is said to be **positively acyclic** if in the dependency graph of $\mathcal{F}$, there is no infinite directed path consisting solely non-assumption nodes.*

Clearly ABA frameworks over finite languages are all finitary but not the reverse. For example, the framework with $\mathcal{R} = \{\neg\alpha_{i+1} \leftarrow \alpha_i \mid i \in \{1, 2, \dots\}\}$ and $\mathcal{A} = \{\alpha_1, \alpha_2, \dots\}$, $\overline{\alpha_i} = \neg\alpha_i$, is finitary but has an infinite language.

Theorem 3 *(Generalizing Theorem 4.4 in [2]). Given a positively acyclic and finitary assumption-based framework $\mathcal{F}$.*

1. *If π is supported by an admissible set S of assumptions, then for any selection strategy there is a successful AB-dispute derivation $\langle \mathcal{P}_0, \mathcal{O}_0, A_0, C_0\rangle, \dots,$ $\langle \mathcal{P}_n, \mathcal{O}_n, A_n, C_n\rangle$ for π where $A_n \subseteq S$.*
2. *There are no infinite AB-dispute derivations for any proposition.*

In non-finitary and/or positively cyclic frameworks, credulously acceptable propositions may not have successful AB-dispute derivations. For example, consider a positively cyclic ABA framework with $\mathcal{R} = \{\neg\alpha \leftarrow \neg\alpha\}$ and $\mathcal{A} = \{\alpha\}$ where $\overline{\alpha} = \neg\alpha$. Clearly α is credulously acceptable but it has no successful AB-dispute derivation. Note that the only AB-dispute derivation for α is

[13] Silence about a component means it remains the same as the previous step. In this case 2.a.i, for example, $A_{i+1} = A_i$ and $C_{i+1} = C_i$.

$\langle\{\alpha\},\emptyset,\{\alpha\},\emptyset\rangle,\langle\emptyset,\{\{\neg\alpha\}\},\{\alpha\},\emptyset\rangle,\langle\emptyset,\{\{\neg\alpha\}\},\{\alpha\},\emptyset\rangle,\ldots$ which is infinite. Similarly, q is credulously acceptable in a non-finitary ABA framework with $\mathcal{R} = \{q \leftarrow \beta\}\cup\{\neg\beta \leftarrow \alpha_i \mid i \in \{1,2,\ldots\}\}\cup\{\neg\alpha_{i+1} \leftarrow \neg\alpha_i \mid i \in \{1,2,\ldots\}\}\cup\{\neg\alpha_1 \leftarrow\}$ and $\mathcal{A} = \{\beta,\alpha_1,\alpha_2,\ldots\}$ where $\overline{x} = \neg x$ for each $x \in \mathcal{A}$. However there are no successful AB-dispute derivation for q.

To facilitate the presentations of our inference procedures in next sections, let $\mathcal{DS}_{\mathcal{F}}(t,sl)$ refer the set of tuples that can *immediately* follow a tuple t of the form $\langle\mathcal{P},\mathcal{O},A,C\rangle$ in some AB-dispute derivation using selection strategy sl. From Definition 10 part 2, $\mathcal{DS}_{\mathcal{F}}(t,sl)$ can be computed by the following procedure.

(a) If sl selects $\sigma \in \mathcal{P}$, then
  i. if σ is an assumption, then $\mathcal{DS}_{\mathcal{F}}(t,sl) = \{\langle\mathcal{P}\setminus\{\sigma\},\mathcal{O}\cup\{\{\overline{\sigma}\}\},A,C\rangle\}$
  ii. if σ is not an assumption, then $\mathcal{DS}_{\mathcal{F}}(t,sl) = \{\langle\mathcal{P}\setminus\{\sigma\}\cup(Bd\setminus A),\mathcal{O},A\cup(\mathcal{A}\cap Bd),C\rangle \mid \sigma \leftarrow Bd \in \mathcal{R}$ and $C\cap Bd = \emptyset\}$

(b) If sl selects $S \in \mathcal{O}$, then $\mathcal{DS}_{\mathcal{F}}(t,sl) = \emptyset$ if $S = \emptyset$. Otherwise, let σ be the sentence selected in S, and
  i. if σ is an assumption, then $\mathcal{DS}_{\mathcal{F}}(t,sl) = \{\langle\mathcal{P},\mathcal{O}\setminus\{S\}\cup\{S\setminus\{\sigma\}\},A,C\rangle\}\cup\delta T$ where δT is computed as follows.
    A. if $\sigma \in A$ then $\delta T = \emptyset$.
    B. if $\sigma \notin A$ and $\sigma \in C$, then $\delta T = \{\langle\mathcal{P},\mathcal{O}\setminus\{S\},A,C\rangle\}$
    C. if $\sigma \notin A$ and $\sigma \notin C$, then
      (C.1) if $\overline{\sigma} \in \mathcal{A}$ then $\delta T = \{\langle\mathcal{P},\mathcal{O}\setminus\{S\},A\cup\{\overline{\sigma}\},\mathcal{C}\cup\{\sigma\}\rangle\}$
      (C.2) otherwise, $\delta T = \{\langle\mathcal{P}\cup\{\overline{\sigma}\},\mathcal{O}\setminus\{S\},A,\mathcal{C}\cup\{\sigma\}\rangle\}$
  ii. if σ is not an assumption, then $\mathcal{DS}_{\mathcal{F}}(t,sl) = \{\langle\mathcal{P},\mathcal{O}',A,C\rangle\}$ where $\mathcal{O}' = \mathcal{O}\setminus\{S\}\cup\{S\setminus\{\sigma\}\cup Bd \mid \sigma \leftarrow Bd$ is a rule in $\mathcal{R}$ s.t. $Bd\cap C = \emptyset\}$

4.2 Cre-Frame Derivation

The following notion of cre-frame derivations extends the notion of AB-dispute derivations to gradually construct complete cre-frames.

Definition 13. *A* ***cre-frame derivation*** *in PABA $\mathcal{P} = (\mathcal{A}_p,\mathcal{R}_p,\mathcal{F})$ using a selection strategy sl is a possibly infinite sequence $\mathcal{T}_0,\mathcal{T}_1,\ldots,\mathcal{T}_i,\ldots$ where*

1. $\mathcal{T}_i$ is a set of pairs of the form (t,ω) where t is a tuple $\langle\mathcal{P},\mathcal{O},A,C\rangle$ as defined in Definition 10 and ω is a partial world.
2. For each i, sl selects one pair $(t,\omega)\in\mathcal{T}_i$ and a proposition σ from the $\mathcal{P}$ component or $\mathcal{O}$ component of t, and $\mathcal{T}_{i+1} = \mathcal{T}_i\setminus\{(t,\omega)\}\cup\Delta\mathcal{T}$ where
  (a) If σ is a probabilistic assumption not occurring in ω[14], then $\Delta\mathcal{T} = \{(t,\omega\cup\{\sigma\}),(t,\omega\cup\{\neg\sigma\})\}$
  (b) Otherwise $\Delta\mathcal{T} = \{(t',\omega) \mid t' \in \mathcal{DS}_{\mathcal{F}_\omega}(t,sl)\}$.

Definition 14. *1. A cre-frame derivation* ***for a proposition*** *π is a cre-frame derivation that begins with $\mathcal{T}_0 = \{(\langle\{\pi\},\emptyset,\mathcal{A}\cap\{\pi\},\emptyset\rangle,\emptyset)\}$[15].*

[14] That is, neither σ nor its complement are elements of ω.
[15] $\mathcal{A}$ is the set of assumptions in $ABA\,\mathcal{F}$.

2. A finite cre-frame derivation $\mathcal{T}_0, \ldots, \mathcal{T}_n$ is said to be ***full*** *if it can not be extended further, or equivalently $\mathcal{T}_n$ contains only pairs of the form $(\langle\emptyset, \emptyset, \_, \_\rangle, \_)$. The set $\{\omega \mid (\langle\emptyset, \emptyset, \_, \_\rangle, \omega) \in \mathcal{T}_n\}$ is called the* ***derived frame****.*

Example 9 (Cont. Example 4). A full cre-frame derivation for $\neg\alpha$ is given in the next page. Note that notation $\underline{x}$ means that x is selected by selection strategy.

Theorem 4 below says that cre-frame derivations provide a sound procedure for generating complete cre-frames.

Theorem 4. *If $\mathcal{D}$ is a full cre-frame derivation for a proposition π, then the frame derived by $\mathcal{D}$ is a complete cre-frame for π.*

So, continue Example 9, Theorem 4 says that $\{\{p_1\}, \{p_1, \neg p_2\}\}$ is a complete cre-frame for $\neg\alpha$. Theorem 5 below says that cre-frame derivations provide a terminating procedure for generating complete cre-frames in a general class of PABA frameworks.

Theorem 5. *In an PABA $\mathcal{P} = (\mathcal{R}_p, \mathcal{A}_p, \mathcal{F})$ where $\mathcal{F}$ is positively acyclic and finitary, there are no infinite cre-frame derivations for any proposition.*

	(t, ω)		By rule
	t	ω	
$\mathcal{T}_0$	$\langle\{\neg\alpha\}, \emptyset, \emptyset, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_1$	$\langle\{\underline{\alpha}, p_0\}, \emptyset, \{\alpha\}, \emptyset\rangle$	$\emptyset$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_2$	$\langle\{p_0\}, \{\{\underline{\neg\alpha}\}\}, \{\alpha\}, \emptyset\rangle$	$\emptyset$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_3$	$\langle\{p_0\}, \{\{\underline{\alpha}, p_0\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\emptyset$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_4$	$\langle\{\underline{p_0}\}, \{\{p_0\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\emptyset$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_5$	$\langle\{\underline{p_0}\}, \{\{p_0\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\{\neg p_0\}$	2.a
	$\langle\{p_0\}, \{\{p_0\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\{p_0\}$	
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_6$	$\langle\{\underline{p_0}\}, \{\{p_0\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\{p_0\}$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_7$	$\langle\emptyset, \{\{\underline{p_0}\}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\{p_0\}$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_8$	$\langle\emptyset, \{\underline{\{\,\}}, \{\beta, p_1\}\}, \{\alpha\}, \emptyset\rangle$	$\{p_0\}$	2.b
	$\langle\{\beta, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	
$\mathcal{T}_9$	$\langle\{\underline{\beta}, p_1\}, \emptyset, \{\beta\}, \emptyset\rangle$	$\emptyset$	2.b
$\mathcal{T}_{10}$	$\langle\{\underline{p_1}\}, \{\{\neg\beta\}\}, \{\beta\}, \emptyset\rangle$	$\emptyset$	2.b
$\mathcal{T}_{11}$	$\langle\{\underline{p_1}\}, \{\{\neg\beta\}\}, \{\beta\}, \emptyset\rangle$	$\{\neg p_1\}$	2.a
	$\langle\{p_1\}, \{\{\neg\beta\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1\}$	
$\mathcal{T}_{12}$	$\langle\{\underline{p_1}\}, \{\{\neg\beta\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1\}$	2.b
$\mathcal{T}_{13}$	$\langle\emptyset, \{\{\underline{\neg\beta}\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1\}$	2.b
$\mathcal{T}_{14}$	$\langle\emptyset, \{\{\underline{\alpha}, p_2\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1\}$	2.b
$\mathcal{T}_{15}$	$\langle\emptyset, \{\{\underline{p_2}\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1\}$	2.b
	$\langle\{\neg\alpha\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{16}$	$\langle\emptyset, \{\{\underline{p_2}\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.a
	$\langle\emptyset, \{\{p_2\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1, p_2\}$	
	$\langle\{\neg\alpha\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{17}$	$\langle\emptyset, \emptyset, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.b
	$\langle\emptyset, \{\{\underline{p_2}\}\}, \{\beta\}, \emptyset\rangle$	$\{p_1, p_2\}$	
	$\langle\{\neg\alpha\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{18}$	$\langle\emptyset, \emptyset, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.b
	$\langle\emptyset, \{\underline{\{\,\}}\}, \{\beta\}, \emptyset\rangle$	$\{p_1, p_2\}$	
	$\langle\{\neg\alpha\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{19}$	$\langle\emptyset, \emptyset, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.b
	$\langle\{\underline{\neg\alpha}\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{20}$	$\langle\emptyset, \emptyset, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.b
	$\langle\{\underline{p_1}\}, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	
$\mathcal{T}_{21}$	$\langle\emptyset, \emptyset, \{\beta\}, \emptyset\rangle$	$\{p_1, \neg p_2\}$	2.b
	$\langle\emptyset, \emptyset, \{\beta\}, \{\alpha\}\rangle$	$\{p_1\}$	

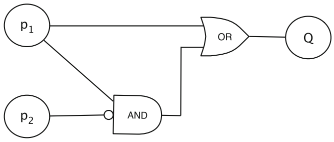
4.3 Computing $P(s_1 \vee \cdots \vee S_n)$

For a set of partial worlds $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$, let $NDF_{\mathcal{S}}$ denote to the DNF formula $\bigvee_{i=1}^{n} \bigwedge_{j=1}^{|s_i|} p_{ij}$ where $p_{ij} \in s_i$. For example, if $\mathcal{S} = \{\{p_1\}, \{p_1, \neg p_2\}\}$ then $NDF_{\mathcal{S}} = p_1 \vee (p_1 \wedge \neg p_2)$.

Lemma 2. *Let $\mathcal{P}$ be an PABA including Bayesian network $\mathcal{N}$. If $\mathcal{S} = \{s_1, \ldots, s_n\}$ is a set of partial worlds of $\mathcal{P}$, then $P(s_1 \vee s_2 \cdots \vee s_n) = Pr_{\mathcal{N}}(NDF_{\mathcal{S}})$ where $Pr_{\mathcal{N}}$ is the probability distribution defined by $\mathcal{N}$.*

Continue Example 4, the lemma says that $P(\{p_1\} \vee \{p_1, \neg p_2\}) = Pr_{\mathcal{N}}(p_1 \vee (p_1 \wedge \neg p_2))$.

So to compute $P(s_1 \vee s_2 \cdots \vee s_n)$, one can compute $Pr_{\mathcal{N}}(NDF_{\mathcal{S}})$ instead using any BN inference algorithms. Doing so on $\mathcal{N}$, one need to translate the query into standard queries on $\mathcal{N}$ using the inclusion-exclusion rule. For example $Pr_{\mathcal{N}}(p_1 \vee (p_1 \wedge \neg p_2)) = Pr_{\mathcal{N}}(p_1) + Pr_{\mathcal{N}}(p_1 \wedge \neg p_2) - Pr_{\mathcal{N}}(p_1 \wedge p_1 \wedge \neg p_2)$. Alternatively, one could construct a new network $\mathcal{N}_{\mathcal{S}}$ by adding to $\mathcal{N}$, for each $i \in \{1, \ldots n\}$, one AND gate to compute $\bigwedge_{j=1}^{|s_i|} p_{ij}$, and one OR gate to compute $\bigvee_{i=1}^{n} \bigwedge_{j=1}^{|s_i|} p_{ij}$ from the output of n AND gates. For example, if $\mathcal{S} = \{\{p_1\} \vee \{p_1, \neg p_2\}\}$, then $\mathcal{N}_{\mathcal{S}}$ contains the following new nodes and edges. Clearly $Pr_{\mathcal{N}}(NDF_{\mathcal{S}})$ equals $Pr_{\mathcal{N}_{\mathcal{S}}}(Q)$, where Q the child of the OR gate.



5 Related Work and Conclusions

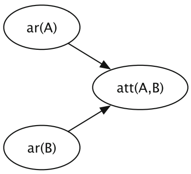
One of the early known models of Probabilistic Argumentation (PA) is the one of Dung and Thang [5] (DT's PAA) defined as a triple $(\mathcal{F}, \mathcal{W}, P)$ where $\mathcal{F} = (AR, Att)$ is a standard AA framework, $\mathcal{W}$ is a set of possible worlds such that each $\omega \in \mathcal{W}$ defines a set of arguments $AR_\omega \subseteq AR$, and P is a probability distribution over $\mathcal{W}$. DT's PAA defines the grounded probability of argument A by the sum of probabilities of possible worlds in which A is groundedly accepted[16], but, following AA's style, abstracts from: (1) the representation of possible world; (2) the construction of AR_ω for each possible world ω; and (3) the representation and computation of $P(\omega)$. So the authors in the same work [5] combine DT's PAA with Assumption-based Argumentation (ABA [2,4]) to introduce PABA where: (1) a possible world represented by a conjunction of so-called probabilistic assumptions; (2) arguments of AR_ω are constructed from inference rules and assumptions (as in ABA) together with facts representing the occurrences of probabilistic assumptions in ω; and (3) $P(\omega)$ is represented by means of so-called probabilistic rules and computed by grounded semantics. Inference procedures for PABA, however, remain unexplored. To our best knowledge, our work is the first developing PABA inference procedures but there have been many works done on other models of PA, focusing on different computational issues. For example, Li et al. in [8] use a Monte-Carlo simulation to *approximate* the probability of a set of arguments consistent with an

[16] That is, $Prob_{gr}(A) \triangleq \sum_{\omega \in \mathcal{W}: (AR_\omega, Att \cap (AR_\omega \times AR_\omega)) \vdash_{gr} A} P(\omega)$.

argumentation semantics in their model of Probabilistic Abstract Argumentation (Li's PAA). The complexity of this problem is recently investigated in [6]. In [1], Doder and Woltran translates Li's PAA frameworks into formulas in a probabilistic logic, so obtaining, as a by-product, a schematic way to compute the above probability precisely using solvers for probabilistic logic. However the authors did not explore this direction further to develop inference procedures for Li's PAA. Interestingly, to use our inference procedures for computing Li's PAA, we just need a simple translator as follows. Consider a Li's PAA framework $(\mathcal{F}, P_{AR}, P_{Att})$. Recall that $\mathcal{F} = (AR, Att)$ is a standard AA framework, $P_{AR} : AR \rightarrow [0, 1]$ and $P_{Att} : Att \rightarrow [0, 1]$ are probability distributions over AR and Att. $P_{AR}(A)$ is interpreted as the probability of the event that A actually occurs as an argument (denoted $ar(A)$); and $P_{Att}((A, B))$ is interpreted as the probability of the event that A attacks B (denoted $att(A, B)$), conditional to a joint event $ar(A) \wedge ar(B)$. Events in $\{ar(A) \mid A \in AR\}$ are assumed to be pair-wise independent, and $att(A, B)$ is assumed to depend only on $ar(A)$ and $ar(B)$. In other words, the probability distribution over all events $\{ar(A) \mid A \in AR\} \cup \{att(A, B) \mid (A, B) \in Att\}$ is defined by a Bayesian network of the pattern in the next page. A possible world is an AA framework (AR', Att') where $AR' \subseteq AR$ and $Att' \subseteq Att \cap (AR' \times AR')$ with probability of the join event $\bigwedge_{A \in AR'} ar(A) \quad \bigwedge_{A \in AR \setminus AR'} \neg ar(A) \quad \bigwedge_{(A,B) \in Att'} att(A, B) \quad \bigwedge_{(A,B) \in Att \setminus Att'} \neg att(A, B)$.

ar(A)	¬ ar(A)
$P_{AR}(A)$	$1\text{-}P_{AR}(A)$

ar(B)	¬ar(B)
$P_{AR}(B)$	$1\text{-}P_{AR}(B)$



		att(A,B)	¬ att(A,B)
ar(A)	ar(B)	$P_{Att}((A,B))$	$1 - P_{Att}((A,B))$
ar(A)	¬ar(B)	0	1
¬ar(A)	ar(B)	0	1
¬ar(A)	¬ar(B)	0	1

Finally, the semantics of Li's PAA is as follows: the probability that argument A is accepted equals the sum of probabilities of possible worlds in which A is accepted. So, in translating a Li's PAA framework $(\mathcal{F}, P_{AR}, P_{Att})$ into an PABA framework $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, (\mathcal{R}, \mathcal{A}, \overline{}))$, we would like $\mathcal{P}$ to be Bayesian and contains an assumption A, for each argument $A \in AR$, such that the probability of the acceptability of assumption A in $\mathcal{P}$ equals the probability that argument A is accepted according to Li's PAA semantics. The readers can easily verify that $\mathcal{P}$ can be defined as follows: $\mathcal{A} = \{A \mid A \in AR\}$ with $\overline{A} = \neg A$; $\mathcal{R} = \{\neg A \leftarrow \neg ar(A) \mid A \in AR\} \cup \{\neg B \leftarrow A, att(A, B) \mid (A, B) \in Att\}$; $\mathcal{A}_p = \{ar(A) \mid A \in AR\} \cup \{att(A, B) \mid (A, B) \in Att\}$ and $\mathcal{R}_p$ represent the described Bayesian network. Readers can also simplify this translation for subclasses of Li's PAA frameworks such as those in [7] where attacks are all certain given the presences of involved arguments (i.e. $P_{Att}((A, B)) = 1$ for any $(A, B) \in Att$).

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