

# A Multi-objective Time Segmentation Approach for Power Generation and Transmission Models

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**Abstract** The complexity of large-scale power system models often necessitates the choice of a suitable temporal resolution. Nowadays, mainly simple heuristic approaches are used. An adequate decision support related to power generation and transmission optimisation in systems with a high RES share, however, requires preserving the complex intra-period and intra-regional links within and between the volatile electricity demand and supply profiles. Focussing on power systems operation, we are able to show that even an amount of less than 300 time segments may be sufficient for the modelling of a whole year, if chosen carefully.

## 1 Introduction

The rapid expansion of renewable energy sources (RES) necessitates an extensive structural rearrangement of the power system. The power grid plays a key role in this context. In order to provide valuable decision support for power systems operation and analyse grid utilisation, especially during times of peak load or generation, models are needed which are able to consider a high regional and temporal input data granularity. This requirement inevitably leads to a target conflict between model complexity and computational intensity on the one hand and model accuracy on the other hand. When simplifying the representation of time, it is therefore crucial to minimise the loss of relevant information. In particular, in case of a combined consideration of power generation and transmission in systems with a high RES share, it is important to preserve the complex intra-period and intra-regional links within and between the volatile electricity demand and supply profiles. So far, however, mainly simple heuristic approaches are used (see e.g., [2–5]).

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We therefore propose a multi-objective optimisation approach for time segment selection in Sect. 2 allowing for an explicit analysis of the sensitivity of the temporal structure. Focussing on operating decisions in this paper, we present selected results in Sect. 3 and show that even an amount of less than 300 time segments may be sufficient for the modelling of a whole year. In Sect. 4, we conclude and indicate needs for future research.

## 2 The Multi-objective Approach for Time Segment Selection

The developed approach is aimed at representing the nodal profiles of electricity demand and supply-dependent volatile RES generation of a specific year through a subset of the initial hourly time structure. Preserving the complex intra-period and intra-regional links within and between the volatile electricity demand and supply profiles is a major challenge. The time segment selection is therefore based on the solution of a two-step multi-objective integer optimisation problem.

An adequate generation and transmission optimisation within load flow constrained power system models is basically determined through local or global bottlenecks. In consequence, the trade-off between selecting the critical hours for the power grid usage and the typical periods for the unit dispatch constitutes a serious challenge, especially as the critical hours are not known in advance. In the following, we therefore choose extreme combinations of load, wind and solar photovoltaic energy as constraints for the time segment selection. This approach is based on [1], where the eight possible combinations of high and low electricity demand and feed-in from wind and photovoltaic energy are used to define critical situations for grid utilisation. For the following selection of time slices, the “*lwp*”-(load-wind-pv) constraint requires that at least one time segment in the reduced set is an element of the critical “*lwp*” sets ( $S^{lwp}$ ).

As mentioned above, our overall target is to provide a time segment selection which accounts for both, typical and extreme demand and supply profiles on a nodal and global level and thus minimises the hourly deviations between the original and reduced profiles. In Sect. 2.1, we therefore introduce our grid impact based error measure. In Sect. 2.2, we subsequently describe the first step of our two-step approach, i.e. the selection of ‘typical’ days by solving a multi-objective binary clustering problem. In Sect. 2.3, we describe the second step of our two-step approach, i.e. the intraday time slice reduction by means of constraint programming.

The implementation of the developed time segment selection approach is based on MATLAB and GAMS. MATLAB is used for the data handling and preprocessing, such as the calculation of the clustering distances and the critical sets as well as for the calculation of the initial MIP start solutions, based on a k-Means clustering. The multi-objective binary clustering problem is implemented in GAMS and solved with CPLEX.

### 2.1 A Grid Impact Based Error Measure

We evaluate the reduction of a time series to its characteristic values based on the resulting error’s impact on the solution space. In direct current optimal power flow approaches (DC-OPF), the solution space is determined by the load flow equations:

$$P_f = \Phi \cdot P_{inj} , \tag{1}$$

where the relation between the bus injection  $P_{inj}$  and the branch flow  $P_f$  is determined through the power transfer distribution factor (PTDF) matrix  $\Phi$ . Splitting the bus injection into a variable and fix part by defining the electricity demand and RES-E feed-in as exogenous parameters, and clustering the original right hand side injection vector along the temporal dimension, the impact of the resulting deviation  $E$  of the underlying values to their representative within the cluster on the original solution can be expressed by:

$$P_f - \Phi \cdot P_{inj}^{var} = \Phi \cdot P_{inj}^{fix} + \Phi \cdot E . \tag{2}$$

The impact of a clustering policy on the solution space of the load flow equations is therefore given by the product of the cluster distance of the right hand side parameters of a specific hour with the PTDF matrix. Reducing the resulting  $(L \times 1)$  vector, where  $L$  corresponds to the number of branches  $l (1 \leq l \leq L)$ , by the  $L_2$  norm, we can define the single hour distance  $c$  for a deviation from the exogenous bus injection:

$$c = \left( \sum_{l \in L} (\Phi \cdot E)^2 \right)^{1/2} . \tag{3}$$

### 2.2 Selection of the ‘Typical’ Days (Step 1)

In the first step, the ‘typical’ days are selected based on the clustering of the 365 daily 24-h vectors of a year subject to a minimisation of a distance function. In order to include constraints for the time segment selection, a multi-objective combinatorial optimisation is chosen. Restricting the definition of typical days to elements of the underlying vector set, an optimal clustering for a given cluster number ( $CL^{lim}$ ) may be obtained based on Eqs.(4)–(6)

$$\sum_{\tilde{d} \in D^{SS}(d)} x_{d,\tilde{d}} = 1 , \tag{4}$$

$$\sum_{\tilde{d} \in D} x_{d,\tilde{d}} \leq M \cdot x_{\tilde{d}} , \tag{5}$$

$$\sum_{\tilde{d} \in D} x_{\tilde{d}} \leq CL^{lim} , \tag{6}$$

where  $x_{\tilde{d}} \in \{0, 1\}$ ,  $x_{d,\tilde{d}} \in \{0, 1\}$  and  $x_{d,\tilde{d}}$  denotes the binary clustering decision of representing day  $d$  through the profile of  $\tilde{d}$  under a certain mapping policy.<sup>1</sup> Demanding that each day is assigned to exactly one typical day (Eq. 4), the selection of the typical  $x_{\tilde{d}}$  is defined by the Big-M method (Eq. 5). The implementation of the preliminary discussed constraints of the time segment selection is achieved by demanding the critical sets  $S^{lvp}$  to be nonempty under the current clustering decision:

$$\sum_{\tilde{d} \in S^{lvp}(\tilde{d})} x_{\tilde{d}} \geq 1. \tag{7}$$

For handling the multiple objectives of selecting ‘typical’ time slices for every energy conversion technology and energy consumption profile on a nodal and global basis, the PTDF based error measure defined above is used. Basically, this approach allows reducing the multiple objectives to a single distance which captures the impact of clustering the residual load bus injection vector from a flow based point of view. In order to avoid a balancing between the different bus injection types (load, wind and photovoltaic energy), which may be undesirable in the further model-based processing, we define the 24h clustering distance for all target dimensions  $\tau \in \{load, wind, pv, residual\}$  as follows:

$$c_{\tau,d,\tilde{d}} = \sqrt{\sum_{h \in \{1, \dots, 24\}} \sum_{l \in L} \left( \sum_{n \in N} \phi_{n,l} \cdot (v_{n,\tau,d,h} - v'_{n,\tau,\tilde{d},h}) \right)^2}, \tag{8}$$

where  $v$  and  $v'$  are the hourly nodal profile vectors defined over the set of 24-h vectors of an underlying year and  $\phi_{n,l}$  is the power transfer distribution factor, determining the impact of an injection at bus  $n$  on the flow over branch  $l$ . Based on the reduced coefficient matrix of the clustering distance and the binary decision variable  $x_{d,\tilde{d}}$ , we obtain the following clustering costs:

$$\gamma_{\tau} = \sum_{d,\tilde{d}} c_{\tau,d,\tilde{d}} \cdot x_{d,\tilde{d}}. \tag{9}$$

For an optimisation of the remaining multiple dimensions of the clustering cost a general goal programming formulation is chosen:

$$\min_{z^{L_1}, z^{L_{\infty}}} z = \alpha \cdot z^{L_1} + (1 - \alpha) \cdot z^{L_{\infty}}, \tag{10}$$

$$\text{with } z^{L_1} = \sum_{\tau \in \{load, wind, pv, residual\}} \omega_{\tau} \cdot \frac{p_{\tau}}{\gamma_{\tau}^{min}}, \tag{11}$$

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<sup>1</sup>Restricting the mapping to a subset of days  $D^{SS}(d)$  may be desirable, e.g., to avoid a mapping of profiles from working days to weekends.

$$z^{L_\infty} \geq \omega_\tau \cdot \frac{p_\tau}{\gamma_\tau^{min}}, \tag{12}$$

$$\gamma_\tau^{min} = \gamma_\tau + p_\tau - n_\tau, \tag{13}$$

where  $z^{L_1}$  and  $z^{L_\infty}$  represent the weighted positive percentage deviations from the minimal cost targets  $\gamma_\tau^{min}$  based on the  $L_1$  and  $L_\infty$  norm for a multi-objective optimisation with a focus on an efficient or balanced solution, respectively.

### 2.3 Intraday Time Slice Reduction (Step 2)

A further reduction of the time slice number on an intraday basis is similarly modelled to the previous clustering of the daily 24-h vectors. Restricting the definition of the optimal time slices of the previously selected typical days to the underlying 24 h of a day and the set of possible aggregations of subsequent hours on a two hour level or three hour level (during the night), an optimal aggregation of time slices for a given upper limit ( $TS^{lim}$ ) may be obtained by:

$$\sum_{ts \in TS(ts,h)} x_{\tilde{d},ts} = 1 \quad \forall h, \tag{14}$$

$$\sum_{ts \in TS(ts,h)} \sum_{h \in TS(ts,h)} x_{\tilde{d},ts} = 24 \quad \forall h, \tag{15}$$

$$\sum_{\tilde{d} \in \tilde{D}} \sum_{ts \in TS(ts,h)} x_{\tilde{d},ts} \leq TS^{lim}, \tag{16}$$

with  $x_{\tilde{d},ts} \in \{0, 1\}$ ,  $TS = \{ts_1, \dots, ts_{54}\}$  and

$$\begin{cases} ts_1 = \{h_1\}, & \dots & ts_{24} = \{h_{24}\} & \text{for 1-hour intervals,} \\ ts_{25} = \{h_1, h_2\}, & \dots & ts_{47} = \{h_{23}, h_{24}\} & \text{for 2-hour intervals,} \\ ts_{48} = \{h_1, h_2, h_3\}, & \dots & ts_{54} = \{h_{22}, h_{23}, h_{24}\} & \text{for 3-hour intervals,} \end{cases} \tag{17}$$

where  $x_{\tilde{d},ts}$  denotes the binary decision of clustering the corresponding hours of the previously selected optimal typical day  $\tilde{d} \in \tilde{D}$ . Similar to the clustering in step 1, Eq. (14) defines that each underlying value is assigned to exactly one cluster, while Eq. (15) demands that the combinations of the time slices need to represent the 24 h of a day. Analogously to step 1, the constraints of the time segment selection are defined by requiring that the critical sets  $S^{lwp}$  should be nonempty under the current clustering decision:

$$\sum_{(\tilde{d},ts) \in S^{lwp}(\tilde{d},ts)} x_{\tilde{d},ts} \geq 1. \tag{18}$$

Given a known clustering policy of 24-h vectors  $\tilde{D}(d, \tilde{d})$ , the distance function for the clustering of time slices is defined as follows:

$$c_{\tau, \tilde{d}, ts} = \sqrt{\sum_{d \in \tilde{D}(d, \tilde{d})} \sum_{h \in TS(ts, h)} \sum_{l \in L} \left( \sum_{n \in N} \phi_{n, l} \cdot (v_{n, \tau, d, h} - v'_{n, \tau, \tilde{d}, h}) \right)^2}. \quad (19)$$

Based on the reduced coefficient matrix of the clustering distance and the binary decision variable  $x_{\tilde{d}, ts}$ , we obtain the following clustering costs:

$$\gamma_{\tau} = \sum_{\tilde{d} \in \tilde{D}, ts \in TS} c_{\tau, \tilde{d}, ts} \cdot x_{\tilde{d}, ts}. \quad (20)$$

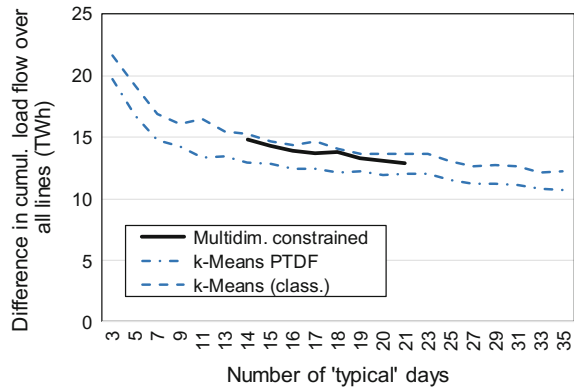
For an optimisation of the remaining multiple dimensions of clustering costs, a constraint programming formulation is chosen, with the goal of finding the efficient supported solutions. Due to computational efficiency, the four target dimensions (load, wind energy, pv, residual load) are reduced to two target dimensions based on the  $L_1$  norm (efficiency objective) and  $L_{\infty}$  norm (balancing objective). The algorithmic implementation of the first phase of the two phase method corresponds to [6]. A detailed explanation is therefore omitted.

### 3 Selected Results

A time segment selection based on simulated transmission grid injection data of 2012 shows that the right hand side error of the load flow restriction (Eq. 2) becomes rather insensitive above a certain temporal resolution. An intraday reduction of the time structure showed no significant increase of the error in the range of 14 to 21 typical days illustrating that the marginal gain of additional time segments decreases after a number of approximately 300 time segments (see Fig. 1).

Obviously, an even lower potential load flow error could be achieved with the same temporal resolution, in the event that the multi-objective nature of the problem or the need of including extreme situations could be ignored. In this case, a less advanced and quicker clustering technique, such as a k-Means clustering approach could be utilised. In our application, however, the multi-objective nature and extreme situations cannot be ignored. Nevertheless, Fig. 1 visualises the impact of the proposed new PTDF-based distance measure. Despite the higher degree of freedom in optimisation, a k-Means clustering is not able to outperform the proposed multi-dimensional approach for time slice selection if the proposed PTDF-based distance measure is not applied.

**Fig. 1** Development of the difference in cumulative load flow over all lines for different numbers of typical days: comparison of our approach ('Multidim. constrained') with two k-Means variants



## 4 Conclusions and Outlook

We proposed a structured, multi-objective approach for time segment selection to handle the complexity of large-scale power system models. In this paper, we focussed on time segment selection for power systems operation optimisation with a special emphasis on power grid utilisation. Our approach is therefore aimed at preserving both, intra-period and intra-regional links within and between the volatile electricity supply and demand profiles. We could show that even an amount of less than 300 time segments may be sufficient for the modelling of a whole year, if chosen carefully. Future enhancements of the approach should include the extension to power generation and transmission expansion planning problems, simulation-based sensitivity analyses, accounting for variations between the importance of load, wind and solar energy profiles respectively, as well as further validation studies for a series of realistic energy economic problems.

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