

Chapter 7

Zero-Error Accessible Information of a Quantum Source

A *quantum source* is an essential component of quantum communication system because it corresponds to the set of quantum symbols that will be used to encode classical messages. In this encoding process, there is a bijective mapping between messages and quantum states, but each quantum state is associated with a certain probability. Differently from classical messages that are completely distinguishable, quantum states may not necessarily be so. A consequence is that the classical information encoded by the quantum source may not be fully recoverable after a measurement.

Considering this intrinsic difficulty to recover information from quantum sources, an information measure, called *accessible information*, has been proposed in the literature [12, Sect. 12.1]. It establishes the maximum amount of classical information that can be retrieved after being encoded by a quantum source. Because all measurement strategies can be used to retrieve information from the quantum system, calculating the accessible information is a hard task in general. Fortunately, there are some useful upper and lower bounds that are easiest to calculate and that give good estimates for the accessible information [3, 4, 7, 8, 16].

Aiming at avoiding errors in decoding messages from quantum sources, this chapter presents some recent results regarding an information measure for quantum sources, called *Zero-Error Accessible Information (ZEAI)*. This quantity represents the maximum amounts of bits per symbol that can be retrieved from a quantum source with no decoding errors. The ZEAI of a quantum source unifies concepts from quantum sources, accessible information, classical zero-error information theory and also from graph theory.

To introduce these results, this chapter is organized as follows. In Sect. 7.1 we revisit some fundamental concepts such as the formal definition of a memoryless quantum source, its entropy, accessible information, and the Holevo bound, which is an upper bound for accessible information. Section 7.2 introduces the ZEAI of a quantum source and its relation with classical zero-error channels. The relation

between ZEI and graph theory is elucidated in Sect. 7.3. After that, some detailed examples are given in Sect. 7.4. The relation of ZEI and other works in the literature is described in Sect. 7.5.

7.1 Accessible Information of Quantum Sources

To study accessible information in the quantum information theory domain, we take into account the canonical communication scheme shown in Fig. 7.1. The quantum source encodes classical messages in quantum states as described in Definition 7.1.

Definition 7.1 (Memoryless Quantum Source). Let $\mathcal{A} = \{0, \dots, \ell\}$ be a set of classical messages. A memoryless quantum source is a device that prepares quantum states according to an ensemble $\{\rho_i, p_i\}$. The set $\mathcal{S} = \{\rho_0, \dots, \rho_\ell\}$ is denoted the *source alphabet*, commonly composed of pure non-orthogonal quantum states ρ_i , called *quantum letters*. The quantum letter ρ_i is associated with the classical message i . The quantum source outputs a letter ρ_i with probability p_i , where $\sum_{i=0}^{\ell} p_i = 1$. For a given sequence a of classical messages, $a = a_1 a_2 \dots a_n$, $a \in \mathcal{A}^n$, the corresponding quantum state prepared by the quantum source, called *quantum codeword*, is given by the tensor product of the corresponding quantum letters, i.e.,

$$\rho(a) = \rho_{a_1} \otimes \rho_{a_2} \otimes \dots \otimes \rho_{a_n}. \quad (7.1)$$

According to Definition 2.7, the ensemble $\{\rho_i, p_i\}$ of a quantum source can also be represented by the corresponding density operator

$$\rho = \sum_{i=0}^{\ell} p_i \rho_i. \quad (7.2)$$

Taking into account such density operator for a quantum source, we can introduce the *entropy of a quantum source*.

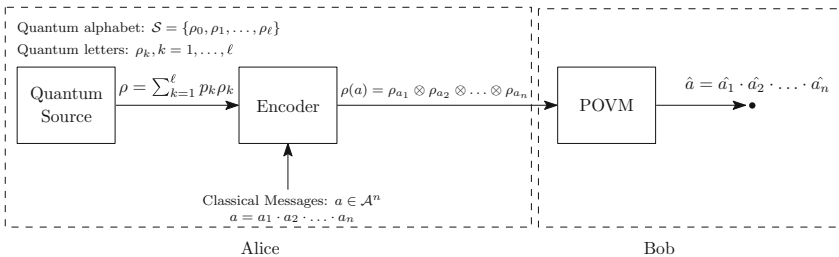


Fig. 7.1 Canonical communication model in which there is a single sender and a single receiver

Definition 7.2 (Entropy of a Quantum Source). The entropy of a quantum source is the von Neumann entropy of the density operator (7.2) that describes the ensemble $\{\rho_i, p_i\}$, i.e.,

$$S(\rho) = -\text{Tr } \rho \log \rho. \quad (7.3)$$

We assume that Alice has a quantum source with the given description and that she prepares the quantum state ρ . Alice gives such quantum state to Bob, who can adopt a POVM measurement scheme aiming at identifying the corresponding message sent by Alice. The measurement outputs are arguments for a decoding function. The decoder must decide which classical message was originally sent by Alice.

A quantum source is *purely classical* if the corresponding source alphabet \mathcal{S} is composed of pairwise orthogonal quantum states, since such states are completely distinguishable at the receiver's end. If the set \mathcal{S} contains nonorthogonal quantum states, then there is no measurement strategy that can extract all the information about the quantum source. A third situation considers that the states of the quantum source are nonorthogonal but with commuting density matrices. In this case we have a *broadcast source* in which given two quantum systems that are not a copy of the source, their partial trace results in the state of the quantum source [2].

We recall the accessible information, presented in Definition 3.18. It is a measure of how well one can infer the state prepared by the source by measuring its output. In quantum information theory, there is no general method to calculate the accessible information of a quantum source. However, several lower and upper bounds for the accessible information were demonstrated; the most important is the *Holevo bound* [12, Chap. 12].

Theorem 7.1 (Holevo Bound [7]). Suppose that Alice has a quantum source F with ensemble $\{\rho_i, p_i\}$. She sends Bob some quantum letters emitted by F . Bob measures the received quantum letters with a POVM $\{M_i\}_{i=0}^m$ and obtains B . The Holevo bound states that for any measurement scheme used by Bob, we have that

$$I_{acc}(F) \leq S(\rho) - \sum_{k=0}^{\ell} p_k S(\rho_k), \quad (7.4)$$

where ρ is the density operator given by (7.2).

The right side of (7.4) is known as the Holevo quantity and it is denoted by χ . Taking into account the concavity of the entropy, we have $I(A; B) \leq \chi \leq H(A)$.

Example 7.1 (Holevo Bound). Suppose that Alice has a quantum source F that emits the states ρ_0 and ρ_1 according to a uniform distribution, where $|\psi_0\rangle = |0\rangle$, $|\psi_1\rangle = \cos \theta |0\rangle + \sin \theta |1\rangle$, $\rho_0 = |\psi_0\rangle \langle \psi_0|$, and $\rho_1 = |\psi_1\rangle \langle \psi_1|$. From Example 3.17, we know that $S(\rho) = H(\frac{1}{2} - \frac{\cos \theta}{2})$. Remember that ρ_0 and ρ_1 are pure states. Again, consider that A is the index of the state emitted by the source and B corresponds to the measurement result. The best measurement strategy to

discriminate between ρ_0 and ρ_1 is given by the projective measurement $\mathcal{M} = \{E_0 = |w_0\rangle\langle w_0|, E_1 = |w_1\rangle\langle w_1|\}$, where

$$|w_0\rangle = \cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)|1\rangle, \quad (7.5)$$

$$|w_1\rangle = \cos\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)|0\rangle + \sin\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)|1\rangle. \quad (7.6)$$

Notice that $\langle w_0|w_1\rangle = 0$ and that $E_0 + E_1 = \mathbb{1}$. The conditional probability that $B = b$ given that $A = a$, for $a, b \in \{0, 1\}$, is calculated as

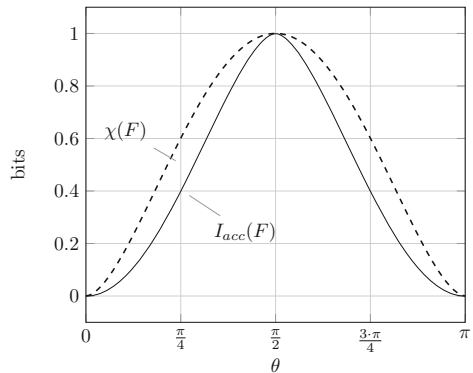
$$p(b|a) = \text{Tr}(E_b |\psi_a\rangle\langle\psi_a|) = |\langle w_b|\psi_a\rangle|^2 = \begin{cases} \frac{1+\sin\theta}{2} & \text{if } a = b \\ \frac{1-\sin\theta}{2} & \text{if } a \neq b \end{cases} \quad (7.7)$$

It is possible to see that we can make an analogy with a binary symmetric channel, where A is the input, B is the output, and the error probability is $\frac{1-\sin\theta}{2}$ (see Example 3.8). The random variable A is uniformly distributed at the channel input and the random variable B is also uniformly distributed. Then, $H(A) = H(B) = 1$. The mutual information is given by

$$\begin{aligned} I(X; Y) &= H(Y) - H(Y|X) \\ &= 1 - H\left(\frac{1 - \sin\theta}{2}\right). \end{aligned} \quad (7.8)$$

Since the measurement scheme is proven to be optimal [1], we have that $I_{\text{acc}}(F) = 1 - H\left(\frac{1-\sin\theta}{2}\right)$. Figure 7.2 shows plots of accessible information $I_{\text{acc}}(F)$ and Holevo quantity $\chi(F)$ versus the parameter θ . For any $0 \leq \theta \leq \pi$, we see that $I_{\text{acc}}(F) \leq \chi(F)$, with equality when $\theta = \pi/2$, i.e., when the states ρ_0 and ρ_1 are completely distinguishable.

Fig. 7.2 Plots of accessible information and Holevo bound for a quantum source F



In the previous example, the parameter $\theta = \pi/2$ led the quantum letters to be orthogonal and the accessible information was maximum. Unfortunately, there are situations where Bob is not able to infer the state given by Alice.

Example 7.2 (Measurement with Inconclusive Results). Suppose that a quantum source can produce two quantum states $|\psi_1\rangle = |0\rangle$ or $|\psi_2\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ with equal probability. Since $\langle\psi_1|\psi_2\rangle \neq 0$, it is not possible to precisely determine what state was emitted by the source. However, it is possible to perform measurements that can distinguish these states most of the time. To do this, define a POVM with the following three elements:

$$\begin{aligned} E_1 &= \frac{\sqrt{2}}{1 + \sqrt{2}} |1\rangle\langle 1|, \\ E_2 &= \frac{\sqrt{2}}{1 + \sqrt{2}} \frac{(|0\rangle - |1\rangle)(\langle 0| - \langle 1|)}{2}, \\ E_3 &= \mathbb{1} - E_1 - E_2. \end{aligned}$$

Suppose that the state $|\psi_1\rangle$ was delivered by the quantum source. After performing a measurement with POVM operators $\{E_1, E_2, E_3\}$, the probability of getting 1 is zero, since $\langle\psi_1|E_1|\psi_1\rangle = 0$. Therefore, if measurement outcomes 1, then we can conclude with certainty that the state emitted by the source was $|\psi_2\rangle$. In a similar way, if the measurement result is 2, then we conclude that the source outputted the state $|\psi_1\rangle$. Certain times, however, the result will be 3 and the measurement is inconclusive. In summary, every time we get outputs 1 and 2 we can infer the quantum state emitted by the source without confusion. Otherwise, we can infer nothing, since $p_1 = p_2 = 1/2$.

7.2 Zero-Error Accessible Information of a Quantum Source

We are going to consider the simplified communication scheme of Fig. 7.1. Alice has a quantum source that emits quantum letters to be measured by Bob using a POVM. When Bob attempts to identify the received message, we assume that *no decoding errors can occur*. In other words, by using measurements, Bob must be 100 % sure about the original message sent by Alice. Next, we define the *Zero-Error Accessible Information* (ZEAI) of a quantum source.

Definition 7.3 (Zero-Error Accessible Information of a Quantum Source).

Let A be a discrete random variable corresponding to message indexes that are associated with quantum letters emitted by a memoryless quantum source F . Now let B be a discrete random variable corresponding to measurement results of quantum states sent by the source. The zero-error accessible information of such quantum source, denoted by $I_{\text{acc}}^{(0)}(F)$, is the maximum amount of messages of length n sent by the source such that $H(A|B) = 0$.

In the original definition of accessible information, vide Definition 3.18, we have that the maximum is taken over the mutual information of the random variables A and B . This turn, in Definition 7.3, we have that the maximum is taken over the set of messages that makes the conditional entropy of A and B equal to zero.

The equality in $H(A|B) = 0$ of Definition 7.3 means that the uncertainty of the random variable A is zero when the random variable B is known, showing the zero-error behavior in this measure of information.

The next step is to obtain a numerical expression for the ZEI of a quantum source.

Theorem 7.2 (Numerical Expression for ZEI). *Let $N(n)$ be the set of quantum codewords of length n which can be sent by a memoryless quantum source F and that can be retrieved with no error with a POVM measurement. The zero-error accessible information of F is given by*

$$I_{acc}^{(0)}(F) \triangleq \sup_{n \rightarrow \infty} \frac{1}{n} \log N(n). \quad (7.9)$$

Proof. To demonstrate the theorem, we consider that both the emission of quantum letters by the quantum source and the subsequent measurement with no decoding errors are equivalent to the zero-error capacity of a discrete memoryless quantum channel.

To do so, we consider that the state produced by the quantum source and the output of the POVM can be written as a discrete memoryless classical channel $W : A \rightarrow B$ with the following stochastic matrix

$$W(a, b) \triangleq \Pr[B = b|A = a] = \text{Tr}(\rho_a M_b), (a, b) \in \mathcal{A} \times \mathcal{B}, \quad (7.10)$$

where ρ_a is the quantum letter emitted by the source; M_b is the operation element of the POVM used for measurement; and \mathcal{A} and \mathcal{B} are the alphabets of the random variables A and B , respectively. When the source emits k quantum letters, we have

$$W^k(a^k, b^k) = \prod_{i=1}^k W(a_i, b_i). \quad (7.11)$$

Considering this interpretation, the maximum amount of messages per number of source emissions that can be sent over the channel W with no decoding errors is corresponding to its zero-error capacity, i.e., $I_{acc}^{(0)}(F) = C_0(W) = \sup_{n \rightarrow \infty} \frac{1}{n} \log N(n)$. Thus, we conclude the proof.

Theorem 7.2 shows an interesting aspect: although zero-error accessible information is a measure of information that characterizes a quantum device, its computation depends on the classical zero-error capacity of a corresponding discrete memoryless channel. This is a counterintuitive result specially because there is no restriction or requirement regarding the quantum letters emitted by the source nor regarding

their probabilities. Note that if the quantum source is purely classical, we have that all quantum letters are distinguishable. In this case, the zero-error accessible information is $\log |\mathcal{S}|$, where \mathcal{S} is the alphabet of the quantum source.

The accessible information of a classical source is not a relevant measure of information, since two classical states can always be distinguished. In contrast, the ZEI of a quantum source is not trivial because quantum information cannot always be distinguished. It is important to emphasize that the definition of ZEI of a quantum source imposes one restriction: *the absence of errors*. As a consequence, we can verify the following inequalities: $I_{\text{acc}}^{(0)}(F) \leq I_{\text{acc}}(F) \leq \chi(F)$ for a quantum source F .

7.3 Representation in Graphs

The zero-error capacity of classical channels has a formulation in terms of graph theory, as shown previously in Sect. 5.2. The problem of finding the zero-error accessible information of a quantum source is equivalent to the problem of obtaining the zero-error capacity of a classical channel. Concepts like orthogonality of input states and characteristic graphs of a quantum channel are straightforwardly defined for a quantum source.

Definition 7.4 (Orthogonality of Quantum Letters). Given two quantum letters $\rho_i = |\psi_i\rangle\langle\psi_i|$ and $\rho_j = |\psi_j\rangle\langle\psi_j|$ belonging to the alphabet \mathcal{S} of a quantum source, we say that ρ_i and ρ_j are non-adjacent, denoted by $\rho_i \perp \rho_j$, if they are orthogonal, i.e., $\langle\psi_i|\psi_j\rangle = 0$.

Orthogonality of quantum letters can be extended to quantum codewords. Two quantum codewords $\rho(i) = \rho_{i_1} \otimes \rho_{i_2} \otimes \dots \otimes \rho_{i_n}$ and $\rho(j) = \rho_{j_1} \otimes \rho_{j_2} \otimes \dots \otimes \rho_{j_n}$ are said to be orthogonal or non-adjacent if there is at least one k , $1 \leq k \leq n$, such that the corresponding quantum letters ρ_{i_k} and ρ_{j_k} are non-adjacent. The characteristic graph can therefore be defined.

Definition 7.5 (Characteristic Graph of a Quantum Source). Let F be a memoryless quantum source according to Definition 7.1. The characteristic graph of F is given by $\mathcal{G}(F) = (V, E)$, where the sets of vertices and edges are given as follows.

1. The vertex set V is given by the classical messages associated with quantum letters of the source alphabet \mathcal{S} ;
2. There is an edge connecting the vertices (i, j) if the corresponding quantum letters ρ_i and ρ_j , $i \neq j$, are non-adjacent.

The graph $\mathcal{G}(F)$ can be generalized for n quantum source outputs. The vertex set of $\mathcal{G}^n(F)$ is given by V^n , and the set of edges is composed by the pairs of vertices whose corresponding codewords of length n are orthogonal.

Vertices of the characteristic graph are connected if the corresponding quantum letters or codewords are fully distinguishable by means of a quantum measurement. A clique on the characteristic graph corresponds to a subset of quantum letters or codewords that are pairwise distinguishable. The zero-error accessible information can be defined in terms of the clique number of the characteristic graph.

Theorem 7.3 (ZEAI in terms of Graph Theory). *Let F be a quantum source with characteristic graph $\mathcal{G}(F)$ according to Definition 7.5. The accessible information of F is given by*

$$I_{\text{acc}}^{(0)}(F) = \sup_n \frac{1}{n} \log \omega(\mathcal{G}^n(F)), \quad (7.12)$$

where $\omega(\mathcal{G}^n(F))$ stands for the clique number of $\mathcal{G}^n(F)$.

Proof. As proved in Theorem 7.2, there is an equivalence between the zero-error accessible information of a quantum source F and the zero-error capacity of a discrete memoryless classical channel W . As we already know, two input letters are non-adjacent at the source output if the corresponding rows of the stochastic matrix $W(b|a)$ are orthogonal. It is straightforward to conclude that the characteristic graph of the quantum source is identical to the characteristic graph of the DMC W , whose zero error-capacity is given by the right side of (7.12).

Upon reducing the zero-error accessible information to the problem of identifying the clique number of a graph, we can see that the calculation of the ZEAI is an \mathcal{NP} -complete problem. The exponential hardness is due to the difficulties of identifying the best POVM for zero-error measurements as well as in determining the length n of quantum codewords that maximize the number of non-confusable codewords per source output.

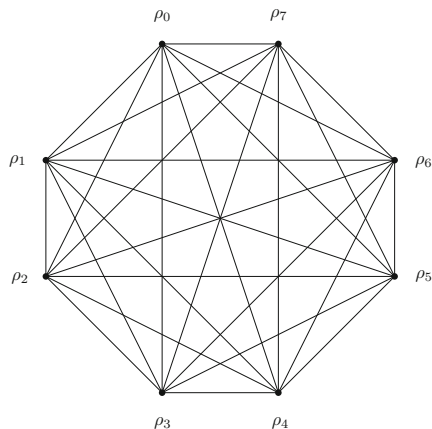
7.4 Examples

This section shows how to obtain the zero-error accessible information for some quantum sources by following the procedures shown previously in Sect. 7.2.

Example 7.3 (Quantum Source With ZEAI Equal to Zero). Let F_1 be a quantum source that emits two states $\rho_1 = |0\rangle\langle 0|$ and $\rho_2 = |+\rangle\langle +| = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|)$ according to the uniform distribution. The source ensemble is given by $\{\rho_i, p_i\}$, where $p_1 = p_2 = 0.5$. According to Definition 7.4, it is easy to verify that the quantum letters ρ_1 and ρ_2 are non-orthogonal, since $\langle 0|+\rangle = \frac{1}{\sqrt{2}}$. Thus, we have that $I_{\text{acc}}^{(0)}(F_1) = 0$.

Example 7.4 (ZEAI of a Purely Classical Quantum Source). Let F_2 be a quantum source that emits states belonging to the computational basis of the 8-dimensional Hilbert space, $\{\rho_0 = |0\rangle\langle 0|, \rho_1 = |1\rangle\langle 1|, \dots, \rho_7 = |7\rangle\langle 7|\}$, each one with

Fig. 7.3 Characteristic graph of the quantum source F_2



probability $1/8$. According to Definition 7.5, the characteristic graph of F_2 is shown in Fig. 7.3.

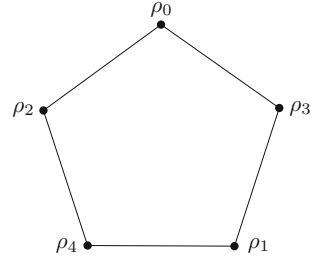
Considering that states emitted by the source are pure and orthogonal, we have that F_2 is a purely classical quantum source. A projective measurement scheme with the POVM $\{M_{2,i} = |i\rangle\langle i|\}_{i=0}^7$ is sufficient to perfectly distinguish all the states. This way, the zero-error accessible information of F_2 is

$$\begin{aligned} I_{\text{acc}}^{(0)}(F_2) &= \sup_n \frac{1}{n} \log \omega(\mathcal{G}^n(F)) \\ &= \frac{1}{1} \log 8 \\ &= 3 \text{ bits per symbol.} \end{aligned}$$

In this example, it is interesting to notice that individual measurements are sufficient to reach the zero-error accessible information of this quantum source. Because $\mathcal{G}(F)$ is a complete graph, $\omega(\mathcal{G}^n(F)) = \omega(\mathcal{G}(F))^n = |\mathcal{S}|^n$. Therefore, $\frac{1}{n} \log |\mathcal{S}|^n = \log |\mathcal{S}|$.

Example 7.5 (ZEAI of a Quantum Source Corresponding to the Pentagon). Let F_3 be a quantum source that emits quantum letters of the alphabet $\mathcal{S} = \{\rho_0, \dots, \rho_4\}$, which is composed only of pure states that are not necessarily orthogonal. These letters are emitted according to the uniform distribution. Figure 7.4 illustrates the characteristic graph of F_3 . This is the pentagon graph of the G_5 channel of Fig. 4.3, whose zero-error capacity was calculated by Lovász and discussed in Sect. 4.3 and in the Example 5.4.

Fig. 7.4 Characteristic graph of the quantum source F_3



To reach the zero-error accessible information of F_3 , we consider quantum codewords of length $n = 2$ and the POVM $\mathcal{M} = \{M_{00}, M_{12}, M_{24}, M_{31}, M_{43}\}$. Then,

$$I_{\text{acc}}^{(0)}(F_3) = \frac{1}{2} \log 5 \approx 1,1609 \text{ bits per symbol.} \quad (7.13)$$

This example illustrates how collective measurements can extract more zero-error information from the quantum source than individual measurements. Moreover, we need more than one source emission in order to reach the zero-error accessible information.

7.5 Related Literature

The accessible information is not known for most quantum sources, and indeed, there is no general method for calculating this quantity. The Holevo bound is the famous and the most important upper bound of the accessible information [7]. The *Holevo χ quantity* is fundamental in proving several results in quantum information theory. Cerf and Adami [3] presented a formal proof for the bound and extended it to consider sequential measurements based on conditional and mutual entropies.

The first lower bound of the accessible information was conjectured by Wootters [16] and proved by Jozsa et al. [8]. Other bounds based on Jensen and Schwarz inequalities and also in purification schemes were also proposed in the literature. A survey can be found in Fuchs [4, Sect. 3.5].

Several numerical methods to calculate the accessible information were developed. In general, these methods seek for the best POVM that maximizes the mutual information $I(A; B)$. Nascimento and de Assis [10, 11] developed a method that is based on genetic algorithms. The open-source tool SOMIM (*Search for Optimal Measurements by an Iterative Method*) also considers this approach and makes use of an iterative method to find the best measurement scheme [9, 13, 15].

Sasaki et al. [14] addressed the problem of obtaining the accessible information of a quantum source, but restricted to the case of real and symmetric quantum

sources. A quantum source is said to be real if the coefficients of the quantum letters are real numbers. The symmetry is verified when the states of the quantum source are equally spaced in a plane $x - z$ of the Bloch sphere. To calculate the accessible information of this kind of quantum source, the authors developed a method based on group theory to identify the best measurement. The resulting POVM has only three elements and can be implemented in a real scenario with existing technology, as discussed by the authors. Unfortunately, the results of Sasaki et al. [14] are restricted only to a certain class of quantum sources.

7.6 Further Reading

In this chapter we introduced the zero-error accessible information, a measure of information that gives the maximum amount of information that can be retrieved from a quantum source without errors. Obtaining the zero-error accessible information of a quantum source can be reduced to the problem of finding the zero-error capacity of an equivalent classical channel. The ZEAI involves concepts from quantum sources, accessible information, classical and quantum information theories and also from graph theory. Some examples illustrated the concepts and the relation with other existing works was discussed.

The zero-error accessible information was first proposed by Guedes and de Assis [6]. Later, on the thesis of Guedes [5], this concept was fully depicted. As discussed in Sect. 7.3, the problem of finding the ZEAI of a quantum source is equivalent to identifying the clique number of a graph, which is \mathcal{NP} -Complete.

Some authors developed heuristics to obtain the best POVM based on iterative methods [9, 13, 15]. Other strategies include genetic algorithms [10, 11].

References

1. Arikan E, Shumovsky A (2002) Holevo's bound. <http://www.ee.bilkent.edu.tr/~qubit/n18holevo.ps>. Accessed 28 May 2016
2. Bennett CH, Shor PW (1998) Quantum information theory. *IEEE Trans Inf Theory* 44(6):2724–2742
3. Cerf NJ, Adami C (1996) Accessible information in quantum measurement. <http://arxiv.org/abs/quant-ph/9611032>. Accessed 17 Feb 2014
4. Fuchs CA (1995) Distinguishability and accessible information in quantum theory. Ph.D Thesis, University of New Mexico, USA
5. Guedes EB (2013) Capacidade quântica de sigilo erro-zero e informação acessível erro-zero de fontes quânticas. Ph.D Thesis, Universidade Federal de Campina Grande, Brazil
6. Guedes EB, de Assis FM (2013) Informação acessível erro-zero de fontes quânticas. In: XXXI Simpósio Brasileiro de Telecomunicações, Fortaleza, Brazil, pp 1–5
7. Holevo AS (1973) Information theoretical aspects of quantum measurements. *Probl Inf Transm* 9(2):110–118

8. Jozsa R, Robb D, Wootters WK (1994) Lower bound for accessible information in quantum mechanics. *Phys Rev A* 49:668–677
9. Lee KL, Shang J, Chua WK, Looi SY, Englert BG (2011) SOMIM: an open-source program code for the numerical search for optimal measurements by an iterative method. <http://www.quantumlah.org/publications/software/SOMIM/>. Accessed 25 May 2013
10. Nascimento EJ, de Assis FM (2006) A numerical solution for the accessible quantum information problem. In: International telecommunications symposium, Fortaleza, Brazil, pp 495–500
11. Nascimento EJ, de Assis FM (2006) Soluções numéricas para o cálculo da informação acessível. In: Workshop-Escola de Computação e Informação Quânticas, Porto Alegre, Brazil, pp 265–274
12. Nielsen MA, Chuang IL (2010) Quantum computation and quantum information. Cambridge University Press, Cambridge
13. Rehacek J, Englert BG, Kaszlikowski D (2005) Iterative procedure for computing accessible information in quantum communication. *Phys Rev A* 71:054303
14. Sasaki M, Barnett SM, Jozsa R, Osaki M, Hirota O (1999) Accessible information and optimal strategies for real symmetrical quantum sources. *Phys Rev A* 59(5):3325–3335
15. Suzuki J, Assad SM, Englert BG (2007) Accessible information about quantum states: an open optimization problem. Chapman & Hall, Boca Raton, pp 309–348
16. Wootters WK (1992) Two extremes of information in quantum mechanics. In: Workshop on physics and computation, Dallas, Texas, pp 181–183