Chapter 1
Introduction

The notion of chaos is so deeply integrated into all fields of science, culture, and other human activities, that it has become fundamental. It is quite probable that each person has his or her own intuitive concept of what is chaos. The first cosmogonical views which explained the origin of the world contained the notion of chaos. For example, the ancient Greeks believed that chaos appeared before everything and then only afterwards the world appeared.

For example, Hesiod says: "In the beginning there was only chaos. Then out of the void appeared Erebus, the unknowable place where death dwells, and Night. All else was empty, silent, endless, darkness. Then somehow Love was born bringing a start of order. From Love came Light and Day. Once there was Light and Day, Gaea, the earth appeared."

The origin of the word chaos itself, $\chi\alpha o\varsigma$, comes from ancient Greek $\chi\alpha\iota\nu\omega$, which means to open wide.

We can find a similar concept concerning the origins of the world in ancient Chinese myth: "In the beginning, the heavens and earth were still one and all was chaos…"

In ancient Indian literature the origins of the world are also associated with chaos: "A time is envisioned when the world was not, only a watery chaos (the dark, "indistinguishable sea") and a warm cosmic breath, which could give an impetus of life." [1]

An ancient Egyptian origin myth holds that in the beginning, the universe was filled with the primeval waters of chaos, which was the god Nun.

Such unanimity of such distant civilizations (in space as well as in time) on the idea of the origination of the world from chaos is quite striking. It seems that this notion is one of the most important to come from ancient times. What is also striking is the fact that the problem of chaos is still topical after some hundreds of years of its study in mathematics and physics. Even a simple list of fields where chaos plays a fundamental part in modern science would take up most of this book. However, a

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new understanding of the origins of chaos was reached quite recently in connection with the discovery of deterministic chaos.

Before the discovery of this phenomenon, all studies of random processes and of chaos were usually conducted within the frame of classical theory of probability, which requires one to define a set of random events or a set of random process realizations or a set of other statistical ensembles. After that, probability itself is assigned and studied as a measure on this set, which satisfies Kolmogorov's axioms [2]. The discovery of deterministic chaos radically changed this situation.

Chaos was found in dynamical systems, which do not contain elements of randomness at all, i.e., they do not have any statistical ensembles. On the contrary, the dynamic of such systems is completely predictable, the trajectory, assigned precisely by its initial conditions, reproduces itself precisely, but nevertheless its behavior is chaotic.

At first sight, this situation does not correspond to our intuitive understanding, according to which chaotic behavior by its very nature cannot be reproduced. A simplified and pragmatic explanation is frequently used to explain this phenomenon. Dynamical chaos appears in non-linear systems with trajectories utterly sensible to minor modifications of initial conditions. In that case any person calculating a trajectory using a computer observes that the small uncertainty of initial conditions engenders chaotic behavior.

This answer does leave some feeling of dissatisfaction. In fact, we know that, for instance, the number $\sqrt{2}$ exists accurately, without any uncertainty. What would happen if the trajectory began precisely from $\sqrt{2}$? The usual answer to this question is that the behavior of trajectory will become more and more complex, because the number $\sqrt{2}$ is irrational and ultimately will be practically indistinguishable from the chaotic, although remaining determined.

However, two questions persist: what do we mean by "complex" and what does "practically indistinguishable from the chaotic" mean? For example, genetic code is complex but not chaotic, while a coin toss is a simple, but chaotic process. Even from the above we can see that the phenomenon of deterministic chaos requires a deeper understanding of randomness, not based on the notion of a statistical ensemble.

Such a theory was developed by Kolmogorov and his disciples even before the discovery of the phenomenon of deterministic chaos. The main principle of this theory will be stated in the next chapter, where we will introduce all its necessary components: algorithms, Turing machine, Kolmogorov's complexity, etc. It is significant that Kolmogorov came to his theory when discussing in articles [3, 4] the limited nature of Shannon's theory of information [5].

As an example, let us question how to understand what is the genetic information, for example, of a tiger or of Mr. Smith. It seems that, since the notion of information is based on the introduction of probabilities, we have to examine a set of tigers with assigned probability. Only after that, one can calculate Shannon information in the tiger's genes. It is clear that something in these considerations provokes anxiety. Above all, the dissatisfaction is caused by the introduction of the set of Smiths, let's say. Obviously, it is more pleasant to consider that Mr. Smith is unique and has individual genetic information like a particular tiger. The limited nature of probabilistic approach becomes even clearer if you consider how much information is contained in the book "War and Peace" by Leo Tolstoy. Then the problem with the introduction of the set "War and Peace" becomes perfectly evident.

The problem is that we are interested in information for the object that is individual. In other words, we have only one specimen of the object and it is impossible to create another one even mentally. Therefore, the theory of probability and the theory of information must be restated in such a way that the individual object get the character of random one. Kolmogorov states [4]: "1) The main concepts of the theory of information must be and can be founded without using the theory of probability and in such a way that notions of "entropy" and "quantity of information" become applicable for individual objects. 2) Notions of the theory of information introduced in this way can underlie the new concept of random, which corresponds to natural thought, that randomness is the absence of laws."

The algorithmic theory of randomness developed by Kolmogorov and his disciples is a natural mathematical basis for understanding the theory of deterministic chaos; we will examine it in the next chapter. This theory gives natural answers to the questions that were put earlier: when complexity turns into randomness and how algorithmic randomness obeys the theory of probability. From the physical point of view, this means that the distinction between dynamical and statistical laws gets erased.

Note that the reader is not obliged to begin the study of the theory of chaos and its applications with Kolmogorov's algorithmic theory of randomness. In other words, the chapter Paradigm for Chaos is to a great extent independent from the other chapters in this book. However, experience has shown that the study of deterministic chaos can feel unfinished and unsatisfying. Therefore, if at any time the reader feels a desire for a deeper understanding of the nature of deterministic chaos, they can turn to "Paradigm for Chaos."

References

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