Multiobjective Interval Transportation Problems: A Short Review

Carla Oliveira Henriques and Dulce Coelho

Abstract The conventional transportation problem usually involves the transportation of goods from several supply points to different demand points and considers the minimization of the total transportation costs. The transportation problem is a special case of linear programming models, following a particular mathematical structure, which has a wide range of potential practical applications, namely in logistic systems, manpower planning, personnel allocation, inventory control, production planning and location of new facilities. However, in reality, the transportation problem usually involves multiple, conflicting, and incommensurable objective functions, being called the multiobjective transportation problem. Several methods have been developed for solving this sort of problems with the assumption of precise information regarding sources, destinations and crisp coefficients for the objective function coefficients. Nevertheless, when dealing with real-life transportation problems, these circumstances may not be verified, since the transportation costs may vary as well as supply and demand requirements. Therefore, different approaches for dealing with inexact coefficients in transportation problems have been proposed in scientific literature, namely with the help of fuzzy and interval programming techniques. This paper is aimed at providing a short critical review of some interval programming techniques for solving this particular type of problems.

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1 Introduction

The transportation problem (TP) might be seen as a particular case of linear programming (LP) models which is generally used to determine the optimal solution for the distribution of certain goods, from different supply points (sources—e.g. production facilities, warehouses) to different demand points (destinations—e.g. warehouses, sales, outlet), considering that there is a certain distance between these points. The objective function in the TP usually represents the total transportation costs, while the constraints are defined by the supply capacity and demand requirements of certain sources or destinations, respectively. However, in real-life situations the TP usually encompasses multiple, conflicting and incommensurate objective functions (e.g. transportation cost, average delivery time, number of goods transported, unfulfilled demand). This type of problem is also known as the multiobjective transportation problem (MOTP). The conventional TP and MOTP follow a particular mathematical structure assuming that the coefficients of the objective functions and the supply and demand values are stated in a precise way, with crisp values. Nevertheless, such assumptions are rarely satisfied (e.g. the unit transportation costs may vary and the supply and demand may also change). Therefore, TP and MOTP for decision support must take explicitly into account the treatment of the inherent uncertainty associated with the model coefficients. Interval programming is one of the approaches to tackle uncertainty in mathematical programming models, which holds some interesting characteristics because it does not require the specification or the assumption of probabilistic distributions (as in stochastic programming) or possibilistic distributions (as in fuzzy programming) or a max-min formulation (as in robust optimization). The use of interval programming techniques is possible providing that information about the range of variation of some (or all) of the parameters is available (Oliveira and Antunes 2007).

This paper is aimed at providing an overview of the different approaches reported in scientific literature regarding interval programming techniques for tackling the uncertainty in TP and MOTP. This paper is structured as follows: Sect. 2 provides the underpinning assumptions of MOTP with interval coefficients (MOITP); Sect. 3 discusses the main approaches found in scientific literature for obtaining solutions to MOITP. Finally, Sect. 4 concludes highlighting the main advantages and drawbacks of the several approaches also suggesting possible future work development in this field of research.

2 Theoretical Underpinning of MOITP

Conventional multiobjective linear programming (MOLP)/LP models usually address practical problems in which all coefficients and parameters are a priori given. However, due to the inexactness and uncertainty aspects of these problems should be explicitly taken into consideration. Uncertainty handling can be dealt with in various ways, namely by means of stochastic, fuzzy and interval programming techniques. In the stochastic approach the coefficients are treated as random variables with known probability distributions. In the fuzzy approach, the constraints and objective functions are regarded as fuzzy sets with known membership functions. However, it is not always easy for the decision-maker (DM) to specify these probability distributions and membership functions. In the interval approach it is considered that the uncertain values are perturbed simultaneously and independently within known fixed bounds, being therefore intuitively preferred by the DM in practice.

2.1 Interval Numbers and Interval Order Relations

Consider that the value x (a real number) is uncertain, knowing that x lies between two real numbers a^{L} and a^{U} forming an interval, where $a^{L} < a^{U}$. All numbers within this interval have the same importance. An interval number A is defined as the set of real numbers x such that $a^{L} \leq x \leq a^{U}$, i.e. $x \in [a^{L}, a^{U}]$, $a^{L}, a^{U} \in \Re$ or,

$$A = \left[a^{L}, a^{U}\right] = \left\{x : a^{L} \leq x \leq a^{U}, \in \Re\right\}. \qquad (1)$$

The width and midpoint of the interval number $A = [a^{L}, a^{U}]$ are $\mathrm{w}[A] = (a^{U} - a^{L})$ and $\mathrm{m}[\mathrm{A}] = \frac{1}{2}(a^{U} + a^{L})$, respectively. There are basically two types of order relations between intervals: one based on the extension of the concept "<" (less than) for real numbers and another based on the extension of the concept of set inclusion. Consider the intervals $A = [a^{L}, a^{U}]$ and $B = [b^{L}, b^{U}]$ given in the set $I(\Re)$ i.e. the set of real interval numbers. Thus, it is considered that $A\ (<)\ B$ if and only if $a^{U} < b^{L}$. Moreover, $A \subseteq B$ if and only if $a^{L} \geq b^{L}$ and $a^{U} \leq b^{U}$. Nonetheless, these interval order relations do not allow comparing overlapped intervals. On the other hand, the extension of set inclusion does not allow ordering intervals in terms of their importance. In this context, several approaches have been suggested which allow comparing two interval numbers. Ishibuchi and Tanaka developed an approach which allows comparing two interval numbers (Ishibuchi and Tanaka 1990). For instance, in a problem where the objective function is minimized, A is better than B, i.e. $A \leq_{LU} B$ if and only if $a^{L} \leq b^{L}$ and $a^{U} \leq b^{U}$.

The interval order relation "$\leq_{LU}$" has the following properties:

(a) If $A \leq_{LU} B$, then $\mathrm{m}[\mathrm{A}] \leq \mathrm{m}[\mathrm{B}]$.
(b) If $a^{L} = a^{U}$ and $b^{L} = b^{U}$, then "$\leq_{LU}$" corresponds to the inequality "$\leq$" used in the set of real numbers.

However, there are interval numbers that cannot be compared through the interval order relation "$\leq_{LU}$". For example, if $A = [85, 95]$ and $B = [80, 100]$, the order relations $A \leq_{LU} B$ and $B \leq_{LU} A$ are not verified. Therefore, Ishibuchi and Tanaka suggested another interval order relation based on the midpoint and width of the interval numbers, called "$\leq_{MW}$" (Ishibuchi and Tanaka 1990). According to this interval order relation, A is better than B, i.e. $A \leq_{MW} B$ if and only if $m[A] \leq m[B]$ and $w[A] \geq w[B]$.

The interval order relation "$\leq_{MW}$" has the following properties:

(a) If $A \leq_{MW} B$ then $a^L \leq b^L$.
(b) If $w[A] = w[B] = 0$, then "$\leq_{MW}$" corresponds to the inequality "$\leq$" used in the set of real numbers.

Ishibuchi and Tanaka have also defined another interval order relation, "$\leq_{LM}$", which allows comparing the previous two (Ishibuchi and Tanaka 1990). According to this interval order relation, A is better than B, i.e. $A \leq_{LM} B$ if and only if $a^L \leq b^L$ and $m[A] \leq m[B]$. On the other hand, $A \leq_{LM} B$ if and only if either $A \leq_{LU} B$ or $A \leq_{MW} B$. Chanas and Kuchta (1996) proposed a generalization of the interval order relations suggested by Ishibuchi and Tanaka (1990) introducing the concept of cutting level—φ_0, φ_1—of an interval. Consider the interval order relations "$\leq_{LU}$", "$\leq_{MW}$", "$\leq_{LM}$" defined as interval order relations of the type 1, 2 and 3, respectively. Let $A = [a^L, a^U]$ and φ_0 and φ_1 any real crisp numbers such that $0 \leq \varphi_0 \leq \varphi_1 \leq 1$. The cutting level—$\varphi_0$, φ_1—of interval A is:

$$A/_{[\varphi_0,\ \ \varphi_1]} = \left[a^L + \varphi_0\left(a^U - a^L\right), a^L + \varphi\left(a^U - a^L\right)\right]. \tag{2}$$

Let $A = [a^L, a^U]$ and $B = [b^L, b^U]$ be two interval numbers, φ_0 and φ_1 any real crisp numbers such that $0 \leq \varphi_0 \leq \varphi_1 \leq 1$ and i any interval order relation of the set $\{1, 2, 3\}$. The interval order relations $\leq_{i/[\varphi_0,\ \ \varphi_1]}$ and $<_{i/[\varphi_0,\ \ \varphi_1]}$ are defined in the following way:

$$A \leq_{i/[\varphi_0,\ \ \varphi_1]} B \quad \text{if and only if} \quad A/_{[\varphi_0,\ \ \varphi_1]} \leq_i B/_{[\varphi_0\ \ \ \varphi_1]}, \tag{3}$$

$$A <_{i/[\varphi_0\ \ \ \varphi_1]} B \quad \text{if and only if} \quad A/_{[\varphi_0,\ \ \varphi_1]} <_i B/_{[\varphi_0\ \ \ \varphi_1]} \tag{4}$$

where $A/_{[\varphi_0,\ \ \varphi_1]}$ and $B/_{[\varphi_0,\ \ \varphi_1]}$ correspond to the cutting levels—φ_0, φ_1—of interval numbers A and B, respectively.

Then, $A \leq_{i/[\varphi_0,\ \ \varphi_1]} B$ if and only if $A \leq_i B$, $i = 1, 2, 3$ and $A \leq_{1/[\varphi_0,\ \ \frac{\varphi_1}{2}]} B$ if and only if $A \leq_{3/[\varphi_0,\ \ \varphi_1]} B$. Nevertheless, there are interval numbers that cannot be compared through the interval order relations defined by Ishibuchi and Tanaka (1990) and Chanas and Kuchta (1996). For example, let us consider $A = [1850, 2215]$ and $B = [1695, 2515]$. Therefore, $m[A] = 2032.5$, $m[B] = 2105$, $w[A] = 365$ and $w[B] = 820$. Therefore it might be concluded that interval A allows obtaining a lower value but less uncertain, whereas interval B allows obtaining a higher value but more uncertain. Thus, it is not possible to know with any of the interval order relations previously established what is the better interval value. In order to overcome this drawback Sengupta and Pal suggested an index which allows comparing any type of interval numbers, taking into account the satisfaction levels of the DM (Sengupta and Pal 2000). Consider the following expanded order relation $\prec$, between the interval numbers A and B. Sengupta and Pal (2000) defined an acceptability function $\mathcal{A} : I(\Re) \times I(\Re) \rightarrow [0, \infty]$, such that $\mathcal{A}(A \prec B)$ or $\mathcal{A} \prec (A, B)$, or even,

$$\mathcal{A} \prec = \frac{(\mathrm{m[B]} - \mathrm{m[A]})}{\left(\frac{\mathrm{w[B]}}{2} + \frac{\mathrm{w[A]}}{2}\right)}, \tag{5}$$

where $\left(\frac{w[B]}{2} + \frac{w[A]}{2}\right) \neq 0$. The index $\mathcal{A} \prec$ can be interpreted has the degree of acceptability of A being inferior to interval B, i.e. $\mathcal{A}(A \prec B)$ or $\mathcal{A} \prec (A, B)$.

The degree of acceptability of $A \prec B$ can be classified by comparing the midpoint and width of interval numbers A and B in the following way:

$$\mathcal{A}(A \prec B) = \begin{cases} 0 & \text{if} \quad \mathrm{m[A] = m[B]}, \\ ]0, 1[& \text{if} \quad \mathrm{m[A] < m[B]} \quad \text{and} \quad \mathrm{a^U > b^L}, \\ [1, \infty[& \text{if} \quad \mathrm{m[A] < m[B]} \quad \text{and} \quad \mathrm{a^U \leq b^L}. \end{cases} \tag{6}$$

If $\mathcal{A}(A \prec B) = 0$ then the premise "A inferior to B" is not accepted; if $0 < \mathcal{A}(A \prec B) < 1$ then the premise "A inferior to B" is accepted with different degrees of satisfaction; if $\mathcal{A}(A \prec B) \geq 1$ then the premise "A inferior to B" is accepted as true. In a minimization problem, if $\mathcal{A}(A \prec B) > 0$, then it should be concluded that A is better than B. The index $\mathcal{A} \prec$ satisfies the DM for any possible judgment regarding the comparison of any pair of interval numbers A and B, since it is always possible to obtain at least one of the following situations: $\mathcal{A}(A \prec B) > 0$, $\mathcal{A}(B \prec A) > 0$ or $\mathcal{A}(A \prec B) = \mathcal{A}(B \prec A) = 0$.

2.2 *MOLP Models with Interval Coefficients and Parameters*

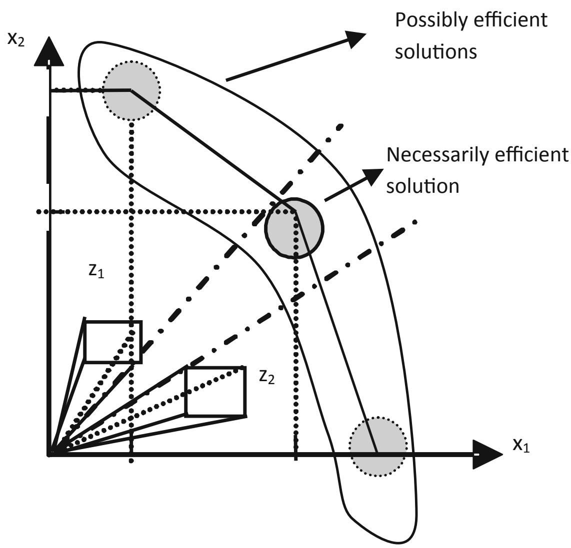
Consider, without loss of generality, the following MOLP model with interval coefficients and parameters and the interval arithmetic operations (Oliveira and Antunes 2007; Moore 1966):

$$\begin{aligned} &\text{Maximize } \mathrm{Z_k}(\mathbf{x}) = \sum_{j=1}^{n} \left[\mathrm{c}_{kj}^{L}, \quad \mathrm{c}_{kj}^{U}\right] \mathrm{x_j} \quad (\mathrm{k} = 1, \ldots, \mathrm{p}), \\ &\text{subject to} \sum_{j=1}^{n} \left[\mathrm{a}_{ij}^{L}, \quad \mathrm{a}_{ij}^{U}\right] \mathrm{x_j} \leq \left[\mathrm{b}_{i}^{L}, \quad \mathrm{b}_{i}^{U}\right] \quad (\mathrm{i} = 1, \ldots, \mathrm{m}), \\ &\mathrm{x_j} \geq 0 \quad (\mathrm{j} = 1, \ldots, \mathrm{n}), \end{aligned} \tag{7}$$

where $\left[c^L_{kj}, \quad c^U_{kj}\right]$, $\left[a^L_{ij}, \quad a^U_{ij}\right]$ and $\left[b^L_i, \quad b^U_i\right]$, $k = 1, \ldots, p$, $j = 1, \ldots, n$, $i = 1, \ldots, m$, correspond to closed intervals.

The interval programming approach has been used to tackle specific issues in MOLP. Some algorithms only tackle the uncertainty in the objective functions, others deal both with the uncertainty in the objective functions and in the right hand side (RHS) of the constraints and others handle with the uncertainty in all the coefficients of the model (Oliveira and Antunes 2007). According to Inuiguchi and Kume and Inuiguchi and Sakawa there are two different approaches to deal with an interval objective function: the satisficing approach and the optimizing approach (Inuiguchi and Kume 1994; Inuiguchi and Sakawa 1995). According to the satisficing approach each interval objective function is transformed into one or several objective functions (the lower bound, the upper bound and the midpoint of the intervals are usually used) in order to obtain a compromise solution. However, the use of this approach may lead to a compromise solution that might not be the most suitable one, if the gradients of the chosen objective functions are highly correlated (Antunes and Clímaco 2000). The optimizing approach extends the concept of efficiency used in traditional MOLP to the interval objective function case [e.g. (Bitran 1980; Ida 1999, 2000a, b, 2005; Inuiguchi and Sakawa 1996; Steuer 1981; Wang and Wang 2001a, b; Oliveira et al. 2014)]. Bitran suggested two types of efficient solutions for the interval MOLP: a "necessarily efficient" if it is efficient for all objective function coefficient vectors within their admissible range of variation (see the vertex with a bold circle obtained with the gradient cones of the two objective functions illustrated in Fig. 1); a "possibly efficient" if it is efficient for at least one of the given objective function coefficient vectors within their admissible range of variation (Bitran 1980). When compared to the "possibly efficient"

Fig. 1 Necessarily and possibly efficient solutions (Oliveira and Antunes 2007)



solutions, the "necessarily efficient" solutions are the most robust (Ida 1999). Although this type of approach allows enumerating all possibly efficient solutions and/or all necessarily efficient solutions, the computational burden involved can be significant. Another issue is that with a large set of solutions, in many cases with just slight differences among the objective function values, the decision problem becomes even more complex (Antunes and Clímaco 2000).

Interactive approaches have also been considered to obtain solutions to the MOLP models with interval coefficients in the whole model. Urli and Nadeau have proposed an interactive algorithm that does not allow the DM to take into account the worst case and the best case "scenarios" (Urli and Nadeau 1992). In the algorithm proposed by Oliveira and Antunes the procedures involved provide a global view of the solutions in the best and worst case coefficients scenario and allow performing the search of new solutions according with the achievement rates of the objective functions, both regarding the upper and lower bounds (Oliveira and Antunes 2009). The main aim is to identify the solutions associated with the interval objective function values which are closer to their corresponding interval ideal solutions. With this approach it is also possible to find solutions with non-dominance relations regarding the achievement rates of the upper and lower bounds of the objective functions, considering interval coefficients in the whole model.

2.3 MOITP Formulation

The conventional MOTP problem can be formulated, without loss of generality, as:

$$
\begin{aligned}
&\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}, \quad \text{where } k = 1, \ldots, p.\\
&\text{subject to}\\
&\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, \ldots, m.\\
&\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, \ldots, n\\
&x_{ij} \geq 0, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n.\\
&\text{with } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,
\end{aligned}
\tag{8}
$$

where x_{ij} is the decision variable which refers to product quantity that has to be transported from supply point i to demand point j; c_{ij}^k, k = 1, …, p, denotes the unit transportation cost from ith supply point to jth demand; a_i, i = 1, …, m, represents the ith supply quantity; b_j, j = 1, …, m, represents the jth demand quantity.

Three major cases can occur in MOITP problems (Das et al. 1999):

1. The coefficients of the objective functions are given in the form of interval values, whereas source and destination parameters are deterministic.
2. The source and destination parameters are in the form of interval values but the objective functions' coefficients are deterministic.
3. All the coefficients and parameters are in the form of interval values.

If all the coefficients and parameters are provided in the interval form, the MOITP can be formulated, without loss of generality, as the problem of minimizing p interval-valued objective functions, with interval source and interval destination parameters, as follows (Das et al. 1999; Sengupta and Pal 2009):

$$\begin{aligned}
&\text{Minimize } Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c^k_{Lij}, c^k_{Uij}\right] x_{ij}, \quad \text{where } k = 1,\ldots,p.\\
&\text{subject to}\\
&\sum_{j=1}^{n} x_{ij} = [a_{Li}, a_{Ui}], \quad i = 1,\ldots,m.\\
&\sum_{i=1}^{m} x_{ij} = \left[b_{Lj}, b_{Uj}\right], \quad j = 1,\ldots,n.\\
&x_{ij} \geq 0, \quad i = 1, \ldots, m, \; j = 1, \ldots, n.\\
&\text{with } \sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{Lj} \text{ and } \sum_{i=1}^{m} a_{Ui} = \sum_{j=1}^{n} b_{Uj}\\
&\text{or with } \sum_{i=1}^{m}\frac{1}{2}(a_{Li} + a_{Ui}) = \sum_{j=1}^{n}\frac{1}{2}\left(b_{Lj} + b_{Uj}\right),
\end{aligned} \tag{9}$$

where x_{ij} is the decision variable which refers to product quantity that has to be transported from supply point i to demand point j; $\left[c^k_{Lij}, c^k_{Uij}\right]$, k = 1, …, p, denotes the unit transportation cost from ith supply point to jth demand comprised between c^k_{Lij} and c^k_{Uij}; $\left[a_{Li,} a_{Ui}\right]$, i = 1, …, m, represents the ith supply quantity within a_{Li} and a_{Ui}; $\left[b_{Lj,} b_{Uj}\right]$, j = 1, …, m, represents the jth demand quantity located between b_{Lj} and b_{Uj}.

3 Solution Methods for MOITP

Abd El-Wahed and Lee classify the solution approaches for MOTP into four categories: interactive approaches [see e.g. (Climaco et al. 1993; Ringuest and Rinks 1987)], non-interactive approaches [see e.g. (Aneja and Nair 1979)], goal programming (GP) approaches [see e.g. (Hemaida and Kwak 1994)] and fuzzy programming approaches [see e.g. (Abd El-Wahed and Lee 2006; Abd El-Wahed 2001; Li and Lai 2000)]. The interactive approaches may present several limitations: the convergence to an efficient solution depends on the DM's reasoning and consistency; unless the interactive method becomes more flexible to facilitate the enumeration and evaluation of the set of efficient solutions in large scale problems, the procedure bears a high computational burden. However, with this sort of approaches the search direction of efficient solutions is controlled by the DM allowing him/her to reach an efficient solution consistent with his/her preferences. The non-interactive approaches depend on the generation set of efficient solutions, and the choice of the preferred compromise solution out of this set is then required from the DM. Therefore, the solution search process may require a large computational burden and the DM may be facing additional difficulties in assessing the tradeoffs between the different solutions. The GP approach allows obtaining a compromise solution according to the established goal levels (aspiration levels). Nevertheless, it is often difficult for the DM to decide the desired aspiration levels for the goals, eventually leading to non-efficient solutions; the choice of the weights in the formulation of the GP problem may also lead to non-satisfying results; erroneous conclusions can be obtained if the achievement function is not correctly formulated; the GP formulation changes the traditional mathematical form of the MOTP problem. Finally, the fuzzy programming approaches are based on the use of fuzzy set theory for solving the MOTP by means of an interactive procedure. The use of fuzzy set theory in solving such MOTP changes the standard form of the TP. Moreover, the computation of an efficient solution is not guaranteed in certain conditions [see e.g. (Li and Lai 2000)]. Although this section is devoted to the MOITP, our analysis will also encompass methods used in the framework of interval TP (ITP). Therefore, by extending the previous classification of solution approaches to MOITP and ITP, it might be concluded that the approaches mainly used are broadly classified into three categories: the composite approach (i.e., they are the result of the combination of two or more approaches), the fuzzy approach (usually used for obtaining a final solution) and non-fuzzy approach. In the framework of the composite approach, Abd El-Wahed and Lee (2006) presented an interactive, fuzzy and goal programming approach to determine the preferred compromise solution for the MOTP. The suggested approach considers the imprecise nature of the input data by implementing the minimum operator, also assuming that each objective function has a fuzzy goal. The approach focuses on minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound of each objective function. The solution procedure controls the search direction by updating both the membership values and the aspiration levels. An important characteristic of the approach is that the decision-maker's (DM's) role is concentrated only in evaluating the efficient solution to limit the influences of his/her incomplete knowledge about the problem domain. The approach controls the search direction by updating both upper bounds and aspiration level of each objective function. In the context of the fuzzy approach, Das et al. suggested a procedure that starts by transforming the MOITP (see problem (8)) into a classical MOTP, where the objectives (the upper bound and midpoint of the interval objective functions) are minimized and the constraints with interval source and destination parameters have been converted into deterministic ones (Das et al. 1999). Finally, the equivalent transformed problem has been solved by the fuzzy programming technique. Sengupta and Pal proposed a methodology for solving an ITP where multiple penalty factors are involved, reflecting the DM's pessimist or optimistic bias in achieving the compromise solution by means of a fuzzy oriented method (Sengupta and Pal 2009). Panda and Das presented a two-vehicle cost varying interval transportation model (TVCVITM) where the cost varies due to the capacity of the vehicles and to the quantity transported (Panda and Das 2013). The source and destination parameters are considered as intervals. Initially, depending on the cost of the vehicles the interval coefficients of the objective function are specified. Then, the problem is converted into a classical ITP. Finally, this model is converted into a bi-objective TP, where the upper bound and the midpoint of the objective function are minimized. The solution to this bi-objective model is obtained with a fuzzy programming technique. Nagarajan et al. suggested a solution procedure for the MOITP problem under stochastic environment using fuzzy programming approach. All source availability, destination demand and conveyance capacities have been taken as stochastic intervals for each criterion. Expectation of a random variable has been used to transform the problem into a classical MOTP where the objectives which are the upper bound and midpoint of the interval objective functions are minimized (Nagarajan et al. 2014). In the context of non-fuzzy approach, Pandian and Natarajan (2010) suggested a new method (the separation method based on the zero point method) for finding an optimal solution for the integer TP where transportation cost, supply and demand are given as interval values. The proposed method has been developed without using the midpoint and width of the interval in the objective function. Joshi and Gupta investigated the transportation problem with fractional objective function when the demand and supply quantities are varying. The method computes the lower and upper bounds of the total fractional transportation cost when the supply and demand quantities are varying. A set of two-level transportation problems is transformed into one-level mathematical programs to obtain the objective value (Joshi and Gupta 2011). Pandian and Anuradha (2011) applied a split and bound method for finding an optimal solution to a fully integer interval TP with additional impurity constraints. This method has been developed without considering the midpoint and width of the intervals and is based on the floating point method [see Pandian and Anuradha (2011b)]. Rakocevic and Dragasevic (2011) presented a parametric TP, in which the coefficients of the objective function depend on a parameter. The main aim of this proposal is to assess the impact on the optimal solution of changes in the coefficients of the objective function, i.e. to find the range of variation of the coefficients of the objective function without affecting the optimal solution obtained. Roy and Mahapatra (2011) dealt with the interval coefficients to the multiobjective stochastic TP, involving an inequality type of constraints in which all parameters (supply and demand) are lognormal random variables and the coefficients of the objective functions are interval numbers. The minimization MOITP is also converted into a bi-objective problem using the order relations which represent the DM's preferences between the interval costs. These interval costs have been defined by the upper and lower bounds and corresponding midpoint and width. The proposed probabilistic constraints are firstly converted into an equivalent deterministic constraint by means of the chance constrained programming technique. Then, a surrogate problem has been solved by the weighted sum method. Guzel et al. (2012) transformed a fractional TP with interval coefficients into a classical TP. Two solution procedures are then proposed for the interval fractional TP. One of them is based on a Taylor series approximation and the other one is based on interval arithmetic. Kavitha and Pandian (2012) presented the sensitivity analysis of supply and demand parameters of an ITP. The method is aimed at determining the ranges of supply and demand parameters in an interval transportation problem such that its optimal basis is invariant. The upper-lower method is used for finding a critical region of the supply and demand parameters at which any change inside the ranges of the region does not affect the optimal basis, while any change outside their ranges will affect the optimal basis. Dalman et al. (2013) considered the Indefinite Quadratic Interval TP (IQITP) in which all the parameters i.e. cost and risk coefficients of the objective function, supply and demand quantities are expressed as intervals. Firstly, a feasible initial point is determined within the Northwest Corner method by means of expressing all the interval parameters as left and right limits. Then the objective function is linearized by using first order Taylor series expansion about the feasible initial point. Thus IQITP is transformed into a traditional LP problem. Then an iterative procedure is presented in such a way that the optimal solution of the last LP problem is selected as the point from which the objective will be expanded into its first order Taylor series in the next iteration step. The stopping criterion of the proposed procedure is obtaining the same point for the last two iteration steps. Fegade et al. (2013) considered TP with and without budgetary constraints, where demand and budget are imprecise. An interval-point method for finding an optimal solution for transportation problems is proposed and compared with zero suffix method. Panda and Das (2014) also presented the two-vehicle cost varying transportation problem as a bi-level mathematical programming model. The Northwest Corner rule is used for determining the initial basic feasible solution and then the unit transportation cost (which varies in each iteration) is established according to the choice of vehicles. These authors also concluded that the two-vehicle cost varying transportation model provides more efficient results than the single objective cost varying transportation problem.

3.1 Advantages and Disadvantages of Solution Methods

From the different methodologies briefly presented it might be concluded that the combination of two or more approaches may reduce some or all of the shortcomings of each individual approach. Another issue refers to the fact that the majority of the approaches herein reviewed transforms the original MOITP/TP into a surrogate crisp problem and then apply some algorithm to solve it (see Table 1). As a result, the uncertainty of the intervals may get lost to a certain extent. In fact, in view of interval order relations, the satisfactory solutions attained sometimes allow obtaining contradictory results. The example suggested by Das et al. and used by Sengupta and Pal helps illustrating this issue (Oliveira and Antunes 2009; Das et al. 1999).

Consider the following example:

$$\text{Minimize } Z^k = \sum_{i=1}^{3}\sum_{j=1}^{4}\left[c^k_{Lij}, c^k_{Uij}\right]x_{ij}, \quad \text{where } k = 1, 2.$$

subject to

$$\sum_{j=1}^{4} x_{1j} = [7, 9], \quad \sum_{j=1}^{4} x_{2j} = [17, 21], \sum_{j=1}^{4} x_{3j} = [16, 18],$$

$$\sum_{i=1}^{3} x_{i1} = [10, 12], \sum_{i=1}^{3} x_{i2} = [2, 4], \sum_{i=1}^{3} x_{i3} = [13, 15], \sum_{i=1}^{3} x_{i4} = [15, 17],$$

$$x_{ij} \geq 0, \; i = 1, 2, 3, \; j = 1, 2, 3, 4.$$

$$\text{where } c^k_{ij} = \left[c^k_{Lij}, c^k_{Uij}\right], \; k = 1, 2,$$

$$c^1_{ij} = \begin{bmatrix} [1, & 2] & [1, & 3] & [5, & 9] & [4, & 8] \\ [1, & 2] & [7, & 10] & [2, & 6] & [3, & 5] \\ [7, & 9] & [7, & 11] & [3, & 5] & [5, & 7] \end{bmatrix} \text{ and}$$

$$c^2_{ij} = \begin{bmatrix} [3, & 5] & [2, & 6] & [2, & 4] & [1, & 5] \\ [4, & 6] & [7, & 9] & [7, & 10] & [9, & 11] \\ [4, & 8] & [1, & 3] & [3, & 6] & [1, & 2] \end{bmatrix}.$$

With the fuzzy programming technique suggested by Das et al. (1999) the following Pareto optimal solution is attained: $x_{12} = 2$, $x_{14} = 5$, $x_{21} = 10$, $x_{23} = 6.01$, $x_{24} = 0.99$, $x_{33} = 6.97$, $x_{34} = 9.01$, with $Z^1 = [113, 204.9]$, $w[Z^1] = 91.9$, $m[Z^1] = 158.95$ and $Z^2 = [129.89, 227.86]$, $w[Z^2] = 97.97$, $m[Z^2] = 178.85$. By means of the fuzzy oriented method proposed by Sengupta and Pal (2009) the following solution is obtained, considering a higher importance of the first objective function and with a more certain result—$x_{12} = 2$, $x_{13} = 5$, $x_{21} = 10$, $x_{24} = 7$, $x_{33} = 8$, $x_{34} = 8$, with $Z^1 = [122, 202]$, $w[Z^1] = 80$, $m[Z^1] = 162$ and $Z^2 = [149, 233]$, $w[Z^2] = 84$, $m[Z^2] = 191$.

Let $Z^{1D} = [113, 204.9]$ and Let $Z^{2D} = [129.89, 227.86]$ be the interval solutions obtained by Das et al. (1999) and $Z^{1S} = [122, 202]$ and $Z^{2S} = [149, 233]$ be the interval solutions reached by Sengupta and Pal (2009). If we compare these interval objective function values using the interval order relations discussed in Sect. 2.1, it can be concluded that:

Table 1 Approaches for obtaining solutions for the MOITP/ITP

Approach	Advantages	Disadvantages
Composite (interactive, fuzzy and GP) (Abd El-Wahed and Lee 2006)	Solves other MOLP problems. Combines fuzzy, GP and interactive approaches, overcoming the conflict among the objectives and the problems of determining the aspiration levels via GP. Provides a preferred compromise solution from the DM's perspective. Determines appropriate aspiration levels of the objective functions	May lead to computation related problems
Fuzzy (Das et al. 1999)	Leads to k non-dominated solutions and one optimal compromise solution with k objective functions	Transforms the MOITP problem into a classical MOTP
Fuzzy (Sengupta and Pal 2009)	Reflects the DM's preferences changing from pessimism to optimism. Requires few steps to yield the compromise solution. Allows flexibility regarding the way the DM reaches his/her desired satisfactory solution	Transforms the problem into a crisp surrogate
Fuzzy (Panda and Das 2013)	Uses two-vehicle cost varying TP	Considers a single objective function
Stochastic/fuzzy (Nagarajan et al. 2014)	Leads to k non-dominated solutions and one optimal compromise solution with k objective functions	Transforms the MOITP problem into a classical MOTP
Non-fuzzy (separation method) (Pandian and Natarajan 2010)	Considers the decision variables are also given as intervals. Uses the separation method without using the midpoint and width of the interval in the objective function of the fully interval TP which is a non-fuzzy method. Extends the approach to fuzzy TP	Considers a single objective function. Transforms the problem into a crisp surrogate
Non-fuzzy (non-interactive) (Joshi and Gupta 2011)	Calculates the total cost bounds of the transportation problem, where at least either the supply or demand is varying	Considers a single objective function. Requires fine-tuning the production strategy or warehouse stocking level for reducing the total transportation cost. The largest total transportation cost may not occur at the highest total quantity shipped

(continued)

Table 1 (continued)

Approach	Advantages	Disadvantages
Non-fuzzy (floating point method) (Pandian and Anuradha 2011a, b)	Considers the decision variables are also given as intervals. Uses the floating point method without using the midpoint and width of the interval in the objective function of the fully interval TP which is a non-fuzzy method. Extends the approach to fuzzy TP	Considers a single objective function. Transforms the problem into a crisp surrogate
Non-fuzzy (parametric) (Rakocevic and Dragasevic 2011)	Allows determining the behavior of the system if the parameters change within the given limits, as well as to set parameters so that, with the optimal solution determined, the greatest possible success is achieved, without significant distortion of the structure of the optimal solution	Considers a single objective function
Non-fuzzy (stochastic) (Roy and Mahapatra 2011)	Reflects the DM's pessimistic or optimistic bias in achieving a non-inferior solution. Facilitates the DM's choice of the desired satisfactory solution by the suitable arrangement of the objective function weights. Only a few steps are required to obtain the non-inferior solution	Transforms the MOITP problem into a classical MOTP
Non-fuzzy (Taylor series approximation and Interval arithmetic) (Guzel et al. 2012)	Introduces two solution procedures for the fractional ITP	Transforms a fractional TP with interval coefficients into a classical TP. Considers a single objective function
Non-fuzzy (sensitivity analysis) (Kavitha and Pandian 2012)	Extends the approach to fuzzy TP. Serves as an important tool for helping DM taking appropriate decisions when handling various types of logistic problems having imprecise parameters	Considers a single objective function.
Non-fuzzy (iterative) (Dalman et al. 2013)	Incorporates the total cost of damaged goods into the problem. Enables the DM to consider tolerances for the model parameters in a more natural and direct way. Uses quadratic functions	Considers a single objective. Transforms the IQITP into a traditional LP problem
Non-fuzzy (Interval-point method) (Fegade et al. 2013)	Enables the DM to evaluate TP with budgetary constraints	Considers a single objective function. Transforms the problem into a crisp surrogate
Non-fuzzy (Non-interactive) (Panda and Das 2014)	Uses two-vehicle cost varying TP	Considers a single objective function

1. According to Ishibuchi and Tanaka (1990) $Z^{1D} \leq_{MW} Z^{1S}$, since $m[Z^{1D}] \leq m[Z^{1S}]$ and $w[Z^{1D}] \geq w[Z^{1S}]$; $Z^{2D} \leq_{MW} Z^{2S}$, since $m[Z^{2D}] \leq m[Z^{2S}]$ and $w[Z^{2D}] \geq w[Z^{2S}]$;
2. According to Ishibuchi and Tanaka (1990) $Z^{1D} \leq_{LM} Z^{1S}$, since the "lower bound of Z^{1D}" ≤ "lower bound of Z^{1S}" and $m[Z^{1D}] \leq m[Z^{1S}]$; $Z^{2D} \leq_{LM} Z^{2S}$, since the "lower bound of Z^{2D}" ≤ "lower bound of Z^{2S}" and $m[Z^{2D}] \leq m[Z^{2S}]$;
3. Using the acceptability index developed by Sengupta and Pal (2000) $Z^{ID} \prec Z^{IS}$, since

$$Z^{1D} \prec = \frac{\left(m\left[Z^{1S}\right] - m\left[Z^{1D}\right]\right)}{\left(\frac{w\left[Z^{1S}\right]}{2} + \frac{w\left[Z^{1D}\right]}{2}\right)} = \frac{(162 - 158.95)}{(40 + 45.95)} = 0.035485748 > 0 \quad \text{and}$$

$$Z^{2D} \prec Z^{2S}, \text{since}\, Z^{2D} \prec = \frac{\left(m\left[Z^{2S}\right] - m\left[Z^{2D}\right]\right)}{\left(\frac{w\left[Z^{2S}\right]}{2} + \frac{w\left[Z^{2D}\right]}{2}\right)} = \frac{(191 - 178.85)}{(42 + 48.985)} = 0.133263725 > 0.$$

In a minimization problem, if both $\mathcal{A}\left(Z^{1D} \prec Z^{1S}\right) > 0$ and $\mathcal{A}\left(Z^{2D} \prec Z^{2S}\right) > 0$ it should be concluded that Z^{1D} is better than Z^{1S} and Z^{2D} is better than Z^{2S}, which allows concluding that from this point of view the solution proposed by Sengupta and Pal (2009) is dominated by the solution obtained by Das et al. (1999). In fact, all the interval order relations indicate the same results.

Table 1 provides a brief overview of the main advantages and drawbacks found in the approaches previously discussed.

4 Conclusions

The mathematical formulation of the traditional MOTP assumes that the coefficients of the objective functions and the supply and demand values are considered as crisp values. However, the unit transportation costs may vary and the supply and demand may also change. In this context, interval programming is one of the approaches used to handle uncertainty in mathematical programming models, which entails some interesting characteristics. In contrast to stochastic programming or to fuzzy programming which start with the specification or the assumption of probabilistic distributions and possibilistic distributions, respectively, and to robustness optimization techniques which inherently consider a max-min formulation (i.e., worst-case), interval programming only requires information about the range of variation of some (or all) of the parameters. In the framework of the critical assessment of the interval programming techniques for solving MOITP revisited in this study, it might be concluded that the combination of two or more approaches in the solution methods applied to MOITP/ITP may reduce the limitations of each individual approach. On the other hand, in general, the approaches herein reviewed transform the original MOITP/TP into a crisp model and then use an algorithm to solve it. Consequently, the uncertainty of the intervals becomes lost to a certain extent. Finally, as it was shown, the satisfactory solutions obtained may lead to contradictory results from the point of view of interval order relations. Therefore, new ways to model the MOITP/TP should be proposed in order to overcome this expected drawback.

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