Negative Terms in Euler Diagrams: Peirce's Solution

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Abstract. We commonly represent a class with a curve enclosing individuals that share an attribute. Individuals that are not predicated with that attribute are left outside. The status of this outer class has long been a matter of dispute in logic. In modern notations, negative terms are simply expressed by labeling the spaces that they cover. In this note, we discuss an unusual (and previously unpublished) method designed by Peirce in 1896 to handle negative terms: to indicate the position of the terms by the shape of the curve rather than by labeling the spaces.

Keywords: Negative term · Euler diagram · Charles S. Peirce · Syllogism

1 Introduction

Traditional Euler diagrams were first introduced to tackle syllogistic problems where only positive terms occur [1]. Hence, they hardly lend themselves to the treatment of negative terms. For instance, the outer space standing for the negation of all the terms in the argument is always shown to exist. Hence, it is not possible to express its absence without further amendments. Early logicians offered several solutions to overcome this difficulty. An obvious trick consists in replacing a negative term by a positive one during the resolution of a problem [2: 63]. For instance, a proposition "Some *x* are *not-y*" might simply be transformed into "Some *x* are *z*" (with *z* = *not-y*) and, consequently, be represented with traditional Euler diagrams. However, this method works merely when a term is not expressed twice with opposite signs in the considered problem.

Another solution would be to enhance Euler diagrams in order to represent actual relations between terms and their opposites, rather than positive terms alone. This can be achieved by representing the universe of discourse and thus devoting a finite space to the outer region of the diagram if existent, or no space at all if absent [3: 170–4]. An advantage of this solution is that it produces true Euler-type diagrams that require no additional conventions for their usage. However, this solution suffers from the complexity and the multiplicity of the figures needed for solving the problems, and thus increases the risk of misusing the diagrams.

This objection disappears in the case of Venn diagrams where all combinations of terms are first represented by compartments before syntactic signs are added to mark them and indicate their status [8]. However, such diagrams are not Euler-type since they do not represent actual information [6]. Therefore, they stand beyond the scope of the present note.

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2 Peirce's Solution

Charles S. Peirce worked on several amendments of Euler diagrams. One particular innovation from 1896 offers an unusual approach to negative terms [7]. Indeed, logicians commonly depict positive terms inside the curves [4, 5]. This usage is conventional and would not operate if we were to draw diagrams on a sphere. To indicate the term's sign, Peirce rather draws closed curves that have convex and concave sides. Then, he demands that positives are found on the concave side of the curve and negatives on the convex one. This does not infringe the common usage of Euler diagrams, since the concave side of a circle is inside it, which means that the positive term is still enclosed in the circle. However, Peirce's idea opens the way to various new shapes where the negative term is found *inside* the curve [Fig. 1a].

This new 'hyperboloid' method greatly simplifies the representation of propositions with negative terms. For instance, proposition "No *not*-A is *not*-B" which denies the existence of any outer region, is depicted with two disjoint curves [Fig. 1b].

Fig. 1. Two examples of Peirce's method: (a) "not-S"; (b) "Everything is either A or B"

Let us observe how this method applies on a syllogism whose premises are: "All S are M" and "No M is P". Since the latter premise can be transformed into "All M are *not*-P", the diagram depicts S inside M which is itself inside *not*-P [Fig. 2a]. Hence, the conclusion is "No S is P". Interestingly, syllogisms with two negative premises might be conclusive if negative terms are introduced. For instance, premises "No S is M" and "No *not*-M is *not*-P" yield the conclusion "All S are P" [Fig. 2b].

Fig. 2. Two examples of syllogisms according to Peirce's method of extending Euler diagrams

Peirce's method represents the same information in different ways, depending on the shape of the curves. This flexibility might prove convenient to represent complex propositions but it complicates their manipulation as it may not be easy to recognise diagrams' equivalences.

3 Conclusion

Modern diagrams represent negative terms along the path laid out by Euler and Venn: a curve produces two regions standing for complementary terms. The identification of the terms is conventional but is conveniently indicated by the label of the regions. Interestingly, Euler, Venn and Peirce appealed to different labeling practices. For a term S, all three would draw a circle, but Euler would put the letter 'S' *inside* the curve, Peirce *on* it and Venn *outside* it [Fig. 3]. Euler's usage is intuitive as it marks the space that stands for the class. Venn's usage is more practical since he demands a single figure for a given number of terms. Hence, the identification of the circles is unambiguous and all regions (except the outer) are kept ready to be marked. Peirce's practice is more challenging: it makes the curve stand for the *differentiæ* that disposes individuals on its both sides, depending on their predication. Hence, the curve acts as a separation line and is the object of the label. Consequently, the location of positive and negative terms is determined by the shape of the curve, not by its label.

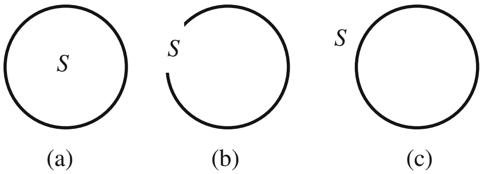

Fig. 3. The labeling conventions of Euler, Peirce and Venn

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