A Multi-direction Prediction Approach for Dynamic Multi-objective Optimization

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Abstract. In the real word, many multi-objective optimization problems are subject to dynamic changing conditions, which may occur in objectives, constraints and parameters. This paper provides a prediction strategy, called multi-direction prediction strategy (MDP), to enhance the performance of multi-objective evolutionary optimization algorithms in dealing with dynamic environments. Besides, the proposed prediction method makes use of multiple directions determined by several representative individuals. Our experimental results indicate that MDP can well tackle dynamic multi-objective problems.

Keywords: Dynamic · Multi-objective optimization · Prediction

1 Introduction

Dynamic multi-objective optimization problems (DMOPs) widely exist in our real world. Practical examples of such situations are design [1], planning [2], scheduling [3], etc. For handling these DMOPs, the optimizer should be capable of tracking the optima whose locations change with time.

Evolutionary algorithms (EAs) have been recognized as one of the most powerful optimizers [8]. The first attempt that utilizes EAs to solve DMOPs was conducted by Fogel and his colleagues in 1966 [9, 11, 12]. Since dynamic multi-objective evolutionary algorithm (DMOEA) is built on the static MOEA, under dynamic environments, converging tendency of a conventional static EA imposes severe limitations on performance of EA. In order to efficiently solve DMOPs, it is better for DMOEA to build its own framework. Moreover, some operators have been added for tackling environmental changes, such as the diversity maintenance technique and the prediction [10].

In this paper, we focus on the prediction method which aims to formulate a new population close to or even cover the PS under the new environment based on the historic information. To achieve this goal, we propose a multiple-direction prediction method for DMOEA. In this approach, several representative individuals are first sought based on the distribution information of the PS in the previous environment; next, several evolutionary directions are estimated by the representative individuals of the previous two PSs; finally, each individual evolves following the direction determined by its corresponding representative individual.

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2 Proposed Method

In this section, we address how to deal with the changes of the environment and the weaknesses of those available prediction approaches utilizing the proposed multiple direction prediction method (MDP), which aims to generate a population close enough to the true Pareto front. The basic idea of MDP is to estimate a set of individuals which have the ability to cover the true PS of the new environment. To achieve this goal, we first record and store several representative individuals which are able to describe the shape and diversity of the Pareto set obtained by the evolutionary algorithm at each time; subsequently, when an environmental change is detected, estimate the evolution directions in terms of information concerning representative individuals at the nearest two former time points (i.e., t and $t-1$). These evolutionary directions are employed to achieve the new locations of the representative individuals and predict the changes of the Pareto front; finally, a certain number of evolutionary individuals are generated around those new locations so as to improve the response of the population concerning the environmental change.

2.1 Determination of Representative Individuals

Due to the fact that representative individuals are designed to describe properties of the Pareto set, it is of great necessity for the selection strategy not only to choose those who are able to have both outperformed convergence and diversity. Therefore, M PF end-points are selected as representative individuals for a DMOP who has M objectives. The calculation equation is as follows.

$$p(m) = \arg \min_{x_j \in s} \left(f_m(x_j)\right) \tag{1}$$

Following that, the PS center point is calculated as one of the representative individuals due to its capability in helping the optimization algorithm converge to the PS under the new environment. In the studies [26–28], researchers utilize PS center points in the current and previous time, to predict the new locations of individuals. The definition of the abstracted PS center point is as follows.

$$C_i^t = \frac{1}{|PS^t|} \sum_{i=1}^{|PS^t|} x_i^t \tag{2}$$

where PS^t is the PS obtained at t, $|PS^t|$ is the number of individuals in PS^t, and x_i^t is the i-th solution in PS^t.

Herein, C^t is utilized to store these $(M + 1)$ representative individuals, where $C_i^t(i = 1,\ldots,M)$ are the M end-points of the PF, and $C_i^t(i = M+1)$ is the center point at t.

2.2 Multiple Direction Estimation

In order to estimate the variation tendency of the Pareto front when an environmental change is detected, a multiple direction estimation strategy is proposed in this section. This estimation strategy utilizes the time series model, connects the two nearest historical information of representative individuals, and predicts multiple directions concerning the representative individuals of the Pareto front. Therefore, the variation trend of the Pareto front is able to be described completely by those multiple directions.

Suppose that $C^t = \{c_1^t, c_2^t \cdots, c_{K'}^t\}$ and $C^{t-1} = \{c_1^{t-1}, c_2^{t-1} \cdots, c_{K''}^{t-1}\}$ are two representative individual sets in accordance with time t and $t-1$, respectively. For one individual c_i^t in C^t, the proposed strategy will first seek for the nearest representative individual to it in the set C^{t-1}, denoted as c_j^{t-1}, which is viewed as the parent individual of c_i^t. In that way, the evolutionary direction of the individual c_i^t can be estimated by the change from the parent individual c_j^{t-1} and the current individual c_i^t as follows.

$$\Delta c_i^t = c_i^t - c_j^{t-1} \tag{3}$$

2.3 Generation of Individuals

For sake of improving the responding speed to the environmental changes, we plan to generate a certain number of evolutionary individuals around the predicted Pareto front. Hopefully, a set of individuals which have the capability to get close to or even cover the true Pareto front with a uniform distribution is likely to be achieved. When a change has been detected, we first employ simple linear model to add the evolutionary directions obtained by means of multiple direction estimation measure in 3.2 to the representative individuals in the nearest historical time. Therefore, the location and shape of the Pareto front under the new environment is able to be predicted. Subsequently, the new individuals will be generated.

Suppose that an individual x^t belongs to the i-th category, which representative individual is c^t. The evolutionary direction of c^t has been calculated by means of Sect. 2.2, denoted as Δc^t. Therefore, a new individual x^{t+1} is generated as follows.

$$x^{t+1} = x^t + \Delta c^t \tag{4}$$

Similarly, several new locations of those representative individuals are generated as follows.

$$c^{t+1} = c^t + \Delta c^t \tag{5}$$

To improve the diversity of the population, a number of individuals are generated randomly in the search space. Therefore, the new population consists of two parts, i.e., a number of predicted individuals and randomly-generated individuals in the decision space.

2.4 Framework of the Prediction Method

The framework of the proposed prediction strategy is incorporated into the particle swarm optimization algorithm. The pseudo code of the dynamic multi-objective particle swarm optimization algorithm based on the multi-direction prediction strategy is depicted as follows (Fig. 1).

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Function MDP-DMOPSO : MDP-DMOPSO ( N_p , N_s , g_max , δ̃ )
//*Input: N_p (size of the swarm), N_s (size of the ar-
chive), g_max (maximum number of iterations), δ̃ ( an envi-
ronment threshold)*//
//*output: Op (the Pareto optimal set )*//
1: g = 0 For i=1 to N_p
initialize(x_i(0))
        pb_i(0) ← x_i(0)

Endfor
1.2  F(pop_0) ← evaluate(pop_0)
1.3  Ar_0 ← non_dominated(pop_0)
2: While g ≤ g_max , do                //*Main cycle *//
2.1  If environment changes
        C^t ← Select_RPoins(ps^t, K)
        ΔC ← Decide(C^t, C^{t-1}, K)
        Reset(Pop_g)
     Endif
2.2  Update the position of each particle in pop_g by PSO
   2.2.1 For i = 1 to N_p
          pbest_i(g) ← GET_pbest()
          gbest_i(g) ← GET_gbest()
          x_i(g+1) ← UP_PARTICLE(gbest_i(g), pbest_i(g), x_i(g))
         Endfor
   2.2.2 F(pop_{+1}) ← EVALUATE(pop_{+1})
   2.2.3 Update the archive
          ps_{g+1} ← NON_DOMINATED(pop_{g+1} ∪ Ar_0)
          If  |ps_{g+1}| > N_S
          Using the crowding-distance to prune the ar-
          chive.
          Endif
   2.3  g ← g + 1
   Endwhile
3:  Op ← ps^t and stop the algorithm
```

Fig. 1. The Pseudo Code of MDP+DMOPSO

3 Results

This section evaluates the performance of the proposed algorithm, MDP-DMOPSO, on several well-known dynamic multi-objective test problems, compared with the particle swarm optimization algorithm with initialize strategy, namely, RIS.

3.1 Experiment Preparations

In this paper seven benchmark problems, FDA1, FDA2, FDA3, DMOP1, DMOP2, DMOP3, are selected. Table 1 shows these functions and their types. The inverted generational distance (IGD) metric has been widely employed to test the performance of stationary MOPs, which can measure both diversity and convergence of non-dominated optimums to true Pareto front.

Let P^{t*} be a set of uniformly distributed Pareto optimal points in the PF^t, and P^t be an approximation of PF^t. The IGD metrics is defined as follows [13].

$$IGD(P^{t*}, P^t) = \frac{\sum_{v \in P^{t*}} d(v, P^t)}{|P^{t*}|} \tag{6}$$

where $d(v, P^t) = \min_{u \in P^t} \|F(v) - F(\mathrm{u})\|$ is the distance between v and P^t and $|P^{t*}|$ is the cardinality of P^{t*}. If P^t gets close enough to PF^t and can cover all parts of the whole PF^t, the IGD value will be low.

For these test functions, the severity of changes is set to be $n_T = 10$. The frequency of the changes is set to be $\tau_T = 20$. The dimensions of the test problems are $n = 10$.

The population size is set to be $N = 150$ for test problems. The archive size (the Pareto set size) is set to be $V = 100$. It is assigned to be 4500 evaluation times for these functions, i.e., the environment changes every 30 generations for all the two algorithms.

3.2 Performance Analysis

Table 2 depicts the IGD values of MDP and RIS on the test instances over 20 runs. For comparing the difference between those two sets of results, t-test is employed, where + (-) supports (fails to support) the hypothesis that there is variance between the two sets of results.

Table 2 suggests that MDP performs better than RIS in almost all cases, especially on those DMOPs whose PS changes. For FDA2, even though $31 \le t < 40$, MDP and RIS can obtain the same mean value of the IGD metric, they have great variance as the t-test result shows. Besides, on DMOP1, although $31 \le t < 40$, the mean value of the IGD metric of MDP is 0.2008, slightly higher than RIS (0.1252), the t-test result fail to support these two sets of IGD values have difference with each other.

Furthermore, Fig. 2 shows the Pareto fronts obtained by the two strategies at t = 8, 25, 35, 40, and 50 with lowest IGD values on FDA2. We can see that MDP has good performance in tracking the changes of the environment, and improving diversity of the algorithm.

Table 1. Test instances

Test instances	Definition	Type
FDA1	$f_1(X_{\text{¢ñ}}) = x_1$ $f_2 = g \cdot h$ $g(X_{\text{¢ò}}) = 1 + \sum_{x_i \in X_{\text{¢ò}}} (x_i - G(t))^2$ $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$ $G(t) = \sin(0.5\pi t),\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: m = 10, X_{\text{I}} = (x_1) \in [0,1];\ X_{\text{II}} = (x_2, \cdots, x_m) \in [-1,1]$	TypeI
FDA2	$f_1(X_{\text{¢ñ}}) = x_1$ $f_2 = g \cdot h$ $g(X_{\text{¢ò}}) = 1 + \sum_{x_i \in X_{\text{¢ò}}} x_i^2$ $h(f_1, g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)^{-1}}$ $H(t) = 0.75 + 0.7\sin(0.5\pi t),\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: X_{\text{¢ñ}} = (x_1) \in [0,1];\ X_{\text{¢ò}}, X_{\text{¢ó}} \in [-1,1],\ \lvert X_{\text{¢ò}} \rvert = \lvert X_{\text{¢ó}} \rvert = 15$	TypeIII
FDA3	$f_1(X_{\text{¢ñ}}) = \sum_{x_i \in X_{\text{¢ñ}}}^{n} x_i^{F(t)}$ $f_2 = g \cdot h$ $g(X_{\text{¢ò}}) = 1 + G(t) + \sum_{x_i \in X_{\text{¢ò}}} (x_i - G(t))^2$ $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$ $G(t) = \lvert \sin(0.5\pi t) \rvert$ $F(t) = 10^{2\sin(0.5\pi t)},\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: \lvert X_{\text{¢ñ}} \rvert = 1, \lvert X_{\text{¢ò}} \rvert = 9, X_{\text{¢ñ}} \in [0,1];\ X_{\text{¢ò}} \in [-1,1]$	TypeII
DMOP1	$f_1(x_1) = x_1$ $f_2 = g \cdot h$ $g(x_2, \cdots, x_m, t) = 1 + 9\sum_{i=2}^{m} x_i^2$ $h(f_1, g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)}$ $H(t) = 0.75\sin(0.5\pi t) + 1.25,\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: m = 10,\ x_i \in [0,1], \forall i = 1, 2, \cdots, m$	TypeII
DMOP2	$f_1(x_1) = x_1$ $f_2 = g \cdot h$ $g(x_2, \cdots, x_m, t) = 1 + \sum_{i=2}^{m} (x_i - G(t))^2$ $h(f_1, g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)}$ $H(t) = 0.75\sin(0.5\pi t) + 1.25,\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: m = 10,\ x_i \in [0,1], \forall i = 1, 2, \cdots, m$	TypeII
DMOP3	$f_1(x_1) = x_1$ $f_2 = g \cdot h$ $g(x_2, \cdots, x_m, t) = 1 + \sum_{i=2}^{m} (x_i - G(t))^2$ $h(f_1, g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)}$ $H(t) = 0.75\sin(0.5\pi t) + 1.25,\ t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $where: m = 10,\ x_i \in [0,1], \forall i = 1, 2, \cdots, m$	TypeII

Table 2. The IGD values on 20 runs

		$1 \leq t < 10$	$11 \leq t < 20$	$21 \leq t < 30$	$31 \leq t < 40$	$41 \leq t \leq 50$
		Mean/std.	Mean/std.	Mean/std.	Mean/std.	Mean/std.
FDA1	MDP	**0.0120/0.0060(+)**	**0.0074/0.0032(+)**	**0.0146/0.0033(+)**	**0.0158/0.0063(-)**	**0.0126/0.0073(+)**
	RIS	0.0311/0.0020	0.0252/0.0021	0.0279/0.0056	0.0230/0.0047	0.0218/0.0027
FDA2	MDP	**0.0036/2.9575e-05 (-)**	**0.00369/1.4205e-05 (+)**	0.0036/2.0682e-05 (-)	**0.0036/1.1720e-05 (+)**	**0.0036/3.4290e-05 (+)**
	RIS	0.0047/0.0030	0.0037/7.53e-05	0.0036/3.8e-05	0.0036/3.35e-05	0.0037/6.35e-05
FDA3	MDP	**0.0036/6.896e-05 (+)**	**0.0036/3.0546e-05 (+)**	**0.0068/0.0048(+)**	**0.0101/0.0088(+)**	**0.0036/4.5897e-05 (+)**
	RIS	0.2707/0.0518	0.2337/0.0305	0.2861/0.0121	0.2874/0.0271	0.2388/0.0237
DMOP1	MDP	**0.0041/0.0001(+)**	**0.0042/0.0001(+)**	0.1760/0.1466(+)	0.2008/0.1488(-)	**0.0041/0.0002(+)**
	RIS	0.0755/0.0283	0.0414/0.0255	**0.0863/0.1067**	0.1252/0.1754	0.0102/0.0008
DMOP2	MDP	**0.0037/ 5.1830e-05(+)**	**0.0037/ 2.8288e-05(+)**	**0.0036/ 1.5351e-05(+)**	**0.0037/ 2.8446e-05(+)**	**0.0037/ 1.4142e-05(+)**
	RIS	0.0316/0.0253	0.0167/0.0095	0.0205/0.0105	0.0150/0.0080	0.0166/0.0067
DMOP3	MDP	**0.0037/ 9.1116e-05(+)**	**0.0037/ 3.4155e-05(+)**	**0.0037/ 3.4269e-05(+)**	**0.0037/ 3.4497e-05(+)**	**0.0037/ 3.2664e-05(+)**
	RIS	0.2622/ 0.2573	0.0617/ 0.0284	0.0758/ 0.0041	0.0419/ 0.0183	0.0466/ 0.0132

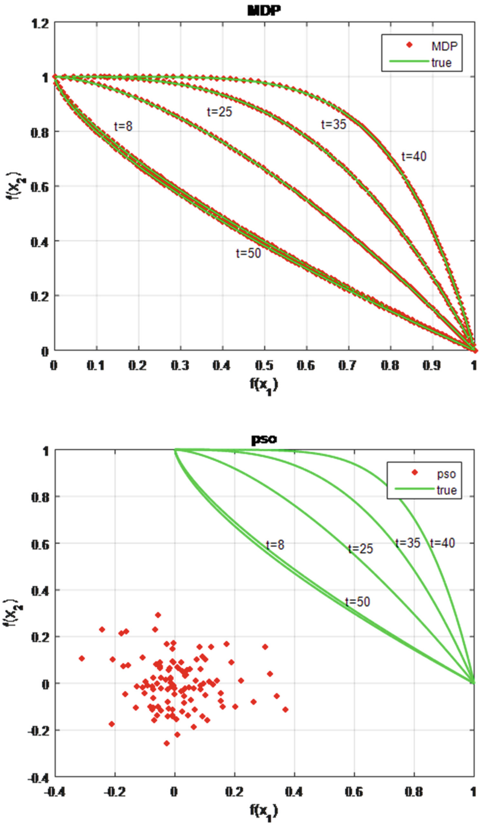

Fig. 2. The Pareto fronts obtained by the two strategies

4 Conclusion

In this paper, a new prediction strategy, MDP is proposed, which employs multiple directions based on representative individuals to provide a guide for the evolution of the population. By comparing with the strategy which generates population randomly when an environmental change is detected, MDP shows great capability in tracing the movement of PF (PS).

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