

Chapter 9

Reduction of the Controller Complexity

9.1 Introduction

The complexity (order of the polynomials R and S) of the controllers designed on the basis of identified models depends upon

- the complexity of the identified model;
- the performance specifications; and
- the robustness constraints.

The controller will have a minimum complexity equal to that of the plant model but as a consequence of performance specifications and robustness constraints, this complexity increases (often up to the double of the size of the model, in terms of number of parameters, and in certain cases even more). In many applications, the necessity of reducing the controller complexity results from constraints on the computational resources in real time (reduction of the number of additions and multiplications).

Therefore one should ask the question: can we obtain a simpler controller with almost the same performance and robustness properties as the nominal one (design based on the plant model)?

Consider the system shown in Fig. 9.1 where the plant model transfer function is given by

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{9.1}$$

and the nominal controller is given by:

$$K(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})} \tag{9.2}$$

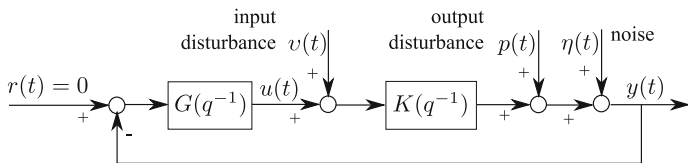


Fig. 9.1 The true closed-loop system

where:

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \dots + r_{n_R} z^{-n_R} \quad (9.3)$$

$$S(z^{-1}) = 1 + s_1 z^{-1} + \dots + s_{n_S} z^{-n_S} = 1 + z^{-1} S^*(z^{-1}) \quad (9.4)$$

Different sensitivity functions have been defined in Sect. 7.1 for the system given in Fig. 9.1.

The system given in Fig. 9.1 will be denoted the “true closed-loop system”. Throughout this chapter, feedback systems which will use either an estimation of G (denoted \hat{G}) or a reduced order estimation of K (denoted \hat{K}) will be considered. The corresponding sensitivity functions will be denoted as follows:

- S_{xy} —Sensitivity function of the true closed-loop system (K, G).
- \hat{S}_{xy} —Sensitivity function of the nominal simulated closed-loop system (nominal controller K + estimated plant model \hat{G}).
- $\hat{\hat{S}}_{xy}$ —Sensitivity function of the simulated closed-loop system using a reduced order controller (reduced order controller \hat{K} + estimated plant model \hat{G}).

Similar notations are used for $P(z^{-1}), \hat{P}(z^{-1})$ when using K and $\hat{G}, \hat{\hat{P}}(z^{-1})$ when using \hat{K} and \hat{G} .

The specific objective will be to reduce the orders n_R and n_S of controller polynomials R and S .

The basic rule for developing procedures for controller complexity reduction is to search for controllers of reduced orders which preserve the properties of the closed-loop as much as possible. A direct simplification of the controller transfer function by traditional techniques (cancellation of poles and zeros which are close, approximations in the frequency domain, balanced truncation, etc.) without taking into account the properties of the closed-loop leads in general to unsatisfactory results (see [1, 2]).

Two approaches can be considered for the controller complexity reduction

1. Indirect Approach

This approach is implemented in three steps

- a. Reduction of the complexity of the model used for design, trying to preserve the essential characteristics of the model in the critical frequency regions for design.

- b. Design of the controller on the basis of the reduced model.
- c. Test of the resulting controller on the nominal model.

2. Direct Approach

Search for a reduced order approximation of the nominal controller which preserves the properties of the closed-loop.

The indirect approach has a number of drawbacks

- Does not guarantee the complexity of the resulting controller (since the robustness specifications will be more severe when using reduced order models).
- The errors resulting from model reduction will propagate in the design of the controller.

The direct approach seems the most appropriate for the reduction of the controller complexity since the approximation is done in the last stage of the design and the resulting performance can be easily evaluated. A combination of the two approaches is also possible (see Chap. 10), i.e., the resulting controller obtained by the indirect approach, after it has been tested on the nominal plant model is further reduced through the direct approach.

9.2 Criteria for Direct Controller Reduction

Two criteria can be considered for direct reduction of the controller complexity

- *Closed-loop input matching (CLIM)*. In this case, one would like that the control generated in closed-loop by the reduced order controller be as close as possible to the control generated in closed-loop by the nominal controller.
- *Closed-loop output matching (CLOM)*. In this case, one would like that the closed-loop output obtained with the reduced order controller be as close as possible to the closed-loop output obtained with the nominal controller.

These two criteria are illustrated in Fig. 9.2, where the nominal controller is denoted by K and is given in (9.2) and the reduced controller is denoted by \hat{K} and is given by

$$\hat{K}(z^{-1}) = \frac{\hat{R}(z^{-1})}{\hat{S}(z^{-1})} \quad (9.5)$$

where:

$$\hat{R}(z^{-1}) = r_0 + r_1 z^{-1} + \cdots + r_{n_R} z^{-n_R} \quad (9.6)$$

$$\hat{S}(z^{-1}) = 1 + s_1 z^{-1} + \cdots + s_{n_S} z^{-n_S} = 1 + z^{-1} \hat{S}^*(z^{-1}) \quad (9.7)$$

The *closed-loop input matching* is equivalent to minimizing the following norm:

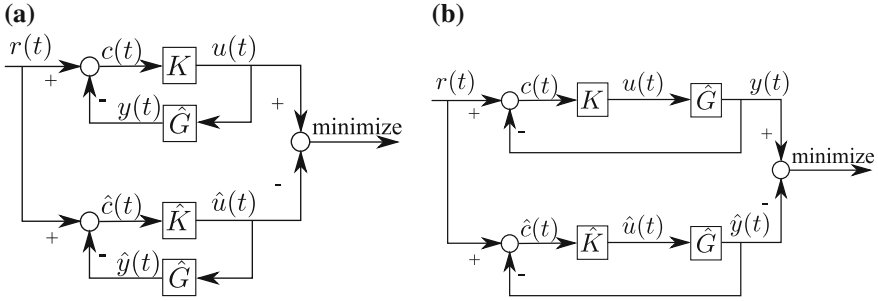


Fig. 9.2 Criteria for controller complexity reduction. **a** Input matching. **b** Output matching

$$\|\hat{S}_{up} - \hat{\hat{S}}_{up}\| = \left\| \frac{K}{1 + K\hat{G}} - \frac{\hat{K}}{1 + \hat{K}\hat{G}} \right\| \quad (9.8)$$

where \hat{S}_{up} is the input sensitivity function of the nominal simulated closed-loop and $\hat{\hat{S}}_{up}$ is the input sensitivity function when using the reduced order controller. Therefore the optimal reduced order controller will be given by

$$\hat{K}^* = \arg \min_{\hat{K}} \|\hat{S}_{up} - \hat{\hat{S}}_{up}\| = \arg \min_{\hat{K}} \|\hat{S}_{yp}(K - \hat{K})\hat{S}_{yp}\| \quad (9.9)$$

As it can be seen, the difference between the two controllers is heavily weighted by the output sensitivity function. The maximum of its modulus corresponds to the critical region for design. Therefore, the reduced order controller will very well approximate the nominal controller in this critical frequency region for design.

If we now consider preservation of performance in tracking using the *closed-loop output matching*, the reduced order controller should minimize the following norm:

$$\|\hat{S}_{yr} - \hat{\hat{S}}_{yr}\| = \left\| \frac{K\hat{G}}{1 + K\hat{G}} - \frac{\hat{K}\hat{G}}{1 + \hat{K}\hat{G}} \right\| \quad (9.10)$$

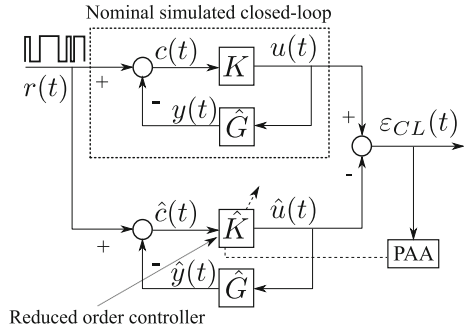
To preserve the performance for output disturbance rejection, the reduced order controller should minimize

$$\|\hat{S}_{yp} - \hat{\hat{S}}_{yp}\| = \left\| \frac{1}{1 + K\hat{G}} - \frac{1}{1 + \hat{K}\hat{G}} \right\| \quad (9.11)$$

Fortunately, these two norms are equal and the reduced order controller can be obtained using the following expression:

$$\hat{K}^* = \arg \min_{\hat{K}} \|\hat{S}_{yp} - \hat{\hat{S}}_{yp}\| = \arg \min_{\hat{K}} \|\hat{S}_{yp}(K - \hat{K})\hat{S}_{yv}\| \quad (9.12)$$

Fig. 9.3 Estimation of reduced order controllers by the closed-loop input matching (CLIM) method. Use of simulated data



Equations (9.9) and (9.12) show that a weighted norm of $K - \hat{K}$ should be minimized.

For closed-loop input matching (Fig. 9.2a) one tries to find a reduced order controller which will minimize the difference between the input sensitivity function of the nominal simulated system and the input sensitivity function of the simulated system using a reduced order controller. This is equivalent to the search for a reduced controller which minimizes the error between the two loops (in the sense of a certain criterion) for a white noise type excitation (like PRBS).

For the tracking of the nominal output (Fig. 9.2b) the principle remains the same, except that in this case one tries to minimize the difference between the nominal complementary sensitivity function (7.8) and the reduced order complementary sensitivity function computed with \hat{K} and \hat{G} .

It can be seen immediately that in both cases the problem of finding a reduced order controller can be formulated as an identification in closed-loop (see Chap. 8) where the plant model is replaced by the reduced order controller to be estimated and the controller is replaced by the available estimated model of the plant (dual problem).

The reduction procedures and the validation techniques for reduced order controllers to be presented next are available in the MATLAB® toolbox REDUC® [3] (to be downloaded from the book website) or in the stand alone software iReg which includes a module for controller complexity reduction.¹

9.3 Estimation of Reduced Order Controllers by Identification in Closed-Loop

9.3.1 Closed-Loop Input Matching (CLIM)

The principle of closed-loop input matching approach is illustrated in Fig. 9.3.

¹ See the website: <http://tudor-bogdan.airimitoiaie.name/ireg.html>.

The upper part represents the simulated nominal closed-loop system. It is made up of the nominal controller (K) and the best identified plant model (\hat{G}). This model should assure the best closeness behaviour of the true closed-loop system and the nominal simulated one. Identification of this plant model in closed-loop can be considered if the nominal controller can be implemented.

The lower part is made up of the estimated reduced order controller (\hat{K}) in feedback connection with the plant model (\hat{G}) used in the nominal simulated system. The parameter adaptation algorithm (PAA) will try to find the best reduced order controller which will minimize the closed-loop input error. The closed-loop input error is the difference between the plant input generated by the nominal simulated closed-loop system and the plant input generated by the simulated closed-loop using the reduced order controller.

The output of the nominal controller is given by

$$u(t+1) = -S^*(q^{-1})u(t) + R(q^{-1})c(t+1) = \theta^T \psi(t) \quad (9.13)$$

where

$$c(t+1) = r(t+1) - y(t+1) \quad (9.14)$$

$$y(t+1) = -\hat{A}^*y(t) + \hat{B}^*u(t-d) \quad (9.15)$$

$$\psi^T(t) = [-u(t), \dots, -u(t-n_S+1), c(t+1), \dots, c(t-n_R+1)] \quad (9.16)$$

$$\theta^T = [s_1, \dots, s_{n_S}, r_0, \dots, r_{n_R}] \quad (9.17)$$

To implement and analyze the algorithm, we need respectively the *a priori* (based on $\hat{\theta}(t)$) and the *a posteriori* (based on $\hat{\theta}(t+1)$) predicted outputs of the estimated reduced order controller (of orders $n_{\hat{s}}$ and $n_{\hat{r}}$) which are given by (see the lower part of Fig. 9.3).

a priori:

$$\begin{aligned} \hat{u}^\circ(t+1) &= \hat{u}(t+1)\hat{\theta}(t) = -\hat{S}^*(t, q^{-1})\hat{u}(t) + \hat{R}(t, q^{-1})\hat{c}(t+1) \\ &= \hat{\theta}^T(t)\phi(t) \end{aligned} \quad (9.18)$$

a posteriori:

$$\hat{u}(t+1) = \hat{\theta}^T(t+1)\phi(t) \quad (9.19)$$

where

$$\hat{\theta}^T(t) = [\hat{s}_1(t), \dots, \hat{s}_{n_{\hat{s}}}(t), \hat{r}_0(t), \dots, \hat{r}_{n_{\hat{r}}}(t)] \quad (9.20)$$

$$\phi^T(t) = [-\hat{u}(t), \dots, -\hat{u}(t-n_{\hat{s}}+1), \hat{c}(t+1), \dots, \hat{c}(t-n_{\hat{r}}+1)] \quad (9.21)$$

$$\hat{c}(t+1) = r(t+1) - \hat{y}(t+1) = r(t+1) + \hat{A}^*\hat{y}(t) - \hat{B}^*\hat{u}(t-d) \quad (9.22)$$

The closed-loop input error is given by

a priori:

$$\varepsilon_{CL}^{\circ}(t+1) = u(t+1) - \hat{u}^{\circ}(t+1) \quad (9.23)$$

a posteriori:

$$\varepsilon_{CL}(t+1) = u(t+1) - \hat{u}(t+1) \quad (9.24)$$

The equation governing the *a posteriori* prediction error becomes (see [4, 5] for details)

$$\varepsilon_{CL}(t+1) = \frac{\hat{A}}{\hat{P}} [\theta - \hat{\theta}(t+1)]^T \phi(t) \quad (9.25)$$

and the parameter adaptation algorithm will be given by

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1) \quad (9.26)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \quad (9.27)$$

$$0 < \lambda_1(t) \leq 1; \quad 0 \leq \lambda_2(t) < 2; \quad F(0) > 0$$

$$\varepsilon_{CL}(t+1) = \frac{\varepsilon_{CL}^{\circ}(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)} = \frac{u(t+1) - \hat{u}^{\circ}(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)} \quad (9.28)$$

As we can see from (9.28), the *a posteriori* closed-loop input error $\varepsilon_{CL}(t+1)$ can be expressed in terms of the *a priori* (measurable) closed-loop input error $\varepsilon_{CL}^{\circ}(t+1)$. Therefore, the right-hand side of (9.26) will depend only on measurable quantities at $t+1$.

Specific algorithms will be obtained by an appropriate choice of the observation vector $\Phi(t)$ as follows:

- CLIM: $\Phi(t) = \phi(t)$
- F-CLIM: $\Phi(t) = \frac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})} \phi(t)$

where

$$\hat{P}(q^{-1}) = \hat{A}(q^{-1})S(q^{-1}) + q^{-d}\hat{B}(q^{-1})R(q^{-1}). \quad (9.29)$$

The introduction of the filtering of ϕ is motivated by the elimination of a positive realness sufficient condition for stability and convergence which, in the case of the CLIM algorithm, depends on \hat{A}/\hat{P} . A detailed analysis of the properties of these algorithms can be found in [5].

The properties of the estimated controller in the frequency domain results from the following expression (bias distribution) [5]:

$$\hat{\theta}^* = \arg \min_{\hat{\theta} \in \mathcal{D}} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 \left[|K - \hat{K}|^2 |\hat{S}_{yp}|^2 \phi_r(\omega) + \phi_{\eta}(\omega) \right] d\omega \quad (9.30)$$

where $\phi_r(\omega)$ is the excitation spectrum and $\phi_{\eta}(\omega)$ is the measurement noise spectrum (it does not have effect upon the minimization of $|K - \hat{K}|$).

Estimation of reduced order controllers is also possible using real-time data (if the prototype of the nominal controller can be implemented on the real system) [5].

9.3.2 Closed-Loop Output Matching (CLOM)

The principle of this method is illustrated in Fig. 9.4. Despite that, the point where the external excitation is applied and the output variable is different with respect to Fig. 9.2b, the transfer function between $r(t)$ and $u(t)$ in Fig. 9.4 is the same as the transfer function between $r(t)$ and $y(t)$ in Fig. 9.2b. This means that in the absence of disturbances (it is the case in simulation) $u(t)$ generated by the upper part of the scheme given in Fig. 9.4 is equal to $y(t)$ generated in Fig. 9.2b. This allows one to use for closed-loop output matching the CLIM (or F-CLIM) algorithm. For effective implementation of the algorithm, the only changes occur in Eqs. (9.13) and (9.18), where $c(t)$ is replaced by:

$$x(t) = \hat{G}(r(t) - u(t)) \tag{9.31}$$

and $\hat{c}(t)$ is replaced by:

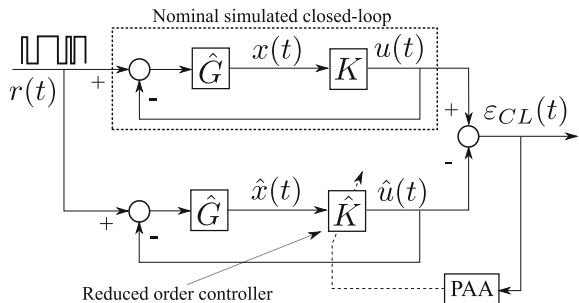
$$\hat{x}(t) = \hat{G}(r(t) - \hat{u}(t)) \tag{9.32}$$

One should note that the order of the blocks in the upper part of Fig. 9.4 can be interchanged (like the upper part of Fig. 9.2b) without affecting the operation of the algorithm.

9.3.3 Taking into Account the Fixed Parts of the Nominal Controller

It is often required that the reduced order controller contains some of the fixed filters incorporated in the nominal controller (for example, model of the disturbance, opening of the loop at $0.5 f_s$ or at other frequency). In order to do this, one first

Fig. 9.4 Estimation of reduced order controllers by the closed-loop output matching (CLOM) method. Use of simulated data



factorizes the nominal controller under the form $K = K_F K'$, where K_F represents all the fixed parts that one would like to be also incorporated in the reduced order controller. The reduced order controller is factorized as $\hat{K} = K_F \hat{K}'$.

One replaces in the CLIM algorithm the input \hat{c} of the controller \hat{K} by the input to the controller \hat{K}' , denoted \hat{c}' , where \hat{c}' is given by

$$\hat{c}'(t) = K_F(q^{-1})\hat{c}(t) \quad (9.33)$$

and in $\Phi(t)$, $\hat{c}(t)$ is replaced by $\hat{c}'(t)$. In the CLOM algorithm, one replaces \hat{x} by \hat{x}' given by

$$\hat{x}'(t) = K_F(q^{-1})\hat{G}(q^{-1})(r(t) - \hat{u}(t)). \quad (9.34)$$

9.3.3.1 Validation of Reduced Order Controllers

Once a reduced order controller has been estimated, it should be validated before considering its implementation on the real system.

It is assumed that the nominal controller stabilizes the nominal plant model (used for controller reduction). One implicitly assumes that the model uncertainties have been taken into account in the design of the nominal controller. The reduced order controller should satisfy the following conditions:

- It stabilizes the nominal plant model.
- The reduced sensitivity functions (computed with the reduced order controller) are close to the nominal sensitivity functions in the critical frequency regions for performance and robustness. In particular, the output and input sensitivity functions should be examined.
- The *generalized stability margin* (see Appendix A) of the system using the reduced order controller should be close to the *generalized stability margin* of the nominal closed-loop. This condition is expressed as

$$|b(K, \hat{G}) - b(\hat{K}\hat{G})| < \varepsilon; \varepsilon > 0 \quad (9.35)$$

where $b(K, \hat{G})$ and $b(\hat{K}\hat{G})$ are the *generalized stability margins* corresponding to the nominal controller and to the reduced order controller respectively and ε is a small positive number. The closeness of the two stability margins allows maintaining the robustness properties of the initial design.

The proximity of the nominal and reduced sensitivity functions can be judged by visual examination of their frequency characteristics. There is however the possibility to make a numerical evaluation of this proximity by computing the *Vinnicombe distance* (v gap) between these transfer functions (see Appendix A). The *Vinnicombe distance* allows with one number (between 0 and 1), to make a first evaluation of the proximity of the reduced and nominal sensitivity functions.

9.4 Real-Time Example: Reduction of Controller Complexity

In Sect. 7.3, a controller based on the open-loop identified model has been designed for the active vibration control system using an inertial actuator (see Sect. 2.2) and tested experimentally. It was shown in Sect. 8.3 that the controller designed on the basis of the model identified in open-loop provides similar performance to that of the controller designed on the basis of the model identified in closed-loop. Therefore in this section, the reduction of the complexity of the controller designed on the basis of the model identified in open-loop (which achieves the specifications) will be considered.

The criterion given in Eq. (9.8) will be considered, which corresponds to CLIM with external excitation added to the input of the controller. The model of the plant identified in closed-loop operation has been used. The excitation used was a PRBS with the following characteristics: $N = 11$ (number of cells) and $p = 2$ (clock frequency divider). The fixed parts of the controller have been preserved (internal model of the disturbance, opening the loop at $0.5 f_S$ and at 0 Hz).

Table 9.1 presents a summary of the controller order reduction results for various values of n_R and n_S . The first column represents the controller number (the controller with number 00 represents the initial nominal controller). The orders of the reduced controllers are indicated in columns n_R and n_S . The next column gives the Vinnicombe gap (Vg) between the initial controller and the reduced order controller. Similarly, the Vinnicombe gaps for the input and output sensitivity functions are also given in columns 5 and 6, respectively. A Vg of 0 indicates perfect matching while a Vg of 1 indicates very important differences between the two transfer functions. The *generalized stability margin* (see Appendix A) is given in column 7. For robustness reasons, it should be close to the value obtained for the nominal controller. The maximum of the output sensitivity function and the frequency in Hz for which it is obtained are given in columns 8 and 9, respectively. Finally, the stability of the closed-loop is indicated in the last column (1 represents a stable closed-loop, 0—unstable).

Only the first 12 reduced order controllers are shown in the table.² For experimental evaluation, controller 11 has been considered ($n_R = 19$, $n_S = 22$).

The output and input sensitivity functions obtained with the nominal and reduced order controllers are shown in Figs. 9.5 and 9.6, respectively. As it can be observed, the differences are very small within the frequency region of interest (except for the input sensitivity function at the 50 Hz—but this does not affect nor the robustness nor the performance). In Fig. 9.7 the transfer functions of the two controllers are shown.

It is important to remind that the comparison of the Bode characteristics of the two controllers does not guarantees that the reduced order controller stabilizes the system or that it assures good performances. It is the comparison of the sensitivity functions and the stability test which gives the right answers.

²These results have been obtained using the software iREG. Similar results are obtained with the *compcn.m* function from the toolbox REDUC.

Table 9.1 Summary of the controller order reduction results

No.	n_R	n_S	$Vg(\frac{R}{S})$	$Vg(S_{up})$	$Vg(S_{yp})$	St-margin	$\max(S_{yp})$	[f _{max}]	stable
00	29	32	0	0	0	0.3297	3.92	[60.0147]	1
01	29	32	0	0	0	0.3297	3.92	[60.0147]	1
02	28	31	0.001	0.003	0	0.3297	3.92	[60.0147]	1
03	27	30	0.0101	0.0284	0.0031	0.3296	3.8742	[60.0147]	1
04	26	29	0.0095	0.0282	0.0035	0.3306	3.8958	[60.0147]	1
05	25	28	0.0096	0.0327	0.004	0.3286	3.8958	[60.0147]	1
06	24	27	0.0103	1	0.0017	0.3263	3.9329	[60.0147]	1
07	23	26	0.0154	0.0498	0.0041	0.3213	3.9459	[60.0147]	1
08	22	25	0.0153	0.0545	0.0048	0.3232	3.9548	[60.0147]	1
09	21	24	0.0159	0.0514	0.0045	0.3232	3.9406	[60.0147]	1
10	20	23	0.0253	0.0972	0.0109	0.3268	3.9676	[60.0147]	1
11	19	22	0.0604	0.2645	0.0328	0.3089	3.9345	[59.3959]	1
12	18	21	1	1	1	0	3.7477	[59.3959]	0

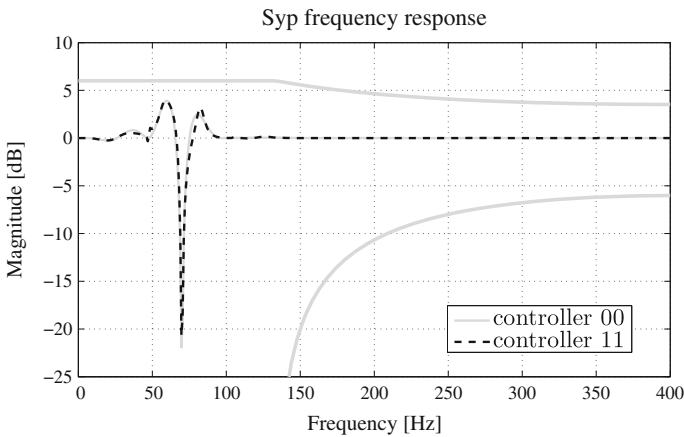


Fig. 9.5 Output sensitivity functions for initial and reduced order controllers

Finally, the controller has been tested in real time in the presence of a 70Hz sinusoidal disturbance. Time domain results in open and in closed-loop operation are shown in Fig. 9.8. The difference between the two power spectral densities for open-loop and closed-loop is shown in Fig. 9.9.³

For the reduced order controller, the following results have been obtained: (1) the global attenuation is 48.2 dB (instead of 48.4 dB for the nominal controller), the

³Figures 9.8 and 9.9 should be compared with Figs. 7.15 and 7.17.

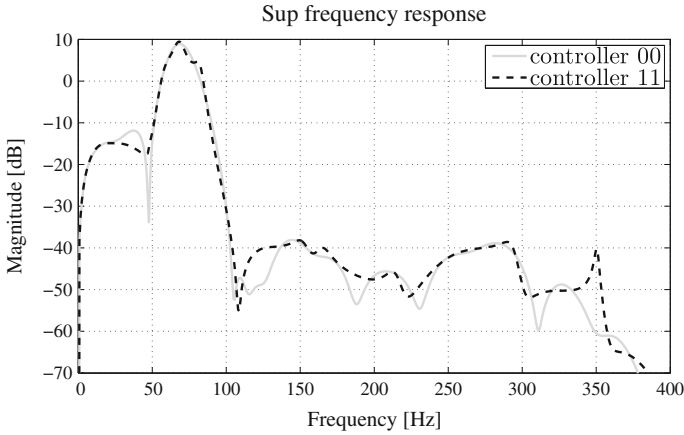


Fig. 9.6 Input sensitivity functions for initial and reduced order controllers

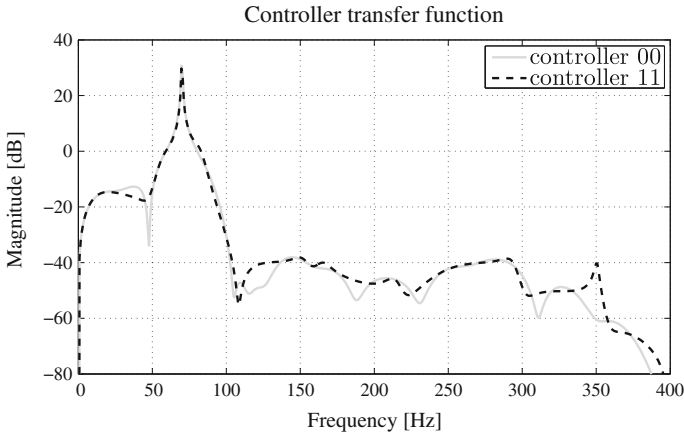


Fig. 9.7 Controller transfer function comparison between initial and reduced order controller

disturbance attenuation is 56.4 dB (instead of 62.4 dB but still much more than the required attenuation) and the maximum amplification is 7.5 dB (instead of maximum 6 dB specified). A small reduction in performance with respect to the initial nonreduced controller is observed but the number of parameters has been reduced from 62 to 44. These results presented above have been obtained using a single trial.

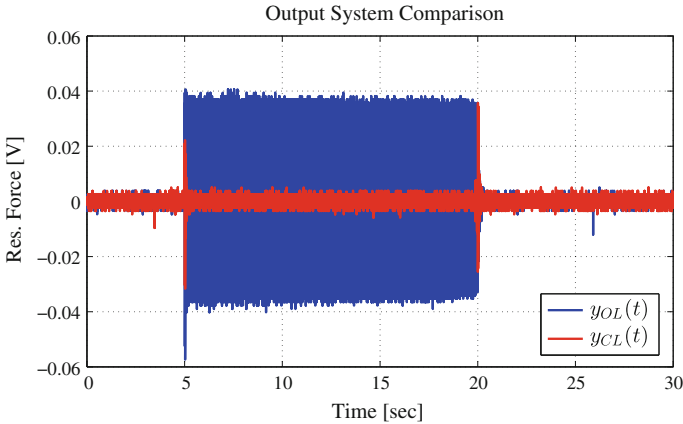
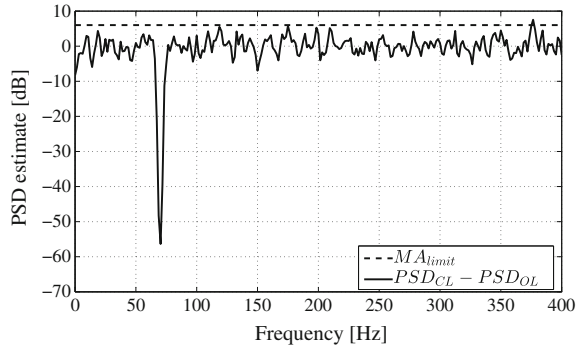


Fig. 9.8 Time response results for a 70Hz sinusoidal disturbance in open- and in closed-loop operation using the reduced order controller

Fig. 9.9 Effective residual attenuation/amplification PSD estimates computed as the difference between the open-loop PSD and the closed-loop PSD (reduced order controller)



9.5 Concluding Remarks

- The objective of controller reduction is to find a controller of reduced complexity such that the characteristics of the closed-loop using the reduced order controller are as close as possible to the characteristics of the closed-loop using the nominal controller.
- Two specific objectives have been considered
 - closed-loop input matching (CLIM); and
 - closed-loop output matching (CLOM).
- The CLOM (CLIM) objective corresponds to the estimation of a reduced order controller such that the error between the output (the control input) of the closed-loop using the reduced order controller and the output (the control input) of the

closed-loop using the nominal controller be minimized in the sense of a certain criterion.

- Controller reduction can be viewed as a dual problem with respect to plant model identification in closed-loop (similar algorithms will be used).
- The reduced order controllers should be validated before their effective use.
- Techniques for validation of the reduced order controllers have been provided in this chapter.

9.6 Notes and References

The problem of controller reduction is clearly presented in [1, 2]. See also [6].

The basic references for the algorithms discussed in this chapter (analysis and evaluation) are [5, 7, 8]. A unified view of identification in closed-loop and controller reduction can be found in [8].

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