Fuzzy Topsis Method Associated with Improved Selection of Machines of High Productivity

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Abstract This study combines Fuzzy Logic and multicriteria TOPSIS method for the selection, from three different alternatives, which machines of high productivity is more convenient to a construction company. The evaluation of each alternative is made through group decision making which identifies the most important criteria according to the requirements presented by the company. To assess the selected criteria in the TOPSIS method is weighted by a group of experts who, based on their experience and knowledge of this type of machinery, assess the relevance of these in the operation and functioning of the hydraulic excavator. Both qualitative and quantitative studies are used in this work, however the experts evaluate, through surveys based on Likert scale all the criteria in which they want to measure the perception. Data provided from the surveys is used for the construction and association of the groups of expert's opinion through the use of fuzzy sets to avoid ambiguity problems of the linguistic variables.

Keywords TOPSIS · Likert · Linguistic labels · Fuzzy sets

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1 Introduction

Multicriteria decision making is a process of finding the best alternative among a set of optimal alternatives. Among the various compensatory methods of multicriteria decision making, it is possible to consider a subgroup that includes cost and benefits aspects. One of them is the TOPSIS (Technique for Order Performance by Similarity to Idea Solution) method [2], technique that allows to sort preferences by similarity to an ideal solution. The TOPSIS method is a model for making decisions with which it is possible to sort requirements in comparison to an ideal solution in order to acquire a hierarchical order of the compared alternatives. The intuitive concept of the ideal alternative would be the one that, without hesitate, would be selected by the decision maker. Similarly, the anti-ideal alternative would be the one, without hesitate, never would be selected by the decider [1]. This paper is organized as follows: in the next section, fuzzy numbers, fuzzy sets and linguistic variables are described. The third section presents TOPSIS and Likert methods. An application example is discussed in the fourth section and finally the main conclusions are presented.

2 Theoretical Framework

2.1 Fuzzy Numbers and Fuzzy Sets

A fuzzy set A in R1 is called a fuzzy number if A is convex and exists in an exact point, M ∈ R1, with mA(M) = 1 (Aα = 1 = M), [3]. The linguistic expression of such fuzzy number would be: "Approximately M". For a better manipulation, the fuzzy numbers are usually defined L-R (left-right). Fuzzy numbers are very useful to process the information in a fuzzy environment and implement a representation [3]. The usage of fuzzy sets is precise with expressions that do not have clear boundaries. A fuzzy set is associated to a linguistic value defined by an expression that in most cases is denominated linguistic label. The vaguely defined sets play an important role in human thought, particularly in fields such as pattern recognition, communication of information and abstraction. Fuzzy sets are used to perform a qualitative evaluation of a physical quantity [4]. For a fuzzy set, the belonging of an element to the set is not a question of all or nothing, but are possible different degrees of membership. Membership functions can take any real value in the interval [0,1]. That is, mA |U → [0, 1] is the membership function of a fuzzy set, if X has a non-empty set. A fuzzy set A in the X domain is characterized by membership function.

2.2 Linguistic Variables

When people use language to convey something they want to communicate, they use expressions that do not relate exactly what they want to express. However, the use of these expressions do not implicate they should be accurate since in the language process the expressions are used collectively between the sender and receiver, without being necessary that they mean the same for both. For example, if it is desired that a group of people describe the temperature of water contained in a container, it is possible to use the expressions such as cold, hot and warm, in that way it is obtained a set of fuzzy elements denominated by Zadeh [4]. The meaning of the linguistic labels is determined by the fuzzy sets.

3 TOPSIS

It is a mathematical programming technique originally used in continuous contexts and has been modified for the analysis of discrete multicriteria problems. This technique was developed by Hwang and Yoon [2] in 1981 and refined by the authors in 1987 and 1992, they also have developed different versions of other authors. In this paper the results of the weighting of the qualitative criteria by the Likert scale with fuzzy sets and quantitative criteria associated to the data sheet of each alternative are used to evaluate these by a multicriteria TOPSIS model. This methodology proposes an algorithm to determine the preferred choice among all possible alternatives.

3.1 Likert Scale

This scale requires the respondents to select a label (answer) to represent their personal perception of each of the statements that are presented [7]. In this paper the scale is used to acquire the opinion or perception of the evaluators to qualify and weigh the selected criteria to evaluate each alternative. The linguistic labels are used to evaluate the importance level of each criteria and aptitude of each alternative. In this surveys linguistic labels are used to calculate the weight of each criteria and for the construction of the decision matrix in the TOPSIS method.

3.2 Defuzzification Method

Among the possible existing defuzzification methods, this paper uses the method proposed by Garcia-Cascales and Lamata [1], because it allows to include bias parameters that affect the fuzzy set's value.

The calculation is made by the following expression:

$$I_{\beta,\lambda}(A_i) = \beta S_M(A_i) + (1-\beta)\lambda S_R(A_i) + (1-\beta)(1-\lambda) S_L(A_i)$$

where :

$S_L(A_i)$ represents the lower medium value associated with the inverse function $g_A^L(x)$

$S_M(A_i)$ is the heart's area of the fuzzy number.

S_R (A_i) represents the high medium value of the fuzzy number associated with the inverse function $f_A^R(x)$.

β ∈ [0,1] is the modality index representing the importance of the central value.

λ ∈ [0,1] is the optimism's degree of the decision maker.

3.3 Methodology

Step 1. Decider-makers group establishment and expert profile definition. The experts and decider-makers groups were integrated as the TOPSIS methodology requires.

Step 2. Planning. Determining the finite set of alternatives to achieve the goal.

Information search. The information of three backhoe loader was collected.

Step 3. Determination and justification of the evaluation criteria. The hydraulic excavators that are compared in this paper have similar physic and technical characteristics.

Step 4. Weighting for each criterion. To calculate the criteria's weight, a double survey is applied to each of the evaluators.

Step 5. Triangular fuzzy sets approximation for each label. The selected criteria for the evaluation of each of the proposed alternatives and parameters for the construction of the triangular fuzzy sets are associated to the linguistic labels: Very important (VI), Moderately important (MI), Important (I), Indifferent (IN) and No importance (NI).

Step 6. Survey Implementation. Likert and Semantic differential scale are used for the application of the surveys applied to the decider and expert groups.

Step 7. Fuzzy triangular sets modification for each label. To modify the parameter's values of each set associated to the linguistic labels, the numeric values that the evaluators related to the linguistic labels were identified and tabulated.

Step 8. TOPSIS implementation. The fuzzy TOPSIS methodology proposed by Garcia-Casales and Lamata is used.

Step 9. Obtaining of positive and negative ideal solution (A+, A–). Ideal solutions were obtained to be compared with each of the proposed alternatives.

Step 10. Calculation of the relative proximity of each alternative to the ideal solutions found.

Step 11. Calculation of the relative proximity to each of the alternatives with respect to the ideal solution it is made.

Step 12. Triangular sets defuzzification. A value that represents the triangular set is calculated and the associated to a linguistic label according to the numerical value obtained.

4 Study Case

This section describes a real case decision. This case is shown to illustrate the Fuzzy Topsis Method. The goal is to decide which is the best productivity machine that should use a constructor company to excavate, when there are floods of land on roads caused by rains, earthquakes or landslides for any other reason. Construction companies have machines as hydraulic excavators that have flaws in some of its parts. These companies need to perform a quality job quickly and to remove the alluvium. This activity should consider the time and cost of repairing failures in key parts as the rotation mechanism and the hydraulic system. Construction companies have machines type hydraulic excavators, with flaws in some of its parts. These companies should perform a quality job and quick to remove the alluvium. This activity should consider the time and cost of repairing failures in the key parts as the rotation mechanism and the hydraulic system. In addition, it is required to consider aspects such as ergonomic interface and load capacity. Some criteria are feasible to quantify, in other evaluation is performed only by using linguistic values. The hydraulic excavators compared in this paper have very similar physical and technical characteristics, they are manufactured by world renowned companies that stand out for their quality and prestige. Figure 4 shows the three different alternatives that are evaluated in this paper, the first one is a 349DL model from CAT, the second one is a 22T is a BC-6225 from SANY and the third one is a HB12LC-1 from KOMATSU.

4.1 Criteria to Evaluate

To derive the characteristics or criteria to be analyzed, opinions and needs of the decision making group were taken. In the next table each criteria is defined, as in table 1.

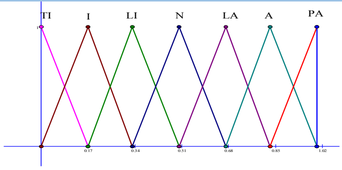
Table 1 Selected criteria

C1	Social Cost	C5	Engine power
C2	Rotation mecanism	C6	Fuel capacity
C3	Hydraulic system	C7	Weight
C4	Ergonomic interface		

4.2 Triangular Fuzzy Sets Approximation for Each Label

The linguistic variables that integrate the fuzzy sets are shown in table 2, wherein also the triangular weighting of each label is shown.

Table 2 Linguistic labels

Linguistic labels	Fuzzy numbers Cj	Fuzzy numbers Cj
Totally unacceptable (TU)	[0, 0, 0.167]	
Unacceptable (U)	[0, 0.167, 0.333]	
Lightly unacceptable (LU)	[0.167, 0.333, 0.5]	
Neutral (N)	[0.333, 0.5, 0.667]	
Lightly acceptable (LA)	[0.5, 0.667, 0.833]	
Acceptable (A)	[0.667, 0.833, 1]	
Perfectly acceptable (PA)	[0.833, 1, 1]	

4.3 Survey Application

For the survey application both, deciding group and experts group, their results were weight it, to obtain an average which became part of the TOPSIS. Likert scale is used to associate a linguistic label to each of the criteria evaluated. This survey is in charge to associate the importance level for each criteria on the personal perception of a group of evaluators. The results of this survey are used to weight and calculate the level of importance of the selected criteria through a double survey. In this paper fuzzy sets are defuzzyfied after the surveys have been applied, weighting the labels by the frequency they were selected.

4.4 Triangular Fuzzy Sets Modification for Each Label

To calculate the new parameter values for each associated fuzzy set to a linguistic label, all numerical values are identified and tabulated according to the linguistic labels the evaluator relate. The core value of the triangular number takes the geometric average value, the lower limit is the least numerical value received for the label, while the upper limit represents the larger label received it.

4.5 TOPSIS Application

Based on TOPSIS methodology with fuzzy numbers proposed by Garcia and Lamata, it starts with the identification of the weight of each criteria for each of the evaluators as is shown in in table 3.

Table 3 Labels associated to weights.

	E1			E2			E3			E4			E5			E6		
C1	0.9	0.967	1	0.9	0.967	1	0.9	0.967	1	0.9	0.967	1	0.9	0.967	1	0.9	0.967	1
C2	0.833	0.85	0.9	0.5	0.73	0.9	0.833	0.85	0.9	0.833	0.85	0.9	0.5	0.73	0.9	0.833	0.85	0.9
C3	0.85	0.89	0.9	0.85	0.89	0.9	0.9	0.967	1	0.85	0.89	0.9	0.85	0.89	0.9	0.9	0.967	1
C4	0.667	0.7	0.833	0.833	0.85	0.9	0.667	0.7	0.833	0.667	0.7	0.833	0.833	0.85	0.9	0.833	0.85	0.9
C5	0.95	0.983	1	0.95	0.983	1	0.95	0.983	1	0.95	0.983	1	0.95	0.983	1	0.95	0.983	1
C6	0.833	0.85	0.9	0.5	0.592	0.8	0.833	0.85	0.9	0.5	0.592	0.8	0.5	0.592	0.8	0.5	0.592	0.8
C7	0.833	0.85	0.9	0.833	0.85	0.9	0.833	0.85	0.9	0.833	0.85	0.9	0.833	0.85	0.9	0.5	0.592	0.8

The obtained normalized vectors for each criteria are shown in (table 4 and table 5).

Table 4 Normalized matrix for each criteria.

	Vector de pesos normalizado		
C1	0.14026	0.16287	0.17930
C2	0.11252	0.13647	0.16137
C3	0.13507	0.15426	0.16735
C4	0.11689	0.13058	0.15537
C5	0.14806	0.16568	0.17930
C6	0.09522	0.11420	0.14942
C7	0.12117	0.13596	0.15838

Table 5 Description of comparative values after initial analysis.

	C1			C2			C3			C4			C5			C6			C7		
A1	0.569	0.833	0.894	0.586	0.658	0.747	0.917	0.950	0.967	0.580	0.783	0.867	0.725	0.820	0.875	0.797	0.822	0.883	0.750	0.824	0.872
A2	0.508	0.576	0.683	0.622	0.705	0.767	0.758	0.865	0.928	0.725	0.820	0.875	0.672	0.823	0.917	0.800	0.839	0.911	0.589	0.700	0.806
A3	0.797	0.822	0.883	0.683	0.749	0.867	0.683	0.730	0.833	0.650	0.728	0.822	0.797	0.822	0.883	0.742	0.839	0.894	0.672	0.823	0.917
Σx^2	1.104	1.304	1.431	1.094	1.221	1.377	1.372	1.478	1.578	1.134	1.347	1.481	1.270	1.423	1.545	1.351	1.444	1.552	1.167	1.359	1.500

1. Construction of the initial matrix with the attribute's weight

2. Normalized decision matrix construction: The matrix normalized values for each criteria, as is shown in (Table 6).

3. Associated normalized weighted matrix construction: is obtained from equation:

$$\overline{v_{ij}} = \underline{w_j}\, \overline{n_{ij}}$$

Table 6 Normalized weighted matrix

	C1			C2			C3			C4			C5			C6			C7		
A1	0.056	0.104	0.145	0.048	0.074	0.110	0.078	0.099	0.118	0.046	0.076	0.119	0.069	0.095	0.124	0.049	0.065	0.098	0.061	0.082	0.118
A2	0.050	0.072	0.111	0.051	0.079	0.113	0.065	0.090	0.113	0.057	0.079	0.120	0.064	0.096	0.129	0.049	0.066	0.101	0.048	0.070	0.109
A3	0.078	0.103	0.143	0.056	0.084	0.128	0.058	0.076	0.102	0.051	0.071	0.113	0.076	0.096	0.125	0.045	0.066	0.099	0.054	0.082	0.124

4. Positive and negative ideal solutions obtainment: are obtained from equations 3 and

$$\bar{A}^{+} = \{\bar{v}_1^{+}, \ldots, \bar{v}_n^{+}\} = \{(\max \bar{v}_{ij}, j \in J), (\min \bar{v}_{ij}, j \in J')\}$$
$$\bar{A}^{-} = \{\bar{v}_1^{-}, \ldots, \bar{v}_n^{-}\} = \{(\min \bar{v}_{ij}, j \in J), (\max \bar{v}_{ij}, j \in J')\}$$

5. Distance calculation of each alternative to the ideal solutions found by and shown in (Table 7)**:**

$$\bar{d}_i^{+} = \{\sum_{j \in J} (\bar{v}_{ij} - \bar{v}_j^{+})^2\}^{\frac{1}{2}}, \quad i = 1, \dots, m$$

Table 7 Distance to the positive ideal solution

	C1			C2			C3			C4			C5			C6			C7		
A1	0.00050	0.00000	0.00000	0.00006	0.00010	0.00031	0.00000	0.00000	0.00000	0.00013	0.00001	0.00000	0.00005	0.00000	0.00003	0.00000	0.00000	0.00001	0.00000	0.00000	0.00004
A2	0.00080	0.00102	0.00118	0.00002	0.00002	0.00022	0.00018	0.00008	0.00002	0.00000	0.00000	0.00000	0.00014	0.00000	0.00000	0.00000	0.00000	0.00000	0.00017	0.00015	0.00023
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00040	0.00053	0.00026	0.00004	0.00008	0.00005	0.00000	0.00000	0.00002	0.00001	0.00000	0.00000	0.00004	0.00000	0.00000

The sum of all the criteria for each of the alternatives is made and equation 5 is used to calculate the distance to the positive ideal solution (Table 8).

Table 8 Distance to the positive ideal solution

	Sum			Distance		
	a	m	b	a	m	b
A1	0.000740147	0.000116566	0.00039239	0.02720564	0.010796573	0.019808831
A2	0.00132409	0.001281679	0.001643481	0.036388053	0.035800544	0.04053987
A3	0.000486779	0.000610494	0.000346002	0.022063059	0.024708181	0.018601126

To calculate the ideal negative solution (Table 9), equation is used:

$$\bar{d}_i^{-} = \{\sum_{j \in J} (\bar{v}_{ij} - \bar{v}_j^{-})^2\}^{\frac{1}{2}}, \quad i = 1, \dots, m$$

Table 9 Distance to the negative ideal solution

	C1			C2			C3			C4			C5			C6			C7		
A1	4E-05	1E-03	1E-03	0E+00	0E+00	0E+00	4E-04	5E-04	3E-04	0E+00	3E-05	4E-05	3E-05	0E+00	0E+00	1E-05	0E+00	0E+00	2E-04	2E-04	8E-05
A2	0E+00	0E+00	0E+00	9E-06	3E-05	8E-06	4E-05	2E-04	1E-04	1E-04	8E-05	5E-05	0E+00	1E-07	3E-05	1E-05	2E-06	9E-06	0E+00	0E+00	0E+00
A3	8E-04	9E-04	1E-03	6E-05	1E-04	3E-04	0E+00	0E+00	0E+00	3E-05	0E+00	0E+00	1E-04	7E-08	1E-06	0E+00	2E-06	1E-06	5E-05	2E-04	2E-04

The sum of all the criteria for each of the alternatives is made and equation is used to calculate the distance to the negative ideal solution, as is shown in (Table 10).

Table 10 Distance calculation to the negative solution

	Suma			Distancia		
	a	m	b	a	m	b
A1	0.000642318	0.001736658	0.001559024	0.025344002	0.04167323	0.039484484
A2	0.00019314	0.0003082	0.000237193	0.013897488	0.017555626	0.015401071
A3	0.001084495	0.001197742	0.001597029	0.032931667	0.034608411	0.039962846

6. Relative proximity calculation for each positive and negative ideal solution through proximity index.

To conclude this study, The ideal alternative "Distance" will be divided by the ideal alternative "Distance" minus the anti-ideal alternative "Distance". In this paper the alternate ranking method is used. To defuzzify the results, the proposed values by Garcia-Cascales and Lamata β=1/2 and λ=1/3 are taken, corresponding to a neutral decision level and a second option is calculated to define the best alternative, because the observed bias during data collection in the surveys, for this reason β=1/2 and λ=1/3 values are proposed, in this case equal importance is given to the right and left areas of the triangular fuzzy numbers obtained by the assigned values by the experts, as is shown in (Table 11).

Table 11 Decision index

	Neutral defuzzified output	**Bias defuzzified output**	**Neutral order**	**Bias order**	
A1	0.729528116	0.707880799	1	1	CAT
A2	0.313698264	0.308572001	3	3	SANY
A3	0.591864834	0.594656304	2	2	Komatsu

5 Results and Recommendations

The analysis of the final classificated data allows to observe that the three alternatives exhibit distant values, however it would be difficult to decide which criteria make a trend to generate a selection without making a analysis to determine which is the best option for the decision making group. Although the three alternatives meet the requirements and satisfy the buyer at some point, the fuzzy TOPSIS method finds which is the optimal alternative, however it is possible to consider the real bias that can be calculated taking the frequency of the surveys.

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