Chapter 1
Fundamentals of Fluidics

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1 Introduction

Microfluidics has had tremendous impact on miniaturization of biological experiments by reducing the reagent volumes, shortening the reaction times, and enabling multiplexed parallel operations by integrating an entire laboratory protocol onto a single chip (i.e., lab-on-a-chip or LOC). Best examples of microfluidic tools in biology are Gene chips, Capillary electrophoresis, CD-based inertial cell separation devices, integrated transcriptome analysis systems, and others. Along with miniaturization comes a tremendous opening at the microscale where slight manipulation in physics can provide unprecedented number of applications for each design. An understanding of the physical processes at microscale and their dynamics can allow biologists to leverage those for performing experiments that are practically not feasible at macroscale. Since microfluidics can allow new processes and experimental paradigms to emerge therefore, here we will focus on fundamentals that predominantly govern the processes at microscales and how we can manipulate those to address problems in the field of biology.

2 Microfluidic Physics

Dimension is the key in understanding the magnitude of a physical event taking place. Prior to discussing physics of microfluidic processes we must first understand that on what we are working. Few important symbols representing physical quantities and the microfluidic scales that are mainly relevant to biologists are mentioned in Tables 1.1 and 1.2, respectively.

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Table 1.1 Common symbols for physical parameters

Greek letter symbols			
α	alpha	λ	lambda
β	beta	μ	mu
γ	gamma	ν	nu
Δ	delta	Π	pi
ε	epsilon	ρ	rho
ζ	zeta	σ	sigma
η	eta	τ	tau
Θ	theta	ω	omega
κ	kappa		

Table 1.2 Length scales for common biological moieties

Sample matrix	Approximate scales
Distance between molecules in a liquid	0.1 nm
Distance between molecules in a gas	3 nm
Mean free path between collision in a gas, air at ambient pressure (λ)	61 nm
Sample	
Protein, lipid molecule of the membrane	1 nm
Virus	10 nm
Cells	1–20 μm

These are few illustrative sample matrices and sample types that are routinely employed in biological analysis. Given the sizes, our focus should be on the phenomenon that can be used to manipulate micron and sub-micron entities. Reagent mixing, reagent delivery, cell capture, and shear-free conditions for biological analysis are few typical applications that are sought by biologists. We will understand physical entities in this chapter with respect to these applications that will allow developing an understanding of microfluidics.

2.1 Hierarchy of Dimensions

Before advancing to the complex physics dominating the micron regimen, we must first review the basic concepts and their respective dimensions. Table 1.3 summarizes few of the most basic scaling entities.

Entity	Dimension
Size	$[l]$
Surface	$[l]^2$
Volume	$[l]^3$
Van der Waals	$[d]^{-3 \text{ to } -7}$
Various Forces	$[l]^{1 \text{ to } 3}$
l is size of an object,	
d is distance between two objects	

Table 1.3 Scaling laws: variation at changing length scales

Quantity	Scaling law
Time	$[l]^0$
Length	$[l]^1$
Area	$[l]^2$
Volume	$[l]^3$
Velocity	$[l]^1$
Acceleration	$[l]^1$
Density	$[l]^{-3}$
Viscosity	$[l]^{-2}$
Diffusion time	$[l]^2$
Reynolds number	$[l]^2$
Peclet number	$[l]^2$
Hydraulic resistance	$[l]^{-4}$

With our previous knowledge of physical processes, we can realize that size, shape, and volume have tremendous impact on the forces acting upon/between bodies. For example, let us consider the force exerted upon a body by earth. This force is called gravitational pull and is represented as the ratio of the product of masses of earth and ours to the squared distance between us. As we realize this force has dimensional dependence on the distance between the two bodies, which is $[l]^2$. Similarly, a body flowing through a water stream will experience some force exerted upon it by the flow. This is dependent on the size and surface of the body and is somewhat close to how biomolecules and cells will feel in the microfluidic channels. Therefore, we must now look few years back in high school physics, which is actually the foundation to our advanced understanding of microfluidics.

2.2 Non-dimensionalization and Dimensionless Numbers

This section is intended to introduce the concept and importance of non-dimensionalization because you will now know terms that will be commonly used throughout the text; if it is hard to understand at this point then these can be revisited once all the basics are learnt. Dimensions are critical in physical analysis as they draw boundaries around a physical quantity by defining them in dimensions. Their importance becomes predominant when we are working at structures in micrometer range where surface area increases drastically relative to volume. This characteristic dependence of physical processes on dimensions must be addressed in such a way that the process can be explained as a function of the intrinsic properties of the fluid rather than the dimensions of those properties. In other words, we must make equations governing these processes without any resultant dimensions. This can be achieved by carefully replacing quantities in

those equations with others, such that their dimensions cancel out each and have no net dimensional dependence. These quantities may be constants and can be employed for understanding the relative importance of entities within the process itself. Thus, non-dimensionalization is known as removal of units from the mathematical expression of a phenomenon by substituting with appropriate variables. This is also termed as *scaling*.

Scaling reduces the dependence of the process on several variables and significantly contributes to understand the relative importance of the physical quantities in the process and to realize the variation in their dimensions. This certainly helps in neglecting the smaller terms from the equation, which simplifies the associated physics. Therefore, it allows understanding physics at smaller scales and thus, is very important in microfluidics.

We will not deal scaling in great detail as it is a complex method but generally non-dimensionalization can be achieved via following steps:

(a) Identify the unit for which scaling is required; developing a scaling law
(b) Identify all the variables dependent and independent to that unit
(c) Identify a set of physically-relevant dimensionless groups and plug them in
(d) Determine the scaling exponent for each one, and
(e) Rewrite the equations in terms of new dimensionless quantities.

Such dimensionless numbers are crucial for exploring fundamentals of the physics governing microfluidics. The essential fluid physics of a system is dictated by a competition between various phenomena. This competition is expressed via a series of dimensionless numbers capturing their relative importance. These dimensionless numbers (Tables 1.4 and 1.5) form a sort of 'parameter space' for microfluidic physics.

2.3   Hydrostatics: Physics of the Stagnant

Fluids, liquids and gases, are defined as a material which will continue to deform with the application of a shear force. These are governed by certain basic rules of physics. Fluids have a special property to mention, they flow but only under the influence of external forces; these are mainly governed by **pressure, field gradients, surface tension, and gravity**. Since we will be mainly dealing with liquids therefore, our main focus is on the concepts of hydrostatic and hydrodynamic fluidics. As the name suggests hydrostatics and hydrodynamics are processes related to static and flowing liquids, respectively. Both these processes are controlled by associated physical parameters that we will discuss in this section.

Table 1.4 Dimensionless numbers in fluid mechanics

Dimensionless number	Details	Formula
Reynolds Number	Inertial force/Viscous force convective momentum/viscous momentum *Forced Convection*	$Re = \rho UL/\eta = UL/\nu$
Prandtl Number (heat) Prandtl-Schmidt Number (mass)	Momentum/Species diffusivity Used to determine fluid or heat or mass transfer boundary layer thickness	$Pr_{heat} = \nu/\alpha = \eta C_P/K$ $Pr_{mass} = Sc = \nu/D = \eta/\rho D$
Péclet Number (heat) Péclet Number (mass)	Convection transport rate/Diffusion transportation rate	$Pe_{heat} = RePr = UL/\alpha$ $\alpha = k/\rho C_P$ $Pe_{Mass} = RePr = UL/D$
Nusselt Number (heat) Nusselt-Sherwood Number (mass)	Length scale/Diffusion boundary layer thickness Used to determine the heat (h) or mass (h_D) transfer coefficient	$Nu = \left[f_\varepsilon Re(Pr)^{1/3}\right]/2$ $Nu_{heat} = hL/k_{fluid}$ $Nu_{Mass} = h_D L/D_{fluid}$ $L = A_s/Pm$
Grashof Number (heat) Grashof Number (mass)	Natural convection buoyancy force/Viscous force Used to calculate Re for buoyant flow Controls the lengthscale to natural convection boundary layer thickness *Natural Convection*	$Gr_{heat} = g\beta(T_s - T_b)L^3/\nu^2$ $Gr_{Mass} = g\beta_C(C_{as} - C_{aa})L^3/\nu^2$ $\beta = -\left[(\partial\rho/\partial C_\alpha)_{T,P}\right]/\rho$
Rayleigh Number (heat) Rayleigh Number (mass)	Natural convection/Diffusive heat or mass transport Used to determine the transition to turbulence	$Ra_{heat} = GrPr = g\beta(\Delta T)L^3/\nu\alpha$ $Ra_{mass} = GrPr = g\beta_C(\Delta C)L^3/\nu D$
Knudsen Number (to analyze extent of continuum)	Slip length/Macroscopic length	$Kn = \beta/L$
Richardson Number	Buoyancy/Flow gradient	$Ri = g(\Delta\rho)/\rho U^2$
Eötvös (Eo) or Bond Number (Bo)	Body forces/Surface tension Used together with Morton Number to determine shape of drops or bubbles in surrounding fluid or continuous phase	$Eo = Bo = \left[(\Delta\rho)gL^3\right]/\sigma$
Capillary Number	Viscous forces/Interfacial forces	$Ca = \eta U/\sigma$
Elasticity Number	Elastic effects/Inertial effects	$El = \theta\eta/\rho R^2 = Wi/Re$
Weissenberg Number	Viscous forces/Elastic forces	$Wi = \gamma'.t_s$
Deborah Number	Stress relaxation time/Time of observation	t_s/t_o

Table 1.5 Common physical entities in fluid mechanics

Physical entity		Unit	Dimension
U	Characteristic velocity	m/s	LT^{-1}
L	Characteristic length	m	L
T	Temperature	K	Θ
T_s	Surface temperature	K	Θ
T_b	Temperature of the bulk	K	Θ
D	Mass diffusivity	m^2/s	L^2T
C_p	Specific heat	J/Kg.K	$L^2T^{-2\Theta-1}$
C_{as}	Concentration of species a at surface	Kg/m^3	ML^{-3}
C_{aa}	Concentration of species a in ambient medium	Kg/m^3	ML^{-3}
A_s	Surface area of the pipe	m^2	L^2
Pm	Perimeter	m	L
η	Dynamic viscosity	Pa.s $=$ Ns/m^2 $=$ Kg/ms	$ML^{-1}T^{-1}$
ν	Kinematic viscosity	m^2/s	L^2t^{-1}
σ	Surface/interfacial tension	$Kg/s^2 =$ N/m	MT^{-2}
ρ	Density	Kg/m^3	ML^{-3}
β	Coefficient of thermal expansion	1/K	Θ^{-1}
α	Thermal diffusivity	m^2/s	L^2T^{-1}
k	Thermal conductivity	W/mK	$MLT^{-3}\Theta^{-1}$
h	Convective heat transfer coefficient	W/m^{2}K	$MT^{-3}\Theta^{-1}$
h_D	Convection mass transfer coefficient	m/s	LT^{-1}
λ	Mean free path	m	L
γ	Specific weight	N/m^3	$ML^{-2}T^{-2}$
R	Radius of the pipe	m	L
Θ	Stress evolution		
t_s	Stress relaxation time for the fluid	s	T
t_o	Time of observation of event	s	T
γ'	Sheer rate	1/s	T^{-1}

Pascal's Law

- Pressure applied anywhere to a fluid transmits the force equally in all directions
- Change in pressure disperses equally throughout the fluid
- Force acts at right angles to any surface in contact with the fluid
- Hydraulic press is the representative example

> **Hydrostatics** is the physics of pressure confined within the definitions of Pascal's law and Archimedes principle constitute hydrostatics

2.3.1 Pressure and Pumping

Consider a cuboidal bottle filled with water to a height of one meter with length and width of the bottle at 5 cm each. The liquid in bottle is not continuous, instead a stack of several individual layers of water molecules, such that each layer is parallel to each other and continuously interacting with each other.

Now, **PRESSURE** is how much force is exerted on a given area and is expressed as

$$P = F/A \tag{1.1}$$

where,

P is pressure, F is force exerted, and A is the surface area on which force is exerted.

SI unit of pressure is atmosphere (atm) and is equivalent to 10^5 Pascals, another unit for pressure and have dimension Nm^{-2}.

By the virtue of the definition of pressure, the top layer of the water molecules must exert a force on the layers beneath it over the surface area of the layer. Similarly, the top layer will do so on the last layer at the bottom. It is crucial to understand that for fluids under gravity, based on (1.1), pressure exerted by an upper layer on the one underneath is directly dependent on the distance between those layers expressed as height. From Fig. 1.1a, the pressure exerted by the liquid on the bottom of the container should be calculated as
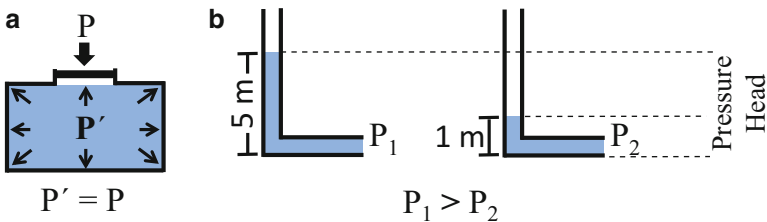

Fig. 1.1 Illustration of Pascal's law. (**a**) Pressure exerted at any point on a continuous fluid is dissipated equally in all directions on that fluid. This concept makes the basis of hydraulic press and brakes. (**b**) An extension of Pascal's law is pressure head driven flows where the height of the liquid exerts a pressure on the lower layers. This concept of height-dependent pressure is used in pumping in microfluidics. As depicted, 5 m head will exert more pressure than 1 m head

$$F = mg \qquad (1.2)$$

where,

m is mass of the liquid, and g is gravity constant.

Since,

$$m = \rho V \qquad (1.3)$$

where,

V is volume of container, and ρ is mass density of the liquid.

Therefore, replacing (1.3) in (1.2) will give us

$$F = V(\rho g) = hA(\rho g) \qquad (1.4)$$

such that volume = height of the liquid (h) * area of the surface (A = length * width)

Similarly, replacing (1.4) in (1.1) will give us the relation of height to the pressure

$$P = hA\rho g/A = h\rho g \qquad (1.5)$$

Continuing with the case that we were discussing, in Fig. 1.1b pressure exerted by a layer on the other separated by certain height within the liquid will be

$$P_2 - P_1 = \Delta \mathbf{P} = (h - h_1)\rho g = \Delta \mathbf{h}.\rho \mathbf{g} \qquad (1.6)$$

Equation (1.6) constitutes the basic of **hydrostatic pressure-based pumping** in **microfluidic systems**. '$\mathbf{\Delta P}$' is known as pressure head.

2.3.2 Buoyancy and the Problem of Microfluidic Mixing

Buoyancy is the apparent loss of weight of a body when submerged in liquid and this is mainly known as Archimedes Principle. This loss is attributed to the resistance offered by the liquid to the body. Buoyancy from Fig. 1.2 can be mathematically expressed as

$$F_{net} = F_B(\text{buoyant force}) - F_g(\text{weight}) \qquad (1.7)$$
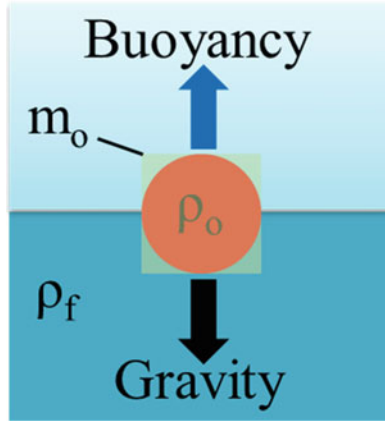
$$= (\rho_f V_f - \rho_o V_o)\, g \qquad (1.8)$$

Fig. 1.2 An illustration of Archimedes principle of buoyancy on a body of density 'ρ_o' dipped in a fluid of density 'ρ_f'. The body of mass m_o experiences two opposite forces on it, gravity acting downwards and buoyancy or thrust acting upwards. Denser the body will be with respect to the fluid, the greater the gravity force will be on it. Thus the body will drown. On the contrary, if it is lighter than fluid, then it will float on the surface

For static liquid, $F_{net} = 0$; therefore, it can be deduced as

$$F_g \rho_o = F_B \rho_f \tag{1.9}$$

Where, 'o' and 'f' denote 'object' and 'fluid', respectively.

Now, based on (1.9) If,

$\rho o > \rho f$, the object will submerge and settle down to the bottom of the fluid. However, an object will float but submerged with $\rho o = \rho f$, and will float on the surface with a $\rho o < \rho f$.

This knowledge becomes the basis of buoyancy-dependent mixing in certain microfluidic set-ups. The best example is introduction of air bubbles from underneath of the static layer of liquid. The air bubbles have lower density than liquid and will move towards the top of the channel thus causing disruption in the solvent layers and introducing mixing Fig. 1.3. We will discuss other details later in this chapter.

2.4 Hydrodynamics: Physics of the Flows

Fluids at motion are governed by a set of variables and these are crucial in understanding the phenomena taking place within confined boundaries in microfluidics. There are few properties we will first acquaint with before looking into other aspects.
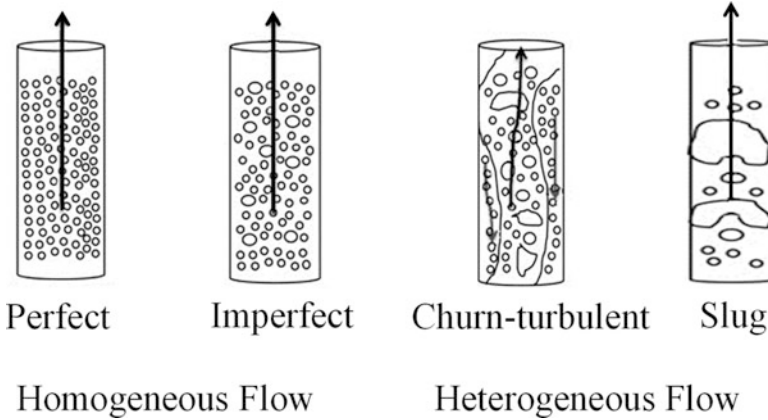
Perfect   Imperfect   Churn-turbulent   Slug

Homogeneous Flow   Heterogeneous Flow

Fig. 1.3 Buoyancy-driven mixing in liquids with various types of bubble flows. Each type of bubble flow introduces mixing which could be diffusive or turbulent

2.4.1 Concept of Continuum

Typically matter is made up of atomic and sub-atomic particles. Thus, when analyzing those at micron level, matter becomes discontinuous in space with inter-atomic separation. However, when considering fluids as a continuum material, assumptions have to be made where we have to neglect that atoms are the smallest unit. And, the matter/fluid must be defined in terms of continuous fields, such as density and force density (defined as per unit volume), rather than discrete physical quantities, such as mass and force. The continuum can be confirmed for fluids by first reducing the sample volume to a very small magnitude and then measuring intrinsic properties, like density, at several points in liquid space. The density should be equal to approve the consideration of fluid continuum.

2.4.2 Important Intrinsic Properties

- Mass density (ρ; Kg/m^3): It is the mass distribution over a unit volume,
- Specific volume (V; m^3/Kg): It is the volume occupied by a unit mass,
- Weight density or Specific weight (γ; N/m^3): It is the force due to gravity on the mass in a unit volume and is expressed as

$$\gamma = g\rho \tag{1.10}$$

- Specific gravity or Relative density (δ): it is the ratio of density of the fluid to the density of water,
- Viscosity: It is the resistance offered by the fluid to gradual deformation by neighboring fluid layers under an external force, namely shear stress or tensile stress. It is also known as thickness of the fluids. This parameter also represents

the interaction of parallel moving fluid plates with each other and with surroundings. Inter-plate collisions in a moving fluid create friction which opposes the motion of the fluid. Therefore, to move a fluid certain external stimulus, such as pressure gradient, is required. A fluid that doesn't offer any intrinsic resistance to shear force is known as an ideal or inviscid fluid while those offering resistance are called viscous or viscid.

Momentum of molecules in each respective layer is considered to be homogeneous. Additionally, due to mixing the molecules from one layers move to the other. In this case, a molecule diffusing to a fast moving layer needs to be accelerated and deaccelerated when travelling to a slow moving layer. During this these molecules carry their respective momentum with them. This is the main reason for introduction of the shear into the layers.

Dynamic/Shear viscosity (η; Poiseuille (Pl); Pa.s; N. s/m^2; Kg/ms):
It is the resistance offered by a fluid layer to adjacent layers where all the layers are moving parallel to each other but at different speeds. Thus, is also called shear viscosity. The simplest understanding can be developed with the explanation of illustration in Fig. 1.4. In panel a, suppose there are three parallel layers moving in same direction, with lowest layer being at rest and top most layer moving at a constant speed 'U', while layers should have no other gradient fields, such as concentration or temperature. For simplifying the condition, we must also assume that the plates (interface of fluid and surface) to be large enough; and the reason is that we want to omit boundary or edge effects where fluids are in contact with the surface. ***However, boundary or edge effects will have to be incorporated in theory in microfluidics***.

Therefore, when top layer is moving slow, then ideally all the layer will be parallel to each other and speed of layers will be 'zero' in the bottom layer and maximum in the top layer. Here, each layer will oppose the forward motion of the layer above it and make the layer beneath it to move forward. In such conditions, an
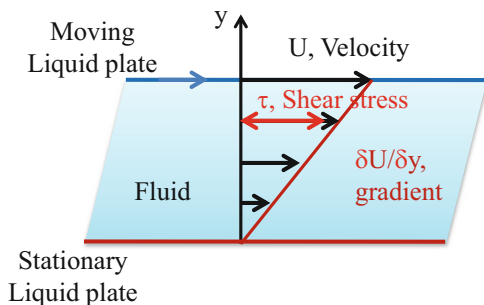

Fig. 1.4 Viscous force opposes the motion of the fluid layer moving faster than it and pushes the layer moving slower than it in its direction of motion. This introduces a resistance in fluids, which tends to exert a net opposite force resisting the fluid motion. This resistance is known as shear stress and is depicted as a function of change in velocity profile of the fluid layers moving from a static layer towards the fastest moving layer

external force will be required to overcome the fluid viscosity and keep it in motion. This force is Newton's viscous/shear force and is given by

$$F = \eta.A.U/y \tag{1.11}$$

where,

F is external force required to overcome viscosity, A is the area of each fluid layer plate, U is the velocity of each layer plate, and y is the separation between them.

U/y is rate of shear deformation of layers or shear velocity along the perpendicular to the fluid motion.

Shear stress (τ) can be written from (1.11) as

$$\tau = F/A = \eta.\partial U/\partial y = \eta.\gamma' \tag{1.12}$$
$$\text{or,} \ \ \eta = \tau/\gamma' \tag{1.13}$$

<u>**Kinematic viscosity**</u> (ν; m^2/s): It is the ratio of dynamic viscosity to the density of fluid

$$\nu = \eta/\rho \tag{1.14}$$

2.4.3 Types of Fluids Based on Intrinsic Properties

An in-depth knowledge of physical properties of fluids is crucial and it is the foremost thing one must know to design an efficient fluidics. Pumping is an integrated part in microfluidics. And, to effectively design the integrated pumping mechanism, knowledge of viscosity, fluid type, and fluidity becomes important. For example, a viscous fluid like honey will not flow easily through micron wide channels, but if the temperature inside channels is high then its viscosity will change making it less viscous and easy to flow. Else, an external pump will be required to force honey through channels. I first case temperature changed the fluid's viscosity while in second pressure has pushed it without affecting its viscosity. If we know this beforehand then we can design the tool to compensate these effects. We will focus in this section the type of fluids and their properties.

Newtonian vs. Non-newtonian: Case of Whole Blood Analysis
in Microfluidics

According to Newton's law of viscosity, the fluid viscosity has proportionality with shear stress and shear rate, as depicted in (1.11) and (1.12). Based on this relation of viscosity (1.13), fluids can be categorized into two broad groups. The first group

that has a constant viscosity for relation (1.13), and second that has changing viscosity with either of the two variables.

Fluids that has a constant η, in other words has a constant ratio of shear stress and shear rate. Such fluids are called *Newtonian* fluids. Water, honey, organic solvents are few examples of this type of fluids. Their viscosity only changes with temperature.

However, majority of the fluids in nature do not follow Newtonian fluid concept, thus called *non-Newtonian*. Their viscosity changes with the change in shearing stress and shearing rate. This is why the viscosity plays crucial role in fluid properties. Now we qualify to classify fluids as Newtonian and non-Newtonian.

Let us consider the case of whole blood. Prior proceeding we must ponder to decide what type of fluid whole blood would be. Considering the composition of whole blood with ~40 % cellular material it can be classified as non-Newtonian. The reason is that if we shear the whole blood by increasing pressure the cell-fraction will not aggregate; thus, will change the blood viscosity making it less viscous. This is what we observe in systolic and diastolic blood where systolic blood is under high pressure flowing at high speed which makes it thin while diastolic blood under decreased pressure is thick where cells tend to come closer to each other and increase viscosity of the blood. On the other hand serum and plasma are Newtonian fluids and their viscosity in independent of the shear, where serum are plasma are essentially cell-free but serum is also free of clotting factors.

This knowledge of whole blood being non-Newtonian can be employed to make several kinds of microfluidic devices ranging from separating plasma to clustering cells.

Compressible and Incompressible Fluids

As the name indicates, the fluids that can be compressed into a smaller volume under an external pressure are called *compressible fluids*. Typically, all the fluids are compressible where gases are highly compressible while liquids are slightly compressible. The fluid compressibility (β_C) is a measure of the relative change in volume due to a pressure change, and is expressed as

$$\beta_C = -1/V \, (\partial V/\partial P) \, = -1/\rho \, (\partial \rho/\partial P) \qquad (1.15)$$

where, $\partial V/\partial P$ and $\partial \rho/\partial P$ are change in the volume and density, respectively. V is the initial volume and ρ is the initial density.

On the contrary, if the fluid volume does not change under an external pressure, then it is considered to be *incompressible*. There are literally no such examples of incompressibility. Incompressibility is used for the convenience of calculation purposes in fluid dynamics where an assumption is made that fluids with small or negligible compressibility are incompressible. It is important in microfluidics to assume so because then the density can be considered constant which significantly simplifies the calculation (see text box).

If the microfluidic channel is too long, there might be a huge pressure drop along the length, say 20 %. Then the density of the fluid at the inlet and outlet will be very different. This difference in density will affect the experimental composition and mathematical modelling of the experiments.

By now, we are able to understand several common terminologies that have been used while describing microfluidic systems. We will now discuss the primary laws that bind these concepts and terms together to create tools for understanding fluidics.

2.4.4 Other Important Properties

- Surface tension (σ; N/m; Kg/s^2): It is primarily a property of an interface, either liquid-air, or liquid–solid. It is the elasticity of a fluid surface to acquire minimum possible surface area. The plausible reason for surface tension is attributed to the unequal distribution of cohesion on the surface molecules due to which they continuously feels an inward pull toward the center of mass (Fig. 1.5).

The amount of surface tension (σ) is given by the force 'F' required to oppose the net inward cohesive force experienced by the top layer of length 'L', such that the top layer stop to sink toward bottom (Fig. 1.5a)

$$\sigma = F/2L \qquad (1.16)$$

where, ½ is introduced in the eq. to equate the force that is acting only on one side of the surface.

Capillarity is an effect of surface tension at a solid–liquid interface, such that the liquid tends to rise in a **tube with small radius (small Bond number)** due to interplay between cohesion and adhesion between liquid molecules at liquid-air and liquid–solid interface.

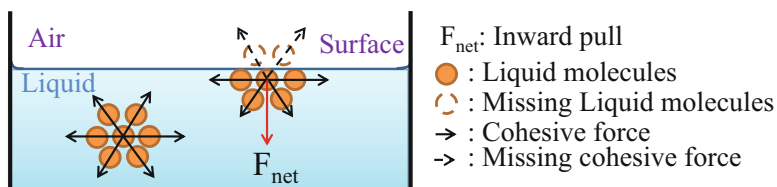The height to which a liquid will go up is expressed as

Fig. 1.5 Illustration of the tension on the surface of a liquid. On the surface layer, cohesion of liquid molecules causes a net inward pull due to which the surface behaves as a continuous film. Contrary to the surface, cohesion is cancelled by the neighboring molecules

$$h = 2\sigma Cos\Theta/\rho gr \tag{1.17}$$

where, Θ is the contact angle, σ is surface tension, ρ is density of the liquid, and r is the radius of the tube. This forms the basis of capillary pumping.

Importance of Surface Tension in Microfluidics

- Capillary pumping
- Droplet formation from a stream
- Contact angle determination
- Bubble generation for mixing

Capillary pumping: Capillary effect is employed regularly in microfluidics for removing physical pumps to minimize the bulky features. Capillary-driven pumps operate under *Young-Laplace law* defining the relation of difference in pressure at the interface of two fluids due to surface tension to the curvature in the surface of the liquid. This partial differential equation of Young-Laplace is expressed as

$$\Delta P = \sigma[1/r_1 + 1/r_2] = 2\sigma/r \text{ (if } r_1 = r_2) \tag{1.18}$$

where, ΔP is capillary pressure in a tube, σ is surface tension, r_1 and r_2 are the principle radii of curvature for internal and external surfaces at the interface/meniscus, and r is the radius of curvature. If r_1 and r_2 are equal then the equation reads as on extreme right.

Now, the actual radius of the tube is related to the meniscus radius by a cosine relation, such that $r = RCos\Theta$ then the (1.18) will read as

$$\Delta P = 2\sigma/RCos\Theta \tag{1.19}$$

Critical Thinking

Ignore the surface wettability for an instance. A single microfluidic channel opened at both the ends. Two drops of water were placed on both ends, such that one drop is smaller than the other drop. What should be the direction of flow?

: σ (tension) for both the liquids given the interface is same. Since r is smaller for small drop therefore, from (1.19) the capillary pressure will be more. Thus, water will move from small drop towards big drop.

In order to compensate for this pressure difference the liquid will move a distance thus giving rise to capillary pumping. The capillary pressure is crucial in

designing an efficient pump because it requires a precise knowledge of the surface wettability, wetting phase and non-wetting phase. This relation is

$$P_c = P_{non-wetting\ phase} - P_{wetting\ phase} \qquad (1.20)$$

Such that non-wetting phase in a typical experiment is air while water or buffer serves as a wetting phase. This is only true when surface is water-wettable, viz. hydrophilic. For hydrophobic surfaces, hydrophobic solvents serve as the wetting phase.

More regarding capillary pumping in paper microfluidics will be discussed in Chap. 2.

2.4.5 Laws Governing Dynamics

Basic Law

To understand the concept, we must first understand it intuitively what the governing principle to this branch of science is. In a general sense, laws governing fluid mechanics can be stated as *the absence of relativistic effects for the conservation of mass, energy, and momentum*. In this process we must first (1) **identify** a system, (2) **identify** boundary of that system, (3) **identify** surroundings of the system, and (4) i**dentify** how it interacts with the surroundings. As described in Fig. 1.6, if 'A' depicts mass, momentum or energy then the influx of any of these entities in the system should be equated with the efflux of equal amount of that respective entity.

This indicates that the total of any of these entities for the system will be a constant and can be written as
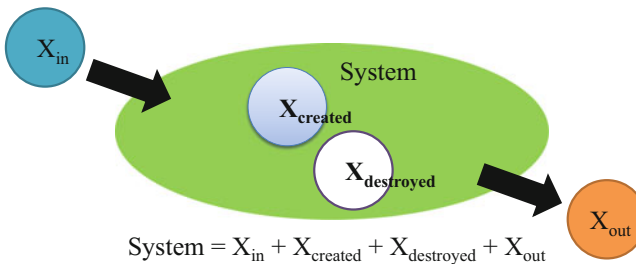

$$System = X_{in} + X_{created} + X_{destroyed} + X_{out}$$

Fig. 1.6 Fundamental to understand fluid mechanics is conservation of fundamental physical components that are mass, momentum, and energy. As a general rule, system has its own mass, energy, and momentum and it has a tendency to conserve that. Any external force exerted to that body then the body should bring change in some form to conserve its mass, energy, and momentum. This is illustrated as system is equal to the total sum of the quantity of all the physical entities applied to it, applied by it, created by it and destroyed by it

$$A_{\text{in}} - A_{\text{out}} = A_{\text{system}} \tag{1.21}$$

Expression (1.21) stands true only if the extensive properties participating in conservation laws are considered, such as mass, volume, length etc. but not for intensive properties, which include pressure or temperature. The extensive properties are those which changes with the change in the amount of fluid; while intensive properties are those that do not change when an amount of fluid changes in the system.

Equations of Motion

A fluid in motion experience following forces

- Fg—gravity force
- Fp—pressure force
- Fv—Force due to viscosity
- Ft—Force due to turbulence
- Fc—Force due to compressibility

Therefore, according to the Newton's 2nd law, the net force (Fx) on a fluid with mass 'm' and acceleration 'a' in x-direction can be expressed as

$$Fx \ = \ Fg \ + \ Fp \ + \ Fv \ + \ Ft \ + \ Fc \tag{1.22}$$

Recalling the previous assumptions we discussed in fluid compressibility, the liquids with limited compressibility are considered incompressible and Fc becomes negligible.

Rewriting (1.22) with Fc = 0

$$Fx \ = \ Fg \ + \ Fp \ + \ Fv \ + \ Ft \tag{1.23}$$

Equation (1.23) is called **Reynold's equation of motion**.

If the system is not turbulent then Ft is negligible and the resultant eq is known as **Navier–Stokes equation of motion** and is expressed as

$$Fx \ = \ Fg \ + \ Fp \ + \ Fv \ + \ Fc \tag{1.24}$$

And can be written as,

$$P(\partial U/\partial t + U. = \Delta U) \ = \Delta.\sigma \ = - \ \Delta P + \eta \Delta^2 U + f \tag{1.25}$$

For real fluids with negligible viscosity, Fv is 0, the eq. is known as **Euler's equation of motion** and is expressed as

$$Fx = Fg + Fp + Ft + Fc \tag{1.26}$$

Conservation of Mass

It can be summarized as a time-dependent mass change over a defined fluid boundary such that mass within that boundary is constant

$$\text{Final mass} = \text{Original mass} + \text{Mass added} - \text{Mass removed} \tag{1.27}$$

or

$$\text{Final mass} - \text{Original mass} = \text{Mass added} - \text{Mass removed} \tag{1.28}$$

Equation (1.28) forms the basis of mass conservation of fluids in microfluidic systems, and can mathematically be written as

$$\text{Rate of change of mass} = \text{Net mass influx} \tag{1.29}$$

or

$$\Delta M/\Delta t = -\; \Delta I_m (\text{mass flux}) \tag{1.30}$$

Left part of (1.30) can be written in terms of *extensive intrinsic properties, such as density and volume* (refer back to the types of fluid section to know why intrinsic properties are used and basic conservation law to know why extensive properties are employed)

$$\Delta M/\Delta t = \partial(M)/\partial t = \partial(\rho \partial V)/\partial t = \partial/\partial t \left[\iint_V \rho.\partial V \right] \tag{1.31}$$

where, ΔM is the change in mass, Δt is time interval of the mass change, differential $\partial(M)/\partial t$ is rate of change of mass, $\rho \partial V$ is the mass change in terms of changing volume

Similarly, right part of (1.30) can be written as

$$I_m = \Delta m.\Delta A \tag{1.32}$$

where, Δm is mass flowing normal to an area ΔA.

Equation (1.32) can further be expressed in terms of extensive intrinsic properties as

$$\Delta m.\Delta A = \int_s \rho.U.\partial S \tag{1.33}$$

where, U is mass flow velocity S is the surface area of the boundary region.

Now, by replacing (1.31) and (1.33) in (1.30), we will have the mass conservation equation for fluids

$$\partial/\partial t \left[\int_v \rho.\partial V\right] = -\int_s \rho.U.\partial S \tag{1.34}$$

Conservation of Linear Momentum/Inertia

It can be defined as the net momentum in a given volume at a given time is constant.

Newton's second law describes the relation of force and momentum with the expression

$$F = m.a = m.\ \partial U/\partial t = \partial(mU)/\partial t = \partial p/\partial t = U.\ \partial m/\partial t \tag{1.35}$$

where, m is mass of the fluid in a given area, p is momentum, U is the flow velocity, a is acceleration, $\partial U/\partial t$ is velocity rate, $\partial m/\partial t$ is mass flow rate. U. $\partial m/\partial t$ is known as *momentum flow*.

Momentum flow can be written in terms of extensive properties

$$\partial p/\partial t = U.\ \partial m/\partial t = U.I_m = U.(mA) = U.\ (\rho VA) \tag{1.36}$$

Now, for momentum on this given mass of fluid to be constant,

External forces(F) = Momentum flow rate + Momentum out − Momentum in

(1.37)

The external forces acting on the fluid in a defined boundary are **body force** (force due to gravity) and **surface forces** (pressure, viscosity)

Thus conservation (1.37) will become

Fg + Fv + Fp = Momentum flow rate + Momentum out − Momentum in

(1.38)

Substituting respective values will give us the conservation of momentum equation

$$\rho g + Fv + \partial P/\partial L = \rho U.\partial V/\partial t + \partial(\rho U)/\partial t \tag{1.39}$$

Conservation of Energy

It is stated as energy within a system remains constant such that energy acting upon the body is continuously changed to other form, such as work. For fluids it is a very complicated equation that considers several forms of energy acting and dissipating out of a defined body. In its simplest form the law can be written as

Fig. 1.7 According to conservation law, volumetric flow rate in the pipe sections depicted as *red circles* must remain same. However, channel diameters, cross-section areas, and flow velocities of the highlighted sections different. Thus, to satisfy $Q_1 = Q_2$, flow velocity of the smaller diameter > the flow velocity of higher diameter
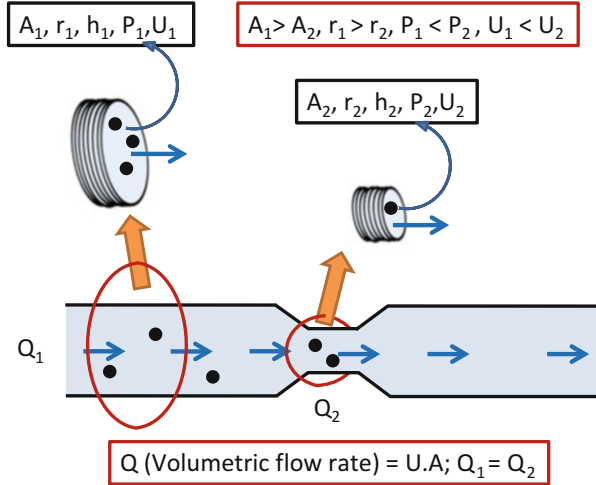
$$\Delta E = \text{constant} \tag{1.40}$$

The simplest example of energy conservation is Bernoulli's equation where fluid flowing through a pipe having different radius as depicted in Fig. 1.7.

Bernoulli's equation for per unit volume for a given area is written as

$$\text{Pressure energy} + \text{kinetic energy} + \text{potential energy} = \text{constant} \tag{1.41}$$

$$P + \left(mU^2\right)/2 + mgh = P + \left(\rho U^2\right)/2 + \rho gh = \text{constant} \tag{1.42}$$

Now, from Fig. 1.7, at two different points with different cross-section areas, Equation (1.42) can be written as

$$P_1 + \left(\rho U_1^2\right)/2 + \rho gh_1 = P_2 + \left(\rho U_2^2\right)/2 + \rho gh_2 \tag{1.43}$$

Important Concepts

- **Stokes law: Friction and Drag on spherical particles**

Frictional force due to viscosity, which is also known as ***Stokes drag*** is given by

$$F_d = 6\Pi\eta rU \tag{1.44}$$

where,

F_d is stokes drag, η is dynamic viscosity, r is hydrodynamic radius of the spherical particle, U is the flow velocity around the particle. $6\Pi\eta r$ is together is called **drag coefficient** ζ.

> **Applications of Stokes Law**
>
> - Hydrodynamic separation of cells
> - Viscous force calculation
> - Calculating shear on cells

This is important because mostly biomolecules and cells are approximately spherical. Thus Stokes law can be employed with approximation.

The **viscous force** experienced by each spherical particle is given by

$$Fv_Z = 3\eta U/2r \tag{1.45}$$

The **gravitational force** experienced by a spherical particle falling in a liquid

$$Fg = 4[(\rho_s - \rho_f)g\Pi r^3]/3 \tag{1.46}$$

where, ρ_s is particle density, ρ_f is fluid density, g is gravity constant, r is radius of particle

Terminal velocity of a spherical particle falling in a liquid under gravity

$$U_{ter} = 2[(\rho_s - \rho_f)gr^2]/9\eta \tag{1.47}$$

Stokes-Einstein law relates kinetics to Stokes law for understanding **Diffusion**

$$D = k_B T/6\Pi\eta r \tag{1.48}$$

where, D is diffusion constant, k_B is Boltzmann constant, T is temperature of the system

Drag force on a particle completely enclosed in fluid is expressed as

$$F_D = \rho U^2 C_D A/2 \tag{1.49}$$

where, F_D is drag force, C_D is drag coefficient, A is the area of reference, U is flow velocity, and ρ is fluid density.

Equation (1.49) can be rewritten as

$$F_D = \rho U^2 CAf(Re)/2 \tag{1.50}$$

where, f(Re) is function operator for Reynolds number

- **Poiseuille principle: Volumetric flow rate and pressure drop**

It describes the relation of pressure drop in a moving fluid enclosed within a tube with the flow resistance and flow rate (Table 1.6). It is also known as **Hagen-Poiseuille law** (Fig. 1.8).

Table 1.6 Volumetric flow rates for common geometries

Shape of cross-section	Volumetric flow rate (Q)	Fabrication approach
Cylindrical	$\Pi r^4 \Delta P/8\eta L$	Isotropic wet etching, Ball-end milling
Rectangular	$\Delta P w/[8(A+1)^2/A]\,\eta$	Photolithogrgaphy
Triangular	$\Delta P(3a^4)^{1/2}/320\eta l$	Anisotropic wet etching

Fig. 1.8 Illustration of the pressure drop 'ΔP' along the channel length 'L' due to the flow resistance offered by the fluid to its motion from point 1 to point 2

Pressure drop along L = $P_2 - P_1$
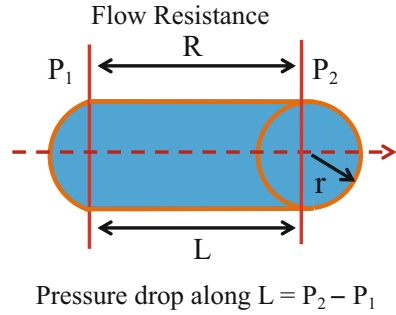
For circular channels the relation is expressed as

$$R = (P_1 - P_2)/Q = \Delta P/Q = 8\eta l/\Pi r^4 \tag{1.51}$$

The (1.51) can be written as

$$\Delta P = RQ = 8\eta l Q/\Pi r^4 = 32\eta l U/r^2 \tag{1.52}$$

where, ΔP is pressure drop, R is the fluidic resistance, and Q is the volumetric flow rate through a cross-section area 'A' (Q = U.A)

For rectangular channels the relation in (1.51) is modified to

$$R_{hd} = C_{nc}\eta l/A^2 \tag{1.53}$$

where, C_{nc} is numerical coefficient and is given as

$$C_{nc} = 8(A+1)^2/A \tag{1.54}$$

where, A is aspect ratio = height of the channel (h)/width of the channel (w)

Replacing (1.53) and (1.54) in (1.52)

$$\Delta P = R_{hd}Q = C_{nc}\eta l Q/A^2 = \left[8(A+1)^2/A\right]\eta l Q/A^2 \tag{1.55}$$

Volumetric flow rate can be expressed in terms of mass flow rate with the relation

$$Q = I_m/\rho \tag{1.56}$$

Replacing (1.56) in (1.55) gives

$$\Delta P = R_{hd}I_m/\rho = C_{nc}\eta lI_m/A^2 \cdot \rho = (\eta/\rho).\left[C_{nc}lI_m/A^2\right] = \nu \cdot C_{nc}l\ I_m/A^2 \tag{1.57}$$

where, I_m is mass flow rate, ν is kinematic viscosity
For circular pipes, (1.57) can be written as

$$\Delta P = RQ = 8\nu l/\Pi r^4 \tag{1.58}$$

An extension to Hagen-Poiseuille law is **Darcy–Weisbach equation**
Darcy–Weisbach equation relates head loss or pressure loss due to friction along a given circular channel and is expressed as

Pressure loss form:

$$\Delta P \text{ (pressure loss)} = f_D l\rho V^2/2D \tag{1.59}$$

Head loss form:
    Replacing (1.6) in (1.59)

$$\rho g\Delta h = f_D l\rho V^2/2D \tag{1.60}$$

$$\Delta h \text{ (head loss)} = f_D lV^2/2gD \tag{1.61}$$

where, f_D is Darcy friction factor from channel wall which is

$$f_D = 64/Re \tag{1.62}$$

Fanning equation relates the ratio of local shear stress to the local fluid kinetic energy and is expressed as

$$f = \tau/\text{Kinetic Energy} = 2\tau/\rho U^2 = 16/Re \tag{1.63}$$

where,
    f is fanning friction factor, τ is shear stress, Re is Reynolds number.

Pressure loss form:

$$\Delta P = \tau A/A' \qquad (1.64)$$

where, A is wall area $(= 2\Pi\, rl)$ and A' is cross-sectional flow area $(= \Pi r^2)$, r is radius of the pipe, l is flow length.

Replacing (1.63) in (1.64)

$$\Delta P = f\rho U^2 A/2A' = f\rho U^2. \left(2\Pi\, rl/2\Pi r^2\right) = f\rho U^2.(l/r) \qquad (1.65)$$

Head loss form:

Replacing (1.6) in (1.63)

$$\Delta h = f\rho U^2. \,(l/rg\rho) = f\, U^2 l/rg \qquad (1.66)$$

- **Coriolis effect: Inertial frame and particle motion**

It is inertial force acting upon bodies relative to a rotating reference frame. For example, if a particle rolls on a static disc as illustrated in Fig. 1.9, by the virtue of inertia, it appears to move in a straight line to the observer in the same frame of reference. When the disc starts to rotate then the particle is still moving in the straight line if observed by someone standing in an inertial frame of reference outside of the rotating disc. However, if the observer is standing on the rotating disc in the non-inertial reference frame, then the particle will look like following a curved path, such that the particle is resisting in the change of its final destination by the virtue of inertia. Thus we can say that *the Coriolis effect is in contrast to the normal inertia which resists the change in body's motion, whereas in this effect body resists the change in displacement*. It is crucial in inertial microfluidics where plasma can be separated from whole blood and cells of different sizes can be separated from each other. The direction of fluids in specific channels in centrifugal microfluidics as a function of inertial forces and Coriolis effect can also be achieved.

The Coriolis effect can be expressed

$$F_c = m.a_c \qquad (1.67)$$

where,

Fc is Coriolis force, m is mass of the fluidic plug or particle, a_c is angular acceleration.

Since,

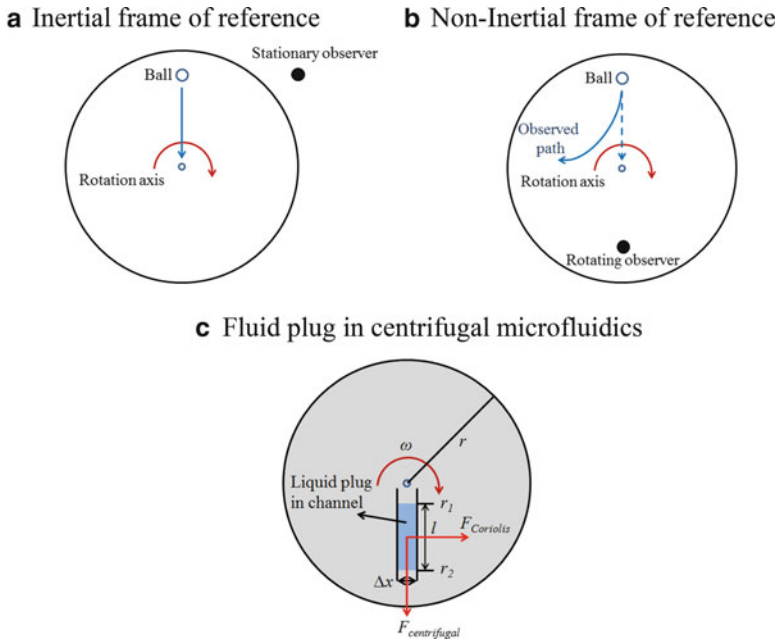$$a_c = -2U\omega \qquad (1.68)$$

where,

a Inertial frame of reference   **b** Non-Inertial frame of reference

c Fluid plug in centrifugal microfluidics

Fig. 1.9 Explanation of the Coriolis effect. (**a**) In an inertial frame of reference the observer is out of the rotating disk. The stationary observer will see the rolled ball following a straight path. (**b**) However, when observer stands on the same rotating disk on which the ball was rolled then to this rotating observer ball will seem to follow a curved path outwards. This perspective of curving of the path of ball is Coriolis effect. (**c**) This effect is used in rotating microfluidics for separating particles in a fluid plug of length 'l'. In this plug, the particle will experience an outward centrifugal force normal to the rotation axis and an outward force normal but opposite to the direction of the rotation, as depicted in '(**c**)'. Due to this effect particles will move to the wall of the channel continuously pulled the disk boundary

U is linear velocity of the liquid plug in the channel/particle (velocity relative to the rotation speed), and ω is angular velocity,

Hence,

$$F_c = -2mU\omega = -2(\rho V)U\omega \qquad (1.69)$$

where,

ρ is density and V is volume of the particle.

The force is also expressed as force density, as expressed below

$$f_c = F_c/V = -2\rho U\omega \qquad (1.70)$$

In centrifugal microfluidics the velocity of the liquid plug or particle (U) in the channel depends on angular velocity (ω), radial location of the fluid reservoir,

channel geometry, and fluid properties, such as dynamic viscosity (η) and density (ρ). This linear velocity component is given as

$$U = D_h^2 \omega^2 \rho r \Delta r / 32 \eta l_c \tag{1.71}$$

where, D_h is hydraulic diameter of the microchannel, r is average distance of the liquid from the center of rotation, Δr is radial extent of the liquid plug (how much it has moved from its initial position), and l_c is the plug length in the channel.

Replacing (1.71) in (1.69),

$$F_c = -2(\rho V)\omega(D_h^2 \omega^2 \rho r \Delta r / 32 \eta l_c) = -D_h^2 \omega^3 \rho^2 r \Delta r V / 16 \eta l_c \tag{1.72}$$

The (1.70) can now be written as,

$$f_c = -D_h^2 \omega^3 \rho^2 r \Delta r / 16 \eta l_c \tag{1.73}$$

Centrifugal and Coriolis forces are related to each other in a sense that they operate together but normally (perpendicular) to each other as depicted in Fig. 1.9.

Centrifugal force is given by

$$F_\omega = -m\omega^2 r = -(\rho V)\omega^2 r \tag{1.74}$$

Where, r is radius of rotation.

The centrifugal force density is given by

$$f_\omega = F_\omega / V = -\rho \omega^2 r \tag{1.75}$$

Now, finding ration of (1.69) and (1.71) will give us relative effect of both the forces acting upon the particle in rotatory frame.

$$F_c / F_\omega = 2U / \omega r \tag{1.76}$$

Inertia circle is the path that moving body in a rotating reference frame will follow. The radius of this circle (r_c) and the time required to travel the edge of the frame (t_c) is given by

$$r_c = U / 2\omega, \text{ and } t_c = \Pi / \omega \tag{1.77}$$

Rossby Number—length scales and Coriolis effect: It is the ratio of inertial and Coriolis forces. We can determine the effect of length scale on the efficiency of rotation in achieving Coriolis effects. The relation is expressed as

$$Ro = U / fcL = D_h^2 \omega^2 \rho r \Delta r / 32 \eta \, fcLl_c \tag{1.78}$$

where,

Ro is Rossby number, U is the relative velocity of the particle, fc is Coriolis factor, and L is the length scale of the motion. Coriolis factor, fc is expressed as

$$fc = 2\omega Sin\Theta \tag{1.79}$$

where, Θ is the angle of the body to the plane of the reference surface. In case of particles in centrifugal microfluidics, Θ will be 90° thus changing the Coriolis factor to 2ω.

- **Dean number: Flows in curved pipes**

It is defined as the product of Reynolds number and the square root of the curvature ratio.

$$De = Re.(d/2r)^{1/2} = (\rho Vd/\mu).(d/2r)^{1/2} \tag{1.80}$$

where,

De is Deans number, Re is Reynolds number, r is curvature radius of the channel/tube, d is travelled length of the liquid, and V is axial velocity.

2.5   Key Dimensionless Numbers Explained

2.5.1   Reynolds Number: Inertial Focusing to Separate Plasma from Whole Blood

The Reynolds number is one of the most crucial dimensionless numbers in fluid mechanics. However, when we discuss it with reference to microfluidics, its relevance is practically limited. The reason is that the fluids employed in microfluidics-related applications have small values for their respective Reynolds numbers that make the inertial effects irrelevant.

Still, importance of the Reynolds number can't be undermined. One best example to explain the importance of inertia in microfluidics is separation of plasma from whole blood. A straight channel, as illustrated in Fig. 1.10, is curved at one end. The liquid flowing through this channel will feel a sudden curve on its path. At the corner, liquid still tends to go straight due to which in the process of changing path, it loses momentum at the corner. In this case,

Time taken for this liquid to turn around the corner is expressed as

$$t_i \sim w/U_0 \tag{1.81}$$

where, t_i is turn time, w is width of the curve, U_0 is velocity of the fluid before turning.
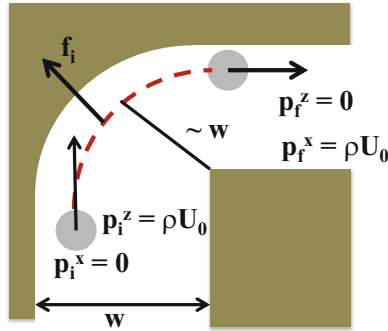
Fig. 1.10 Inertial separation of the particles moving with a velocity U_0 and following a curved path approximately equal to the width of the channel. Due to the conservation of momentum, particle will experience an outward push known as inertial centrifugal force with a density f_i as described in (1.84). P_i is inertial flow pressure where x represents in x direction and z represents in z direction. Initially, prior to the curve, along x axis, the momentum is 0 while all the momentum is focused along z axis. At the curve particle will lose z momentum that will translate to the x-momentum. During this transition, the particle will experience f_i normally outwards towards the putter wall of the microchannel

Now, the liquid during this time from (1.81) will lose a fraction of its linear momentum density, which can be given as

$$p_i \sim \rho U_0 \tag{1.82}$$

The lost fraction of momentum (p_d) in (1.82) will be transferred as a force, named inertial/centrifugal force, which is directed outwards in the same direction the liquid was initially flowing prior entering the curve. This inertial force density can be calculated as

$$f_i \sim p_i/t_i \tag{1.83}$$

By replacing (1.81) and (1.82) in (1.83) for p_d and T_i we will get

$$f_i \sim \rho U_0/ti \sim \rho U_0^2/w \tag{1.84}$$

These three equations form the basis of particle separation in non-circulating fluidic chips.

2.5.2 Péclet Number: Diffusivities Across Channel Width and No-Membrane Dynamic Filtering

In day-to-day life turbulent fluid mixing is crucial. To elaborate the time scale vs length scale in the absence of this mixing, let us consider that we are holding a cup

Table 1.7 Diffusivities of common biological elements

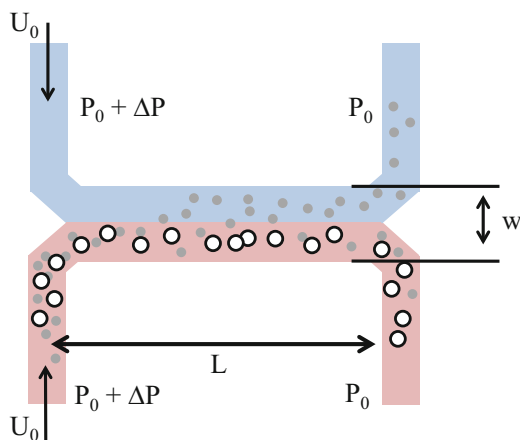| Biological element	Solute	Proteins	Virus	Bacterium	Mammalian Cell
Size	0.1 nm	5 nm	100 nm	1 μm	10 μm
Diffusivity ($\mathbf{\mu m^2/s}$)	2000	40	2	0.2	0.02

Fig. 1.11 Illustration of the working mechanism of a membrane-less dynamic particle separator. The working principle depends on the different diffusivities of different particles and the time and length scale required for diffusion in transverse direction according to their respective diffusivities. Two liquids enter through a T-junction and are left to diffuse over a length L in the arm of the H-filter. However, the liquids will never mix due to laminar regimen. A solution with different particles is injected through one channel. During the flow in the arm section, where both the fluids stay in contact, smaller particles with higher diffusivity and lower Pèclet number will move to the other layer and thus separate from the rest of the initial solution

of coffee; in this case we will not be able to smell the aroma of coffee held in our hands for next several days. If real world fluids were low Reynolds number then due to laminarity in flow, diffusion will be the only means of mixing, as we have already seen the relation between time and length scales. On the contrary, in microfluidics, due to low-Reynolds number regimen, mixing is predominantly by diffusion; this could be lengthy on a time scale. The common diffusivities for few biological moieties are enlisted in Table 1.7. Now, there are several applications requiring rapid mixing, and this is where Pèclet number helps us understanding the extent of mixing in our micro-devices.

Let us discuss the case of the 'H' filter. It is named so due to its geometrical appearance (Fig. 1.11). Left T-junction is used as inlets keeping those at pressure $P_0 + \Delta P$; while T-junction on the right is used as outlet keeping it at pressure P_0. The arm of the 'H' is the central channel where diffusion takes place. The H-filter works on the basis of diffusion, which is the only mean of transverse movement of particles between two parallel moving fluid layers.

According to the mass transport and Fick's law,

Table 1.8 Conditions for particle separation in no-membrane H-filter

	t_{conv} (time of particle convection)	t_{diff} (time of particle diffusion)	Pe_{Mass} (Pěclet number)
	L/U_0	$w^2/4D$	$RePr = U_0L/D$
For diffusion	$t_{conv} \leq t_{diff}$		Low Pe,
For no diffusion	$t_{conv} \gg t_{diff}$		High Pe
			Either longer channel length or slower flow velocity or both are required to achieve diffusion

$$D(\text{diffusion coefficient}) = w^2(\text{width to diffuse})/2t_{diff}(\text{time to diffuse}) \quad (1.85)$$

The time to diffuse is dependent on the particle velocity in the fluid (U_0) and the length (L) it will travel before diffusing completely to the adjacent stream. Thus, eq. (1.85) can be written as

$$D = w^2U_0/2L \quad (1.86)$$

However, the particle has to diffuse through only half of the total width of the H-connecting arm because the other half is filled with different liquid. Due to this w becomes w/2 and eq. (1.86) will be

$$D = w^2U_0/4L \quad (1.87)$$

Also, particles in liquid flow experience convection along with the diffusion. The time of particle convection become important as referred in Table 1.8. Thus the particle diffusivity becomes crucial as well. Typical diffusivities for common biological moieties are mentioned in Table 1.7.

As mentioned in Table 1.8, for particle to diffuse into the other half across the channel width filled with different liquid, then the prerequisite must be the diffusivity of the particle should be high; the aspect ratio of channel length to its width should be smaller along with flow velocity. This all will result in low Pěclet number and smaller diffusion time. The opposite will be true for a particle not to diffuse into the adjacent fluid across its width.

3   Conclusion

This chapter provides basic information, such as basic principles and related theory, for developing a fundamental background to understand the so-looking complex physics of microfluidics. On the contrary, the foundation of fluid mechanics starts

dating back to high school physics. In this chapter, we have brushed all those concepts we studied back then and added some advanced theoretical knowledge built upon that. In principle, this chapter covers everything that one should get acquainted to for understanding microfluidics. This will allow non-physicists and engineers to interface with microfluidic engineers and thus, can improve communication, which is the biggest challenge when non-physical scientists and doctors interact with engineers.

Further Reading

Recommended reviews and articles

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3. Beebe DJ, Mensing GA, Walker GM (2002) Physics and applications of microfluidics in biology. Annu Rev Biomed Eng 4:261. doi:10.1146/annurev.bioeng.4.112601.125916
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