Discontinuous Galerkin Method for the Solution of Elasto-Dynamic and Fluid-Structure Interaction Problems

Miloslav Feistauer, Martin Hadrava, Adam Kosík, and Jaromír Horáček

Abstract This paper is concerned with the numerical solution of dynamic elasticity by the discontinuous Galerkin (dG) method. We consider the linear and nonlinear St. Venant-Kirchhoff model. The dynamic elasticity problem is split into two systems of first order in time. They are discretized by the discontinuous Galerkin method in space and backward difference formula in time. The developed method is tested by numerical experiments. Then the method is combined with the space-time dG method for the solution of compressible flow in a time dependent domain and used for the numerical simulation of fluid-structure interaction.

1 Description of the Dynamic Elasticity Problem

Let us consider an elastic body represented by a bounded polygonal domain $\Omega^b \subset \mathbb{R}^2$. We assume that $\partial\Omega^b = \Gamma_D^b \cup \Gamma_N^b$ and $\Gamma_D^b \cap \Gamma_N^b = \emptyset$. On Γ_D^b and Γ_N^b we prescribe the Dirichlet boundary condition and the Neumann boundary condition, respectively. The deformation of the body is described by the displacement $\boldsymbol{u} : \Omega^b \times [0,T] \to \mathbb{R}^2$ and the deformation mapping $\boldsymbol{\varphi}(\boldsymbol{X},t) = \boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X},t)$, $\boldsymbol{X} \in \Omega^b$, $t \in [0,T]$, where $[0,T]$ with $T > 0$ is a time interval. Further, we introduce the deformation gradient $\boldsymbol{F} = \nabla\boldsymbol{\varphi}$, the Jacobian $J = \det \boldsymbol{F} > 0$ and the Green strain

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tensor $\boldsymbol{E} \in \mathbb{R}^{2\times 2}$,

$$\boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I}\right), \quad \boldsymbol{E} = (E_{ij})_{i,j=1}^2, \tag{1}$$

with the components

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) + \frac{1}{2}\sum_{k=1}^{2}\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}. \tag{2}$$

We set $\mathrm{tr}(\boldsymbol{E}) = \sum_{i=1}^2 E_{ii}$ and by $\boldsymbol{I}$ we denote the unit tensor. By the symbol $\mathrm{Cof}\boldsymbol{F}$ we denote the cofactor of the matrix $\boldsymbol{F}$ defined as $\mathrm{Cof}\boldsymbol{F} = J(\boldsymbol{F}^{-1})^T$. Further, we introduce the first Piola-Kirchhoff stress tensor $\boldsymbol{P}$. Its form depends on the chosen elasticity model.

The general dynamic elasticity problem is formulated in the following way: Find a displacement function $\boldsymbol{u} : \Omega^b \times [0,T] \to \mathbb{R}^2$ such that

$$\rho^b\frac{\partial^2\boldsymbol{u}}{\partial t^2} + C_M\rho^b\frac{\partial\boldsymbol{u}}{\partial t} - \mathrm{div}\boldsymbol{P} = \boldsymbol{f} \qquad \text{in } \Omega^b \times [0,T], \tag{3}$$

$$\boldsymbol{u} = \boldsymbol{u}_D \quad \text{in } \Gamma_D^b \times [0,T], \tag{4}$$

$$\boldsymbol{P}\boldsymbol{n} = \boldsymbol{g}_N \quad \text{in } \Gamma_N^b \times [0,T], \tag{5}$$

$$\boldsymbol{u}(\cdot,0) = \boldsymbol{u}_0, \quad \frac{\partial\boldsymbol{u}}{\partial t}(\cdot,0) = \boldsymbol{z}_0 \qquad \text{in } \Omega^b, \tag{6}$$

where $\boldsymbol{f} : \Omega^b \times [0,T] \to \mathbb{R}^2$ is the density of the acting volume force, $\boldsymbol{g}_N : \Gamma_N^b \times [0,T] \to \mathbb{R}^2$ is the surface traction, $\boldsymbol{u}_D : \Gamma_D^b \times [0,T] \to \mathbb{R}^2$ is the prescribed displacement, $\boldsymbol{u}_0 : \Omega^b \to \mathbb{R}^2$ is the initial displacement, $\boldsymbol{z}_0 : \Omega^b \to \mathbb{R}^2$ is the initial deformation velocity, $\rho^b > 0$ is the material density and $C_M \geq 0$ is the damping coefficient.

We consider two elasticity models (see [2]).

St. Venant-Kirchhoff material. In this case we set

$$\boldsymbol{\Sigma} = \lambda^b\mathrm{tr}(\boldsymbol{E})\boldsymbol{I} + 2\mu^b\boldsymbol{E}, \quad \boldsymbol{P} = \boldsymbol{F}\boldsymbol{\Sigma}, \tag{7}$$

where $\boldsymbol{\Sigma}$ is the second Piola-Kirchhoff stress tensor. The Lamé parameters λ^b and μ^b are expressed with the aid of the Young modulus E^b and the Poisson ratio ν^b:

$$\lambda^b = \frac{E^b\nu^b}{(1+\nu^b)(1-2\nu^b)}, \quad \mu^b = \frac{E^b}{2(1+\nu^b)}. \tag{8}$$

Linear elasticity model is the simplest elasticity model obtained by the assumption of small deformations. By this assumption the second term in (2) is neglected

and the linear approximation of $\boldsymbol{E}$ (linear with respect to the gradient $\boldsymbol{F}$) is denoted by $\boldsymbol{e}$ and called the small strain tensor. Then $\boldsymbol{E} = \boldsymbol{e} = (e_{ij})_{i,j=1}^2$ and

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right). \tag{9}$$

In this case we write

$$\boldsymbol{P} = \lambda^b \mathrm{tr}(\boldsymbol{e})\boldsymbol{I} + 2\mu^b \boldsymbol{e}. \tag{10}$$

As we see, in general, $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{u}), \boldsymbol{E} = \boldsymbol{E}(\boldsymbol{u}), \boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{u}), \boldsymbol{P} = \boldsymbol{P}(\boldsymbol{u})$.

For the time discretization of problem (3), (4), (5) and (6) we rewrite the dynamic elasticity problem as the following system of first-order in time for the displacement $\boldsymbol{u} : \Omega^b \times [0,T] \to \mathbb{R}^2$ and the deformation velocity $\boldsymbol{z} : \Omega^b \times [0,T] \to \mathbb{R}^2$:

$$\rho^b \frac{\partial \boldsymbol{z}}{\partial t} + C_M \rho^b \boldsymbol{z} - \mathrm{div}\boldsymbol{P} = \boldsymbol{f}, \quad \frac{\partial \boldsymbol{u}}{\partial t} - \boldsymbol{z} = 0 \quad \text{in } \Omega^b \times [0,T], \tag{11}$$

$$\boldsymbol{u} = \boldsymbol{u}_D \quad \text{in } \Gamma_D^b \times [0,T], \tag{12}$$

$$\boldsymbol{P}\boldsymbol{n} = \boldsymbol{g}_N \quad \text{in } \Gamma_N^b \times [0,T], \tag{13}$$

$$\boldsymbol{u}(\cdot,0) = \boldsymbol{u}_0, \quad \boldsymbol{z}(\cdot,0) = \boldsymbol{z}_0 \quad \text{in } \Omega^b. \tag{14}$$

2 Discrete Problem

The discretization of the dynamic elasticity problem will be carried out by the dG method in space and the backward difference formula (BDF) method in time.

Let $\mathscr{T}_h^b$ be a partition of the closure $\overline{\Omega}^b$ formed by a finite number of closed triangles with disjoint interiors.

Let us consider a partition of the time interval $[0,T]$ formed by time instants $t_k = k\tau, k = 0,\dots,M$, where M is a sufficiently large positive integer and $\tau = T/M$ is the time step. (The generalization to a nonuniform partition is possible.)

Let $p > 0$ be an integer. By S_{hp} we denote the space of piecewise polynomial functions on the triangulation $\mathscr{T}_h^b$,

$$S_{hp} = \left\{v \in L^2(\Omega_h^b); v|_K \in P^p(K)\ \forall K \in \mathscr{T}_h^b\right\}, \tag{15}$$

where $P^p(K)$ denotes the space of polynomial functions of degree $\le p$ on the element K. The approximate solution will be sought in $\boldsymbol{S}_{hp} = S_{hp} \times S_{hp}$ at each time level.

By $\mathscr{F}_h^b$ we denote the system of all faces of all elements $K \in \mathscr{T}_h^b$ and $\mathscr{F}_h^{bB}, \mathscr{F}_h^{bD}, \mathscr{F}_h^{bN}$ and $\mathscr{F}_h^{bI}$ will denote the sets of all boundary, Dirichlet, Neumann and inner faces, respectively. We set $\mathscr{F}_h^{bID} = \mathscr{F}_h^{bI} \cup \mathscr{F}_h^{bD}$. Further, for each $\Gamma \in \mathscr{F}_h^{bI}$

there exist two neighbouring elements $K_\Gamma^{(L)}, K_\Gamma^{(R)} \in \mathscr{T}_h^b$ such that $\Gamma \subset \partial K_\Gamma^{(L)} \cap \partial K_\Gamma^{(R)}$. For each $\Gamma \in \mathscr{F}_h^b$ we define a unit normal vector $\boldsymbol{n}_\Gamma$. We assume that for $\Gamma \in \mathscr{F}_h^{bB}$ the normal $\boldsymbol{n}_\Gamma$ has the same orientation as the outer normal to $\partial\Omega^b$. We use the convention that $\boldsymbol{n}_\Gamma$ is the outer normal to $\partial K_\Gamma^{(L)}$ and the inner normal to $\partial K_\Gamma^{(R)}$. For $\boldsymbol{v} \in \boldsymbol{S}_{hp}$ we introduce the following notation: $\boldsymbol{v}|_\Gamma^{(L)} =$ the trace of $\boldsymbol{v}|_{K_\Gamma^{(L)}}$ on Γ, $\boldsymbol{v}|_\Gamma^{(R)} =$ the trace of $\boldsymbol{v}|_{K_\Gamma^{(R)}}$ on Γ, $\langle \boldsymbol{v} \rangle_\Gamma = \frac{1}{2}\left(\boldsymbol{v}|_\Gamma^{(L)} + \boldsymbol{v}|_\Gamma^{(R)}\right)$, $[\boldsymbol{v}]_\Gamma = \boldsymbol{v}|_\Gamma^{(L)} - \boldsymbol{v}|_\Gamma^{(R)}$, where $\Gamma \in \mathscr{F}_h^{bI}$. If $\Gamma \in \mathscr{F}_h^{bB}$, then there exists an element $K_\Gamma^{(L)} \in \mathscr{T}_h^b$ such that $\Gamma \subset K_\Gamma^{(L)} \cap \partial\Omega_h^b$ and we set $\boldsymbol{v}|_\Gamma^{(L)} =$ the trace of $\boldsymbol{v}|_{K_\Gamma^{(L)}}$ on Γ, $\langle \boldsymbol{v} \rangle_\Gamma = [\boldsymbol{v}]_\Gamma = \boldsymbol{v}|_\Gamma^{(L)}$. Finally, we set $h_\Gamma = (h_{K_\Gamma^{(L)}} + h_{K_\Gamma^{(R)}})/2$.

In the derivation of the space discretization by the dG method the following process is essential. We multiply the governing system by a test function $\boldsymbol{v} \in \boldsymbol{S}_{hp}$, integrate the resulting relations over elements $K \in \mathscr{T}_h^b$, apply Green's theorem to the term containing $\boldsymbol{P}$, add some mutually vanishing terms, use boundary conditions and sum over all elements. In this way we get the following forms:

$$a_h^b(\boldsymbol{u}, \boldsymbol{v}) = \sum_{K \in \mathscr{T}_h^b} \int_K \boldsymbol{P}(\boldsymbol{u}) : \nabla \boldsymbol{v} \, \mathrm{d}\boldsymbol{x} - \sum_{\Gamma \in \mathscr{F}_h^{bID}} \int_\Gamma (\langle \boldsymbol{P}(\boldsymbol{u}) \rangle \boldsymbol{n}) \cdot [\boldsymbol{v}] \, \mathrm{d}S, \tag{16}$$

$$J_h^b(\boldsymbol{u}, \boldsymbol{v}) = \sum_{\Gamma \in \mathscr{F}_h^{bID}} \int_\Gamma \frac{C_W^b}{h_\Gamma} [\boldsymbol{u}] \cdot [\boldsymbol{v}] \, \mathrm{d}S, \tag{17}$$

$$\ell_h^b(\boldsymbol{v}) = \sum_{K \in \mathscr{T}_h^b} \int_K \boldsymbol{f} \cdot \boldsymbol{v} \, \mathrm{d}\boldsymbol{x} + \sum_{\Gamma \in \mathscr{F}_h^{bN}} \int_\Gamma \boldsymbol{g}_N \cdot \boldsymbol{v} \, \mathrm{d}S + \sum_{\Gamma \in \mathscr{F}_h^{bD}} \int_\Gamma \frac{C_W^b}{h_\Gamma} \boldsymbol{u}_D \cdot \boldsymbol{v} \, \mathrm{d}S, \tag{18}$$

$$A_h^b = a_h^b + J_h^b, \tag{19}$$

$$(\boldsymbol{u}, \boldsymbol{v})_{\Omega^b} = \int_{\Omega^b} \boldsymbol{u} \cdot \boldsymbol{v} \, \mathrm{d}\boldsymbol{x} = \sum_{K \in \mathscr{T}_h^b} \int_K \boldsymbol{u} \cdot \boldsymbol{v} \, \mathrm{d}\boldsymbol{x}, \tag{20}$$

where $\boldsymbol{u}, \boldsymbol{v} \in \boldsymbol{S}_{hp}$ and $C_W^b > 0$ is a sufficiently large constant.

For $k = 0, \ldots, M$ we use the approximations $\boldsymbol{u}(t_k) \approx \boldsymbol{u}_h^k \in \boldsymbol{S}_{hp}$ and $\boldsymbol{z}(t_k) \approx \boldsymbol{z}_h^k \in \boldsymbol{S}_{hp}$. A general backward difference formula approximating the time derivative reads

$$\frac{\partial \boldsymbol{u}}{\partial t}(t_{k+1}) \approx \frac{1}{\tau} \sum_{j=0}^{l} c_l \boldsymbol{u}_h^{k+1-j}, \tag{21}$$

where l is the order of the method and c_j, $j = 0, \ldots, l$, are the coefficients.

The BDF-dG approximate solution of problem (11)–(14) is defined as a couple of sequences $\{\boldsymbol{u}_h^k\}_{k=0}^M$, $\{\boldsymbol{z}_h^k\}_{k=0}^M$ such that

$$
\begin{aligned}
&\text{a) } \boldsymbol{u}_h^k, \boldsymbol{z}_h^k \in \boldsymbol{S}_{hp}, \quad k = 0, \ldots, M, \qquad (22)\\
&\text{b) } \left(\frac{\rho^b}{\tau} \sum_{j=0}^{l} c_l \boldsymbol{z}_h^{k+1-j}, \boldsymbol{v}_h \right)_{\Omega_h^b} + \left(C_M \rho^b \boldsymbol{z}_h^{k+1}, \boldsymbol{v}_h \right)_{\Omega_h^b} + A_h^b(\boldsymbol{u}_h^{k+1}, \boldsymbol{v}_h)\\
&\quad = \ell_h^b(\boldsymbol{v}_h)(t_{k+1}) \quad \forall \boldsymbol{v}_h \in \boldsymbol{S}_{hp},\\
&\text{c) } \left(\frac{\rho^b}{\tau} \sum_{j=0}^{l} c_l \boldsymbol{u}_h^{k+1-j}, \boldsymbol{v}_h \right)_{\Omega_h^b} - \left(\boldsymbol{z}_h^{k+1}, \boldsymbol{v}_h \right)_{\Omega_h^b} = 0 \quad \forall \boldsymbol{v}_h \in \boldsymbol{S}_{hp},\\
&\quad k = 0, \ldots, M-1,\\
&\text{d) } (\boldsymbol{u}_h^0 - \boldsymbol{u}_0, \boldsymbol{v}_h)_{\Omega_h^b} = 0, \quad (\boldsymbol{z}_h^0 - \boldsymbol{z}_0, \boldsymbol{v}_h)_{\Omega_h^b} = 0 \quad \forall \boldsymbol{v}_h \in \boldsymbol{S}_{hp}.
\end{aligned}
$$

The initial values $\boldsymbol{u}_h^k, \boldsymbol{z}_h^k$, $k = 1, \ldots, l$ are obtained by k-step BDF schemes.

In the first order BDF method we have $l = 1$, $c_0 = 1$, $c_1 = -1$ and in the second order BDF method $l = 2$ and $c_0 = 3/2$, $c_1 = -2$, $c_2 = 1/2$.

The discrete nonlinear problems are solved on each time level by the Newton method. For the solution of linear subproblems either direct UMFPACK solver or GMRES method with block diagonal preconditioning are used.

3 Numerical Experiments

3.1 A Benchmark Problem

The applicability and accuracy of the BDF-dG method is tested by the comparison with the benchmark denoted by CSM3 proposed by J. Hron and S. Turek in [4], where they used a different solution approach. We consider a 2D domain formed by the rigid cylinder with an attached elastic beam, as is shown in Fig. 1.

The following data are used: $\boldsymbol{f} = \left(0, -2\rho^b\right)^T$ [m.s^{-2}], $\rho^b = 10^3$ [kg.m^{-3}], on the left part Γ_D^b of the boundary connected with the rigid body we prescribe

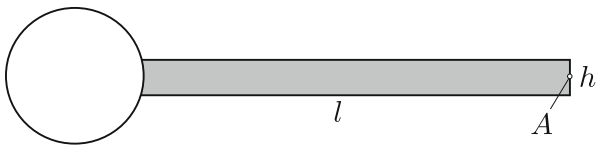

Fig. 1 Rigid cylinder with an elastic beam of the nonlinear elasticity benchmark problem

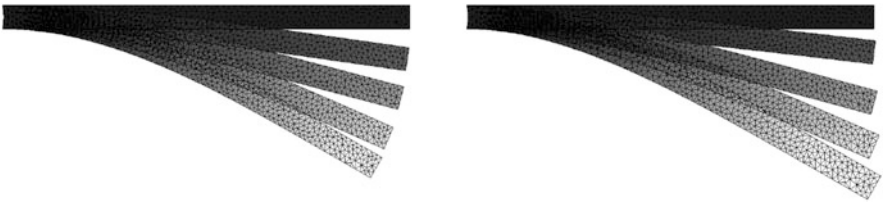

Fig. 2 The deformation of the beam in case CSM3: St. Venant-Kirchhoff model (*left*), linear elasticity model (*right*)

Table 1 CSM3: comparison of the position of the point A for BDF2, St. Venant-Kirchhoff material and different time steps τ. The values are written in the format "*mean value* $\pm$ *amplitude [frequency]*"

Method	τ	$u_1\,[\times 10^{-3}]$		$u_2\,[\times 10^{-3}]$	
Ref		-14.305 ± 14.305	[1.0995]	-63.607 ± 65.160	[1.0995]
BDF2	0.04	-10.566 ± 9.963	[1.0675]	-64.866 ± 45.218	[1.0675]
BDF2	0.02	-13.477 ± 13.462	[1.0850]	-64.133 ± 61.177	[1.0850]
BDF2	0.01	-14.119 ± 14.111	[1.0900]	-63.905 ± 64.212	[1.0900]
BDF2	0.005	-14.454 ± 14.453	[1.0925]	-64.384 ± 64.939	[1.0925]

homogeneous Dirichlet boundary condition $\boldsymbol{u}_D = \mathbf{0}$ and on the rest part Γ_N^b of the boundary we prescribe the Neumann boundary condition with no surface traction $\boldsymbol{g}_N = \mathbf{0}$. The initial conditions $\boldsymbol{u}_0 = \boldsymbol{z}_0 = 0$. The material is characterized by the Young modulus $E^b = 1.4 \cdot 10^6$ and the Poisson ratio $\nu^b = 0.4$.

Figure 2 shows the deformation of the beam at several time instants computed by the linear model and St. Venant-Kirchhoff model. The linear model does not give results correct from the physical point of view in contrast to the nonlinear case. Table 1 presents the comparison between the reference results of the benchmark with our computation carried out by the second-order BDF2 time discretization with several time steps on a relatively coarse mesh with 722 elements and polynomial degree $p = 1$. According to [4], the time dependent displacement is represented by its mean value mean $= 1/2(\max + \min)$, amplitude $= 1/2(\max - \min)$ and frequency. Table 1 shows a good agreement with the reference data from [4].

3.2 Example of Fluid-Structure Interaction

The BDF-dG method described above is combined with the solution of compressible flow in a time dependent domain Ω_t and the resulting coupled problem is applied to the simulation of fluid-structure interaction. The boundary of Ω_t is formed by three disjoint parts: $\partial\Omega_t = \Gamma_I \cup \Gamma_O \cup \Gamma_{W_t}$, where Γ_I is the inlet, Γ_O is the outlet and Γ_{W_t} represents impermeable time-dependent walls. The time dependence of the domain

Ω_t is taken into account with the aid of the Arbitrary Lagrangian-Eulerian (ALE) method (see, e.g., [3], Chap. 10). It is based on a regular one-to-one ALE mapping of the reference configuration Ω_0 onto the current configuration Ω_t. The compressible Navier-Stokes system transformed to the ALE formulation is discretized by the space-time discontinuous Galerkin method, see [1] or [3].

In the FSI simulation the common interface between the fluid and structure at time t is defined as $\tilde{\Gamma}_{Wt} = \{\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X}, t); \boldsymbol{X} \in \Gamma_N^b\} \subset \Gamma_{W_t}$. The flow and structural problems are coupled by the transmission conditions

$$\boldsymbol{v}(\boldsymbol{x}, t) = \frac{\partial \boldsymbol{u}(\boldsymbol{X}, t)}{\partial t}, \ \boldsymbol{P}(\boldsymbol{u}(\boldsymbol{X}, t))\boldsymbol{n}(\boldsymbol{x}) = \boldsymbol{\sigma}^f(\boldsymbol{x}, t)\mathrm{Cof}(\boldsymbol{F}(\boldsymbol{u}(\boldsymbol{X}, t)))\boldsymbol{n}(\boldsymbol{x}), \tag{23}$$

$$\boldsymbol{X} \in \Gamma_N^b, \ \boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X}, t), \quad \sigma_{ij}^f = -p\,\delta_{ij} + \lambda\delta_{ij}\mathrm{div}\boldsymbol{v} + 2\mu d_{ij}(\boldsymbol{v}),$$

$$d_{ij}(\boldsymbol{v}) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

Here $\boldsymbol{v}$ denotes the fluid velocity, p is the fluid pressure and $\mu > 0$ and $\lambda = -2\mu/3$ are the fluid viscosity coefficients. The fluid-structure interaction problem is solved with the aid of a coupling procedure, see [3], Chap. 10.

As an example of the FSI problem we present the simulation of vibrations of vocal folds model excited by the airflow in a simplified geometry of vocal tract and vocal folds shown in Fig. 3. Figure 4 presents the velocity field containing a number of vortices in the deformed vocal tract at time instants $t = 0.0336, 0.0360, 0.0384, 0.0408$ s. The light shades correspond to low velocity, whereas the dark shades represent higher velocity. The pressure is in the range between 88200 and 99950 Pa. The prescribed outlet pressure is 97611 Pa. The inlet velocity is 4 ms^{-1}. The deformation of the vocal folds was computed with the use of St. Venant-Kirchhoff model. The Young modulus and the Poisson ratio have values $E^b = 12000$ Pa and $\nu^b = 0.4$, respectively, the structural damping coefficient $c_M = 1\ \mathrm{s}^{-1}$ and the material density $\rho^b = 1040\,\mathrm{kg\,m}^{-3}$.

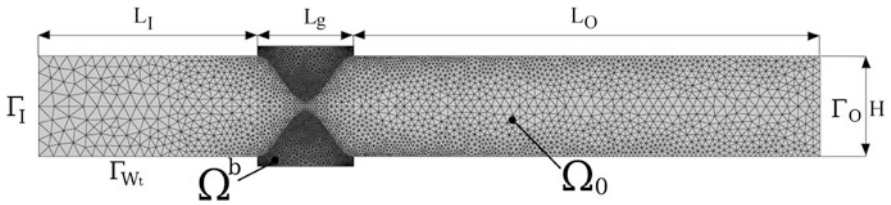

Fig. 3 Computational domain with the mesh at time $t = 0$ and the description of its size: $L_I = 50$ mm, $L_g = 15.4$ mm, $L_O = 94.6$ mm, $H = 16$ mm. The width of the channel in the narrowest part is 1.6 mm

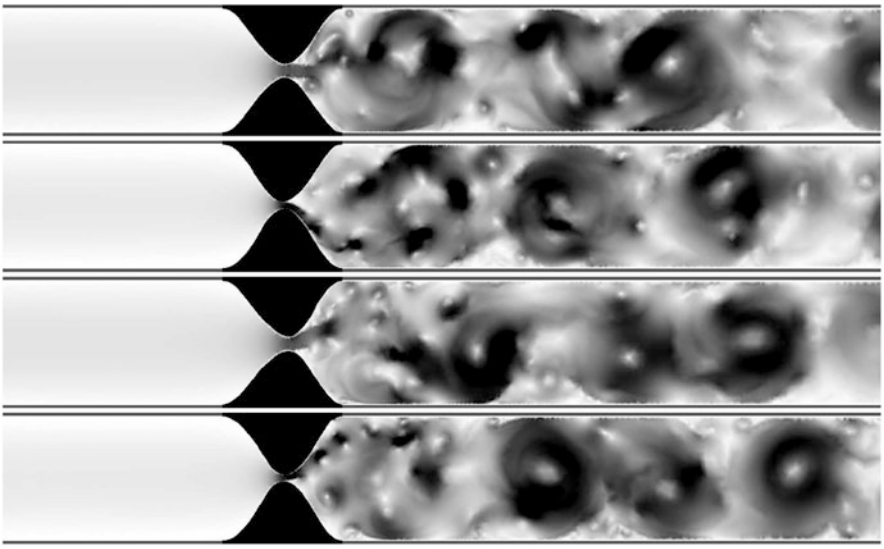

Fig. 4 The velocity field. The values of velocity magnitude (*white* to *black*) at time instans $t = 0.0336, 0.0360, 0.0384, 0.0408$ s

4 Conclusion

This paper is concerned with the application of the discontinuous Galerkin method in space combined with the BDF time discretization to the numerical solution of dynamic nonlinear elasticity problems using St. Venant-Kirchhoff material model. The method was tested on the benchmark proposed by J. Hron and S. Turek with satisfactory results. It is also shown that the method can be successfully applied to fluid-structure interaction.

Our further goal is a deeper analysis of the vocal folds vibrations using more complex geometry of vocal tract. Moreover, theoretical analysis of the developed method will be carried out.

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