

Geometry Explorer: Facilitating Geometry Education with Virtual Reality

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Abstract. A key to learning high-level geometry concepts is an individual's ability to understand spatial dimensions and to mentally transform 3D shapes. However, prior research indicates that many students do not have adequate spatial understanding skills. On the other hand, immersive virtual reality (VR) has been shown to afford greater spatial understanding. In this paper, we present the Geometry Explorer, a VR system designed to facilitate geometry education by leveraging increased spatial understanding. The system uses a Samsung Gear VR to allow users to view and manipulate the dimensions of 3D shapes. We have created a game based on the goal of manipulating shapes to reach target volumes. We informally evaluated our initial prototype using the Rapid Iterative Testing and Evaluation (RITE) approach. In this paper, we present our modifications based on the usability issues discovered during RITE testing. We conclude by discussing the future of the Geometry Explorer.

Keywords: Virtual reality · Geometry education · Head-mounted display

1 Introduction

Geometry is important and exists in every aspect of people's daily lives. Analysis of two-dimensional (2D) and three-dimensional (3D) shapes and the study of geometric relationships are used in fields ranging from architecture to manufacturing. As a fundamental component of mathematics [1], geometry provides a means of applying mathematical ideas to the concrete things that we can see in the real world. According to a study on high school students [2], a good knowledge of geometry plays a critical role in developing expertise in STEM¹ domains. Being knowledgeable of geometry helps people to understand the world better and to think in different ways.

Although geometry is so important in everyday life, many people are not equipped with essential geometry knowledge and ability. This is due to the fact that geometry is difficult to learn and to teach [3]. One critical aspect to geometry education is the ability to understand and convey spatial relationships. Spatial understanding is not an ability that people are born with. Like many other skills, it requires practice to develop [4]. Formative experiences constructing and actively manipulating shapes and models play an important role in the development of spatial understanding and geometric knowledge [3].

¹ STEM is the acronym of Science, Technology, Engineering and Math.

However, traditional methods of teaching geometry do not support these types of activities. Instead, texts and 2D figures are used as attempts to illustrate 3D shapes and convey 3D concepts. Considering these issues, new technologies provide opportunities to improve geometry education.

One technology that has potential to drastically improve geometry education is immersive virtual reality (VR). Immersive VR provides computer graphics that appear to surround the user, in turn allowing the user to naturally look around a virtual environment [5]. This capability, combined with stereoscopic graphics, allows immersive VR to greatly facilitate spatial understanding [6]. Additionally, VR can allow the user to interact with and manipulate 3D shapes. These advantages make VR an ideal technology for improving geometry education.

By taking advantage of modern immersive VR technologies, we have built a VR system for facilitating geometry education that we call the *Geometry Explorer*. Currently, the system focuses on teaching the user how to calculate the volume of different 3D shapes by directly manipulating the dimensions of the shapes to achieve desired volumes. Prior to creating the Geometry Explorer, we identified several system requirements in order for it to be a usable and feasible solution to geometry education problems. With these requirements in mind, we used the Samsung Gear VR head-mounted display (HMD) to implement the system and a game based on the objective of manipulating 3D shapes to achieve target volumes. We used the Rapid Iterative Testing and Evaluation (RITE) approach [7] to informally assess the efficacy of the system and to identify problems with the preliminary prototype. After several improvements, we are now ready to conduct a summative evaluation to compare the effectiveness of the Geometry Explorer to traditional geometry education methods.

2 Related Work

Several technologies have been proposed to aid in teaching and learning geometry. Most of these are either desktop-based applications or immersive systems.

2.1 Desktop-Based Geometry Applications

Numerous desktop-based applications have been created for teaching geometry. Dynamic geometry systems (DGS) are the most common type among these and include applications such as *Cabri*, *Cinderella*, and *GeoGebra* [8]. These DGS applications allow users to carry out a set of operations on different geometry shapes in order to create and manipulate geometric constructions. According to prior research studies [9, 10], such applications work effectively in improving students' understanding of planar geometry and 2D geometric concepts. But they have not been demonstrated to significantly improve students' understanding of 3D geometry and concepts [10].

While the above-mentioned systems are helpful for improving some aspects of geometry education, particularly for 2D shapes, they are less effective for conveying 3D geometry concepts. There are several potential reasons why. First, these desktop-based systems do not usually support stereoscopic viewing, which deprives the user of an

important depth cue. Second, motion parallax cues are only supported through virtual camera movements controlled with the mouse and keyboard. Finally, due to limitations of the mouse and keyboard, 3D manipulations are only supported through complex modal interactions, like those found in 3D modeling applications [5].

2.2 Immersive Geometry Systems

As mentioned before, immersive systems provide several advantages that desktop-based interfaces do not. These advantages include large fields of regard (FOR; i.e., the total size of the visual field surrounding the user [6]), stereoscopy, and head tracking for natural viewing. Furthermore, these advantages yield additional benefits, such as improved spatial understanding [6] and a greater sense of presence (i.e., the sense of “being there” in a virtual environment [11]).

Realizing the potential benefits of using VR to promote geometry education, Kaufmann and his colleagues have developed an immersive augmented reality system called *Construct3D* [12]. The Construct3D system uses an optical see-through HMD to provide stereoscopic views of virtual objects within the real-world space. The system uses a two-handed 3D interaction tool called the Personal Interaction Panel (PIP), which acts as a handheld tablet. Using the PIP, users can place predefined solids, planes, lines, and points within the virtual environment. These open-ended construction features make Construct3D similar to a 3D modeling application.

In a series of usability evaluations conducted with teachers, Kaufmann identified three key strengths of the Construct3D system [13]. First, the construction of 3D dynamic geometry is a major educational asset for explorative learning. Second, the ability to dynamically place and modify shapes allows users to visual objects that are composites of traditional shapes. Third, users can actively walk around objects, which supports better spatial understanding. However, Kaufmann acknowledged that Construct3D was not a feasible classroom solution due to the expensive cost of the Construct3D hardware, the technical complexity required to set up the system, and its inability to support more than a few students [13].

With these limitations in mind, we set out to develop an immersive VR system that would provide benefits similar to Kaufmann’s Construct3D, but in a form that was both usable and feasible for classroom instruction. Additionally, we chose to focus on a specific learning objective for geometry education, in the form of calculating the volume of various 3D shapes.

3 System Requirements

The primary purpose of our research was to develop a VR system that would facilitate learning how to calculate the volumes of various 3D shapes and how manipulating a 3D shape would affect its volume. Prior research has indicated that students frequently encounter difficulties in understanding volume formulas and measurements [14]. One noted issue is that students tend to memorize volume formulas without understanding their meaning [3]. This is likely due to a lack of spatial understanding skills [4]. Hence,

one system requirement was that it could facilitate spatial understanding and support the development of such skills. An additional learning requirement was that the system should be interesting to use, as research has shown that students are intrinsically motivated to learn when they find the process interesting [15].

With regard to helping students learn how to calculate volumes, we had to decide on which 3D shapes, and their corresponding volume formulas, to focus. We consulted the Common Core State Standards (CCSS), an educational initiative in the United States that details what students should know about English and mathematics at the end of each grade, kindergarten through high school [16]. According to the CCSS for math, the first shape that students should learn how to calculate its volume is the right rectangular prism (5.MD.C.5.A), in 5th grade. Later on, during high school geometry, students should learn how to use the volume formulas for cylinders, pyramids, cones, and spheres (HSG.GMD.A.3). Hence, the system needed to support learning of all five shapes.

Outside of learning requirements, we identified several requirements in order for the system to be feasible for use in an educational environment. The first of these was universal usability (i.e., the system must be usable for every user) [17]. A particular concern related to universal usability was accommodating users with visual impairments, as many VR systems are not usable with glasses [5]. Ergonomics was another concern, as some VR systems cause fatigue when the user must sustain uncomfortable positions or weights [18]. One feasibility concern was whether schools could afford the system hardware. Some schools struggle to maintain in-class computers, especially those in poorer socioeconomic districts. Hence, the solution needed to be inexpensive. Finally, the system had to be easy to use, as any good interface should be [17].

4 System Design

To address our system requirements, we designed a game for the Samsung Gear VR HMD that involves manipulating 3D shapes to achieve target volumes.

4.1 Hardware: Samsung Gear VR

Immersive VR has been shown to facilitate greater spatial understanding by providing more depth cues, such as stereopsis, motion parallax, perspective, and occlusion [6]. Hence, we decided to utilize immersive VR to address our requirement of facilitating spatial understanding. In turn, we expected VR to better support understanding volume formulas, as opposed to just memorizing them.

Considering all of our requirements, we decided to use the Samsung Gear VR for our system hardware (see Fig. 1). With a 96° field of view (FOV), 360° field of regard (FOR [6]), a resolution of 1280 × 1440 per eye, and an internal tracker for rotational head tracking, the Gear VR provides an immersive user experience. Unlike most other HMDs, the Gear VR provides a focal adjustment that accommodates nearsightedness and farsightedness. The device also only weighs 318 g and provides adjustable head straps for an ergonomic form factor. These two features make the Gear VR more universally usable than other VR hardware options. The Gear's integrated touchpad and back

button afford easy-to-use interactions without the need for additional devices, such as a game controller. Finally, the Gear VR is inexpensive and costs less than most classroom desktop computers.



Fig. 1. High school students using the Gear VR to interact with the Geometry Explorer

4.2 Interactions: 3D Volume Manipulations

To facilitate learning how manipulating a shape affects its volume, we designed the Geometry Explorer to directly provide the ability to manipulate a shape’s dimensions and, in turn, its volume. As seen in Fig. 2, context-sensitive widgets afford scaling along each of the three individual axes of a shape. Using head orientation to control a cursor, the user first selects one of the context-sensitive widgets with a tap of the touchpad. Once selected, the widget moves along its indicated axis to remain centered in the user’s view as the user rotates his or her head. As the widget moves, the scale of the corresponding dimension changes accordingly. At any point, the user can tap the touchpad again to release the widget and stop manipulation of the dimension.

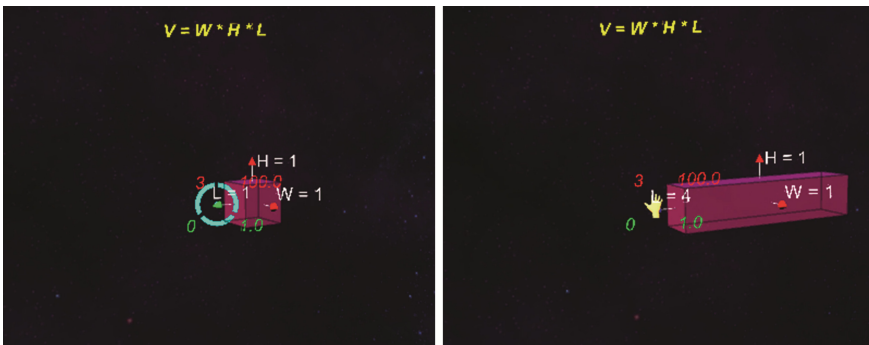


Fig. 2. The Geometry Explorer uses context-sensitive widgets for 3D volume manipulations.

While every shape has three dimensions, we developed the Geometry Explorer in a manner to provide manipulations that correspond to the shape's volume calculation. For both the right rectangular prism and pyramid, all three dimensions (XYZ) could be independently manipulated. However, for the cylinder and cone, the widgets for the horizontal axes (XZ) were linked. Hence, manipulating the X -axis widget would result in the Z -axis widget being manipulated in the same manner. Finally, all three widgets were linked for the sphere, as dragging one would manipulate all three.

4.3 Motivation: Manipulation Game

In order to make the Geometry Explorer interesting to use, we decided to create a game around the concept of manipulating the volumes of the shapes. Other researchers have had success with making mathematics interesting by developing intrinsically motivating games [15], so we took a similar approach. Our game's objective was to manipulate a unit-sized shape to a randomly generated target volume as quickly as possible. Users would receive higher scores for achieving the target volume faster and lower scores for solving the volume problem slower. To further motivate the importance of scoring high (i.e., efficiently solving the volume problem), we retain a leaderboard of the ten highest scores and indicate if the user's score is among them.

5 RITE-Based Modifications

After developing a preliminary prototype of the Geometry Explorer, we used the RITE approach [7] to determine the potential efficacy of our system and to quickly improve its overall design. We conducted a series of RITE sessions with high school students, high school teachers, college students, and college educators over the course of three months. We identified and fixed several issues with our original prototype through these sessions. The following subsections are a summary of our major RITE-based modifications.

5.1 Multiplication Look-up Matrix

The most important issue that we identified during our RITE sessions is that mentally calculating the volume of a 3D shape is extremely challenging. The high school and college students were not able to successfully calculate target volumes while immersed, due to the inability to write down interim calculations. Instead, they resorted to manipulating the shapes in an ad hoc fashion, comparing the current volume to the target volume. Even the high school and college educators struggled with the mental calculations, likely due to the relatively small capacity of working memory [19].

Because we were interested in helping students learn what the volume formulas represented and not how to multiple numbers, we have added a multiplication look-up matrix. The concept of the matrix is similar to a two-factor multiplication look-up table, in which the user can find a target result and determine the two numbers that yield it

when multiplied. However, because volume calculations require at least three factors, we had to create a three-factor look-up matrix.

Seen in Fig. 3, the matrix consists of a single plane of cubes with 12 columns and 12 rows, representing the X and Y-axis values, respectively. At the top of the matrix, the current Z value is also provided, which is initially set to 1. Each cube contains the numerical result of multiplying its row number, column number, and the Z value. In order to find results in which Z is greater than 1, the user can swipe forward on the Gear VR touchpad to increase the value of Z. Similarly a backward swipe decreases Z. This matrix now allows users to quickly determine three factors that can be multiplied to find a target volume and to focus on learning what the formulas represent, as opposed to using cognitive resources to mentally make the calculations.

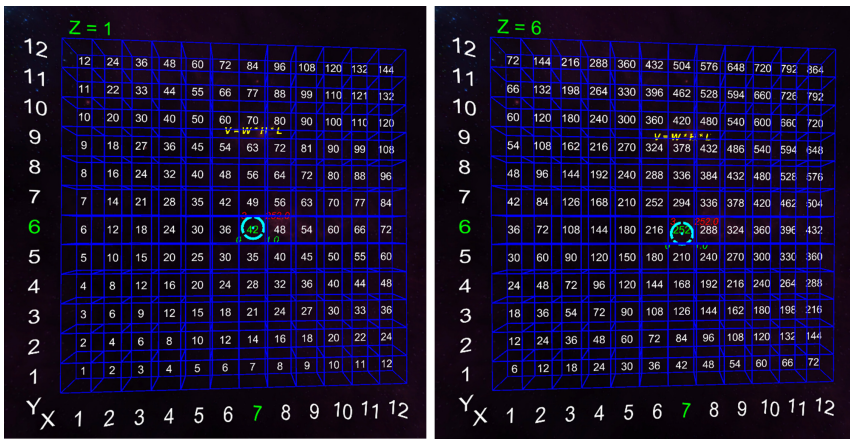


Fig. 3. The new multiplication look-up matrix allows students to focus on learning the volume formulas, as opposed to mentally multiplying numbers.

5.2 Head-Referenced Formula

In our original prototype, we positioned the shape’s volume formula within the world to initially appear in the center of the user’s FOV. If the user looked left or right, the formula would no longer be visible. We initially chose this world-referenced positioning to avoid the occlusion issues that accompany head-referenced objects [5]. Additionally, we thought it would force users to memorize the formula. However, during the RITE evaluations, we discovered that users struggled and became frustrated with not being able to see the formula and manipulate the shape at the same time. This was particularly the case for users not familiar with the formulas.

To remedy this issue, we modified the formula to be head-referenced instead of world-referenced. Now, the formula is always visible just above the center of the user’s FOV, as seen in Fig. 2. Though we originally avoided making the formula head-referenced to adhere to view-occlusion guidelines [5], our modification now adheres to Shneiderman’s guideline on minimizing short-term memory loads [17].

5.3 Target Number of Manipulations

As described in Sect. 4.3, our game was originally focused on only how quickly the user could achieve the target volume. However, during RITE sessions, we found that many users would only solve for the easiest calculation. For example, if the target volume for a rectangular prism was 12, many users would only scale one dimension to a length of 12, leaving the other two dimensions at their initial value of 1 unit. Few users would manipulate all three dimensions, such as $X = 2$, $Y = 2$, and $Z = 3$.

We have since included a target number of manipulations for each shape to address this issue. To randomly generate target volumes, we were already randomly selecting an integer value between 1 and 12 for each dimension. We used this information to set a target number of manipulations based on the number of dimensions that were not randomly set to 1. For example, if X and Y were randomly set to numbers greatly than 1, but Z was set to 1, the target number of manipulations would be two.

We also devised a new scoring mechanism to reinforce the importance of performing the exact number of target manipulations. In addition to decreasing with every second used to achieve the target volume, the user’s score is also now based on the ratio of performed manipulations to target manipulations. Hence, performing too few or too many manipulations will decrease the overall score. This rewards those users that manipulate all three dimensions in a pre-calculated manner.

5.4 Informative Cursor

In our original implementation, the cursor was simply a crosshair, as seen in Fig. 4 on the left. We provided the target and current volume information as head-referenced objects in the lower periphery of the user’s FOV to avoid occlusions. However, during the RITE sessions, we found that many users struggled to read the peripheral text.

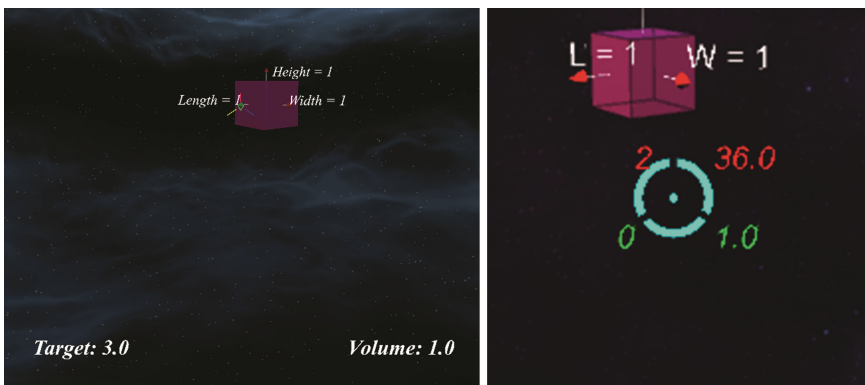


Fig. 4. Comparison of the original information layout (left) and the new cursor (right)

To solve this readability issue and to support the inclusion of the new information regarding the target and current number of manipulations, we designed an informative

cursor, as seen in Fig. 4 on the right. We placed the volume information in the right corners of the cursor, with the target volume above the shape's current volume. We placed the manipulation information in the left corners of the cursor, again with the target number being above the current number of manipulations. To further visually separate the numbers and avoid confusion, we colored the target information values red and the current information values green. Altogether, this new cursor ensures that the information needed for the game is readily available in the user's fovea.

5.5 Shape Rendering

In our initial prototype, we used the standard shader and directional lighting provided by Unity 5, our software development platform, for rendering the 3D shapes. This can be seen in Fig. 4 on the left. However, during the evaluations, some users reported difficulties the dimensions of the shapes at certain dimensions. To fix this issue, we have switched to using a wireframe shader to highlight the dimensions of each shape, as seen in Fig. 4 on the right.

5.6 Environment Change

Another seemingly minor aspect to the Geometry Explorer game that became evidently important was the surrounding environment. Originally, we chose to a vibrant space-related scene with an asteroid belt as the environment for our shape manipulations (see Fig. 5). However, during RITE testing, most of the high school students were more interested in looking around the environment instead of focusing on the shape-manipulation task. To remedy this issue, we switched to another space-related scene, but without distracting asteroids or vibrant colors.

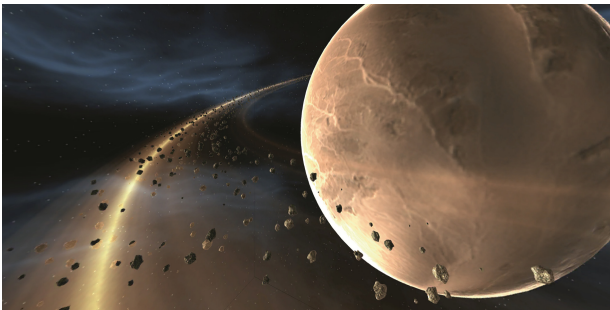


Fig. 5. This environment was deemed too distracting during our RITE sessions.

5.7 In-Game Tutorial

A major issue that we encountered during RITE testing was the inability to see what the user was viewing on the Samsung Gear VR. With computer-based HMDs, such as the Oculus Rift, the user's point of view (POV) can be duplicated on another screen.

However, due to the Gear VR's graphics running on a mobile phone screen, we were unable to view the user's POV on another display.

This issue was most problematic for explaining how to use the interface to play the game. Firstly, while immersed, the users could not see the touchpad or back button, and had to rely on tactile sensations to distinguish the two. Often, we had to physically place the user's hand over these inputs. Secondly, we found it difficult to effectively describe how to interact with the widgets, as we did not know where the user was looking. While most users would eventually figure it out, a few users became frustrated with the disconnection in communication and quit.

To alleviate the above issues, we have created an in-game tutorial. The tutorial first covers the inputs on the Samsung Gear VR and has the user practice using them, similar to the setup tutorial that accompanies the Gear VR. Next, the tutorial explains the assets seen within the environment, starting with the informative cursor, what each number represents, the formula displayed above, and finally the context-sensitive widgets. The tutorial then explains to the user how to solve a rectangular prism volume calculation by manipulating all three widgets to specified dimensions. We have found the system much easier to introduce to new users with this in-game tutorial (Fig. 6).



Fig. 6. Our new in-game tutorial covers both the hardware features of the Gear VR and the mechanics of the volume-manipulation game.

6 Conclusions and Future Work

Understanding geometry is an important skill, especially for STEM fields. However, many students struggle to learn geometry due to a lack of spatial understanding. VR offers the potential to drastically improve geometry education by enhancing spatial understanding and allowing students to interact with and manipulate 3D shapes. As such, we have developed a VR system for facilitating geometry education that we call the Geometry Explorer. The system uses a Samsung Gear VR HMD to view and manipulate the dimensions of 3D shapes in order to learn how such manipulations affect the volumes of the shapes. To make learning volume formulas interesting, we created a game that assesses how quickly a user can manipulate a shape to correctly obtain a target volume. Using the RITE approach with students and educators, we identified several issues with

our initial Geometry Explorer prototype and made modifications to address them. In this paper, we have shared those insights and modifications, in hopes to inform future research on using VR for educational purposes.

Currently, the Science and Engineering Education Center (SEEC) at the University of Texas at Dallas is using the Geometry Explorer as an exhibit in its Contact Science Program. As part of the program, the Geometry Explorer is taken to public libraries on a regular basis for youth from the general population to interact with the system. We are also working with local school districts to integrate the system into their geometry curriculum.

Furthermore, we are currently in the process of conducting a summative evaluation to compare the efficacy of the Geometry Explorer to traditional classroom materials and to online materials, such as Khan Academy. We are using a between-subjects design to compare the efficacies of the three conditions. We hypothesize that the Geometry Explorer will yield greater increases in accurately remembering the volume formulas and applying them to given problems.

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